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RESEARCH ARTICLE

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Electrodynamic radiation reaction and general relativity

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Abstract We argue that the well-known problem of the instabilities that are associated with the self-forces (radiation reaction forces) in classical electrodynamics are possibly stabilized by the introduction of gravitational forces via general relativity.

Keywords Equations of motion for charged bodies, Radiation reaction forces

1 Introduction

The problems and difficulties associated with the motion of charged particles interacting with both an external electromagnetic field and its own self-field have not been resolved even after over a century of investigation [1; 2; 3; 4; 5]. The problems arise in many different contexts: the difficulties in giving appropriate initial conditions, infinite self-energy problems, model building, Lorentz invariance difficulties and perhaps the most serious, the instabilities (or pre-acceleration) in the solutions to the equations of motion.

The best known equations of motion, coming from a point structureless particle, are the Abraham–Lorentz equations and the relativist generalization the Abraham–Lorentz–Dirac equations. They are given, respectively, by [3]

$$m\dot{\overrightarrow{v}} = q\overrightarrow{E} + q\overrightarrow{B}\times\overrightarrow{v} + \frac{2q^2}{3c^3}\ddot{\overrightarrow{v}}, \tag{1}$$

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and [2]

$$m\dot{v}^a = qF^{ab}v_b + \frac{2q^2}{3c^3}\left(\ddot{v}^a + \frac{1}{c^2}v^a\dot{v}^b\dot{v}_b\right), \tag{2}$$

where F^{ab} (or $\overrightarrow{E}, \overrightarrow{B}$) are an external field and are derived by a variety of means, but always with severe approximations.

It is generally acknowledged that there are fundamental difficulties with this issue. And there seems to be a variety of different reasons, explanations and suggested remedies for these difficulties. They range from: quantum theory is the resolution, to the approximations leading to these equations are wrong or even that there is no real problem. The author, Jackson [1], summarizes the situation in his well known graduate text as:

"The difficulties presented by this problem touch one of the most fundamental aspects of physics, the nature of the elementary particle. Although partial solutions, workable within limited areas, can be given, the basic problem remains unsolved. One might hope that the transition from classical to quantum-mechanical treatments would remove the difficulties. While there is still hope that this may eventually occur, the present quantum-mechanical discussions are beset with even more elaborate troubles than the classical ones. It is one of the triumphs of comparatively recent years (˜ 1948 - 1950) that the concepts of Lorentz covariance and gauge invariance were exploited sufficiently cleverly to circumvent these difficulties in quantum electrodynamics and so allow the calculation of very small radiative effects to extremely high precision, in full agreement with experiment. From a fundamental point of view, however, the difficulties remain."

The purpose of this note is to describe what is basically a new point of view towards this problem. (There had been earlier attempts along the lines of this work [6; 7] but some of the essential new insights were missing.) This view is completely classical, with no reliance on quantum theory. It however does rely heavily on the Einstein–Maxwell equations of general relativity.

The basic situation that we address is to first consider an arbitrary compact gravitating-electromagnetic system which is taken to be the 'particle' whose motion we want to describe. The system is given by local mass and charge densities and currents. There are no external fields acting on it. It is an isolated system with arbitrary internal degrees of freedom. The program is to solve the Einstein–Maxwell equations in the future null asymptotic region and, from the asymptotic field (the asymptotic Weyl and Maxwell tensors), determine a center of mass and center of charge and their laws of motion. Aside from the conditions that the total charge Q be non-vanishing and the important requirement that the complex centers of mass and charge should coincide, there is no further model building. This latter condition is a severe restriction on the source distribution: the local mass and charge densities and currents.

Since our detailed calculations, which were done in the language of the spin-coefficient formalism, are long and complicated and have appeared elsewhere [8], we will just summarize the ideas and results. Basically the calculations are done to 2nd order in deviations from Reissner–Nordstrom. The nature of our approximations is essentially heuristic and informal: we have no small parameter with a related truncated power series. Instead we have the zero order mass and charge (from the Reissner–Nordstrom metric) with all further variables treated as small, i.e., as first order quantities. More specifically, the Bondi shear, $\sigma(u)$, the complex electromagnetic dipole moment, $\phi_{0i}^{0}(u)$, the stereographic angle field, $L(u)$, are all first order. A parametrized complex world-line, $z^a = \xi^a(\tau)$, is introduced which has the form, $\xi^a(\tau) = \tau\delta_0^a$ for Reissner–Nordstrom; we treat deviations from this as first order. Quadratic products of first order quantities are retained while higher order products are ignored.

In the spherical harmonic expansions, with frequent use of Clebsch-Gordon products, only terms up to the $l = 2$ harmonics are kept.

Later for comparison with conventional physical notation we change the time variable from the Bondi u to the conventional retarded time, w, via $w = \sqrt{2}uc^{-1}$. Derivatives with respect to w are denoted by prime, ($'$).

2 The complex center of charge: its identification

The basic starting idea in this work is essentially simple. It is in the generalizations and implementations where difficulties arise.

Starting in Minkowski space in a given Lorentzian frame with spatial origin, the electric dipole moment $\overrightarrow{D}_E$ is calculated from an integral over the (localized) charge distribution. If there is a shift, $\overrightarrow{R}$, in the origin, the dipole transforms as

$$\overrightarrow{D_E^*} = \overrightarrow{D}_E - Q\overrightarrow{R}. \tag{3}$$

If $\overrightarrow{D}_E$ is time dependent, we obtain the center of charge world-line by taking $\overrightarrow{D_E^*} = 0$, i.e., from $\overrightarrow{R} = \overrightarrow{D}_E/Q$. It is this idea that we want to generalize and extend to gravitational fields.

First, however, we want to discuss other dipole issues in flat-space.

On the time-like world-line at the spatial origin, we construct the family of future directed light-cones, $\mathfrak{C}_u$, each labeled by the time at the origin, u. On each cone the null generators (null geodesics), $\mathfrak{g}$ are labeled by the complex stereographic coordinate at the apex, $(\zeta,\overline{\zeta})$. The affine parameter, r, 'measures' the distance along $\mathfrak{g}$ from the apex. The natural tetrad $(l,n,m,\overline{m})$, associated with $\mathfrak{C}_u$, is chosen. The vector field l^a is the tangent field to the null geodesic generators of the null cones $\mathfrak{C}_u$. At $\mathfrak{I}^+$, n^b is the tangent field to the null generators of $\mathfrak{I}^+$ while $(m^a,\overline{m}^b)$ are (the complex conjugate pair) tangent to the two surface, S^2, the intersection of each $\mathfrak{C}_u$ with $\mathfrak{I}^+$ and parallel propagated backwards along $\mathfrak{g}$.

Using this coordinate-tetrad system we investigate behavior of the Maxwell field in the limit as null infinity is approached, i.e., at Penrose's $\mathfrak{I}^+$. Using the null tetrad formalism and where the Maxwell pair ($\overrightarrow{E}$, $\overrightarrow{B}$ or F_{ab}) is replaced by the complex vector $\overrightarrow{E} + i\overrightarrow{B}$, we define their tetrad components, (ϕ_0,ϕ_1,ϕ_2), by [9]

$$\begin{aligned}\phi_0 &= F_{a'b'}l^{a'}m^{b'}\\ \phi_1 &= \frac{1}{2}F_{a'b'}(l^{a'}n^{b'} + m^{a'}\overline{m}^{b'})\\ \phi_2 &= F_{a'b'}\overline{m}^{a'}n^{b'}.\end{aligned} \tag{4}$$

(The indices, both here and later, with primes, e.g., $l^{a'}, F_{a'b'}$, are to be treated as abstract indices just describing the tensor character of the object.)

The asymptotic (peeling) behavior of these fields for a compact source is given by

$$\begin{aligned}\phi_0 &= \frac{\phi_0^0}{r^3} + O(r^{-4}) \\ \phi_1 &= \frac{\phi_1^0}{r^2} + O(r^{-3}) \\ \phi_2 &= \frac{\phi_2^0}{r} + O(r^{-2}).\end{aligned} \tag{5}$$

The r independent quantities $(\phi_0^0, \phi_1^0, \phi_2^0, ...)$ are functions 'living' on $\mathfrak{I}^+$, i.e., functions of the retarded time, u, and $(\zeta, \overline{\zeta})$, the complex stereographic coordinates of $\mathfrak{I}^+$. The components of their spherical harmonic decomposition,

$$\phi_0^0 = \phi_0^{0i} Y_{1i}^1 + \phi_0^{0ij} Y_{2ij}^1 + \cdots, \tag{6}$$

$$\phi_1^0 = Q + \phi_1^{0i} Y_{1i}^0 + \phi_1^{0ij} Y_{2ij}^0 + \cdots, \tag{7}$$

$$\phi_2^0 = \phi_2^{0i} Y_{1i}^{-1} + \phi_2^{0ij} Y_{2ij}^{-1} + \cdots, \tag{8}$$

are the asymptotically defined multipole moments and their time derivatives.

For example, the $l = 0$ harmonic component of ϕ_1^0 is proportional to the total source charge, Q. For us the important quantity is ϕ_0^{0i}, the $l = 1$ component of ϕ_0^0 : ϕ_0^{0i}
is proportional to the (*asymptotically defined*) complex dipole moment, $\mathcal{D}_C = \mathcal{D}_E + i\mathcal{D}_M$, where $\mathcal{D}_M$ is the magnetic dipole moment.

The problem now is: how does the $\mathcal{D}_C$ transform under an origin shift to an arbitrary world-line? With an origin shift there will be new light-cones and a new null vector field, l^{*a}, obtained from the old one, l^a, by a null rotation at $\mathfrak{I}^+$. This can be expressed explicitly by [10]

$$\begin{aligned} l^* &= l + \frac{L}{r}\overline{m} + \frac{\overline{L}}{r}m + O(r^{-2}) \\ m^* &= m + O(r^{-1}) \\ n^* &= n \end{aligned} \tag{9}$$

where $L = L(u,\zeta,\overline{\zeta})$ is a stereographic angle field given on $\mathfrak{I}^+$ (still to be described) that determines the new null geodesic field, l^*.

The transformation law for ϕ_0^{0i}, which is [8; 10]

$$\phi_0^{*0i} = (\phi_0^0 - 2L\phi_1^0 + L^2\phi_2^0)|^i,$$

becomes (small origin shift, with the cubic term omitted[1])

$$\phi_0^{*0i} \simeq (\phi_0^0 - 2L\phi_1^0)|^i \tag{10}$$

If we are given a Minkowski space world-line, $x^a = \xi^a(s)$, for the apex of the new light cones, then $L = L(u,\zeta,\overline{\zeta})$ is given in the parametric form

$$\begin{aligned} L(u,\zeta,\overline{\zeta}) &= \xi^a(s)m_a(\zeta,\overline{\zeta}), \\ u &= \xi^a(s)l_a(\zeta,\overline{\zeta}), \end{aligned} \tag{11}$$

with

$$\begin{aligned} l_a(\zeta,\overline{\zeta}) &= \frac{\sqrt{2}}{2}\left(1, \frac{\zeta+\overline{\zeta}}{1+\zeta\overline{\zeta}}, -i\frac{\zeta-\overline{\zeta}}{1+\zeta\overline{\zeta}}, \frac{-1+\zeta\overline{\zeta}}{1+\zeta\overline{\zeta}}\right) \\ &= \frac{\sqrt{2}}{2}\left(Y_0^0,0,0,0\right) - \frac{1}{2}(0,Y_{1i}^0), \end{aligned} \tag{12}$$

$$m_a(\zeta,\overline{\zeta}) = \frac{\sqrt{2}}{2(1+\zeta\overline{\zeta})}(0, 1-\overline{\zeta}^2, -i(1+\overline{\zeta}^2),\ 2\overline{\zeta}) = (0, Y_{1i}^1(\zeta,\overline{\zeta})), \tag{13}$$

By the appropriate choice of $\xi^a(s)$, from Eq. (10), with the use of Eq. (11), one can force the real part of ϕ_0^{*0i} to vanish, thereby making $x^a = \xi^a(s)$ the (real) center of charge. If however we generalized the choice of $L(u,\zeta,\overline{\zeta})$ and allowed it to be defined parametrically by

$$\begin{aligned} L(u,\zeta,\overline{\zeta}) &= \xi_C^a(\tau)m_a(\zeta,\overline{\zeta}), \\ u &= \xi_C^a(\tau)l_a(\zeta,\overline{\zeta}), \end{aligned} \tag{14}$$

where $z^a = \xi_C^a$ is a complex analytic world in complex Minkowski space, then by setting $\phi_0^{*0i} = 0$, in Eq. (10) the complex world-line is determined. This complex curve (which is purely formal) defines the *complex center of charge*. Using this "curve" as the origin, both the electric *and* magnetic dipoles vanish.

[1] The description of the finite transformation is considerably more complicated and is postponed for a later publication.

Remark We stress, for later use, that this formal construction has a deeper geometric meaning. Null vector fields, l^a, constructed as tangent fields to the null cones with apex on a real world-line have three characteristics, they are tangent to *null geodesic congruences*, they are *twist-free* (*i.e., null-surface forming*) and are *shear-free*. More generally, null geodesic congruences and their null tangent fields, that are *shear-free* but are *twisting* can be (formally) constructed in the following fashion: choose a one-complex parameter family of complex light cones with apex on a complex (analytic) curve in complex Minkowski space and then project the complex tangent vectors of the cone into the (real) Minkowski space. They form a *twisting, shear-free null geodesic congruence*. In other words, congruences of this type are generated by complex analytic curves [11]. This construction is further generalized to *asymptotically* flat space-times by considering *asymptotically* shear-free null geodesic congruences, where again they are generated by complex analytic curves [12; 13].

3 The complex center of mass

For asymptotically flat Einstein–Maxwell space-times the situation is totally analogous: the shear-free null geodesics originating from light-cones from world-lines (real or complex) are replaced by (regular) asymptotically shear-free null geodesic congruences generated by a complex world-line [12], in the space of the complex Poincare translation subgroup of the BMS group. The Maxwell asymptotic dipole transforms exactly as in the flat space case, i.e., as in Eq. (10) with however a slight change in the parametric description of the function $L(u,\zeta,\overline{\zeta})$:

$$L = \xi^i(\tau)Y^1_{1i}(\zeta,\overline{\zeta}) - 6\xi^{ij}(\tau)Y^1_{2ij}(\zeta,\overline{\zeta}), \tag{15}$$

$$u = \frac{1}{\sqrt{2}}\xi^0(\tau) - \frac{1}{2}\xi^i(\tau)Y^0_{1i}(\zeta,\overline{\zeta}) + \xi^{ij}(\tau)Y^0_{2ij}(\zeta,\overline{\zeta}) + \cdots, \tag{16}$$

where the extra terms come from the existence of a non-vanishing Bondi shear, given up to $l = 2$ terms, by

$$\sigma = 24\xi^{ij}(u)Y^2_{2ij} + \cdots \tag{17}$$

Note: Both $\xi^i(\tau)$ and $\xi^{ij}(\tau)$ are first-order quantities.

For later use we define $(\xi^i_R, \xi^i_I, v^i_R, v^i_I)$ by

$$\xi^i = \xi^i_R + i\xi^i_I,$$
$$v^i = v^i_R + iv^i_I \equiv \xi^{i\,\prime}.$$

Turning to the gravitational behavior, the relevant (for us) tetrad components of the Weyl tensor [8; 9; 14]

$$\psi_1 = -C_{a'b'c'd'}l^{a'}m^{b'}l^{c'}n^{d'},$$
$$\psi_2 = -C_{a'b'c'd'}\overline{m}^{a'}n^{b'}l^{c'}m^{d'},$$

have the asymptotic form (the peeling theorem)

$$\psi_1 = \frac{\psi_1^0(u,\zeta,\overline{\zeta})}{r^4} + O(r^{-5}),$$
$$\psi_2 = \frac{\psi_2^0(u,\zeta,\overline{\zeta})}{r^3} + O(r^{-4}).$$

The leading terms have the harmonic expansion:

$$\psi_2^0 = \Upsilon + \psi_2^{0i}Y_{1i}^0 + \psi_2^{0ij}Y_{2ij}^0 + \cdots, \tag{18}$$
$$\psi_1^0 = \psi_1^{0i}Y_{1i}^1 + \psi_1^{0ij}Y_{2ij}^1 + \cdots \tag{19}$$

The mass aspect, defined by

is *real* and has the expansion

$$\Psi = \overline{\Psi} = \Psi^0 + \Psi^i Y_{1i}^0 + \Psi^{ij} Y_{2ij}^0 + \cdots$$

The Bondi mass and linear momentum (four-momentum) is obtained from the $l = (0,1)$ harmonic components of Ψ by:

$$\Psi^0 = -\frac{2\sqrt{2}G}{c^2}M \tag{20}$$
$$\Psi^i = -\frac{6G}{c^3}P^i. \tag{21}$$

The *complex gravitational dipole moment* (roughly, *mass-dipole* + i *angular-momentum*) is identified as being proportional to the $l = 1$ harmonic of ψ_1^0, i.e., as ψ_1^{0i}. (Many authors add further terms that are quadratic in the shear and its derivatives to ψ_1^{0i} for this identification and we might have expected an ambiguity. However because of our harmonic expansion assumptions they all agree with our identification [8].)

The transformation (to second order) of ψ_1^{0i} to an arbitrary (complex) world-line, analogous to Eq. (10), using Eq. (15), is [8]

$$\psi_1^{*0i} = (\psi_1^0 - 3L\psi_2^0 + 3L^2\psi_3^0. - L^3\psi_4^0)|^i \tag{22}$$
$$\psi_1^{*0i} \simeq (\psi_1^0 - 3L\psi_2^0)|^i. \tag{23}$$

Setting $\psi_1^{*0i} = 0$, thereby defining the complex center or mass, $\xi^i(u)$, yields after a lengthy calculation,

$$\psi_1^{0i} = -\frac{6\sqrt{2}G}{c^2}M\left[\xi^i(w) + i\frac{1}{2}\varepsilon_{kjl}\delta^{il}v^k\xi^j\right] + G^i, \tag{24}$$

where G^i is a known non-linear function of quadrupole terms.

At this point we make our only assumption on the physical system being considered. We saw that we could determine a complex center of charge or a complex gravitational center of mass by setting either φ_0^{*0i} or ψ_1^{*0i} to zero. We now assume, for the rest of this work, that the two complex world-lines coincide, i.e., the L, used in Eqs. (23) and the Einstein–Maxwell version of the flat-space Eq. (10), are the same. As we said earlier, this is a restriction on the source distribution. An example of this occurs in the Kerr–Newman metric. Aside from taking $Q \neq 0$, there are no other conditions on the internal structure of our source (particle).

We now turn to the dynamics, which are contained in the asymptotic Bianchi identities which can be written as:

$$\tag{25}$$

$$\Psi^{\cdot} = \sigma^{\cdot}\overline{\sigma}^{\cdot} + k\phi_2^0\overline{\phi}_2^0 \tag{26}$$

$$k = 2Gc^{-4} \tag{27}$$

From these two equations, (25) and (26), we extract the equations of motion with the radiation reaction term. Rather than going through the details (long with rather unattractive calculations) we will describe what we did in words and then give the results.

We first extract from Eq. (25) its $l = 1$ part and then decompose it into its real and imaginary parts. This yields two results: the imaginary part determines the dynamics of the total angular momentum, i.e., the conservation of angular momentum. Other than remarking that we identify $S^i = Mc\xi_I^i$ as the intrinsic spin (with ξ_I^i the imaginary part of ξ^i), this is not our interest here and will not be discussed any further. The real $l = 1$ part can be solved for the linear momentum P^i that was sitting in the $l = 1$ part of Ψ:

$$P^k = Mv_R^k - \frac{2Q^2}{3c^3}v_R^{k\prime} + W^k. \tag{28}$$

This is a major result that comes from our identification of the complex centers of mass and charge. First of all we see *kinematical* expressions for the Bondi 3-momentum, the *mv* term and then the *radiation reaction contribution to the momentum*. The W contains further kinematical terms involving spin and quadrupole interactions that are known but not displayed here [8].

Extracting the $l = (0,1)$ harmonics from Eq. (26) yields the Bondi mass and momentum loss equations [8]:

$$M' = -\frac{G}{5c^7}(Q^{ij'''}_{Mass}Q^{ij'''}_{Mass} + Q^{ij'''}_{Spin}Q^{ij'''}_{Spin}) - \frac{2Q^2}{3c^5}(v^{i\prime}_R v^{i\prime}_R + v^{i\prime}_I v^{i\prime}_I)$$
$$-\frac{1}{180c^7}(D^{ij'''}_E D^{ij'''}_E + D^{ij'''}_M D^{ij'''}_M) \tag{29}$$

$$P^{k\prime} = F^k \equiv \frac{2G}{15c^6}(Q^{lj'''}_{Spin}Q^{ij'''}_{Mass} - Q^{lj'''}_{Mass}Q^{ij'''}_{Spin})\varepsilon_{ilk}$$
$$-\frac{Q^2}{3c^4}(v^{l\prime}_I v^{i\prime}_R - v^{l\prime}_R v^{i\prime}_I)\varepsilon_{ilk} + \frac{Q}{15c^5}(v^{j\prime}_R D^{jk'''}_E + v^{j\prime}_I D^{jk'''}_M)$$
$$+\frac{1}{540c^6}(D^{lj'''}_E D^{ij'''}_M - D^{lj'''}_M D^{ij'''}_E)\varepsilon_{ilk} \tag{30}$$

with the mass and spin quadrupoles related to the ξ^{ij} by

$$\xi^{ij} = (\xi^{ij}_R + i\xi^{ij}_I) = \frac{G}{12\sqrt{2}c^4}(Q^{ij''}_{Mass} + iQ^{ij''}_{Spin}). \tag{31}$$

The mass loss equation is thus exactly the usual quadrupole energy loss plus the classical dipole and quadrupole electromagnetic energy loss.

In equations (29) and (30), to avoid too many new symbols, we have slightly cheated on the notation: the repeated indices are to be treated as Euclidian scalar products.

It is however Eq. (30) that is of most interest to us. By substituting the kinematical expression for the momentum, Eq. (28) into Eq. (30) we obtain our generalized Abraham–Lorentz equations of motion:

$$Mv^{k\prime}_R + v^k_R M' - \frac{2Q^2}{3c^3}v^{k''}_R + R^k = F^k. \tag{32}$$

Note that though it is similar to the Abraham–Lorentz equations there are many differences that are hidden in the known but complicated expressions for M', R^k and F^k. The F^k is the Bondi recoil (or rocket) force due to the momentum loss, while R^k can be considered to be a gravitational radiation reaction force depending on internal degrees of freedom spin and quadrupole moments. M' has exactly the classical dipole energy loss term plus two additional terms from quadrupole energy losses. Though it is very hard to directly see if the solutions to Eq. (32) are well behaved, in the conclusion we will discuss this issue in more general terms. To even talk about solutions, one must assume that ξ^{ij} is an arbitrary but known function different from zero only on a finite range of u.

4 Conclusions

We have considered the situation of a gravitating - electromagnetic source of compact support viewed from future null infinity. The only restriction made on the distributions is that the total charge is non-vanishing and that the complex centers of charge and mass coincide. Though it is not clear how severe this condition is, it certainly is a serious restriction. It has been shown that for this situation the gyromagnetic ratio, the ratio of the spin-angular momentum to the magnetic moment, is that of Dirac's, namely $g = 2$. We showed that in a manner completely analogous to the flat space Maxwell case, one could determine the transformation laws for the two dipole moments and thereby go to the center of mass/charge, determining a unique complex world-line. Then, using the Bianchi Identities, which play the role of dynamical equations, we were able to give kinematical significance to the Bondi linear momentum, in the sense that we had

$$\overrightarrow{P} = M\overrightarrow{v} - \frac{2Q^2}{3c^3}\overrightarrow{a}' + \cdots \tag{33}$$

Remark The Bondi momentum $\overrightarrow{P}$ can be compared with the conjugate momentum $\overrightarrow{P} = M\overrightarrow{v} + q\overrightarrow{A}$ arising from a Lagrangian formulation of particle dynamics. If one computes the asymptotic value of the Lienard-Wiechert potential, it gives $-\frac{2Q}{3c^3}\overrightarrow{a}'$. Thus, our asymptotic formulation agrees with the Lagrangian or Hamiltonian formulation of a charged radiating particle.

From the Bondi momentum loss equation it immediately follows that we have a generalized version of the Abraham–Lorentz equations of motion for an isolated massive charged particle. It should be emphasized that the quadrupole quantity, $\xi^{ij}(u)$, is arbitrary and in most cases it is taken as non-vanishing in a finite interval so that the gravitational radiation exists also in a finite interval. If, however, *the motion,* from Eq. (32)*, is unstable,* the particle acceleration will be unbounded and there will be an infinite energy loss via the *electromagnetic dipole radiation*. This would be a physically unacceptable situation, indicating that something is seriously wrong with the Einstein–Maxwell equations.

The question then is: does the general relativity (gravitational) contributions to the equations of motion stabilize the equations? Though we do not see any immediate prospects for a direct proof, we make a few comments. Looking at Eq. (32), we see that the term $M'v$ has the same form as in the Abraham–Lorentz equation but now is more negative because of the extra radiation terms (electromagnetic and gravitational quadrupoles) and has the correct sign *to try* to stabilize the motion. Whether or not it does stabilize is an open question and probably can not be directly answered, even if the omitted higher order terms were included.

However there is an alternative approach to the issue: it is known that the vacuum Einstein equations are stable in the neighborhood of Minkowski space. If the same were true of the Einstein–Maxwell equations with compact sources, that would constitute a proof that our physical system was indeed stable and the run-away behavior was prevented by the inclusion of classical general relativity. The reason for this is that runaway behavior would force infinite electromagnetic dipole radiation and hence an infinite Bondi energy loss. Stable Einstein–Maxwell solutions would not allow this.

Unfortunately, the stability of the Einstein–Maxwell equations with compact sources is a difficult question and, to our understanding, the answer is unknown. For either case, however, the results would be of potential physical significance. In some sense, the issue of the stability of the Abraham–Lorentz equation is turned into the stability of the Einstein–Maxwell equations.

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References

1. J.D. Jackson (1975) Classical electrodynamics, 2nd edn Wiley Hoboken
2. Thirring, W.: Principles of quantum electrodynamics. Academic Press, New York, p. 25 (1958)
3. L. Landau E.M. Lifschitz (1962) Classical theory of fields Addison-Wesley Reading
4. S. Herbert (2004) Dynamics of charged particles and their radiation field Cambridge University Press Cambridge
5. F. Rohrlich (2007) Classical charged particles, 3rd edn World Scientific London
6. E.T. Newman R. Posadas (1969) *Phys. Rev.* **187** 1784 – 1791
7. R. Lind J. Messmer E.T. Newman (1972) *Phys. Rev. Lett.* **28** 857 – 858
8. Kozameh, C., Newman, E.T., Silva-Ortigoza, G.: On extracting physical content from asymptotically flat space–time metrics. Class. Quantum. Gravit. (submitted, 2008)
9. T. Ezra (1962) Newman and Roger penrose J. Math. Phys. **3** 566 – 768
10. C. Kozameh E.T. Newman G. Silva-Ortigoza (2007) *Class. Quantum. Gravit.* **24** 5479 – 5493
11. R. Lind E.T. Newman (1974) *J. Math. Phys.* **15** 1103
12. C. Kozameh E.T. Newman (2005) *Class. Quantum Gravit.* **22** 4659 – 4665
13. B. Aronson E.T. Newman (1972) *J. Math. Phys.* **13** 1847 – 51
14. Newman, E.T., Tod, K.P.: In: Alan, H. (ed.) General Relativity and Gravitation, vol. 2. Plenum, New York (1980)