Interacting Dark Energy in Brane-Cosmological Perspective

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Abstract: The exotic fluid dubbed as dark energy that stands for the theoretical explanation of observationally confirmed late time acceleration of the universe is itself the fertile ground for research. One of the most demanding explanations from dark energy model is its crossing behaviour of equation of state. Out of different proposals to have crossing, interacting model that postulates interaction between dark energy and dark matter also deserves attention from various reasons. We consider interacting dark energy model in the bulk. And then getting an approximate solution of modified Friedmann equation, find some points for discussion.

Keywords: Dark energy, state parameter, Brane-cosmology

1 Introduction

The late time accelerated expansion of the universe is an observational fact. Though the finding is still surprising, it is usually expected that theoretical explanation will be searched in the context of general theory of relativity(GTR). Einstein equation, $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G t_{\mu\nu}$ gives its spatial component and time component respectively as

$$\dot{\rho} + 3H(\rho + p) = 0$$

And

$$H^2 + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\rho$$

Now it is straight forward to have the equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad (1)$$

So the universe might be in accelerating phase when $\rho + 3p < 0$. Having negative pressure, the candidate triggering the unusual acceleration is rightfully dubbed as Dark energy echoing its exotic nature. But that is not the last story of its peculiarities to be surprised of. The recent analysis of the observed data set favours the time dependent dynamical behaviour of dark energy. What is found in the equation of state of this exotic fluid is the state parameter w (defined as the ratio of pressure to energy density) that crosses -1 at some region with low redshift. This crossing behavior (so called phantom divide) appears as signal for paradigm shift in fundamental physics[1].To resolve this issue, 2-field model, interacting mode and possibilities of failure of general relativity at the cosmological scale are at hand. Here, our work is solely focused in interacting mode.

Dynamics comes from some kind of interaction; it is simple but well understood teaching of physics. So postulation of some yet unknown interaction term might be responsible to cross the phantom divide. However experimental tests strongly stand against the non-minimal coupling between the dark energy and ordinary matter [2]. But the prevailing dark sector (dark matter and dark energy) itself may be the player in the stage of interaction. Generally, here, interaction term is put by hand merely on phenomenological basis. It seems resolving in crisis-overcome but not very appealing in real understanding. In this scenario, an encouraging finding comes from the context of low energy limit of string / M theory involving interacting chaplygin gas model [3]. The effective low energy string theory research is marked as crucial to solve the dark energy problem.

Modified Chaplygin gas (also termed as generalised chaplygin gas) has been extensively pursued in the different context of dark energy related research works. At present, not only in standard cosmology, different constraints on chaplygin gas model have been explored, so to say, in non standard cosmology also. One of the recent considerations, is its candidature as bulk candidate. Bulk field is restricted only to correspond to string fields, though on phenomenological ground one is free to consider any matter in the bulk [4]. We also consider modified chaplygin gas as bulk candidate having some interaction with itself only, that is not with dark matter but with dark energy. In this context, it is mention worthy that the analysis of the deviation from the virial equilibrium for two relaxed clusters with standard density profile is still consistent with the absence of interaction between dark energy and dark matter [5].

We plan the work as follows. The preliminary section contains the brief mathematical structure already worked out[6]. On this structure, our work is carried on. Next is the penultimate section which is completely involved with our own work. The last section is for discussion.

2 Preliminaries

Five dimensional spacetime (5-D) is the usual feature for Brane-world scenario. Its mathematical representation is provided as [6]

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\delta_{ij}dx^i dx^j + b^2(t,y)dy^2 \quad (2)$$

Where as 5-D Einstein equation for gravity comes in the form

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu} \quad (3)$$

Total energy-momentum tensor $T_{\mu\nu}$ has two parts:

(i) The energy-moentum tensor of bulk

$$T^{\mu}{}_{\nu}|_B = diag\left(-\rho_B, P_B, P_B, P_B, P_5\right) \quad (4)$$

Here ρ_B, P_B and P_5 are respectively energy density and pressure component which are independent of fifth coordinate.

(ii) The energy-momentum tensor of brane.

Considering brane internal geometry as homogeneous and isotropic, it is obtained

$$T^{\mu}{}_{\nu}|_{br} = \frac{\delta(y)}{b} diag\left(-\rho_b, p_b, p_b, p_b, 0\right) \quad (5)$$

Where ρ_b and p_b are the energy density and pressure in brane, and the followed assumption is there is no energy transfer from bulk to brane.

Following usual mathematical steps with the above mentioned equations and imposing Z_2 - symmetry, conservation relation on the bulk comes in the form of

$$\dot{\rho_B} + 3\frac{\dot{a}}{a}\left(\rho_B + P_B\right) + \frac{\dot{b}}{b}\left(\rho_B + P_B\right) = 0 \quad (6)$$

Considering modified chaplygin gas for bulk, other some important relations so obtained are

$$\rho_b = \frac{\rho_0}{a_b(t)} \quad (7)$$

Here ρ_0 is an integration constant, and $a_b(t)$ stands for the brane scale factor.

Again

$$\dot{a^2}_b = \frac{\kappa^4}{36}{\rho_0}^2 + \frac{\kappa^2}{6}\rho_B a_b^2 - \frac{\zeta}{a_b^2} - \kappa \quad (8)$$

With these equations (7) and (8), we can obtain

$$H^2 = \left(\frac{\dot{a}_b}{a_b}\right)^2 = \frac{\kappa^4}{36}\left(\frac{\rho_0^2}{a_b^2}\right) + \frac{\kappa^2}{6}\rho_B - \frac{\zeta}{a_b^4} - \frac{\kappa}{a_b^2} \quad (9)$$

3 Present Work

3.1 Density evolution in bulk

For the interaction term, we follow the arguments of the work Izquierdo and Pavon [7]. However, our consideration is only the interaction of dark energy with itself, that is, with dark energy. So the interaction term is $\Gamma = 3\epsilon H\rho_B$, where ϵ ought to be non-negative and small. Now with interaction term, conservation relation on the bulk is

$$\dot{\rho}_B + 3\frac{\dot{a}}{a}\left(\rho_B + P_B\right) + \frac{\dot{b}}{b}\left(\rho_B + P_B\right) = 0 \quad (10)$$

After simplification, we obtain

$$\dot{\rho}_B + 3\frac{\dot{a}}{a}\left[\left(1+\epsilon\right)\rho_B + P_B\right] + \frac{\dot{b}}{b}\left(\rho_B + P_B\right) = 0 \quad (11)$$

Since in our consideration, bulk candidate is modified chaplygin gas, we have

$$P_B = \gamma\rho_B - \frac{A}{\rho_B^{\alpha}} \quad (12)$$

Where γ and A are two positive constant, and $0 < \alpha \le 1$

On substitution of this equation of state in the conservation equation (), we obtain

$$\rho_B^{\alpha+1} = \frac{A}{1+\epsilon+\gamma} + \frac{C}{\left(a^3 b\right)^{(\alpha+1)(1+\epsilon+\gamma)}} \quad (13)$$

Where C is the constant of integration.

As modified chaplygin gas has the status perfect fluid, so

$$w = \gamma - \frac{A}{\rho_B^{\alpha+1}}$$

Where w is the state parameter. Taking the present magnitude of the state parameter as w_0, and already obtained relation in the basis of the mathematical structure given in the preliminaries

$$\frac{\dot{a}(t,y)}{a(t,y)} = \frac{\dot{b}(t,y)}{b(t,y)} = \frac{\dot{a}_b(t)}{a_b(t)} \quad (14)$$

we have the expression

$$\rho_B^{\alpha+1} = \frac{\rho_{B0}^{\alpha+1}}{A+(1+\epsilon+w_0)}\left[A+\frac{(1+\epsilon+w_0)\rho_{B0}^{\alpha+1}}{a_b^{4(\alpha+1)(1+\epsilon+\gamma)}}\right] \tag{15}$$

Now, let us suppose

$$A = n\rho_{B0}^{\alpha+1}$$

Where n some numerical number only. Substituting this for A in equation (15), we obtain

$$\rho_B^{\alpha+1} = \frac{1}{1+\epsilon+n+w_0}\left[n+\frac{1+\epsilon+w_0}{a_b^{4(\alpha+1)(1+\epsilon+n+w_0)}}\right]\rho_{B0}^{\alpha+1} \tag{16}$$

Let $\frac{1}{1+\epsilon+n+w_0} = K$, some constant, we can have

$$\rho_B^{\alpha+1} = K\left[n+\frac{1+\epsilon+w_0}{a_b^{4(\alpha+1)(1+\epsilon+n+w_0)}}\right]\rho_{B0}^{\alpha+1} \tag{17}$$

Therefore

$$\rho_B = K^{\frac{1}{\alpha+1}}\left[n+\frac{1+\epsilon+w_0}{a_b^{\frac{4(\alpha+1)}{K}}}\right]^{\frac{1}{\alpha+1}}\rho_{B0} \tag{18}$$

3.2 Solution of approximate modified Friedmann equation

The modified Friedmann equation (9) is considered in the approximate form as

$$H^2 = \left(\frac{\dot{a}_b}{a_b}\right)^2 = \frac{\kappa^4}{36}\left(\frac{\rho_0^2}{a_b^2}\right)+\frac{\kappa^2}{6}\rho_B \tag{19}$$

The other two terms are ignored as we are focused to late-time universe. The retained first term is very crucial as it radically changes the standard cosmology.

Now from equation (18) and (19)

$$H^2 = \left(\frac{\dot{a}_b}{a_b}\right)^2 = \frac{\kappa^4}{36}\left(\frac{\rho_0^2}{a_b^2}\right)$$

$$+\frac{\kappa^2}{6}K^{\frac{1}{\alpha+1}}\left[n+\frac{1+\epsilon+w_0}{a_b^{\frac{4(\alpha+1)}{K}}}\right]^{\frac{1}{\alpha+1}}\rho_{B0} \tag{20}$$

Now, let us follow some simplification:

Assuming, $\frac{4(\alpha+1)}{K} \cong 4(\alpha+1)$, we have

$$K^{\frac{1}{\alpha+1}}\left[n+\frac{1+\epsilon+w_0}{a_b^{\frac{4(\alpha+1)}{K}}}\right]^{\frac{1}{\alpha+1}}$$

$$= K^{\frac{1}{\alpha+1}}\left[\frac{na_b^{4(\alpha+1)}+1+\epsilon+w_0}{a_b^{4(\alpha+1)}}\right]^{\frac{1}{\alpha+1}}$$

Then $mod\left|\frac{1+\epsilon+w_0}{a_b^{4(\alpha+1)}}\right| \ll 1$, ultimately

$$K^{\frac{1}{\alpha+1}}\left[n+\frac{1+\epsilon+w_0}{a_b^{\frac{4(\alpha+1)}{K}}}\right]^{\frac{1}{\alpha+1}} = (Kn)^{\frac{1}{\alpha+1}}$$

Considering this simplification, we arrive from equation (20)

$$(\dot{a_b})^2 = \frac{\kappa^4}{36}\rho_0^2+\frac{\kappa^2}{6}(Kn)^{\frac{1}{\alpha+1}}a_b^2 \tag{21}$$

So, we can write

$$\int_{t_*}^{t}\frac{da}{\sqrt{(P^2+Q^2a_b^2)}} = \int_{t_*}^{t}dt \tag{22}$$

Where

$$\frac{\kappa^4}{36}\rho_0^2 = P^2$$

and

$$\frac{\kappa^2}{6}(Kn)^{\frac{1}{\alpha+1}} = Q^2$$

So, from equation (), we can arrive

$$Sinh^{-1}\left(\frac{a_b}{\frac{P^2}{Q^2}}\right)\Bigg|_{t_*}^{t} = \left(\frac{1}{1+n+\epsilon+w_0}\right)^{\frac{1}{2(\alpha+1)}}$$

$$\left[\frac{\kappa^2}{6}n^{\frac{1}{\alpha+1}}\right]^{\frac{1}{2}}(t-t_*) \tag{23}$$

And in the other form

$$log\left[\frac{a_b}{\frac{P^2}{Q^2}}+\sqrt{1+\left(\frac{a_b}{\frac{P^2}{Q^2}}\right)^2}\right]\Bigg|_{t_*}^{t} = \left(\frac{1}{1+n+\epsilon+w_0}\right)^{\frac{1}{2(\alpha+1)}}$$

$$\left[\frac{\kappa^2}{6}n^{\frac{1}{\alpha+1}}\right]^{\frac{1}{2}}(t-t_*) \tag{24}$$

4 Discussion

From equation (18), we see when $\epsilon+w_0 = -1$, that is, effective state parameter corresponds to cosmological constant, there is really no dynamic behaviour of dark energy. We have $\rho_B = \rho_{B0}$. In other cases, dark energy shows scale parameter dependency. Equation (23) shows exponetial time behaviour, and for $\epsilon+w_0 = -(1+n)$, there is singular behavior of scale factor. Moreover, $(\epsilon+w_0)$ ought to be negative, so if n is positive, $|(\epsilon+w_0)| < (1+n)$.

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