

## Mass Shift of $D_{sJ}^*(2317)$ by Coupled Channel Effect

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**Abstract.** The resonance  $D_{sJ}^*(2317)$  which is considered to be the  $1^3P_0$  state composed of charm and strange quarks has been discovered recently. The measured mass, which is about 160 MeV lower than the mass of the  $1^3P_0$  state obtained from the potential model calculation by Godfrey and Isgur, was considered surprisingly low and attracted a lot of theoretical investigations. We calculate the mass shift of the  $1^3P_0$  state by using the coupled channel effect. Our result shows that the coupled channel effect naturally explains the observed mass of  $D_{sJ}^*(2317)$ .

The BaBar Collaboration [1] recently discovered a narrow resonance in  $D_s^+\pi^0$ , which is known as the  $D_{sJ}^*(2317)$ , and it was confirmed by the CLEO [2] and the Belle Collaborations [3]. Its decay patterns suggest a quark-model  $0^+$  classification. The measured mass,  $2317.4 \pm 0.9$  MeV [4] which is  $40.9 \pm 1.0$  MeV below the threshold of  $D^0K^+$ , was considered surprisingly low compared to the predictions of the potential model calculations. For example, the prediction of the  $1^3P_0$  mass by Isgur and Godfrey [5] was 2.48 GeV, and that by Eichten and Di Pierro [6] was 2.487 GeV, which are about 160 and 170 MeV higher than the measured mass of  $D_{sJ}^*(2317)$ . Barnes et al. [7] considered a mixing between two molecular states  $|D^0K^+\rangle$  and  $|D^+K^0\rangle$  and pointed out the importance of a very strong coupling between the  $c\bar{s}$  bound and  $DK$  continuum states, as required to induce binding. Van Beveren and Rupp [8] described  $D_{sJ}^*(2317)$  as a quasibound scalar  $c\bar{s}$  state in a unitarized meson model, owing its existence to the strong coupling to the nearby  $S$ -wave  $DK$  threshold. Browder et al. [9] proposed a mixing between the  $q\bar{q}$  and 4-quark states and assigned a linear combination with less mass as  $D_{sJ}^*(2317)$ . In this paper we show that the coupled channel effect naturally explains the observed mass of  $D_{sJ}^*(2317)$  [10]. The work in this paper provides an explicit quantitative realization of the proposals presented in Refs. [7, 8, 9].

The Cornell group studied the effect of OZI allowed decay channels [11]. They proposed that the following interaction hamiltonian is responsible for the decay as well as the binding of quark–antiquark bound states.

$$H_I = \frac{1}{2} \sum_{a=1}^8 \int d^3x d^3y : \rho_a(\mathbf{x}) V(\mathbf{x} - \mathbf{y}) \rho_a(\mathbf{y}) :, \quad (1)$$

where

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a^2}. \quad (2)$$

In (1)  $\rho_a(\mathbf{x}) = \psi^\dagger(\mathbf{x}) \frac{1}{2} \lambda_a \psi(\mathbf{x})$  are the color densities of quark fields. This model corresponds

to the vector coupling since (1) is the leading term of the vector coupling hamiltonian in the nonrelativistic expansion.

One can calculate the transition amplitude between two-quark bound and two-meson continuum states using the interaction hamiltonian (1). Then the effective hamiltonian from the coupled channel effect becomes  $H_{\text{eff}} = M^{\text{bare}} + \Omega(W)$  with

$$\Omega_{nL, mL'}(W) = \sum_I \int P^2 dP \frac{H_{nL, mL'}^I(P)}{W - E_1(P) - E_2(P) + i\epsilon}, \quad (3)$$

where

$$H_{nL, mL'}^I(P) = f^2 \sum_l C(Js, LL', J_1 J_2, l) I_{nL}^l(P) I_{mL'}^l(P), \quad f^2 = \frac{2}{3\pi^2 a^4 m_q^2} \frac{1}{\beta^3}. \quad (4)$$

In (4),  $I_{nL}^l(P)$  is the momentum dependent factor of the transition amplitude. The superscript  $I$  in (3) denotes the participating coupled channels which are composed of two mesons. The angular momenta and energies of these two mesons are denoted by  $J_1, J_2$ , and  $E_1(P), E_2(P)$ , respectively.  $H_{nL, mL'}^I(P)$  depends on the coupled channel, and it also depends on  $J$  and  $s$  (the total angular momentum and spin of the quark-antiquark bound state).

For the vector coupling,  $I_{nL}^l(P)$  in (4) is given by

$$I_{nL}^l(P) = \int_0^\infty dt \Theta(t) R_{nL}\left(\frac{t}{\sqrt{\beta}}\right) j_l\left(\frac{\rho_Q P t}{\sqrt{\beta}}\right) \quad (5)$$

with

$$\Theta(t) = [te^{-t^2} + (t^2 - 1)e^{-t^2/2} \sqrt{\frac{\pi}{2}} \text{erf}\left(\frac{t}{\sqrt{2}}\right)] + 4\beta a^2 \kappa [-te^{-t^2} + e^{-t^2/2} \sqrt{\frac{\pi}{2}} \text{erf}\left(\frac{t}{\sqrt{2}}\right)], \quad (6)$$

where  $R_{nL}(r)$  is the radial wave function of the quark-antiquark bound state,  $j_l(t)$  is the spherical Bessel function, and  $\rho_Q = m_Q/(m_q + m_Q)$ . The first and second terms in (6) come from the linear and Coulombic parts of (2), respectively.

Isgur and Godfrey obtained the mass of the  $1^3P_0$  state composed of  $c$  and  $\bar{s}$  quarks as 2.48 GeV from the interaction between two quarks. Their result corresponds to the bare mass  $M^{\text{bare}}$ . We calculate the mass shift of the  $1^3P_0$  state using  $H_{\text{eff}}$  from the coupled channel effect.

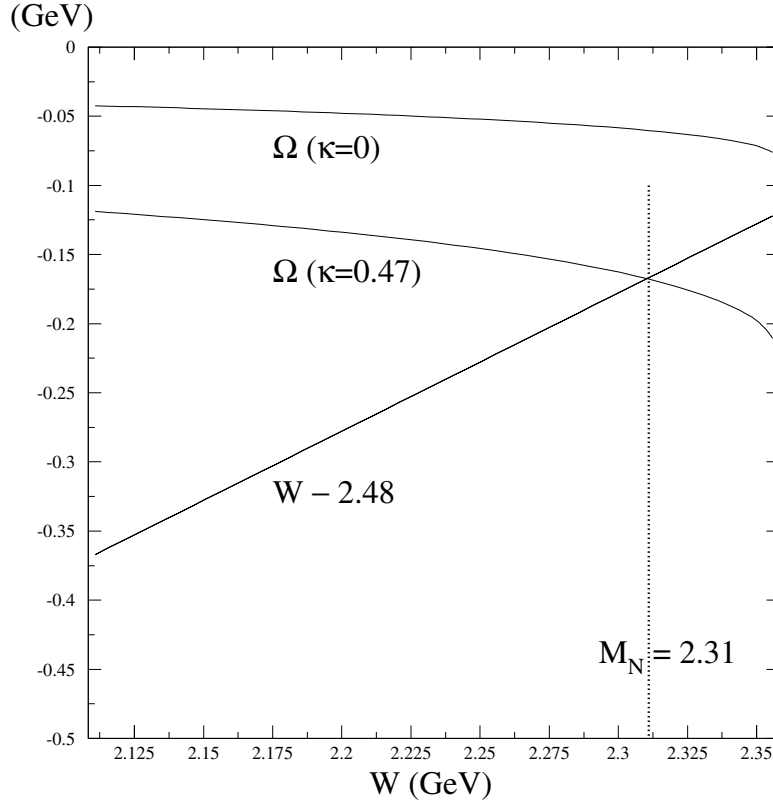
For  $V(r)$  in (2) we use the following two parameterizations in our calculation.

(A) Eichten et al. [11]:  $\kappa=0.517$ ,  $a=2.12$  GeV $^{-1}$ ,  $m_c=1.84$  GeV.

(B) Hagiwara et al. [12]:  $\kappa=0.47$ ,  $a = 1/\sqrt{0.19}$  GeV $^{-1}$ ,  $m_c=1.32$  GeV.

For the masses of  $s$  and  $u, d$  quarks, we use  $m_s=0.55$  GeV,  $m_{u,d}=0.33$  GeV.

Following [5], we use the radial wave functions of  $c\bar{u}$ ,  $c\bar{d}$ ,  $\bar{s}u$ ,  $\bar{s}d$  states with  $L = 0$  which are given by the ground state harmonic-oscillator wave functions  $(2\mu_a^{3/2}/\pi^{1/4}) \exp(-\mu_a^2 r^2/2)$ , and that of  $c\bar{s}$  state with  $L = 1$  by the first excited harmonic-oscillator wave function  $(8^{1/2}\mu_b^{5/2}/3^{1/2}\pi^{1/4}) r \exp(-\mu_b^2 r^2/2)$ .  $\beta$  in (4) and  $\mu_a$  here are related by  $\beta = \mu_a^2/2$ . For the value of  $\beta$  in the calculations of (4) and (5), we take  $\mu_a=0.61$  GeV which is the average value of  $\mu_a(K)=0.61$  GeV and  $\mu_a(D)=0.60$  GeV given in [13]. For the value of  $\rho_Q$  in (5) defined as  $m_Q/(m_q + m_Q)$ , we take  $m_Q$  as  $(m_c + m_s)/2$ . The formula for  $I_{nL}^l(P)$  in (5) was derived for the case where two mesons in the coupled channel have the same heavy quark such as  $DD$  [11]. However, in our present study two mesons are  $D$  and  $K$  mesons, and (5) is not exact in our system. So, we approximate our system by using the formula for  $I_{nL}^l(P)$  given in (5) with the above-mentioned values of  $\beta$  and  $\rho_Q$ . For the value  $\mu_b$  which comes into the  $c\bar{s}$  state radial wave function  $R_{nL}(r)$  in (5), we take two prototype values:  $\mu_b=0.46$  GeV given in [13] for  $D_{sJ}(2632)$  and  $\mu_b=0.57$



**Figure 1.** The result when the potential B and  $\mu_b = 0.46$  GeV were used. The upper first and second lines are  $\Omega$  when the Coulombic part is not included and when it is include, respectively, and the lower diagonal line is  $W - M^{\text{bare}}$  where  $M^{\text{bare}} = 2.48$  GeV given in [5]. As we see here,  $\Omega$  without the Coulombic part does not meet with  $W - M^{\text{bare}}$ , which means that there does not exist a solution of the eigenvalue equation  $W - [M^{\text{bare}} + \Omega(W)] = 0$  below thresholds. On the other hand,  $\Omega$  with the Coulombic part meets with  $W - M^{\text{bare}}$  below thresholds and the value of  $M_N = 2.31$  GeV at the meeting point is the eigenvalue.

GeV in [14] for  ${}^3P_0(c\bar{s})$ . In the calculation in this paper we also adopt the approximation of taking only one  $c\bar{s}$  bound state of  $1^3P_0$  for the quark-antiquark bound state sector, whereas we take all the two-meson continuum sector of  $DK$ ,  $D^*K$ ,  $DK^*$  and  $D^*K^*$ . That is, we do not include in our calculation the higher  $c\bar{s}$  bound states of  $n^3P_0$  whose masses are located above the threshold. If we include those higher  $c\bar{s}$  bound states in the calculation, the physical mass eigenvalue would be modified. However, we expect the modification is not large since the  $1^3P_0$  is very close to the threshold and then the coupling channel effect to this state is dominant. There is also a merit of this approximation that one can see clearly how the coupled channel effect gives rise to the mass shift.

The statistical coupling coefficient  $C(Js, LL', J_1 J_2, l)$  in (4) is 1 for  $l = 0$  in the  $DK$  channel,  $1/3$  for  $l = 0$  and  $8/3$  for  $l = 2$  in the  $D^*K^*$  channel, and other coefficients are zero. We calculate  $I_{nL}^l(P)$  using (5). Ref. [11] ignored the Coulombic part of  $V(r)$  in (1) in order to simplify their calculations. They argued that this is justified because small quark separations are not important in hadronic decays. However, it was first found by Zambetakis [15, 16] that if one includes the Coulombic part in (1), he finds a significant modification of  $I_{nL}^l$ . In fact, we find in this paper that the contribution to  $I_{nL}^l$  from the Coulombic part is about 60 % of that

Parameterizations		$M_N$ (GeV)	$\Delta M_N$ (GeV)	$Z$
Potential	$\mu_b$ (GeV)			
A	0.46	2.29	-0.19	0.67
	0.57	2.27	-0.21	0.70
B	0.46	2.31	-0.17	0.67
	0.57	2.29	-0.19	0.71

**Table 1.** The results of  $M_N$ ,  $\Delta M_N$  and  $Z$  for each parameterization of the potential and the  $\mu_b$  value, when  $\rho_Q(\equiv m_Q/(m_Q + m_{u,d}))$  with  $m_Q = (m_c + m_s)/2$  was used.  $Z$  is the probability that the physical mass eigenstate be in the quark-antiquark bound state sector [10].

from the linear part, and then the magnitude of  $\Omega$  resulting when both linear and Coulombic parts are included is about 2.6 times that resulting when only the linear part is considered [10].

The fact that the inclusion of the Coulombic part in (1) increases the magnitude of  $\Omega$  about 2.6 times is very important in the analysis of this paper. Fig. 1 is the result which we get when the potential B and  $\mu_b = 0.46$  GeV are used. The upper first and second lines are  $\Omega$  when the Coulombic part is not included (that is, when only the linear part is considered) and when it is included, respectively, and the lower diagonal line is  $W - M^{\text{bare}}$  where  $M^{\text{bare}} = 2.48$  GeV given in [5]. As we see the figure,  $\Omega$  without the Coulombic part does not meet with  $W - M^{\text{bare}}$ , which means that there does not exist a solution of the eigenvalue equation below thresholds. On the other hand,  $\Omega$  with the Coulombic part meets with  $W - M^{\text{bare}}$  below thresholds and the value of  $W = 2.31$  GeV at the meeting point is the eigenvalue  $M_N$ . The experimentally measured value of  $D_{s,J}^*(2317)$  mass is  $2317.4 \pm 0.9$  MeV. For all the cases of the parameterizations given in Table 2, we have similar situations as that shown in Fig. 1. In Table 1 we present the physical mass eigenvalue  $M_N$  and the mass shift  $\Delta M_N \equiv M_N - M^{\text{bare}}$  (2.48 GeV). Therefore, we showed that the coupled channel effect explains why the mass of  $D_{s,J}^*(2317)$  is about 160 to 170 MeV lower than the bare mass of the  $1^3P_0$  bound state obtained from the most potential model calculations.

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