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IS ABSORPTION A CONSEQUENCE OF UNITARITY?*

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ABSTRACT

Absorptive corrections to scattering and production in a family of explicitly unitary models is studied. We find that in a particular model in which quantum number constraints are ignored, all the results concerning absorptive corrections found in the Reggeon calculus model of Abramovskii, Kancheli, and Gribov are reproduced. In extended models, however, the effects due to quantum numbers and/or alternative unitarization schemes are shown to have a significant effect on the form of absorptive corrections. In particular, explicitly unitary models are given in which (i) the absorptive corrections enhance all cross sections, and (ii) isospin is introduced in a simple way and is found to produce quite different counting rules from that assumed in the Reggeon calculus. It is shown that the internal quantum number structure of the Pomeron affects the amount of absorption in inclusive cross sections and hence can be experimentally studied.

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I. INTRODUCTION

During the past few years there has been considerable interest in absorptive corrections to both inclusive and exclusive cross sections; in particular, the question of their sign relative to the Born term.[1-7] A popular approach has been the use of models based on field theory (in particular weak coupling $\lambda\phi^3$ perturbation theory). These models suggest that the absorptive corrections to the single particle inclusive cross section cancel out when the trigger particle is in the central region,[1,3] and that the leading absorptive correction to the elastic amplitude, the two Pomeron cut, contributes negatively to the total cross section via the optical theorem.[1,6,7] In this paper we introduce a family of models based on the models of Auerbach, Aviv, Sugar and Blankenbecler.[8,9] These are constructed so that the S-matrix explicitly satisfies the unitarity condition $S^+S = 1$. They will be used to check if the results stated above, or indeed if any of the recipes for calculating absorptive corrections suggested in Refs. 1-7, can be said to be a consequence of unitarity.

Abramovskii, Kancheli and Gribov,[1] hereafter known as AKG, first showed that in a Reggeon calculus model the absorptive corrections to the single particle inclusive cross section in the central region cancelled out, so that this cross section was given by the Mueller diagram of Fig. 1. Interest in absorptive corrections to inclusive cross sections was stimulated when Einhorn and Savit[10] showed that the colored quark parton model, together with the Drell-Yan formula[11] was incompatible with the BNL data[12] for inclusive lepton pair production in hadronic collisions, the calculated cross section being much smaller than the observed one.[13] The inclusive cross section, using the Drell-Yan model, is given by the M^2 discontinuity of the Mueller diagram of Fig. 2a. Landshoff and Polkinghorne had earlier suggested that the Mueller diagram of Fig. 2b may also be important (in this diagram the ladder represents any absorptive correction, possibly the Pomeron). If this diagram were to explain the discrepancy between the Drell-Yan model and the data, its M^2 discontinuity would have to be positive. Henyey and Savit[2] claimed that this discontinuity is in fact negative, but, as pointed out in Refs. 3,5, these authors had not included all the possible contributions. Einhorn and Henyey,[5] in a particular unitary model (their model is similar to that studied in Sec. II), find that the diagram of Fig. 1b gives a zero contribution to the inclusive cross section. This result was also obtained earlier in field theory models by Cardy and Winbow[3] and DeTar, Ellis and Landshoff.[4]

Investigations of the two Pomeron contribution to the elastic amplitude in field theoretical models[1,6] have shown that in these models the contribution is negative. Moreover using the unitarity sum to evaluate the two Pomeron contribution it is found that three kinds of contribution are important, those in which no Pomeron is cut (Fig. 3a), those in which one Pomeron is cut (Fig. 3b), and those in which both Pomerons are cut (Fig. 3c). In these field theoretical models the relative magnitudes of these contributions are (1,-4,+2) which clearly add up to be negative. In the AKG model[1] analogous counting rules can be easily calculated for the exchange of an arbitrary number of Pomerons.

In this paper we study absorptive corrections to the elastic amplitude, the single particle inclusive cross section and exclusive amplitude in a family of unitary models of production, based on the model of Auerbach, Aviv, Sugar and Blankenbecler.[8,9] These models have the advantage over the Reggeon calculus model and the Regge eikonal model in that they naturally include a detailed description of production channels. The unitary model introduced in Refs. 8,9 is found to be equivalent to the model of AKG in the sense that all the prescriptions for calculating absorptive corrections are the same in both models. We find however that simple modifications to the AASB model, still preserving the unitarity of the S-matrix, drastically change these prescriptions.

The plan of the paper is as follows. In Section II we review briefly the model of Auerbach, Aviv, Sugar and Blankenbecler[8,9] and show that in this model the absorptive corrections to the inclusive cross section indeed cancel as was shown in the original papers, that the elastic amplitude is given by the usual Regge-eikonal formula, and that the exclusive amplitude for the production of one particle is given by the elastic S-matrix element multiplied by the Regge Born amplitude in accordance with usual ideas. In order to facilitate comparison with field theory models, we will <u>restrict our discussion to ladder diagrams only</u>, checkerboard diagrams involving interactions between three or more exchanges are neglected. In Section III we show that the counting rules of AKG[1] for the elastic amplitude can be exactly reproduced in this model. In Section IV we present various modifications to the AASB model, and find that each modification changes the counting rules for the elastic and production amplitudes and also the inclusive cross section. In Section V we present an extreme family of models, each of which is explicitly unitary, in which not only is the two Pomeron cut positive, but the exclusive production amplitude is also enhanced by absorptive corrections. Finally in Section VI we present our conclusions.

II. ABSORPTION IN A UNITARY MODEL

In this section we study absorptive effects in a particular unitary model, that of AASB.[8,9] In this model there are two kinds of particles, firstly the initial state leading particles of mass m, which can neither be created nor destroyed which will be called nucleons, and secondly particles of mass μ, which can be created and destroyed which will be called mesons. Here we shall treat only spinless particles with no quantum numbers, we shall introduce a model with isospin in Section IV. Mesons are created and destroyed off chains, which are exchanged between the nucleons. Each chain is represented by a hermitian operator Z. The unitarity of the S-matrix is then ensured by writing

$$S = e^{iZ} \quad , \tag{2.1}$$

where Z is defined by a sum over the number of produced mesons

$$Z = \sum_{n=1}^{\infty} Z_n \tag{2.2}$$

and

$$Z_n = \frac{1}{2s}\frac{1}{n!}\int \prod_{i=1}^{n} \frac{d^3q_i}{2E_i(2\pi)^3} W_n(Y,\underset{\sim}{B};q_i,y_i) : \prod_{i=1}^{n}\left(a(\underset{\sim}{q}_i,\underset{\sim}{y}_i) + a^+(-\underset{\sim}{q}_i,y_i)\right): \tag{2.3}$$

where $Y \equiv \ell n\, s/m^2$, $\underset{\sim}{B}$ is the conjugate variable to $\underset{\sim}{\Delta} \equiv \frac{1}{2}(\underset{\sim}{p}'_a - \underset{\sim}{p}_a) - \frac{1}{2}(\underset{\sim}{p}'_b - \underset{\sim}{p}_b)$; $y_i, \underset{\sim}{q}_i$ are respectively the rapidity and transverse momentum of particle i and the other variables are as defined in Fig. 4. a and a^+ are the usual destruction and creation operators and we use the normalization of AASB,[9]

$$\left[a(\underset{\sim}{q},y), a^+(\underset{\sim}{q}',y')\right] = 2(2\pi)^3\,\delta(\underset{\sim}{q}-\underset{\sim}{q}')\,\delta(y-y') \tag{2.4}$$

W_n is a c-number function of the kinematic variables. Most of the results which are of interest here do not depend on its specific form. When we shall need its detailed form we shall use the solvable model of AASB,[9] which is equivalent to a multi-Regge model:

$$\frac{1}{2s} W_n(Y, \underset{\sim}{B}; q_i, y_i) = e^{-Y} f(\underset{\sim}{B}) \prod_{i=0}^{n} e^{\alpha(y_i - y_{i-1})} n! \theta(y_i - y_{i+1}) \times \prod_{j=1}^{n} g(q_j) \qquad (2.5)$$

We also impose the condition[9] that W_n vanishes unless $|y_i| \leq \frac{1}{2}(1-\epsilon)Y$ where ϵ is an arbitrarily small positive number. At high energies this requirement forces the final state nucleons to have energies of order $\frac{1}{2}\sqrt{s}$, so that as long as the meson multiplicity does not grow as fast as $s^{\epsilon/2}$, the meson variables can be dropped from the energy and longitudinal momentum conservation δ-functions. This defines the model. The more general AASB models that contain transverse momentum correlations can be treated as below but with a considerable increase in notational complexity.

A simple and interesting quantity to study in this model is the single particle inclusive cross section where the trigger particle is a meson in the central region. Then we can write

$$\frac{d\sigma_{incl}}{d^2\underset{\sim}{q}\, dy} = \frac{1}{2(2\pi)^3} \int d^2\underset{\sim}{B} \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} dq_i \frac{1}{n!} |\langle \underset{\sim}{q}, y, \underset{\sim}{q}_1, y_1 \ldots, q_n, y_n | S(Y, \underset{\sim}{B}) | 0 \rangle|^2$$

$$= \frac{1}{2(2\pi)^3} \int d^2\underset{\sim}{B} \langle 0 | [S^+(Y, \underset{\sim}{B}), a^+(\underset{\sim}{q}, y)] [a(q, y), S] | 0 \rangle \qquad (2.6)$$

where the states are defined in the meson Hilbert space so that $|0\rangle$ denotes a state with no mesons but with two leading particles present. With the W_n defined in (2.5) it has been shown in Ref. 9 that

$$Z = f(\underset{\sim}{B})\, e^{(\alpha - 1)Y} : e^{(\lambda Y)^{1/2}(c + c^+)} : \qquad (2.7)$$

where

$$c = [\lambda Y]^{-1/2} \int \frac{d^2\underset{\sim}{q}}{(2\pi)^2} \int_{-(1-\epsilon)Y/2}^{(1-\epsilon)Y/2} \frac{dy}{4\pi}\, g(\underset{\sim}{q})\, a(\underset{\sim}{q}, y) \qquad (2.8)$$

and

$$\lambda = \frac{(1-\epsilon)}{4\pi} \int \frac{d^2\underset{\sim}{q}}{(2\pi)^2} g^2(\underset{\sim}{q}) \quad . \tag{2.9}$$

Using these expressions it can be seen that

$$[a, Z] = g(\underset{\sim}{q})\, Z \tag{2.10}$$

and hence

$$[a, S] = ig(\underset{\sim}{q})\, ZS \quad . \tag{2.11}$$

Substituting (2.11) into (2.6) we obtain the result

$$\frac{d\sigma_{incl}}{d^2\underset{\sim}{q}\, dy} = \frac{1}{2(2\pi)^3} \int d^2\underset{\sim}{B}\, \rho_{in} \tag{2.12}$$

where

$$\rho_{in} = g^2(\underset{\sim}{q}) <0\,|Z^2(Y,B)\,|0> \quad .$$

Hence we see that the single particle inclusive cross section is determined fully by Mueller diagrams with only two chains (i.e., one ladder) exchanged (see Fig. 5). All diagrams with the exchange of more than two chains have cancelled out. As shown in Ref. 9, this even includes all the checkerboard diagrams. Thus in this model there are no absorptive corrections to inclusive meson production in the central region in agreement with Ref. 1. The ansatz (2.5) for W_n will be modified later to include the production of photons (and hence lepton pairs) as well as mesons, with the result that there are no absorptive corrections to the inclusive lepton pair cross section in agreement with Refs. 3,4,5. The model of Einhorn and Henyey[5] is of this type.

This cancellation of absorptive corrections to the inclusive cross section is certainly more general than the ansatz (2.5) for W_n, for example it also occurs in the multiperipheral model of Ref. 9. For models having the operator structure defined by (2.3) such a cancellation will occur provided that $[a, S]$ is proportional to $[a, Z]S$.

Another interesting quantity to study is the elastic amplitude, and in particular, contributions to the elastic amplitude arising from multiple ladder exchange. This has been studied in $\lambda\phi^3$ perturbation theory by Cicuta and Sugar,[15] and multi-Pomeron exchange contributions to the elastic amplitude have been studied in the Reggeon calculus by AKG.[1] We define the scattering amplitude operator as in Ref. 9 by

$$\begin{aligned} M(Y, \underset{\sim}{\Delta}) &= \int d^2\underset{\sim}{B}\, e^{-i\underset{\sim}{\Delta}\cdot\underset{\sim}{B}}\, M(Y, \underset{\sim}{B}) \\ &= 2is \int d^2\underset{\sim}{B}\, e^{-i\underset{\sim}{\Delta}\cdot\underset{\sim}{B}} \left[1 - S(Y, \underset{\sim}{B})\right] \end{aligned} \tag{2.13}$$

The amplitude for the ladder diagram is then given by

$$M_{\text{ladder}}(Y, \underset{\sim}{B}) = \frac{2is}{2!} f^2(\underset{\sim}{B})\, e^{Y[2(\alpha-1)+\lambda]} \equiv 2isA \tag{2.14}$$

We now calculate the contribution to the elastic amplitude due to the exchange of n ladders, we denote this by $\widetilde{M}_n$. The full contribution from the exchange of 2n chains is given by

$$M_{el}^{(2n)} = -2is \frac{(i)^{2n}}{(2n)!} < 0 | Z^{2n} | 0 > \tag{2.15}$$

where, as above, the states are defined in the meson Hilbert space. Contributing to $M_{el}^{(2n)}$ there are $(2n-1)!!$ ladder exchange diagrams since the first chain can link up with $(2n-1)$ other chains, the next one can link up with $(2n-3)$ chains and so on. Hence

$$\begin{aligned} \widetilde{M}_n(Y, \underset{\sim}{B}) &= -2is \frac{(-1)^n}{(2n)!} (2n-1)!! \left[<0|Z^2|0>\right]^n \\ &= -2is \frac{(-1)^n}{2^n n!} \left[<0|Z^2|0>\right]^n \\ &\equiv -\frac{2is}{n!} (-A)^n \quad . \end{aligned} \tag{2.16}$$

Summing over n we have

$$\widetilde{M}(Y, \underset{\sim}{B}) = \sum_{n=1}^{\infty} \widetilde{M}_n(Y, \underset{\sim}{B}) = 2is\left[1 - e^{-A}\right] \tag{2.17}$$

so that

$$\widetilde{M}(Y, \underset{\sim}{\Delta}) = 2is \int d^2\underset{\sim}{B}\ e^{-i\underset{\sim}{\Delta}\cdot\underset{\sim}{B}} \left[1 - e^{-A}\right] \tag{2.18}$$

which is the usual Regge eikonal formula.[16] Unlike the Regge eikonal model however, the present model contains production channels naturally built in. Eikonalization of ladder exchange diagrams in $\lambda\phi^3$ perturbation theory has been discussed in Ref. 15.

An alternative derivation of the above result follows from using a coherent state representation which diagonalizes the hermitian operator Z and only allows ladder diagrams to contribute. The meson vacuum expectation value of Z^{2n} is given by

$$\begin{aligned} <0|Z^{2n}|0> &= \int_{-\infty}^{\infty} \frac{dZ}{(4\pi A)^{1/2}}\ e^{-Z^2/4A}\ Z^{2n} \\ &= \frac{(2n)!}{n!}\ A^n \end{aligned} \tag{2.19}$$

which agrees with the previous result. This technique will prove very useful below, particularly when we introduce isospin into the model in Section 4.

Now, the amplitudes for the exclusive production of a single pion will be computed. We still restrict the absorption to ladder exchanges. Defining $\widetilde{M}^{(1)}$ to be the single particle exclusive amplitude where the absorptive corrections are restricted to be ladder exchanges, we find

$$\widetilde{M}^{(1)}(Y, \underset{\sim}{B}; \underset{\sim}{q}, y) = <0|[a(\underset{\sim}{q}, y), S]|0>$$
$$= ig(\underset{\sim}{q}) <0|ZS|0>$$
$$= ig \int_{-\infty}^{\infty} \frac{dZ}{(4\pi A)^{1/2}} e^{-Z^2/4A} Ze^{iZ}$$
$$= -2g(\underset{\sim}{q})A\, e^{-A} \qquad (2.20)$$

and we see that since only even powers of Z contribute, the produced meson is attached to the projectile and target via Reggeons only, as required for the leading terms in the central or pionization region. Production from the original chain, which is nonleading because it does not have the full Regge behavior, has been dropped. Formulae such as Eq. (2.20) have often been adopted as a phenomenological way of taking absorptive corrections into account.

Let us now extend the above discussion to a generalized Drell-Yan model for heavy photon (or lepton pair) production. The S-matrix will be written as before, but Z will be written as $Z = Z_0 + ez$, where Z_0 is the purely strong interaction chain while z produces a photon with coupling e as well as any number of possible mesons. The photon destruction operator $a(\gamma)$ has the commutation relation

$$[a(\gamma),\ ez] = e\tilde{z} \quad ,$$

where $\tilde{z}$ does not contain any photon operators and it commutes with Z_0. The necessary meson vacuum expectation values are given by

$$<0|\tilde{z}|0> = D \qquad <0|(\tilde{z}-D)^2|0> = 2R$$
$$<0|Z|0> = 0 \qquad <0|Z\tilde{z}|0> = 2R_1$$
$$<0|Z^2|0> = 2A$$

where D can be identified with the usual Drell-Yan production matrix element ($q+\bar{q}\rightarrow$ "γ") and R is its fully Regge behaved counterpart ($R_1+R_2\rightarrow$ "γ"). It is the expected leading term in Mueller-Regge theory. These quantities are represented diagramatically in Fig. 6. To first order in e, the results are

$$<0|S|0> = e^{-A}$$

$$\tilde{M}^{(1)} = <0|S\ ie\tilde{z}|0> = ie(D+2i\ R_1)\ e^{-A}$$

and

$$\rho_{in} = e^2 <0|\tilde{z}^2|0> = e^2\left[D^2+2R\right] \qquad (2.21)$$

If the term 2R is small, ρ_{in} agrees with the result of Drell-Yan. Since R contains a form factor coupling of the virtual photon to two Reggeons, it should indeed be small for large mass production. Note again that inclusive production is not absorbed but the exclusive production amplitude has an explicit factor of $<0|S|0>$.

We have seen that the simple unitary model of AASB has provided us with many features concerning absorption that may have been expected in view of the arguments presented in the introduction. In the next section we shall present a remarkable similarity of this model with the Reggeon calculus of AKG. [1]

III. COUNTING RULES FOR MULTILADDER/POMERON EXCHANGE

Abramoskii, Kancheli and Gribov[1] have presented a technique for calculating the imaginary part of the elastic amplitude, corresponding to multi-Pomeron exchange, by evaluating the terms which contribute to the unitarity sum. They argue that the relevant contributions are those in which each Pomeron is either left totally uncut (i.e., is completely included in either M or M^+ in the unitarity equation) or is totally cut. Hence the contribution of Fig. 7a is important in the evaluation of the leading behavior of the two Pomeron term, whereas that of Fig. 7b is not. In this section, we will show that the AKG counting rules are exactly reproduced in the exponential model of Section II.[17]

The amplitude for the exchange of ν Pomerons is given by (see Fig. 8)

$$iA^{(\nu)}(s,t) = s \int N_\nu \left[(iD_1) \ldots (iD_\nu)\right] N_\nu \, d\Omega_\nu \tag{3.1}$$

where $N_\nu(\underset{\sim}{k}_1, \underset{\sim}{k}_2 \ldots \underset{\sim}{k}_\nu)$ are real vertices of Reggeon emission, $D(\xi, k^2)$ are the complex Green's functions of the Pomerons and $\xi = \log s$. Taking the Pomeron to be a simple pole of positive signature we have

$$D(\xi, \underset{\sim}{k}^2) = -e^{\xi\left(\alpha(\underset{\sim}{k}^2)-1\right)} \frac{e^{i\pi\alpha(\underset{\sim}{k}^2)/2}}{\sin \pi\alpha(\underset{\sim}{k}^2)/2} \tag{3.2}$$

The Reggeon phase space is given by

$$d\Omega_\nu = \frac{1}{\nu!} \delta^{(2)}\left(\underset{\sim}{Q} - \sum_1^\nu \underset{\sim}{k}_i\right) \prod_{i=1}^{\nu} \frac{d^2 \underset{\sim}{k}_i}{2(2\pi)^2} \tag{3.3}$$

where $\underset{\sim}{Q}^2 = t$. The prescription given by AKG is the following. The vertices N_ν are unchanged by cutting the diagram, hence for the purposes of evaluating the imaginary part of $A^{(\nu)}$ by the unitarity equation, we may write

$$A^{(\nu)} \simeq -i(iD_1)(iD_2) \ldots (iD_\nu) \tag{3.4}$$

For a particular term in the unitarity sum we write (iD) for every Pomeron in M, (iD)* for every Pomeron in M^+ and 2 Im D for every cut Pomeron. We take the intercept of the Pomeron to be 1, and neglect its real part. Consider then a diagram with m+n+r Pomerons, where m Pomerons are cut, r are in M, and n are in M^+. Using the above prescription, and noting that there are (m+n+r)!/m!n!r! such terms we find

$$\begin{aligned} 2\,\mathrm{Im}\,A_{r,m,n} &= \frac{(m+n+r)!}{m!\,n!\,r!}\,(iD)^r\,(iD)^{*n}\,(2\,\mathrm{Im}\,D)^m \\ &= \frac{(m+n+r)!}{m!\,n!\,r!}\,2^m(-1)^{n+r}\,(\mathrm{Im}\,D)^{n+r+m} \end{aligned} \tag{3.5}$$

As a specific example we can consider the case of two Pomeron exchange, m+n+r=2. The m=0 contribution is seen to be $2!\,(\mathrm{Im}\,D)^2$, the m=1 contribution is $(-4)\times 2!\,(\mathrm{Im}\,D)^2$ and the m=2 contribution is $(2)\times 2!\,(\mathrm{Im}\,D)^2$. This (1, -4, +2) counting is also true in the Mandelstam diagram in $\lambda\phi^3$ perturbation theory.[6]

We now evaluate the analogous quantity to (3.5) in the model of AASB.[9] In this model we now have 2r+m chains in M and 2n+m chains in M^+ giving us a factor

$$\frac{(iZ)^{2r+m}}{(2r+m)!} \times \frac{(-iZ)^{2n+m}}{(2n+m)!} \tag{3.6}$$

In M we can arrange the m "open" chains in the 2r+1 spaces in

$$\frac{1}{2r!}\,\frac{1}{m!}\,(2r+m)! \quad \text{ways}$$

Similarly in T^+ we can arrange the m "open" chains in the 2n+1 spaces in

$$\frac{1}{2n!}\,\frac{1}{m!}\,(2n+m)! \quad \text{ways}$$

We can link up the m open chains of M with the m open chains in M^+ in m! ways. In M we can draw the r closed ladder diagrams (2r-1)!! ways and in M^+ we can draw the n closed ladder diagrams (2n-1)!! ways. Hence

$$2\,\mathrm{Im}\,M_{n,m,r} \simeq \frac{\left[<0|Z^2|0>\right]^{r+m+n}}{(2r+m)!\ (2n+m)!}\ \frac{(2r+m)!\ (2n+m)!}{2n!\,2r!\,m!\,m!}$$

$$\times\ (-1)^{n+r}\ m!\,(2r-1)!!\,(2n-1)!!$$

$$= \frac{\left[<0|Z^2|0>\right]^{r+m+n}}{2^n 2^r\ n!\,m!\,r!}\ (-1)^{n+r}$$

$$= A^{r+m+n}\ (-1)^{n+r}\ \frac{2^m}{n!\,m!\,r!} \qquad (3.7)$$

Thus we see that the counting is the same in both models.

We end this section by calculating explicitly some lower order diagrams to see how the (1, -4, +2) counting in the two ladder exchange contribution to the elastic amplitude and the cancellation of absorptive corrections in the inclusive cross section arises. For simplicity we take a model in which only a single meson can be created or destroyed off each chain, i.e., W_1 is as defined in Eq. (2.5), $W_i=0$ for $i \geq 2$. The three types of contribution to the unitarity sum for the two ladder elastic amplitude are shown in Fig. 9.

The contribution in which no ladder is cut (Fig. 9a) is proportional to

$$\frac{i^2}{2!} \times \frac{(-i)^2}{2!}\ \left(<0|Z^2|0>\right)^2 = \frac{1}{4}\ \left(<0|Z^2|0>\right)^2$$

$$= A^2 \qquad (3.8a)$$

There are six diagrams such as that of Fig. 9b in which one ladder is cut and their total contribution is proportional to

$$6 \times \frac{i^3}{3!} \times \frac{(-i)}{1!}\ \left(<0|Z^2|0>\right)^2 = -\left(<0|Z^2|0>\right)^2$$

$$= -4A^2 \qquad (3.8b)$$

Finally there are two diagrams such as that in Fig. 9c in which both ladders are cut and their total contribution is proportional to

$$2 \times \frac{i^2}{2!} \times \frac{(-i)^2}{2!} \left(<0|Z^2|0>\right)^2 = \frac{1}{2}\left(<0|Z^2|0>\right)^2$$

$$= 2A^2 \tag{3.8c}$$

Thus we arrive at the (1, -4, +2) counting mentioned above.

The cancellation in the inclusive cross section is now straightforward to see. To order A^2, there are only two types of terms which contribute, those of Fig. 9b and 9c. However both diagrams of the type shown in Fig. 9c contribute twice to the inclusive cross section since either of the two mesons could be the trigger particle. Hence the total contribution of order A^2 to the inclusive cross section is proportional to

$$-4A^2 + 2 \cdot 2A^2 = 0 \tag{3.9}$$

demonstrating the cancellation.

IV. EXTENDED MODELS

Let us now generalize the previous model in order to study the effects of the isospin of the produced particles, alternative unitary prescriptions, and possible excited states of the projectile.

A. Isospin Example

Consider a model in which in addition to the purely isoscalar chain, producing isoscalar mesons, there is a chain which can emit a particle with unit isospin called a pion which can be emitted along with an arbitrary number of I=0 mesons such as illustrated in Fig. 10. The projectile has I=1/2 and the target I=0. Our model S-matrix can be written in the unitary form

$$S = e^{i\vec{\tau}\cdot\vec{\chi} + iZ} \tag{4.1}$$

where Z is as before and $\vec{\tau}$ is the vector isospin matrix for the projectile. If a_j is the destruction operator for a pion with index j, then we have

$$\left[a_j, Z\right] = 0$$

and (4.2)

$$\left[a_j, \chi^k\right] = h\,\tilde{\chi}\,\delta_{jk} \ ,$$

where χ^j is given by

$$\chi^j = \int \frac{d^2q\,dy}{2(2\pi)^3}\, h\left[a^j(q,y) + a^{j+}(q,y)\right] Z_1\left(\tfrac{1}{2}Y - y\right) Z\left(y + \tfrac{1}{2}Y\right) \ ,$$

and Z_1 is the analogue of Z for the I=1 chain. Also, χ^j and Z commute since they are both functions of $(a + a^+)$. The eikonal form for S has not yet been derived in cases with isospin, but it is not clear that such general forms as the above have been considered. In any case, we are primarily interested in the unitarity property of S.

We shall be interested in elastic scattering, inclusive scattering and exclusive production of one pion. With the definition $\chi \equiv (\vec{\chi} \cdot \vec{\chi})^{1/2}$, the S-matrix and its commutators with the single pion destruction operator can be written as

$$S = \left(\cos\chi + i\vec{\tau} \cdot \vec{\chi}\,\frac{\sin\chi}{\chi}\right) e^{iZ} \tag{4.3}$$

and

$$\left[a_j, S\right] = ih\,\tilde{\chi}\ \ e^{iZ}\,Q_j \ , \tag{4.4}$$

where

$$Q_j = \frac{\chi^j \vec{\tau}\cdot\vec{\chi}}{\chi^2}\, e^{i\vec{\tau}\cdot\vec{\chi}} + \frac{\sin\chi}{\chi^3}\left(\tau_j \chi^2 - \chi^j \tau\cdot\chi\right) \quad . \tag{4.5}$$

In order to retain the ladder approximation and avoid checkerboard graphs, we note that the only nonzero vacuum expectation values are defined by

$$\begin{aligned} A &= \frac{1}{2} < 0 | Z^2 | 0 > \\ A_1 &= \frac{1}{6} < 0 | \vec{\chi}\cdot\vec{\chi} | 0 > \\ R &= \frac{1}{2} < 0 | \tilde{\chi} \quad \tilde{\chi} \quad | 0 > \\ R_1 &= \frac{1}{2} < 0 | Z \tilde{\chi} \quad | 0 > \end{aligned} \tag{4.6}$$

which are illustrated graphically in Fig. 11. The lack of cross terms between Z and $\vec{\chi}$ considerably simplifies the following analysis as we shall see.

The elastic S-matrix element is the meson and pion vacuum expectation value of S;

$$\begin{aligned} < 0 | S | 0 > &= \int \frac{d^3\chi}{(4\pi A_1)^{3/2}}\, e^{-\chi^2/4A_1}\, e^{i\vec{\tau}\cdot\vec{\chi}} < 0 | e^{iZ} | 0 > \\ &= e^{-(A+A_1)}\,(1 - 2A_1) \end{aligned} \tag{4.7}$$

The exclusive production amplitude is

$$\begin{aligned} < 0 | \left[a_j, S\right] | 0 > &= ih < 0 | e^{iZ}\, \tilde{\chi}^j | 0 > \int \frac{d^3\chi}{(4\pi A_1)^{3/2}}\, e^{-\chi^2/4A_1} \left[Q_j\right] \\ &= -2h\, R_1 \tau_j H \end{aligned} \tag{4.8}$$

where the integral is performed by introducing $\vec{\chi} = 2A_1^{1/2}\vec{r}$, and the result is

$$H = e^{-(A+A_1)}\left(1 - \frac{2}{3}A_1\right) . \tag{4.8a}$$

The resultant cross section for the production of only one pion will be proportional to the square of the integral of this quantity over impact space. Note that the absorption factors are not related to the elastic S-matrix:

$$<0|\left[a_j, S\right]|0> \neq -2hR_1\tau_j <0|S|0> \quad . \tag{4.9}$$

The inclusive production cross section of neutral pions is given by the vacuum expectation value of $\left|\left[a_3, S\right]\right|^2$ as before, and the result is

$$\rho^{o}_{in} = 2h^2 RF \quad , \tag{4.10}$$

where

$$F = \frac{1}{3}\left[1 + \frac{1}{2A_1}\left(1 - e^{-4A_1}\right)\right] . \tag{4.10a}$$

It should be noted that the absorption due to A has cancelled but not that due to A_1. The result for charged pions is similar:

$$\rho^{\pm}_{in} = 2h^2 R\left\{F \pm \tau_3 G\right\} \tag{4.11}$$

where

$$G = \frac{1}{3}\left[2\, e^{-4A_1} + \frac{1}{4A_1}\left(1 - e^{-4A_1}\right)\right] \tag{4.11a}$$

Note that in the limit $A_1 \to \infty$, $G \to 0$, $F \to 1/3$ and $\rho^{\pm}_{in} = \rho^{o}_{in}$. It is clear that these inclusive cross sections are still shadowed because of the isospin structure of the production matrix element. The commutation properties of the isospin operators of the leading particles change the counting and the absorption no longer cancels.

In the opposite limit, $A_1 \to 0$, $F = G = 1$, and the result is the expected Mueller-Regge limit with R as shown in Fig. 11.

B. K-Matrix

It has been pointed out by R. Sugar[18] that the absorption does not cancel in the single particle inclusive cross section, for the K-matrix form for satisfying unitarity. To see this, write

$$S = \left(1 + \frac{i}{2} Z\right)\left(1 - \frac{i}{2} Z\right)^{-1} \tag{4.12}$$

and the elastic matrix element can then be written as

$$<0|S|0> = 1 + 2 \sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{n!} \left(\frac{A}{4}\right)^n = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx\, e^{-x^2} (1-Ax^2)(1+Ax^2)^{-1}. \tag{4.13}$$

The particle production operator is

$$[a, S] = igZ \left(1 - \frac{i}{2} Z\right)^{-2} \tag{4.14}$$

and the inclusive cross section is proportional to the vacuum matrix element of

$$g^2 Z^+ Z \left(1 + \frac{1}{4} Z^+ Z\right)^{-2} \quad , \tag{4.15}$$

which clearly shows that absorption effects are still present.

C. Projectile Excitations

The effect of excited states of the projectile will now be investigated by considering the S-operator as a matrix in this state space:

$$S = e^{iGZ} \tag{4.16}$$

where Z is the chain meson creation operator and G is the symmetric, real coupling matrix among the projectile states. Using the commutator

$$[a, S] = SG\, i[a, Z] = igSGZ \quad , \tag{4.17}$$

the effects of absorption are again seen to cancel in the meson inclusive cross section for any choice of G.

The choice of G can affect the explicit form of the S-matrix, however, in unusual ways. If the elastic matrix element of G^n is defined by

$$(G^n)_{11} \equiv g(n) \tag{4.18}$$

then S becomes

$$(S)_{11} = \sum \frac{i^n}{n!} g(n) Z^n \tag{4.19}$$

which may or may not be an exponential series.

For example if one chooses G equal to

$$G_0 = \begin{pmatrix} 1 & 1 & 0 & 0 & \cdots & \\ 1 & 3 & 2 & 0 & \cdots & \\ 0 & 2 & 5 & 3 & 0 & \cdots \\ 0 & 0 & 3 & 7 & 4 & \cdots \\ \vdots & \vdots & 0 & 4 & 9 & \cdots \\ & & \vdots & \vdots & \vdots & \end{pmatrix} \tag{4.20}$$

the reader may be interested in proving that $g(n) = n!$, and

$$\begin{aligned} (S)_{11} &= (1 - iZ)^{-1} \\ (S)_{12} &= iZ(1 - iZ)^{-2} \quad . \end{aligned} \tag{4.21}$$

If one cuts off the finite G matrix at any point, g(n) still equals n! until n exceeds the value of the last diagonal element of G. A choice with better convergence properties is to set $G^2 = G_0$ so that $g(n) = (n/2)!$.

Using our previous results, the general elastic S-matrix element is

$$< 0 \,|\, S_{11} \,|\, 0 > = \sum \frac{(-1)^n}{n!} A^n g(2n) \tag{4.22}$$

A similar result holds for the exclusive production amplitude. By choosing different forms for G, the nature of the convergence of this series can be changed at will from exponential (G=1), to geometric (G^2=G_0), to superficially divergent (G=G_0), for example.

V. ANTISHADOWING

Since we have now seen that an exponential form for the S-matrix is not the only possible one, it is amusing to consider the following form. Write S as

$$S = e^{i(1-2a)Z} \, (1+iaZ)\,(1-iaZ)^{-1} \tag{5.1}$$

where a is a parameter. As a varies from zero to one-half, S changes from the exponential form to the K-matrix form. Unitarity is preserved for any value of a. It is convenient to introduce an expansion for S in terms of the coefficients $u_\ell(a)$:

$$S = \sum \frac{(iZ)^\ell \, u_\ell(a)}{\ell!} \tag{5.2}$$

where

$$u_\ell(a) \equiv (1-2a)^\ell + 2 \sum_{m=0}^{\ell-1} \frac{\ell!}{m!} (1-2a)^m a^{\ell-m} \tag{5.3}$$

Some explicit forms for low ℓ are $u_0 = u_1 = u_2 = 1$, $u_3 = 1+4a^3$, $u_4 = 1+16a^3$, $u_5 = 1+8a^3\,(5+6a^2)$, ...

The elastic S-matrix is

$$\langle 0|S|0\rangle = \sum \frac{(-1)^n}{n!} A^n \, u_{2n}(a) \tag{5.4}$$

and the inclusive cross section is proportional to

$$\rho_{in} = g^2 A \sum_0^\infty \frac{A^n}{(n+1)!} U_n \tag{5.5}$$

where

$$U_n = \sum_{k=0}^{2n} \frac{(-1)^{k-n}\,(2n)!}{k!\,(2n-k)!} u_{k+1}(a)\, u_{2n+1-k}(a) \tag{5.6}$$

A few sample values are $U_0 = 1$, and $U_1 = 2(1-u_3) = -8a^3$. The exclusive production cross section is proportional to

$$\rho_{ex} = 4g^2A^2\left[\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} A^n\, u_{2n+1}(a)\right]^2 \tag{5.7}$$

and only if a=0 is the coefficient of the Born term $4g^2A^2$ equal to $<0|S|0>^2$, as would be expected in simple absorption models. Note that the inclusive cross section is proportional to A whereas the exclusive one is proportional to A^2 as would be expected in a fully Reggeized theory.

Let us now examine these quantities only to the order of two Pomeron exchange. An expansion of the above results yields

$$1 - <0|S|0> = P - \frac{1}{2}P^2\, u_4(a) \tag{5.8a}$$

$$\rho_{in} = g^2P\left[1 + P(1 - u_3(a))\right] \tag{5.8b}$$

$$\rho_{ex} = 4g^2P^2\left[1 - P\, u_3(a)\right] \tag{5.8c}$$

where $P = \frac{1}{2} <0|Z^2|0>$. These formulas, which hold in this explicitly unitary theory, have several interesting and surprising properties. Note that ρ_{in} is absorbed in general. However, if a<0, the absorption actually <u>enhances</u> the inclusive cross section. Similarly, if $u_4(a)$ is negative, the two Pomeron cut in the elastic cross section is positive, also an enhancement. This will occur for sufficiently small a, $a < -16^{-1/3} \simeq -0.40$. Finally, if $u_3(a)$ is negative, exclusive production is enhanced as well, and this occurs if $a < -4^{-1/3} \simeq -0.63$. We reiterate that this unexpected behavior is true in an explicitly unitary theory and hence the sign of absorption effects cannot be said to follow from s-channel unitarity alone. However, it should be noted that this type of unitary S-matrix cannot arise from the Lippman-Schwinger or Bethe-Salpeter type of equation used as starting points in the proof of the negative sign for diffractive effects near threshold.

VI. CONCLUSION

The simple models that were discussed above can only be used to suggest possible behavior in more complicated and physically realistic situations. They should be used to increase one's physical intuition and hence to aid in constructing more sensible theories. A few salient points should be stressed which were gleaned from studying the above examples:

(1) The exponential operator form of the S-matrix is a very convenient approach to use in formulating the Reggeon-calculus of AKG and the Regge-eikonal model. It is especially useful in those cases in which s-channel unitarity must be implemented. It may prove to be more basic than Reggeon calculus in the sense that details of the inelastic states can be built into the form of Z which are lost or hidden when only Reggeons are considered.

(2) However, t-channel unitarity is not guaranteed by the model but it can be enforced to any desired order by appropriately modifying the choice of Z to contain the appropriate nonplanar graphs and rapidity gaps.

(3) It was found that the internal quantum numbers and possible excited states of the projectile, which may or may not be discrete states in the continuum, can change the counting rules for higher order diagrams. For example, the counting rules of AKG seem to be true only in a model of the Pomeron which neglects the isospin character of the couplings. These possibilities can also superficially change the convergence property of the S-matrix, from exponential to geometric.

(4) These effects change the counting of higher order diagrams and can strongly modify the shadowing. They may have a large influence on the discussions of nuclear multiplicity using the AKG formulas depending on details of the model of the Pomeron.

(5) The extreme example in Section V that had antishadowing clearly shows that s-channel unitarity does not specify or determine absorption, not even its sign. However, such models may be inconsistent with t-channel unitarity since, with certain analyticity assumptions, White[19] has demonstrated that the full two Pomeron contribution is definitely negative (normal shadowing). The model in the text, however, possesses a positive contribution from the disconnected two Pomeron ($< 0 | Z^2(Y,B) | 0 >^2$) cut only, and hence may or may not be consistent with this theorem.

(6) These models point out the importance of measurements that study absorption effects in elastic, and in inclusive and exclusive production processes. It is possible that one can learn about the basic nature of the Pomeron (such as its isospin content) from detailed studies of the momentum transfer and transverse momentum dependence of these processes.

We find it very surprising that two models, one constructed to satisfy s-channel unitarity (that of AASB[8,9]) and the other satisfying t-channel unitarity (that of AKG[1]) should give identical prescriptions for unitarity corrections. However, even if these counting rules should be correct in the case of no quantum numbers, it is clear, from the discussion in Section IV, that they may be altered by the presence of internal symmetries or excited states. The models presented in Sections IV and V demonstrate that s-channel unitarity alone does not place any relevant constraints on the magnitude or sign of absorptive corrections. Clearly, before the properties of the models reviewed in the introduction can be safely assumed to be general, much more ambitious models must be examined.

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FIGURE CAPTIONS

1. Mueller Regge diagram for the single particle inclusive cross section, where the trigger particle is in the central region.
2. Mueller diagrams for the inclusive production of massive lepton pairs (a) given by the Drell-Yan model; (b) absorptive correction to the Drell-Yan model.
3. Dominant contributions to the unitarity sum for the two Pomeron cut.
4. Basic form of Z_n.
5. Mueller diagram which determines the single particle inclusive cross section in the model of AASB.
6. Definition of quantities which appear in massive lepton pair production.
7. (a) A leading order contribution to the unitarity sum for the two Pomeron cut and (b) a nonleading contribution.
8. Elastic amplitude corresponding to the exchange of ν Pomerons.
9. Three leading contributions to the unitarity sum for the term of order Z^4 in the elastic amplitude.
10. (a) A chain with only I=0 mesons produced and (b) a chain with I=0 mesons produced together with an I=1 pion.
11. Definition of quantities which appear in the production of a pion with isospin 1.

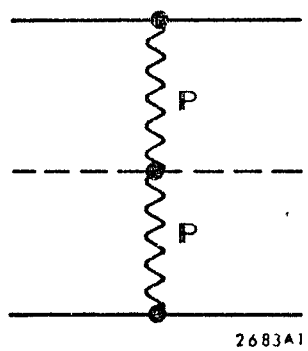


Fig. 1

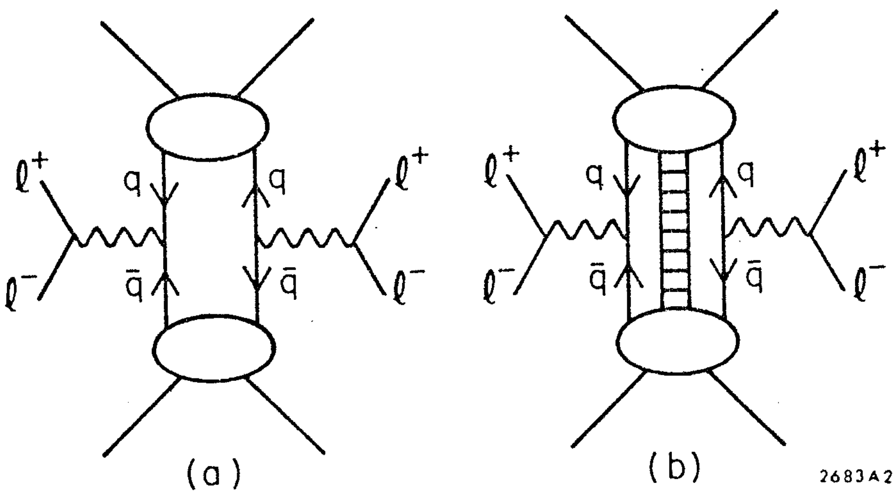
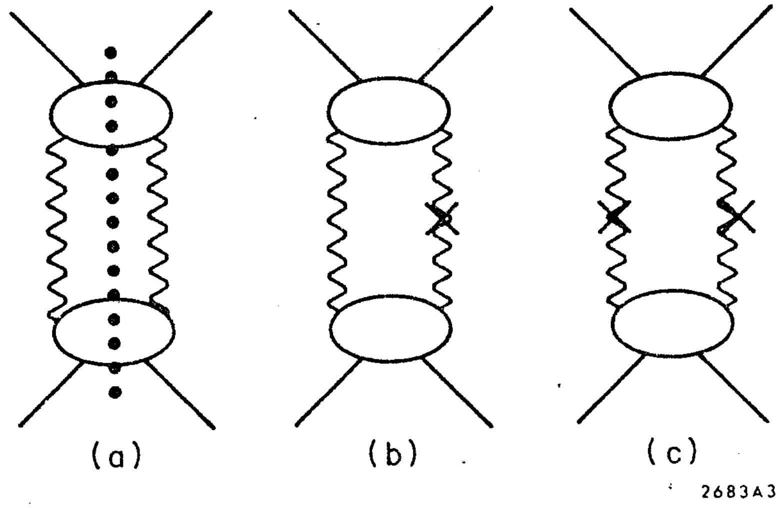


Fig. 2



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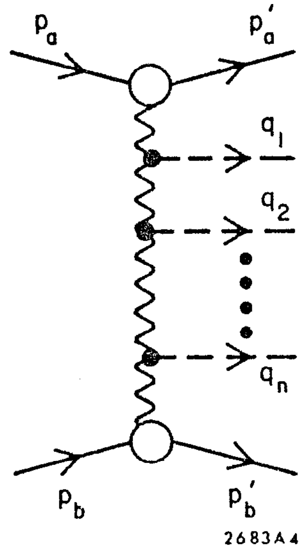

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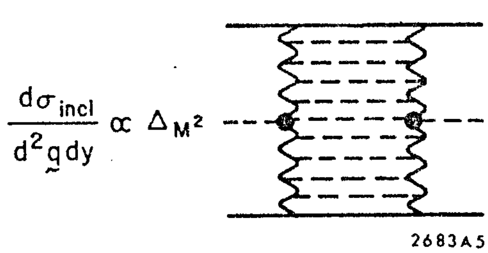

Fig. 5

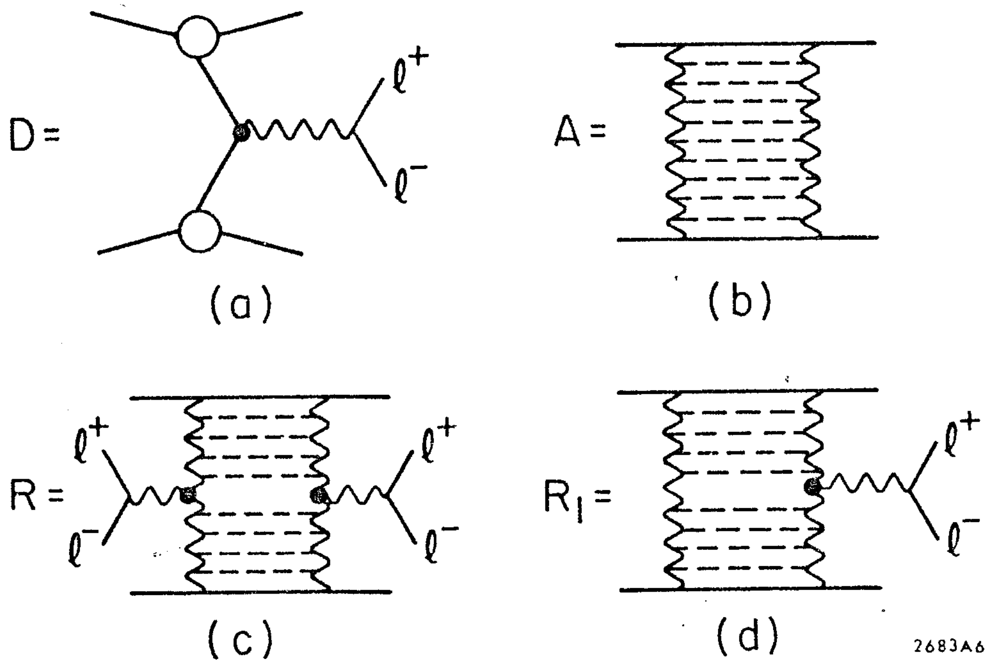
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Fig. 6

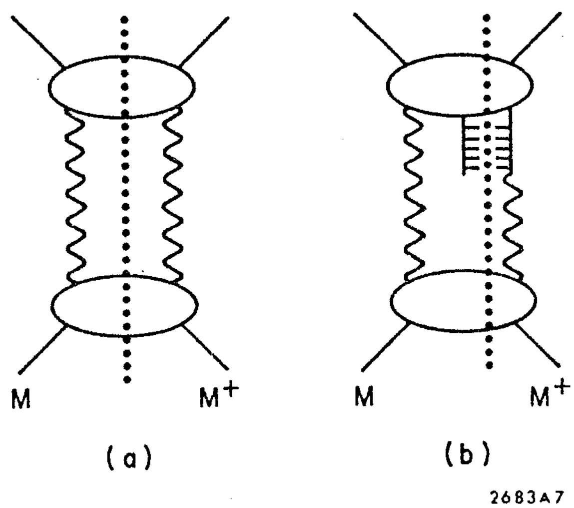
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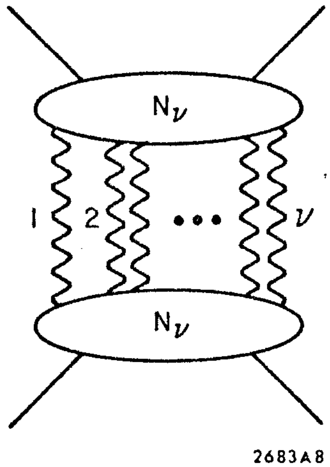
Fig. 7



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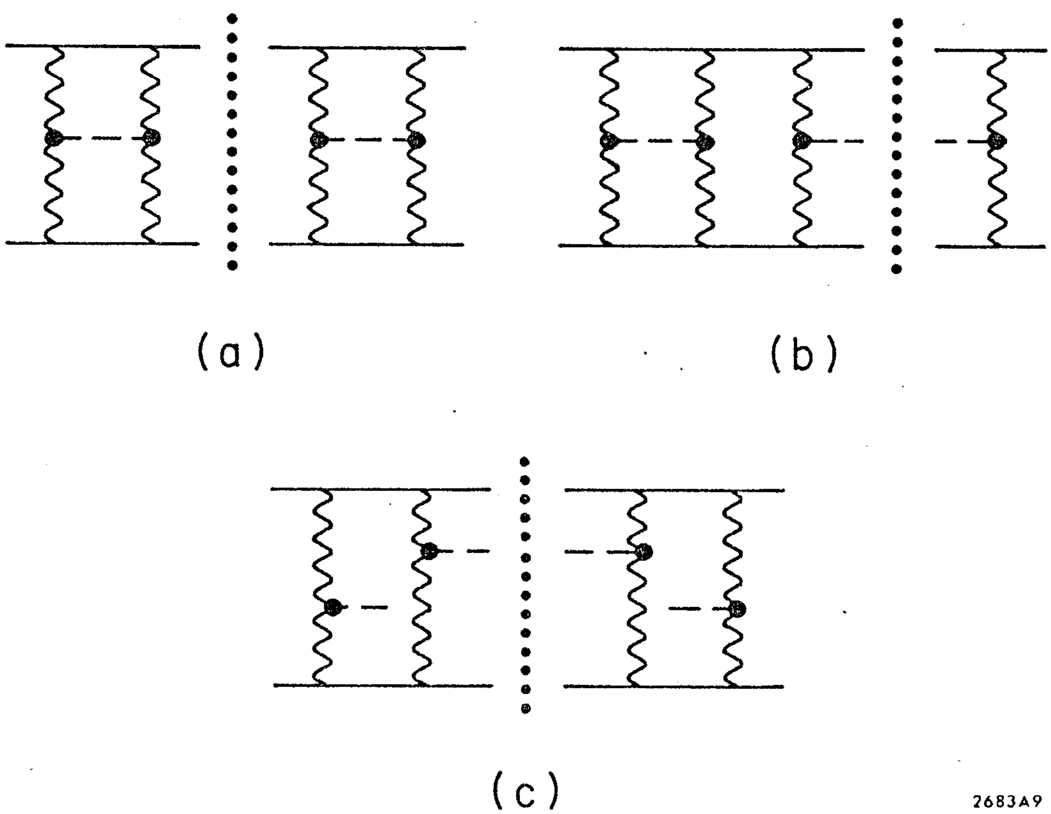

Fig. 9

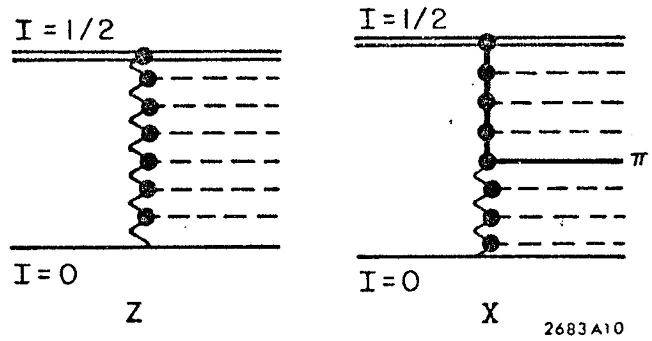

Fig. 10

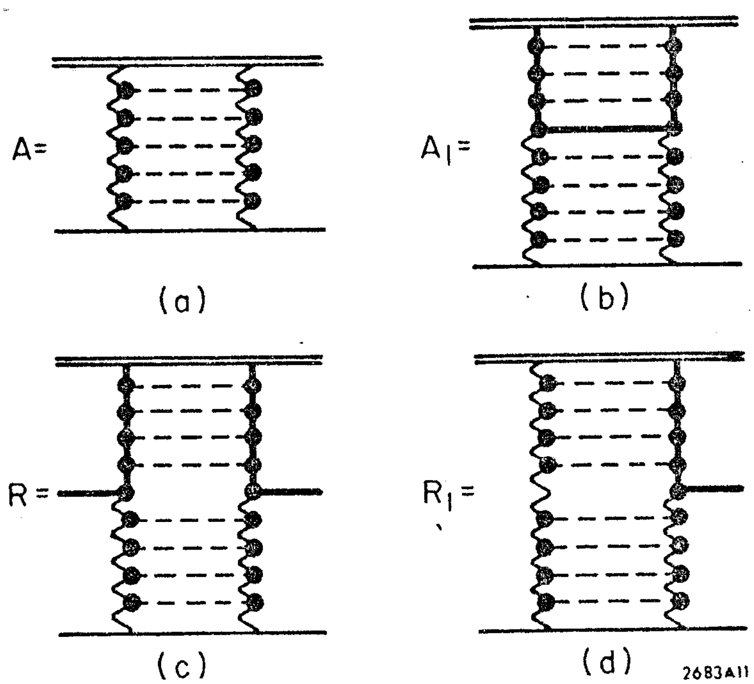

Fig 11