

Constraints on the total mass and mass hierarchy of neutrinos from cosmological observations with bayesian analysis

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Abstract. In the standard model neutrinos consist of three species (flavors) and are massless. However, solar neutrino problem that was solved by neutrino oscillations requires that neutrinos have non-zero masses. Another support for non-zero neutrino masses comes from kinematic experiments. Oscillation experiments give rise to a further problem regarding the order of neutrino masses, known as the neutrino mass hierarchy problem, i.e. whether the hierarchy is normal or inverted. On cosmological scales, massive neutrinos affect Cosmic Microwave Background (CMB) and Large Scale Structure (LSS) formation. The impact of massive neutrinos is imprinted in the power spectrum of CMB, and in the LSS matter power spectrum as well. Since neutrinos affect events in the universe, it is possible to constrain neutrino properties based on cosmological observations. This work is aiming at constraining neutrino total mass from cosmological observations in the two hierarchy scenarios, i.e. normal and inverted hierarchies, and also at determining which neutrino mass hierarchy is favoured by cosmological observations. To constrain neutrino total mass we employed CosmoMC and performed Bayesian analysis with MCMC algorithm, while neutrino mass hierarchy was determined based on Bayes factors, in which likelihoods of the two hierarchy scenarios were compared. In addition, neutrino individual masses were estimated by combining results of oscillation experiments. Depending on the dataset used, different total mass constraints were obtained. The tightest constraint comes from a combination of CMB + LSS + BAO + Supernova data, i.e. $\sum m_\nu < 0.183$ eV (normal hierarchy) and $\sum m_\nu < 0.188$ eV (inverted hierarchy) with 95% C.L. The Bayes factors analysis prefers normal hierarchy, although with weak evidence.

Keywords: *Mass hierarchy of neutrinos, Cosmological observation, and Bayesian analysis*

1. Introduction

Neutrinos are fermions that come in three flavours (electron, muon and tau neutrinos) and are massless in the standard model. However, oscillation experiments (solar and atmospheric) show that neutrinos change flavours from one to another during propagation, meaning that they have non-zero masses. The experiments give squared mass difference instead of absolute masses, which lead to another problem known as neutrino mass hierarchy problem. The neutrino mass hierarchy is called normal (NH) if $m_1 \ll m_2 < m_3$ and inverted (IH) if $m_3 \ll m_1 < m_2$. The total mass of neutrinos depends on the hierarchy. [1] for example, found a lower limit of 0.06 eV if the mass hierarchy is normal, and 0.1 eV if it is inverted. Beside oscillation experiments, kinematic experiments give us evidence for non-zero neutrino masses and set upper bounds on the individual mass of each neutrino, i.e. $m_{\nu_e} < 0.2$ eV, $m_{\nu_\mu} < 190$ keV and $m_{\nu_\tau} < 18.2$ MeV [2]. Massive neutrinos play important roles in the history of the universe. Neutrinos have significant impacts on the two most important observables: Cosmic Microwave Background (CMB) and large scale



structures (LSS). Prior to last scattering, there were two important epochs influenced by the total mass of neutrinos: matter-radiation equality and recombination. If the total mass of neutrinos is greater than ~ 0.5 eV, they would contribute to the matter component, otherwise to the radiation component. The total mass of neutrinos affects the heights of the peaks in the power spectrum of CMB, especially the heights of the acoustic peaks ($200 < l < 1000$), and the integrated Sachs-Wolfe (ISW, $20 < l < 200$) region. It also shifts the position of the acoustic peaks. If neutrinos contribute to the matter density, the height of the acoustic peaks in the power spectrum of CMB would decrease with the increasing total mass of neutrinos. On the contrary, if neutrinos contribute to the energy density of radiation, the height of the acoustic peaks would increase with the increasing total mass of neutrinos. The ISW region is affected only if neutrinos contribute to the matter density. The height of the power spectrum in this scale would decrease with the increasing neutrino total mass. As aforementioned, neutrino total mass has also impact on the position of the acoustic peaks. The higher the neutrino total mass, the more to the left (to the smaller multipoles) would the peaks be shifted. This effect can be seen either neutrinos contribute to radiation or matter density. Massive neutrinos also play important roles in the formation of large scale structures. On small scales, massive neutrinos would suppress structure growth. This effect shows up on the small scales of the matter power spectrum.

Since massive neutrinos influence the history of the universe, constraining the neutrino total mass can be done by analysing cosmological data. In this work, we used data from CMB, LSS, Baryon Acoustic Oscillation (BAO) and Supernova Ia observations. By combining these data, we aimed at better constraints on other cosmological parameters, so that we in turn get tighter constraint on the total mass of neutrinos. Here we have focused on constraining the total mass of neutrinos from cosmological data with Bayesian statistics by employing CosmoMC [3]. We discussed two cases of the neutrino mass hierarchy, i.e. normal hierarchy and inverted hierarchy. We also calculated likelihood probabilities for both cases and compared their Bayes factors, in order to choose which neutrino mass hierarchy is preferred by the observational data. Thereafter, we calculated neutrino individual masses by following the method described in [1]. In this project, we used the standard cosmological model (Λ CDM model).

This paper is organized as follows: we explain data and method used in this project in section 2, in section 3 we show and discuss the results, and we conclude our work in section 4.

2. Data and Method

We used Bayesian statistics to constrain cosmological parameters, especially the total mass of neutrinos. Mathematically, Bayes theorem is based on conditional probabilities:

$$P(\theta|d) = \frac{P(d|\theta)P(\theta)}{P(d)} \quad (1)$$

where θ is the parameter we search for, d is data, $P(d|\theta)$ is likelihood probability, $P(\theta)$ is prior probability, $P(d)$ is evidence and $P(\theta|d)$ is posterior probability.

Bayesian inference can be carried out by using Markov Chain Monte Carlo (MCMC) algorithm. We used “efficient sampling of fast and slow” MCMC algorithm [4] that converges faster than other algorithms. This algorithm is included in CosmoMC, a widely used code to explore cosmological parameters. For each dataset, we run CosmoMC for two scenarios of neutrino mass hierarchy, i.e. normal and inverted. The priors we chose were based on Λ CDM model, as are summarized in Table 1. The first six parameters come from Planck Collaboration 2015 for Λ CDM model and the total mass of neutrinos is from Particle Data Group (PDG) results in 2011-2016.

Table 1. Priors for Bayesian analysis with CosmoMC

Parameter	Physics	Prior
$\Omega_c h^2$	Density parameter of cold dark matter	0.001 – 0.239
$\Omega_b h^2$	Density parameter of baryon	0.005 -0.035
$100\Theta_s$	Ratio between diameter of angular distance to sound horizon	0.5 – 5
τ	Thomson's optical depth at reionization	0.01 – 0.5
n_s	Scalar spectral index	0.8 -1.2
$\log A_s$	Amplitude of primordial power spectrum	2 – 4
$\sum m_\nu$ (eV)	Total mass of neutrinos	0 – 1.5

After calculating the posterior probability of each mass hierarchy for each dataset, we stepped further with comparing the two neutrino mass hierarchies:

$$\frac{P(\theta|d)_1}{P(\theta|d)_2} = \frac{P(d|\theta)_1 P(\theta)_1}{P(d|\theta)_2 P(\theta)_2} = B_{1,2} \frac{P(\theta)_1}{P(\theta)_2} \quad (2)$$

Here $B_{1,2}$ is a Bayes factor,

$$B_{1,2} = \frac{P(d|\theta)_1}{P(d|\theta)_2} \rightarrow \ln B_{1,2} = \ln P(d|\theta)_1 - \ln P(d|\theta)_2 \quad (3)$$

To decide which hierarchy is favoured by the observational data, we used Jeffrey's scale as summarized in Table 2.

Table 2. Jeffrey's scale [5] for interpretation of Bayes factors.

$\ln B_{1,2}$	Odds	Strength of evidence
<1.0	< 3 : 1	inconclusive
1.0 – 2.5	(3 – 12) : 1	weak
2.5 – 5.0	(12 – 150) : 1	moderate
> 5.0	> 150 : 1	strong

The datasets used in this project are summarized in Table 3. CMB [6] and LSS (WiggleZ [7], SDSS LRG DR 4 [8]) data come from processes that are directly influenced by the total mass of neutrinos. We included additional data that are less directly connected to the total mass of neutrinos, but would tighten constraints on other cosmological parameters, which in turn have impact on the neutrino total mass. For example, we used CMB lensing data [6] to constrain A_s and σ_8 , that are related more directly to the total mass of neutrinos. We included SDSS BAO DR12 Final consensus data [9] to constrain matter density, since neutrinos contribute to matter density if they are nonrelativistic. Pantheon Supernova data [10] were included to constrain H_0 , that would affect distance scales.

Having decided on the favoured mass hierarchy from the analysis of Bayes factors, we proceeded to the determination of individual mass of neutrinos by using hierarchy parameters defined in [1] as

$$\Delta = \frac{m_3 - m_1}{m_3 + m_1} \quad (4)$$

combined with oscillation experiment results from Nu-Fit 2018:

$$\begin{aligned}\Delta m_{21}^2 &= 7.40_{-0.20}^{+0.21} \times 10^{-5} \text{eV}^2 \\ \Delta m_{31}^2 &= 2.494_{-0.031}^{+0.033} \times 10^{-3} \text{eV}^2\end{aligned}\quad (5)$$

3. Results and Analysis

We summarize our results in Table 3 for each mass hierarchy and dataset. The triangle plots of the derived parameters are shown in Figure 1, for the case of normal hierarchy. Correlations of other cosmological parameters with the total mass of neutrinos are shown here only for σ_8 , Ω_m , and H_0 . The figure shows that the higher the total mass of neutrinos, the higher is the matter density, but the lower is the Hubble constant, because higher matter density would slow down the expansion of the universe. In the small scales, the correlation between σ_8 (the fluctuation on a scale of $8\text{Mpc}/h$) and the total mass of neutrino is negative, because neutrinos would obstruct structure growth in small scales due to free-streaming of neutrinos. We found that adding lensing to CMB data worsen the constraint on the neutrino total mass, which is not expected and we are still working on the explanation for this. Adding BAO SDSS BAO DR12 Final consensus data to CMB + lensing data improves the constraint significantly, better than adding LSS data. Even the combination of CMB + lensing + LSS + BAO does not differ from CMB + lensing + BAO. Finally, adding supernova data improves the constraint only slightly. The constraints on the total mass of neutrinos we obtained from cosmological observations are therefore stringent than those obtained from kinematic experiments, such as in [2]. This is expected, since a kinematic experiment measures the sum of the rest mass and the kinetic energy of neutrino, whereas from cosmology we get the rest mass, albeit the total rest mass.

Our Bayes factor calculations (see Table 4) show that we are not yet able to distinguish between normal and inverted mass hierarchies with strong evidence. It is expected to be solved by the next missions of LSS survey, such as EUCLID, that will provide us with higher precision data. Although we do not have strong evidence to decide which neutrino mass hierarchy is favoured by the observational data, we still have moderate evidence that favours normal hierarchy from the Bayes factors of CMB + lensing and CMB + lensing + LSS datasets (see Table 4). We therefore chose normal hierarchy and proceeded to the determination of the individual masses of neutrinos. The results are summarized in Table 5. As expected, the most stringent constraint on the individual mass comes from the combination of all data. The upper limit of each neutrino mass eigenstate is given for 95% C.L.

Table 3. Result of total mass of neutrinos of each mass hierarchy from six dataset

Data set	Hierarchy	Σm_ν (eV) (68% CL)	Σm_ν (eV) (95% CL)
CMB TT, TE, EE + LowTEB (CMB)	normal	< 0.258	< 0.593
	inverted	< 0.160	< 0.404
CMB TT, TE, EE + LowTEB + lensing (CMB + lensing)	normal	< 0.335	< 0.591
	inverted	< 0.318	< 0.540
CMB TT, TE, EE + LowTEB + lensing+ WiggleZ + SDSS LRG DR4 (CMB + lensing + LSS)	normal	< 0.253	< 0.434
	inverted	< 0.267	< 0.504
CMB TT, TE, EE + LowTEB + lensing + SDSS BAO DR12 Final consensus (CMB + lensing + BAO)	normal	< 0.112	< 0.205
	inverted	< 0.109	< 0.202
CMB TT, TE, EE +	normal	< 0.104	< 0.195

LowTEB + lensing+ WiggleZ + SDSS LRG DR4 + SDSS BAO DR12 Final consensus (CMB + lensing + LSS + BAO)	inverted	< 0.110	< 0.203
CMB TT, TE, EE + LowTEB + lensing+ WiggleZ + SDSS LRG DR4 + SDSS BAO DR12 Final consensus +Pantheon (CMB + lensing + LSS + BAO +SN)	normal	< 0.098	< 0.183
	inverted	< 0.105	< 0.188

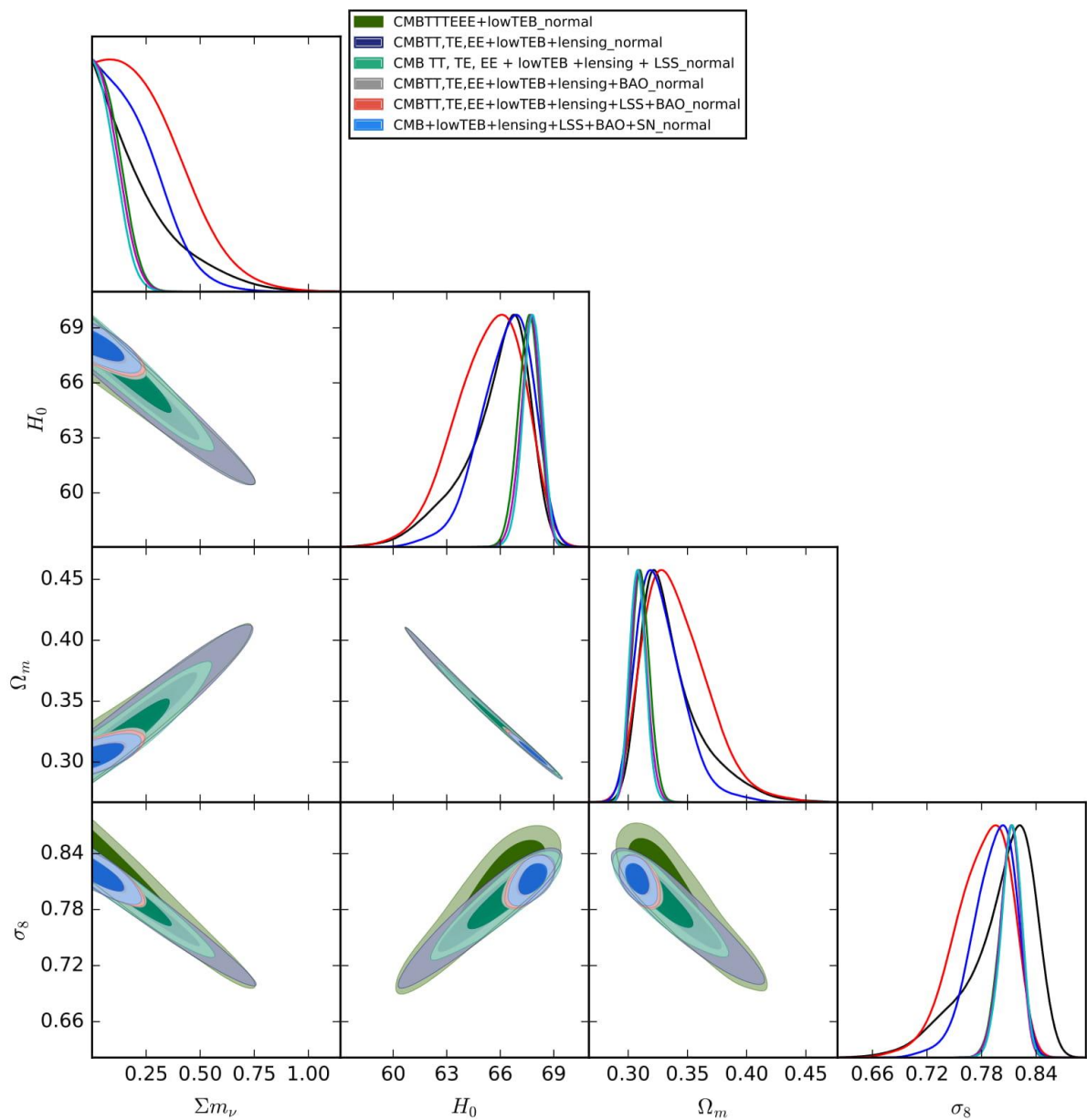


Figure 1. Triangle plots that show correlations between some derived parameters with total mass of neutrino for the case of normal hierarchy.

Table 4. Bayes factors calculated for different dataset

Data Set	Hierarchy	Best-fit (log(likelihood))	$\ln B_{\text{NH,IH}}$	$\ln B_{\text{IH,NH}}$
CMB	normal	-6472.011	1.050	-1.050
	inverted	-6472.467	(weak)	(inconclusive)
CMB +lensing	normal	-6477.220	3.829	-3.829
	inverted	-6478.883	(moderate)	(inconclusive)
CMB +lensing + LSS	normal	-6717.594	2.745	-2.745
	inverted	-6718.786	(moderate)	(inconclusive)
CMB +lensing + BAO	normal	-6480.983	1.418	-1.418
	inverted	-6481.599	(weak)	(inconclusive)
CMB +lensing + LSS +BAO	normal	-6721.289	-0.120	0.120
	inverted	-6721.237	(inconclusive)	(inconclusive)
CMB +lensing + LSS +BAO+SN	normal	-7239.403	-0.281	0.281
	inverted	-7239.281	(inconclusive)	(inconclusive)

Table 5. Individual masses of neutrinos derived from different datasets

Data set	$< m_1(\text{eV})$	$< m_2(\text{eV})$	$< m_3(\text{eV})$
CMB	0.19552 ± 0.00437	0.19571 ± 0.00437	0.20180 ± 0.00426
CMB+lensing	0.19484 ± 0.00437	0.19503 ± 0.00437	0.20114 ± 0.00426
CMB+lensing+LSS	0.14174 ± 0.00434	0.14200 ± 0.00433	0.15028 ± 0.00426
CMB+ lensing+BAO	0.06229 ± 0.00464	0.06288 ± 0.00463	0.07984 ± 0.00364
CMB+lensing+ LSS + BAO	0.05867 ± 0.00468	0.05929 ± 0.00468	0.07704 ± 0.00359
CMB+ lensing +LSS +BAO + SN	0.05428 ± 0.00475	0.05496 ± 0.00474	0.07376 ± 0.00352

4. Conclusion

Cosmology gives us constraints on neutrino total mass. In our project, we found that the most stringent constraint is obtained if we include SDSS BAO DR12 Final consensus. Cosmological data cannot yet distinguish between the two neutrino mass hierarchies with strong evidence, but we found moderate evidence that normal hierarchy is favoured from the Bayes factor calculated for the combination of CMB + lensing + LSS. A global analysis that combine different observations of neutrinos, i.e. cosmology, oscillations and kinematics would do the task better. In the future, observations such as EUCLID (cosmology) and KATRIN and MARE (kinematics experiments) will provide us with higher precision data, so that we can better improve the constraints on neutrino mass and hierarchy.

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