Cosmic-ray Electron Spectrum Estimated from Synchrotron Emissions

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DOI: 10.7529/ICRC2011/V06/0931

Abstract: High energy cosmic-ray electrons in the Galaxy are probably produced in supernova remnants (SNRs), which spectrum is estimated from the non-thermal components of radio and X-ray emissions. Such electron spectra in SNRs indicate a power-law shape with a cutoff, which suggests the upper limit of observed electron spectrum in the Galaxy. Further, cosmic-ray electrons propagating in the Galaxy lose their energy by the synchrotron radiation, and the diffuse radio data indicate the interstellar electron spectrum. We show the analytical formula of synchrotron spectrum from the electron spectrum with several breaks, and apply it to several synchrotron data in the Galaxy. The results show the expected maximum energy in electron observations and the Galactic magnetic field strength near the Earth.

Keywords: synchrotron radiation, electron spectrum

1 Introduction

The synchrotron radiations from relativistic electrons are observed from supernova remnants(SNRs) as the non-thermal emission, and also as background emissions in the Galaxy. The estimates of electron spectrum from those emissions is important because the observed electron intensity below 100 GeV is reduced by the solar activity and the correction is needed for estimates of the interstellar intensity. The diffuse gamma-rays also give us the information of electron spectrum, which include the Bremsstrahlung and the component from nuclear interactions, so that the estimate of the electron flux is limited above TeV. On the other hand the synchrotron data gives us an electron spectrum in a wide range below TeV directly, which may be the same as in SNRs.

In this paper we show the analytical calculation of the radiation spectrum emitted by electrons which have a power-law spectrum with several break energies. In the uniform magnetic field, the electron spectrum with the simple index γ emits the synchrotron spectrum of $(\gamma-1)/2$. If the electron spectrum changes the index from γ to γ', the synchrotron spectrum also bends from $(\gamma-1)/2$ to $(\gamma'-1)/2$. A similar argument is developed in the case of the electron spectrum with several break points. The electron spectrum is represented by the Mellin transforms of the unit step functions and is determined from the least square method between the calculated synchrotron spectrum and the observed data.

Such procedures give us the clear shape of the break points of electron spectrum and the important information about the propagation of cosmic rays. We apply this method to the synchrotron spectrum from SNRs and estimate the upper limit of electron energy. Another application is the background radio emission, and estimation of the Galactic magnetic field near the Earth.

1.1 General formulas

The relativistic electron of energy E which moves at the angle θ to the direction of the magnetic field H emits the electromagnetic energy between the frequency ν and $\nu+d\nu$ [10],

$$s(E,\nu)d\nu \;=\; kH\sin\theta\, F(\frac{\nu}{\nu_s})\, d\nu \quad [\mathrm{W}]\,,$$

where

$$\begin{aligned}
k &= \sqrt{3}\frac{e^3}{mc^2} \;=\; 2.34\times10^{-29} \quad [\mathrm{WHz^{-1}G^{-1}}]\,,\\
F(\frac{\nu}{\nu_s}) &= \frac{\nu}{\nu_s}\int_{\nu/\nu_s}^{\infty} K_{\frac{5}{3}}(x)dx\,,\\
\nu_s &= aE^2H\sin\theta\,,\\
a &= \frac{3}{4\pi}\frac{e}{mc}(\frac{1}{mc^2})^2 \;=\; 1.61\times10^{13}\\
&\qquad [\mathrm{HzGeV^{-2}G^{-1}}]\,.
\end{aligned} \tag{1}$$

The electron simple power-law spectrum with the index γ

$$n(E)dE = n_0(\frac{E_0}{E})^{\gamma}\, d(\frac{E}{E_0})$$

radiates the synchrotron spectrum,

$$S(\nu)d\nu \;=\; \int_0^{\infty} s(E,\nu)n(E)dE\, d\nu \tag{2}$$

$$
\begin{aligned}
&= kn_0 H \sin\theta \frac{1}{\gamma+1}(\frac{2aE_0^2 H\sin\theta}{\nu})^{\frac{\gamma-1}{2}} \\
&\quad \cdot \Gamma(\frac{3\gamma-1}{12})\Gamma(\frac{3\gamma+19}{12})\, d\nu\ , \quad (3)
\end{aligned}
$$

and the spectrum averaged over the angle θ is given by

$$
\begin{aligned}
\overline{S}(\nu)d\nu &= \frac{1}{2}\int_0^\pi S(\nu)\sin\theta d\theta d\nu \\
&= kn_0 H \frac{\sqrt{\pi}}{2}\frac{1}{\gamma+1}(\frac{2aE_0^2H}{\nu})^{\frac{\gamma-1}{2}} \\
&\quad \cdot \frac{\Gamma(\frac{\gamma+5}{4})\Gamma(\frac{\gamma}{4}-\frac{1}{12})\Gamma(\frac{\gamma}{4}+\frac{19}{12})}{\Gamma(\frac{\gamma+7}{4})}\, d\nu \quad (4)
\end{aligned}
$$

[11] . If the electron spectrum has a power-law index γ and an exponential cutoff at E_1,

$$
n(E)dE = n_0(\frac{E}{E_1})^{-\gamma}\exp(-\frac{E}{E_1})d(\frac{E}{E_1})\ ,
$$

the synchrotron spectrum $S(\nu)d\nu$ is given by

$$
\begin{aligned}
S(\nu)d\nu = &\ \frac{\sqrt{3}}{2}\pi^{3/2}kn_0H\{ \\
&\sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}\Gamma(\frac{3}{4}(-n+\gamma-\frac{1}{3}))\frac{\frac{-n+\gamma}{4}+\frac{7}{12}}{3^{3(-n\gamma-1/3)/4}} \\
&(\frac{2aHE_1^2}{\nu})^{\frac{-n+\gamma-1}{2}}/\Gamma(\frac{-n+\gamma+7}{4}) \\
&+\sum_{n=0}^{\infty}\frac{2(-1)^n}{3n!}\Gamma(\frac{-4n+1}{3}-\gamma)\frac{(-n+2)/3}{3^{-n}} \\
&(\frac{2aHE_1^2}{\nu})^{\frac{-2n-1}{3}}/\Gamma(\frac{-2n+11}{6})\}d\nu\ .
\end{aligned}
$$

2 Calculations

The synchrotron spectrum radiated from the electron spectrum with several break energies is formulated.

If the electron spectral index changes from γ_1 to γ_2 at an energy E_0, the spectrum is represented by combinations of step-like spectra as

$$
\begin{aligned}
n(E)dE &= n_0[(\frac{E_0}{E})^{\gamma_1}\mathbf{1}(E_0-E) \\
&\quad +(\frac{E_0}{E})^{\gamma_2}\mathbf{1}(E-E_0)]d(\frac{E}{E_0})\ , \quad (5)
\end{aligned}
$$

where $\mathbf{1}(x)$ is a unit function and the Mellin transforms of the unit functions are given by

$$
\begin{aligned}
\mathbf{1}(x-1) &= \frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty}-\frac{x^{-s}}{s}ds \quad \epsilon<0\ , \\
\mathbf{1}(1-x) &= \frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty}\frac{x^{-s}}{s}ds \quad \epsilon>0\ .
\end{aligned}
$$

The synchrotron spectrum of eq. (2) becomes

$$
\begin{aligned}
S(\nu)d\nu &= kn_0H\sin\theta\frac{(u/\sin\theta)^2}{2} \\
&\quad \int_0^\infty dx\,[x^{\frac{\gamma_1-1}{2}}\mathbf{1}(x-1)+x^{\frac{\gamma_2-1}{2}}\mathbf{1}(1-x)] \\
&\quad \times\int_x^\infty K_{\frac{5}{3}}(\frac{uy}{\sin\theta})dy\,d\nu\ , \quad (6)
\end{aligned}
$$

where $x=(E_0/E)^2$ and $u=\nu/aHE_0^2$. The first term of eq. (6) except the coefficient is calculated from the Mellin transform substituted, and the integrals are performed as

$$
\begin{aligned}
&\frac{1}{2\pi i}\int-\frac{ds}{s}\int_0^\infty dx x^{\frac{\gamma_1-1}{2}-s}\int_x^\infty K_{\frac{5}{3}}(\frac{uy}{\sin\theta})dy \\
&= \frac{1}{4\pi i}\int_C d\mu\frac{1}{\mu-\frac{\gamma_1+3}{2}}\frac{1}{\mu-1}(\frac{2\sin\theta}{u})^\mu \\
&\quad \times\Gamma(\frac{\mu-\sigma}{2})\Gamma(\frac{\mu+\sigma}{2})\ ,
\end{aligned}
$$

where $\mu=(\gamma_1+3)/2-s$ and $\sigma=5/3$. The contour C is $Re(\mu)>(\gamma_1+3)/2$, and the poles are given by

$$
\mu=\frac{\gamma_1+3}{2}\ ,\ 1\ ,\ -2n+\sigma\ ,\ -2n-\sigma \quad n=0,\,1,\,2,\,3,\cdots
$$

The residue of $\mu=(\gamma_1+3)/2$ results in the term $A(\nu,\gamma)$ of the eq. (8).

The result of the synchrotron spectrum is given by

$$
\begin{aligned}
S(\nu)d\nu &= kn_0H\sin\theta \\
&\quad [A(\nu,\gamma_1)-B(\nu,\gamma_1)+B(\nu,\gamma_2)]\,d\nu \quad (7) \\
A(\nu,\gamma) &= \frac{1}{\gamma+1}(\frac{2\nu_{0\perp}}{\nu})^{\frac{\gamma-1}{2}} \\
&\quad \Gamma(\frac{\gamma+3-2\sigma}{4})\Gamma(\frac{\gamma+3+2\sigma}{4}) \quad (8) \\
B(\nu,\gamma) &= \frac{1}{\gamma+1}(\frac{\nu}{2\nu_{0\perp}})\Gamma(\frac{1-\sigma}{2})\Gamma(\frac{1+\sigma}{2}) \\
&\quad -\sum_{n=0}^{\infty}\{\frac{(-1)^n}{2n-\sigma+\frac{\gamma+3}{2}}\frac{1}{2n-\sigma+1} \\
&\quad (\frac{\nu}{2\nu_{0\perp}})^{2n-\sigma+2}\frac{\Gamma(-n+\sigma)}{n!} \\
&\quad +(\sigma\to-\sigma)\}\ , \quad \sigma=5/3 \quad (9)
\end{aligned}
$$

where the break frequency $\nu_{0\perp}\equiv aE_0^2H\sin\theta$. The eq. (8) is equivalent to the eq. (3).

If the electrons move randomly in the magnetic field, the synchrotron spectrum averaged over the angle θ becomes

$$
\begin{aligned}
\overline{S}(\nu)d\nu &= kHn_0 \\
&\quad [\overline{A}(\nu,\gamma_1)-\overline{B}(\nu,\gamma_1)+\overline{B}(\nu,\gamma_2)]d\nu\ , \\
\overline{A}(\nu,\gamma) &= \frac{\pi^{\frac{3}{2}}}{16}(\gamma+\frac{7}{3})(\frac{2\nu_0}{3^{3/2}\nu})^{\frac{\gamma-1}{2}} \\
&\quad \times\Gamma(\frac{3\gamma-1}{4})\Gamma(\frac{\gamma+7}{4})^{-1}\ , \quad (10) \\
\overline{B}(\nu,\gamma) &= -\frac{\sqrt{3}}{12}\pi^{\frac{3}{2}}\sum_{n=0}^{\infty}\frac{n-2}{n+\frac{3\gamma-1}{4}}\frac{(-3)^n}{n!} \\
&\quad \times(\frac{\nu}{2\nu_0})^{\frac{2n+1}{3}}\Gamma(\frac{11}{6}-\frac{n}{3})^{-1}\ , \quad (11)
\end{aligned}
$$

Electron Spectrum $n(E)dE$	Synchrotron Spectrum $kHn_0 \int s(E,\nu)n(E)dE$	Approximations ($\nu_0 = aHE_0^2$)
$n_0(\frac{E_0}{E})^\gamma d(\frac{E}{E_0})$	$A(u,\gamma)$	$\nu^{-(\gamma-1)/2}$
$n_0(\frac{E_0}{E})^\gamma \mathbf{1}(E_0 - E)d(\frac{E}{E_0})$	$A(u,\gamma) - B(u,\gamma)^*$	$\nu \ll \nu_0$ $\nu^{-(\gamma-1)/2}$ $\nu \gg \nu_0$ $\nu^{-1/2}e^{-\nu}$
$n_0(\frac{E_0}{E})^\gamma \mathbf{1}(E - E_0)d(\frac{E}{E_0})$	$B(u,\gamma)$	$\nu \ll \nu_0$ $\nu^{1/3}$ $\nu \gg \nu_0$ $\nu^{-(\gamma-1)/2}$
$n_0\{(\frac{E_0}{E})^\gamma \mathbf{1}(E_0 - E)$ $+(\frac{E_0}{E})^{\gamma'} \mathbf{1}(E - E_0)\}\ d(\frac{E}{E_0})$	$A(u,\gamma)$ $-B(u,\gamma) + B(u,\gamma')$	$\nu \ll \nu_0$ $\nu^{-(\gamma-1)/2}$ $\nu \gg \nu_0$ $\nu^{-(\gamma'-1)/2}$

* $A(u,\gamma)$ and $B(u,\gamma)$ are given by eq. (8) and eq. (9), where $u = \nu/aE_0^2 H \sin\theta$.
If those are averaged over θ, eq. (10) and eq. (11) are adopted with $u = \nu/aE_0^2 H$.

Table 1: The synchrotron spectrum from the electron spectrum with a break energy

where the frequency $\nu_0 \equiv aE_0^2 H$. The eq. (10) is completely equivalent to eq. (4). The results and the approximations are summarized in Table 1.

If the electron spectrum has N break energies, the synchrotron radiation spectrum also has N breaks. In the case of $N = 2$, the electron spectrum with the break energy E_1 and E_2, is represented by

$$\begin{aligned} n(E)dE &= n_0\{(\frac{E_1}{E})^{\gamma_1}\mathbf{1}(E_1 - E) \\ &+(\frac{E_1}{E})^{\gamma_2}\mathbf{1}(E - E_1)\mathbf{1}(E_2 - E) \\ &+(\frac{E_1}{E_2})^{\gamma_2}(\frac{E_2}{E})^{\gamma_3}\mathbf{1}(E - E_2)\}\frac{dE}{E_1} \end{aligned} ,(12)$$

and the new term is introduced as

$$\begin{aligned} \overline{B}(\nu/\nu_i,\gamma_1,\gamma_2) &\equiv \overline{B}(\nu/\nu_i,\gamma_1) - \overline{B}(\nu/\nu_i,\gamma_2) \qquad i = 1,\ 2 \\ &= -\sqrt{3}\pi^{\frac{3}{2}}\sum_{n=0}^{\infty}\frac{(n-2)(\gamma_2-\gamma_1)}{(4n+3\gamma_1-1)(4n+3\gamma_2-1)} \\ &\times\frac{(-3)^n}{n!}(\frac{\nu}{2\nu_i})^{\frac{2n+1}{3}}\Gamma(\frac{11}{6}-\frac{n}{3})^{-1} , \end{aligned}$$

with the $\nu_1 = aHE_1^2$ and $\nu_2 = aHE_2^2$, the synchrotron spectrum from eq. (12) becomes

$$\begin{aligned} \overline{S}(\nu)d\nu &= kHn_0[\overline{A}(\nu/\nu_1,\gamma_1) - \overline{B}(\nu/\nu_1,\gamma_1,\gamma_2) \\ &-(\nu_1/\nu_2)^{(\gamma_2-1)/2}\overline{B}(\nu/\nu_2,\gamma_2,\gamma_3)]d\nu . \end{aligned}$$

In conclusion, electron spectrum having N break energies of $E_1, E_2, \cdots, E_N$ in the range of $E_0 \sim E_{N+1}$,

$$\begin{aligned} n(E)dE &= n_0\sum_{n=0}^{N}(\prod_{i=1}^{n-1}(\frac{E_i}{E_{i+1}})^{\gamma_{i+1}})(\frac{E_n}{E})^{\gamma_{n+1}} \\ &\mathbf{1}(E-E_n)\mathbf{1}(E_{n+1}-E)\frac{dE}{E_1} , \end{aligned}$$

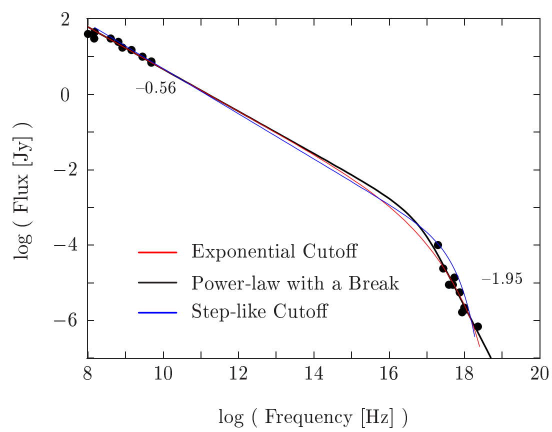

Figure 1: Synchrotron spectra of a typical SNR estimated from electron spectra with different cutoff types

radiates the synchrotron spectrum of

$$\begin{aligned} \overline{S}(\nu)d\nu &= kHn_0[\overline{A}(\nu/\nu_1,\gamma_1) \\ &-\sum_{n=1}^{N}(\prod_{i=1}^{n-1}(\frac{\nu_i}{\nu_{i+1}})^{(\gamma_{i+1}-1)/2})\overline{B}(\nu/\nu_n,\gamma_n,\gamma_{n+1})]d\nu , \end{aligned} \quad (13)$$

where $\nu_n = aHE_n^2$ and $\nu_i/\nu_{i+1} = (E_i/E_{i+1})^2$.

3 Applications and Results

The above formulas are applied to non-thermal radiation from SNRs and the Galactic background radiation.

The electron spectra with different cutoff types are examined from the non-thermal radiation data of a typical SNR. For an example, SN1006 has been adopted as non-thermal

X-rays have been observed [6], though it is too far from the Earth to observe electrons. The estimated synchrotron spectra are shown in Figure 1 with synchrotron data [8]. One type of an electron spectrum is given by eq.(7) with the change of index –2.1 to –4.9, and a break energy of 13 TeV. The other types are the exponential cutoff at 14 TeV, and the step-like cutoff at 30 TeV. Each synchrotron spectrum is calculated at the magnetic field of 20 μG, and has almost the same break energy, though the step-like shape has a little larger than others. The electron total energy corresponds to 5×10^{47} erg at the distance of 2.2 kpc [3].

The radio background radiation of the polar and the anticenter direction in the Galaxy have been transformed to the electron spectrum, which is compared with the observed spectrum for estimation of the Galactic magnetic field. The radiation intensity relates to the flux of eq. (13) as $I(\nu)d\nu = \overline{S}(\nu)(L/4\pi)d\nu$, where L means the effective length of the line-of-sight. For anti-center direction, the radiation data from 10 MHz to 10 GHz are adopted because the data below 10 MHz include the free–free absorption effect [9]. The radio data from the polar direction are given in the range of 0.7 MHz $\sim$ 2 GHz [12][7] in which the extra-galactic component is excluded. For estimation of the local magnetic field, the data from the polar direction is more reliable than that from anticenter direction because it does not include the flux of other arms and inter arm regions. Therefore the anti-center data are multiplied by 0.48, and are fitted to the polar data at 2 GHz. The both data are consistent as shown in Figure 2. The corresponding range of the electron spectrum is $E = 0.5 \sim 10$ GeV from the relationship of eq. (1) at several μG .

The result is that the electron spectrum has two break energies and the index changes from -2.2 to -2.5 at 1.7 GeV and to -3.0 at 5.7 GeV. The electron intensity calculated from $j(E)dE = (c/4\pi)n(E)dE$ includes the parameters of H and L, and the comparison with observed electron data gives the relationship of L and H. If the electron intensity at 10 GeV is given by $(0.4 \sim 0.5)\ \mathrm{m^{-2}sec^{-1}sr^{-1}GeV^{-1}}$ from consideration of the solar modulation [4][5], the comparison with the calculated spectrum gives the result of $L[\mathrm{kpc}]H_{\perp}[\mu\mathrm{G}]^2 = 35 \pm 5$. The local magnetic field strength becomes $H_{\perp} = 5.4 \pm 0.4\ \mu$G in the case of $L = 1.2$ kpc at the polar direction [12].

4 Discussions

The synchrotron spectrum from the electron spectrum with several breaks has been formulated, which will be applied to a complex observed spectrum. As an application, the electron spectra with different cutoff types in SN1006 have been shown in this paper. The recent measurements of gamma-rays [1] and the X-ray break frequency [2] have reported the lower limit of the magnetic field of 25 μG and the maximum energy of electrons around 10 TeV . The estimation in this paper is consistent with their results. In general, if the magnetic field in SNRs is strong as 100 μG, the electron cutoff energy might be measured at the Earth.

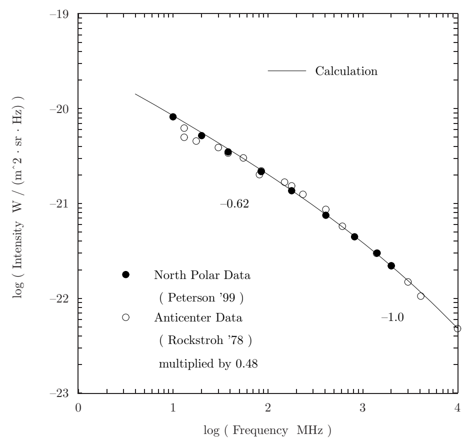

Figure 2: The calculation of Galactic radio emissions compared with the polar data and the anticenter data multiplied by 0.48.

The estimate of the Galactic magnetic field near the solar system is considerably higher than the typical mean value of 3 μG. The discrepancy may come from the uncertainty of the path length of the radio observation and the irregularity of the magnetic field. The Galactic magnetic field consists of the regular component parallel to the arm and the irregular component caused by molecular clouds, old SNRs, etc. which is pointed out to be equivalent to or larger than the regular one [13][14].

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