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CURRENT COMMUTATORS, REPRESENTATION MIXING AND MAGNETIC MOMENTS*

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A detailed analysis of the various sum rules which have been recently derived from the chiral U(3) x U(3) algebra of currents[1] indicates that the exact sum rules may be approximated by sums over a few intermediate states which fall into a relatively simple <u>reducible</u> representation of the current algebra. In a previous paper[2] (hereafter denoted by I) we have shown that the positive helicity state of the nucleon can be properly described as having components in the

$$\{(\underline{6},\underline{3})L_z=0\},\ \{(\underline{\bar{3}},\underline{3})L_z=0\}\ \text{and}\ \{(\underline{3},\underline{\bar{3}})L_z=1\}\ \text{representations of}\ U(3)\times U(3),$$

and that by adjusting one free mixing angle one can then correctly predict the experimental values of G_A - the axial vector coupling constant in β decay, G^* - the strength of the axial vector transition between the nucleon and the $N^*(1238)$ resonance and the $\frac{d}{f}$ ratio for the axial vector current between states of the baryon octet.[3] In the present paper we show that the same assumptions and the same mixing angle lead, in addition, to a prediction for the ratio between

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the anomalous magnetic moment of the neutron and the strength of the magnetic transition between the nucleon and $N^*(1238)$. The predicted value for this ratio is in excellent agreement with the experimental data. Furthermore, by adjusting one additional free parameter, we can obtain the correct ratio between the anomalous moments of the proton and the neutron.

Following the approach we used in I we consider the $U(3) \times U(3) \times U(3) \times U(3)$ algebra[4] generated by the equal time commutators of the z and t components of the vector and axial vector currents, evaluated between particle states moving with infinite momentum in the z direction. Since at infinite momentum the matrix elements of the z components are equal to those of the time components (both for the vector and the axial vector current) it is sufficient to discuss a $U(3) \times U(3)$ algebra, which can then be identified either as the chiral or as the collinear current algebra.[4] In view of the difficulties which follow from the assignment of the positive and negative helicity states of the nucleon to the $(\underline{6},\underline{3})$ and $(\underline{3},\underline{6})$ representations, respectively, it has been suggested that an appreciable amount of representation mixing is present. This was mainly motivated by the following observations:

1. The analysis of the Adler-Weisberger sum rules for both the strangeness conserving[5] and the strangeness changing[6] currents clearly indicates that the decuplet states do not saturate the integrals of

meson-nucleon cross sections and that the contributions of higher resonances (mostly with negative parities) cannot be neglected. Furthermore, the decuplet dominance assumption turns out to be inadequate in a few other cases[7-9] although it is not clear, in these cases, whether the exact sum rules are verified.[10]

2. If the nucleon belongs to any pure U(6) or U(3) x U(3) representation having $L_z = 0$, its anomalous magnetic moment is predicted to vanish[4] (L_z is defined here as $J_z - S_z$ where J_z and S_z are the z components of the total angular momentum and the "intrinsic quark spin", respectively). The prediction is easily obtained by observing that the anomalous moment operator transforms under the current algebra like the z (or time) component of the electromagnetic current but it changes L_z by one unit and therefore cannot connect two $L_z = 0$ states. It is interesting to add, in this connection, that the sum rule derived by Fubini, Furlan and Rossetti[9] for the anomalous moments of the baryons leads to $\mu_A(B) = 0$ if we require SU(3) symmetry <u>and decuplet dominance</u>.[11] This result is, of course, intimately related to the observation that any pure $L_z = 0$ representation for the baryon octet (such as the $(\underline{6},\underline{3})$ of U(3) x U(3) or the 56 of SU(6)) implies a vanishing μ_A.

According to the results of I, the positive helicity state of the nucleon is given by:

$$| N, J_z = \frac{1}{2} \rangle = \cos\theta \, | (\underline{6},\underline{3}) L_z = 0 \rangle + \qquad (1)$$

$$+ \sin\theta \left\{ \sqrt{\frac{1}{3}} \, | (\overline{\underline{3}},\underline{3}) L_z = 0 \rangle - \sqrt{\frac{2}{3}} \, | (\underline{3},\overline{\underline{3}}) L_z = 1 \rangle \right\}$$

Whereas the $J_z = +\frac{1}{2}$ component of $N^*(1238)$ is purely in the $\{(\underline{6},\underline{3}) L_z = 0\}$ multiplet. The magnetic transition operator transforms under the algebra like $\{(8,1) + (1,8);\ L_z = \pm 1\}$ and its matrix elements between two nucleons are given by two independent transition strengths: $\{(\underline{6},\underline{3}) L_z = 0\} \longleftrightarrow \{(\overline{\underline{3}},\underline{3}) L_z = -1\}$ and $\{(\overline{\underline{3}},\underline{3}) L_z = 0\} \longleftrightarrow \{(\overline{\underline{3}},\underline{3}) L_z = -1\}$. Since the $(\overline{\underline{3}},\underline{3}) \longleftrightarrow (\overline{\underline{3}},\underline{3})$ transition is a pure F transition, it does not contribute to the neutron moment. The magnetic $N \to N^*$ transition is therefore simply related to $\mu_A(n)$ and we find:

$$\frac{\mu^*}{\mu_A(n)} = -\frac{\sqrt{2}}{\cos\theta} \qquad (2)$$

where μ^* is the matrix element of the z component of the transition moment between the $J_z = \frac{1}{2}$ states of N and N^*. In a recent analysis of photoproduction data in the neighborhood of the 3-3 resonance Dalitz and Sutherland[12] have obtained (in nucleon magnetons):

$$\mu^* = (1.28 \pm 0.02) \times \frac{2\sqrt{2}}{3} \times \mu_{tot}(p) = 3.36 \pm 0.05$$

By inserting the $\cos\theta$ value obtained in I from the axial vector transitions, we obtain from equation (2):

$$\mu^* = -\mu_A(n) \times \frac{\sqrt{2}}{\cos 37^o} = 3.40$$

The agreement is remarkably good. In fact it is better than what one should expect, in view of the approximations introduced in the model.

The ratio between the anomalous moments of the proton and neutron cannot be uniquely expressed in terms of the mixing angle θ without additional assumptions. It depends on the ratio k between the reduced matrix elements of the magnetic transitions $(\underline{\bar{3}},3) \leftrightarrow (\underline{\bar{3}},3)$ and $(\underline{6},\underline{3}) \leftrightarrow (\underline{\bar{3}},\underline{3})$. This ratio is a free adjustable parameter of the theory and it can always be fitted to the $\mu_A(p)/\mu_A(n)$ ratio. We obtain

$$\frac{\mu_A(p)}{\mu_A(n)} = -1 + k\tan\theta \qquad (3)$$

and for $k = 0$:

$$\mu_A(p) = -\mu_A(n) \tag{4}$$

We find that the $(\bar{3},3) \leftrightarrow (\bar{3},3)$ transition should be smaller than the $(6,3) \leftrightarrow (\underline{\bar{3}},\underline{3})$ transition, at least by an order of magnitude. In order to check this we can roughly estimate the photoproduction amplitude of the second nucleon resonance $N^{**}(1512,\ J^P = \frac{3}{2}^-)$ assuming that its $J_z = +\frac{1}{2}$ states is mostly in the $(\underline{\bar{3}},\underline{3})$ and $(\underline{3},\bar{3})$. We find (for $k = 0$):

$$\mu^{**} \sim \frac{1}{2}\,\mu^{*}\cot\,\theta \sim 2 \tag{5}$$

In the absence of a reliable detailed analysis of the photoproduction amplitudes in the $N^{**}(1512)$ region and in view of the difficulties in separating the background and the E1, L1 and M2 contributions it is hard to compare this with the data, but the order of magnitude of the result is reasonable in the sense that it is smaller than μ^* but not negligible. We cannot expect to do any better with our crude assumptions about the N^* and N^{**} classification.

A by-product of the present model of representation mixing is the predicted zero anomalous moment for the decuplet resonances (including Ω^-). This is hard to test, of course, but we might add that SU(3) predicts that the total magnetic moment of any state in the decuplet is proportional to its charge and that is, of course, consistent with our present result.[13]

We can now summarize the situation as follows: By assuming a simple model of U(3) × U(3) representation mixing for the nucleon and using only one free mixing angle we are able to calculate four transition matrix elements which can be directly or indirectly (via PCAC) compared with experiment. The agreement, presented in Table 1, is excellent. A fifth quantity, $\frac{\mu_A(P)}{\mu_A(n)}$, remains undetermined and is expressed in terms of an additional adjustable parameter. Our simple mixing scheme cannot be incorporated in any simple way into the larger $U(6)_W$ current algebra which includes, in addition to the usual vector and axial vector currents, some components of tensor currents. The suggestion of Gatto et al.[3] that the nucleon has components in the $\underline{56}$, L = 0 and $\underline{20}$, L = 1 representations of SU(6) leads to $\mu_A(n) = 0$, $\mu^* = 0$ and is clearly in contradiction with experiment. If we insist on having some $SU(6)_W$ interpretation, we should probably assign our $(\underline{\bar{3}},\underline{3})$ and $(\underline{3},\underline{\bar{3}})$ to the $\underline{70}$, and find ourselves with a nucleon having both $W = \frac{1}{2}$ and $W = \frac{3}{2}$ components. Another amusing possibility may be to assign the $(\underline{\bar{3}},\underline{3})$ and $(\underline{3},\underline{\bar{3}})$ components of the $J_z = \frac{1}{2}$ nucleon to different $SU(6)_W$ representations which cannot be connected by a $\underline{35}$, such as $\underline{20}$ and $\underline{700}$. In this case we automatically get $\mu(P) = -\mu(n)$, without any additional assumptions. We prefer, however, both the simplicity and our better physical understanding of the $U(3) \times U(3)$ current algebra and we suggest that this is the appropriate framework for studying the problem.

All the results of this paper as well as of I, except the $\frac{d}{f}$ ratio for the axial vector transitions can be derived by using the (chiral or collinear) $U(2) \times U(2)$. The $J_z = \frac{1}{2}$ nucleon will then be in a combination of $\{(1,1/2)\ L_z=0\}$, $\{(0,1/2)\ L_z=0\}$ and $\{(1/2,0)\ L_z=1\}$ and in order to obtain $\mu_A(P) = -\mu_A(n)$ we have to assume that the $\{(\frac{1}{2},0)\ L_z = 0\} \longleftrightarrow \{(\frac{1}{2},0)\ L_z = 1\}$ transition can be neglected either because of a $U(3) \times U(3)$ selection rule or because the reduced matrix element is negligible. We also observe that our specific mixture of $(1, \frac{1}{2})(0,\frac{1}{2})$ and $(\frac{1}{2},0)$ is the only combination which fits the data and does not include $|L_z| > 1$ or $I \geq \frac{5}{2}$ states.

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Table 1: Comparison with Experiment

	Theoretical Expression	Theoretical value for $\theta = 37^{\circ}$	Experimental value
G_A	$\frac{1}{3}(4\cos^2\theta + 1)$	1.18	1.18 ± 0.02 [a]
G^*	$\frac{4}{3}\cos\theta$	1.06	1 ± 0.2 [b]
$\alpha = \frac{d}{d+f}$	$\frac{2\cos^2\theta+1}{4\cos^2\theta+1}$	0.64	0.665 ± 0.018 [c]
$\frac{\mu^*}{\mu_A(n)}$	$-\frac{\sqrt{2}}{\cos\theta}$	1.78	1.76 ± 0.03 [d]

References to Table

a) C. S. Wu, unpublished.

b) The estimate $G^* \sim 1$ is obtained by using PCAC and the contribution of the N^* region to the integral over πP cross sections which appears in the Adler-Weisberger formula. The error of 20% includes the 10%-15% expected discrepancy for PCAC and the ambiguities of subtracting the resonance background.

c) N. Brene, M. Roos et al.; private communication from M. Roos.

d) R. H. Dalitz and D. G. Sutherland, to be published.

References and Footnotes

1. M. Gell-Mann, Phys. Rev. 125, 1067 (1962); M. Gell-Mann, Physics 1, 63 (1964).
2. H. Harari, "Current Commutators and Representation Mixing", to be published.
3. R. Gatto, L. Maiani and G. Preparata, Phys. Rev. Letters 16, 377 (1966), have independently obtained results similar to those of Ref. 2. They used, however, the 20, L = 1 representation which leads to the disastrous predictions for the magnetic transitions, discussed at the end of the present paper. Some other U(3) × U(3) and SU(6) aspects of the problem were studied by H. J. Lipkin, H. Rubinstein and S. Meshkov, to be published, and by N. Cabibbo and H. Ruegg, to be published.
4. R. F. Dashen and M. Gell-Mann, "Algebra of Currents at Infinite Momentum" to be published in the Proceedings of the Third Coral Gables Conference.
5. S. L. Adler, Phys. Rev. Letters 14, 1051 (1965). W. I. Weisberger, Phys. Rev. Letters 14, 1051 (1965).
6. C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters 15, 715 (1965); D. Amati, C. Bouchiat and J. Nuyts, Phys. Letters 19, 59 (1965); L. Pandit and J. Schechter, Phys. Letters 19, 56 (1965) and W. I. Weisberger, Phys. Rev. 143, 1302 (1966).

References and Footnotes (Continued)

7. In the magnetic moment sum rule which was derived by Adler, Bjorken and Cabibbo and Radicati (Ref. 8) the decuplet dominance assumption leads to a wrong sign for the integral over intermediate states. The amount of success of the same assumption in the case of the photoproduction sum rule of Fubini et al. (Ref. 9) is still unsettled (S. L. Adler, private communication).

8. S. L. Adler, Phys. Rev. 143, 1144 (1966); J. D. Bjorken, unpublished; N. Cabibbo and L. A. Radicati, Phys. Letters 19, 697 (1966) and R. F. Dashen and M. Gell-Mann, Ref. 4.

9. S. Fubini, G. Furlan and C. Rossetti, to be published.

10. Since the sum rules derived in references 8 and 9 involve current densities and not only integrated currents, one cannot rule out the possibility of nontrivial divergence terms in the commutator.

11. This can be proved by evaluating the sum rule between Λ states and obtaining from decuplet dominance: $g_{\Lambda\Lambda\eta} \cdot \mu(\Lambda) = 0$. Since the $\frac{d}{f}$ ratio for the BBM coupling (or the axial vector current) does not vanish, $g_{\Lambda\Lambda\eta} \neq 0$ and $\mu(\Lambda) = 0$. This implies $\mu(n) = 0$. Decuplet dominance also leads to $\mu_A^S(N) = 0$ and combined with $\mu(n) = 0$, to $\mu(P) = 0$. We also obtain $\mu^* = 0$, for all octet-decuplet transitions.

References and Footnotes (Continued)

12. R. H. Dalitz and D. G. Sutherland, to be published.

13. As long as there is no experimental determination of the decuplet moments, the theoretical ambiguities in defining an "anomalous moment" for a spin $\frac{3}{2}$ object are not very important for our purposes.