B-meson mixing from full lattice QCD with physical u, d, s and c quarks

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We present the first lattice QCD calculation of the B_s and B_d mixing parameters with physical light quark masses. We use MILC gluon field configurations that include u, d, s and c sea quarks at 3 values of the lattice spacing and with 3 values of the u/d quark mass going down to the physical value. We use improved NRQCD for the valence b quarks. Preliminary results show significant improvements over earlier values.

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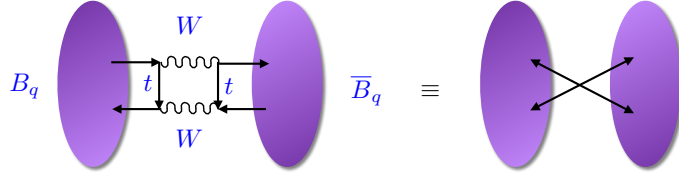
Figure 1: The box diagram and its reduction to a 4-quark operator whose matrix element can be calculated in lattice QCD.

1. Introduction

The Standard Model rates for B_d and B_s oscillations are determined by hadronic parameters obtained from the matrix element between B and $\overline{B}$ states of 4-quark effective operators derived from the box diagram (see Figure 1). The 4-quark operator matrix elements can only be determined by lattice QCD calculations. The accuracy with which this can be done is the limiting factor in the constraint on the Cabibbo-Kobayashi-Maskawa matrix elements that can be obtained from the very precise experimental results.

We study the matrix elements of 3 Standard Model 4-quark operators:

$$\begin{aligned}
O_1 &\equiv (\bar{b}^\alpha \gamma_\mu L q^\alpha)(\bar{b}^\beta \gamma_\mu L q^\beta) \\
O_2 &\equiv (\bar{b}^\alpha L q^\alpha)(\bar{b}^\beta L q^\beta) \\
O_3 &\equiv (\bar{b}^\alpha L q^\beta)(\bar{b}^\beta L q^\alpha).
\end{aligned} \tag{1.1}$$

Here the superscripts are colour indices and L is the 'left' projection operator. O_1 is the key operator for B_s and B_d oscillations, O_2 is needed for the renormalisation of O_1 and all 3 appear in the calculation of the B width difference. It is conventional to express the matrix element of O_1 as:

$$\langle O_1(\mu) \rangle = \frac{8}{3} f_B^2 B_B(\mu) M_B^2 \tag{1.2}$$

where B_B is the 'bag parameter', f_B the decay constant and the factor of 8/3 ensures that B_B is 1 in the 'vacuum saturation approximation'. This is a convenient parameterisation to use since, as we shall see, the bag parameter has very simple behaviour with almost no dependence on light quark mass, although the value is not necessarily 1. The factor of 8/3 becomes -5/3 for O_2 and 1/3 for O_3.

The determination of the matrix elements in lattice QCD is standard [1, 2]. Here we use NRQCD for the b-quark, superseding previous calculations by the use of our radiatively improved NRQCD action [3, 4]. We work on 'second-generation' MILC gluon field configurations [5] that use an improved gluon action [6] and include the effect of u, d, s and c HISQ [7] sea quarks. The parameters of the gluon configurations are given in Table 1. We determined f_{B_s} = 224(5) MeV and f_B = 186(4) MeV on these configurations in [8] and in the same calculation obtained $M_{B_s} - M_B$ = 85(2) MeV and M_{B_s} = 5.366(8) GeV [9], in good agreement with experiment. This shows the accuracy now achievable with second-generation lattice QCD analysis.

To calculate the 4-quark operator matrix elements we set up a '3-point' calculation as in Figure 2. The NRQCD b and light-quark propagators start from local sources at O_n. We then arrange

Table 1: Details of the gauge ensembles used in this calculation. a_Υ is the lattice spacing as determined by the $\Upsilon(2S-1S)$ splitting in [3], where the three errors are statistics, NRQCD systematics and experiment. am_l, am_s and am_c are the sea quark masses, $L \times T$ gives the spatial and temporal extent of the lattices and n_{cfg} is the number of configurations in each ensemble. The ensembles 1,2 and 3 will be referred to as "very coarse", 4,5 and 6 as "coarse" and 7,8 as "fine". We use 16 time sources on each configuration.

Set	a_Υ (fm)	am_l	am_s	am_c	$L \times T$	n_{cfg}
1	0.1474(5)(14)(2)	0.013	0.065	0.838	16×48	1020
2	0.1463(3)(14)(2)	0.0064	0.064	0.828	24×48	1000
3	0.1450(3)(14)(2)	0.00235	0.0647	0.831	32×48	1000
4	0.1219(2)(9)(2)	0.0102	0.0509	0.635	24×64	1052
5	0.1195(3)(9)(2)	0.00507	0.0507	0.628	32×64	1000
6	0.1189(2)(9)(2)	0.00184	0.0507	0.628	48×64	1000
7	0.0884(3)(5)(1)	0.0074	0.037	0.440	32×96	1008
8	0.0873(2)(5)(1)	0.0012	0.0363	0.432	64×96	621
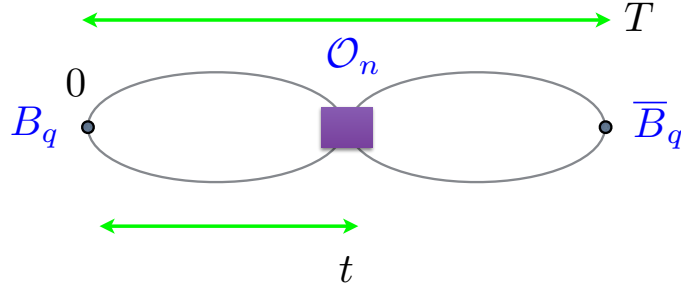
Figure 2: Sketch of the 3-point arrangement of lattice QCD quark propagators for calculating 4-quark operator matrix elements.

results, as shown in the figure, so that we can fit as a function of t and T to standard 3-point correlator forms (see, for example, [10]), simultaneously with appropriate 2-point functions.

The 4-quark operator constructed from NRQCD b-quarks and HISQ light quarks must be matched to the continuum operator, for a physical matrix element. For O_1 this matching takes the form

$$\langle O_1 \rangle_{\overline{MS}}(m_b) = [1 + \alpha_s z_1]\langle O_{1,NRQCD} \rangle + \alpha_s z_2 \langle O_{2,NRQCD} \rangle \qquad (1.3)$$

with similar expressions for O_2 (involving O_2 and O_1) and O_3 (with O_3 and O_1). The NRQCD operators include leading and next-to-leading terms (at tree-leve) in a non-relativistic expression. The next-to-leading terms contain spatial derivatives on the b quark field divided by the b quark mass. The z_i are easily constructed from the results calculated in [11]. To determine the bag parameters, we divide the matrix element by the square of the decay constant determined by a similar matching procedure for the temporal axial current

$$\langle 0|A_0|B \rangle = [1 + \alpha_s z_0]\langle 0|A_{0,NRQCD}|B \rangle. \qquad (1.4)$$

(Note that, in determining f_B in [8] we also included $\alpha_s \Lambda/m_b$ current matching contributions which are not calculated here.)
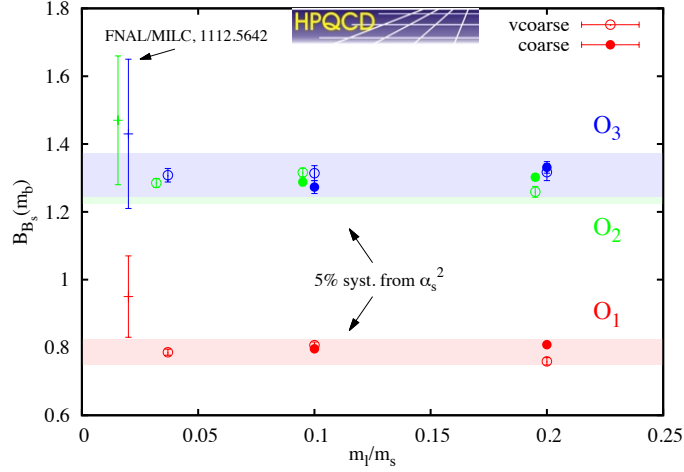
Figure 3: Bag parameters for operators O_1, O_2 and O_3 for the B_s meson calculated on very coarse (sets 1, 2 and 3) and coarse (sets 4 and 5) 2+1+1 gluon configurations. The points marked with a plus at the left-hand side of the plot are from continuum and chiral extrapolation on 2+1 gluon field configurations by the Fermilab Lattice/MILC collaborations [12]. The coloured bands shows the size of a 5% systematic error from missing α_s^2 terms in the matching between lattice NRQCD and the continuum (they are *not* fits to the results).
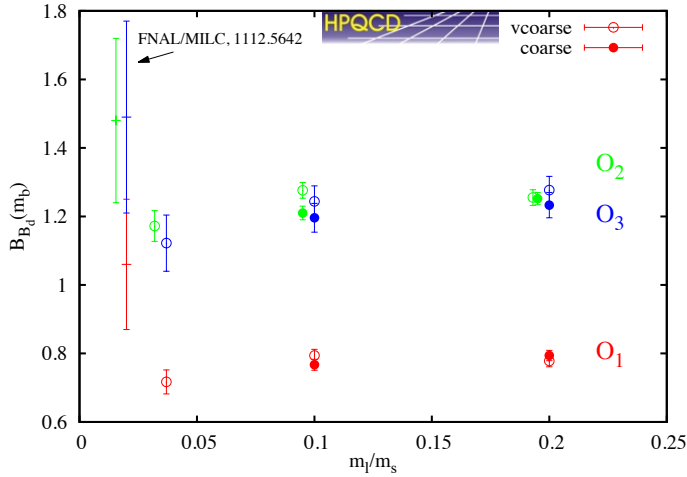

Figure 4: As above, for the B_d. A 5% systematic uncertainty also applies here, correlated with that for the B_s, but, for clarity, it is not shown on the plot.

2. Results

Results from gluon field configurations 1, 2 and 3 (very coarse) and 4 and 5 (coarse) are shown above. Calculations on sets 6, 7 and 8 are not yet complete. Figure 3 shows the bag parameter for B_s for operators O_1, O_2 and O_3. Very little dependence is seen on lattice spacing or sea quark mass. A 5% systematic error from missing α_s^2 matching terms dominates any extrapolation uncertainty. For Figure 4 for the B_d, this is less true, and the results there may show more light quark mass dependence.
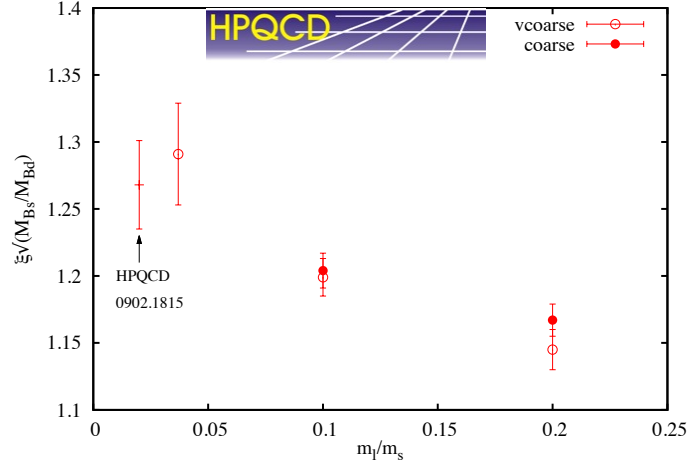
Figure 5: Our new 2+1+1 results for ξ (the ratio $f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$) multiplied by $\sqrt{M_{B_s}/M_{B_d}}$ and plotted against the u/d quark mass in units of the physical s quark mass. Our results include a value calculated at the physical u/d quark mass on a very coarse lattice (set 3). The point marked with a plus is from our previous work on 2+1 gluon field configurations after chiral extrapolation [1].
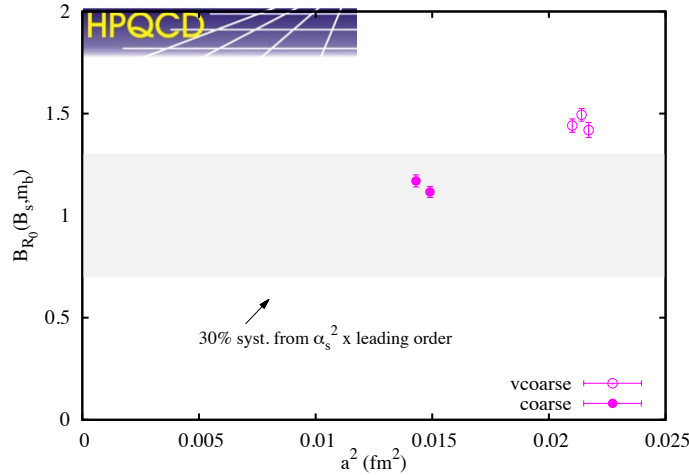
Figure 6: The bag parameter for R_0 [13], a $1/m_b$ operator that appears in the Standard Model calculation of $\Delta\Gamma$ for the B_s meson. The grey band shows the size of the systematic error from missing α_s^2 terms that mix in leading order operators in the continuum and on the lattice.

Figure 5 shows the ratio $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$ multiplied by $\sqrt{M_{B_s}/M_{B_d}}$. Our previous result obtained on the MILC 2+1 asqtad configurations after extrapolation to physical light quark masses [1] is also shown. With the further 2+1+1 results at physical light quark masses that are underway we should be able to improve significantly on our previous value.

Finally, we show values for the bag parameter for R_0, a combination of O_1, O_2 and O_3 which gives a $1/m_b$ suppressed operator that appears in the width difference between eigenstates, $\Delta\Gamma$ [13]. Mixing with leading operators has been corrected at $\mathcal{O}(\alpha_s)$ but a large ($\mathcal{O}(30\%)$) systematic error remains from mixing at $\mathcal{O}(\alpha_s^2)$ *both* in the continuum and on the lattice. As is clear from the figure (given for the B_s), this is much larger than any error from the lattice determination of the raw matrix

elements.

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