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Hot Dense Matter and Random Fluctuation Walk

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The critical point in particle physics at high temperature is studied through the ideal gas of scalars, the dilatons, in the model that implies the spontaneous breaking of an approximate scale symmetry. We found the critical temperature as a function of a dilaton mass, and the fluctuation of particle density grows up very sharply at critical point. Our results also suggest that the critical point may be identified through the fluctuation in yield of primary direct photons induced by conformal anomaly of strong and electromagnetic sectors.

KEYWORDS: conformal anomaly, dilaton, critical point, primary direct photons.

1. Introduction

The spontaneous breaking of symmetry has a major role in particle physics and cosmology where the phase transitions (PT) can occur at extreme conditions (e.g., high enough temperature, baryonic density, chemical baryonic potential). The critical point (CP) of PT corresponds to an initial thermal state (of matter) which is invariant under the conformal group. The critical phenomena if occurred are considered here through quantum PT with Bose-Einstein condensation (BEC) of the scalar field in a single zero mode of an ideal Bose-gas that suggests the breaking of conformal symmetry. One can imply the existence of Goldstone-like modes: the scale symmetry is broken explicitly resulting with an appearance of a scalar, the dilaton σ, in the spectrum, accompanying by π-mesons as a consequence of chiral symmetry breaking [1].

Quantum effects, e.g. gluon fluctuations, break conformal (scale) invariance. It is seen through the anomaly in the trace of energy-momentum tensor $\theta^{\mu}_{\mu} = \partial_{\mu} S^{\mu} \neq 0$, where the dilatation current $S^{\mu} = \theta^{\mu\nu} x_{\nu}$ does not conserved itself with respect to the scale transformations of coordinates $x_{\mu} \to \omega x_{\mu}$ (ω is an arbitrary constant). Because of the presence of strong gluon fields, the QCD vacuum is disordered and scale invariance is destroyed by the appearance of the dimensional scale $M = M_{UV} \exp\left[-8\pi^2/(b_0 g^2)\right]$, with M_{UV} being the ultra-violet (UV) scale, g is the bare gauge coupling constant and b_0 is the first coefficient in the QCD β-function, $\beta(M) \neq 0$. The breaking of conformal invariance assumes that all the processes are governed by the conformal anomaly (CA) resulting from running coupling constant $g(M)$ in β -function related to divergence of S_{μ}. There could be an approximate scale (dilatation) symmetry if $\beta(g)$ is small enough and g is slowly running with M. Theory becomes conformal in the infra-red (IR) with the non-trivial solution $\alpha_s^{\star} = -2\pi b_0/b_1$ (IR fixed point (IRFP)) in the perturbative domain if $b_0 = (11N_c - 2N_f)/3$ is small (N_c and N_f are the numbers of colors and flavors, respectively). The dilatons are unstable, they decay into two photons, where CA acts as a source of primary direct photons. The signature of CP is non-monotonous behavior of observable fluctuation where the latter increases very crucially.

We suggest the novel approach to an approximate scale symmetry breaking with the challenge phenomenology where the primary direct photons induced by CA are in fluctuating regime. The latter is an indicator of CP if the fluctuation length grows to become very large.

2. Dilatons as quantum statistical states

We start with the dilaton-pion model given by the Lagrangian density (LD)

$$L = \frac{1}{2} f_{\sigma}^{2} \left(\partial_{\mu} e^{\sigma}\right)^{2} + \frac{1}{2} f_{\pi}^{2}\, e^{2\sigma} \partial_{\mu}\pi + ... \tag{1}$$

which is scale invariant under transformations of coordinates $x_{\mu} \to \omega x_{\mu}$ if $\pi(x)$ transforms as $\pi(x) \to \pi(xe^{\omega})$ and the dilaton field σ transforms non-linearly

$$\sigma(x) \to \sigma(xe^{\omega}) + \omega. \tag{2}$$

LD (1) has the more suitable form $L = (\partial\chi)^{2}/2 + (f_{\pi}/f_{\chi})^{2}\,\chi^{2}\,\partial\pi + ...$, if one makes redefinition of the dilaton field $\sigma(x) \to \chi(x) = f_{\chi}\, e^{\sigma(x)}$, which transforms non-linearly under (2), $\langle\chi\rangle = f_{\chi}$. S_{μ} acting on the vacuum $|0\rangle$ defines χ: $\langle 0|S^{\mu}|\chi(p)\rangle = i\,p^{\mu}\,f_{\chi}$, $\partial_{\mu}\langle 0|S^{\mu}(x)|\chi(p)\rangle = \langle 0|\theta_{\mu}^{\mu}(x)|\chi(p)\rangle = -f_{\chi}\,m_{\chi}^{2}\,e^{-i\,p\,x}$, where for on-shell case $\langle 0|\theta^{\mu\nu}(x)|\chi(p)\rangle = f_{\chi}(p^{\mu}p^{\nu} - g^{\mu\nu}p^{2})e^{-i\,p\,x}$, $p^{2} = m_{\chi}^{2}$. Here, m_{χ} is the mass of the dilaton, p_{μ} is the momentum conjugate to x_{μ}. We suppose that at the scale larger than that of the confinement scale Λ the dilaton is formed as the bound state of two gluons, the glueball $\chi = O^{++}$, with the mass $m_{\chi} \sim O(\Lambda)$. The characteristic feature of the CP is sharp increasing of the fluctuations of the order parameter field χ. They act as a regulator in the IR with correlation length $\xi = m_{\chi}^{-1}$. In the vicinity of CP, ξ is much higher that that of a size of the particle interacting region at early times.

Consider the system containing dilatons as almost ideal weakly interacting glueball gas. In the state of statistical equilibrium at temperature $T = \beta^{-1}$ the partition function for N particles is

$$Z_{N} = S\,p\,e^{-H\beta}, \tag{3}$$

where H is the Hamiltonian $H = \sum_{1\leq j\leq N} H(j)$, and β in (3) differs from those of the QCD β - function. For the system of regular dilaton functions $\sigma_{f}(x)$ in f representation one has the equation $H(j)\,\sigma_{f}(x_{j}) = F(f)\,\sigma_{f}(x_{j})$, where $H = \sum_{f} F(f)\,b_{f}^{+}\,b_{f} = \sum_{f} F(f)\,n_{f}$ in terms of operators of creation b_{f}^{+} and annihilation b_{f}; n_{f} is an occupation (particle) number. Here, $F(f) = E(f) - \mu\,Q(f)$ with $E(f)$ being the energy, μ is the chemical potential, $Q(f)$ is the conserved charge. Interactions between glueballs should lead to thermal equlibrium, and in case of large n_{f} to the formation of BEC. In principle, operators b_{f} can be distorted by random quantum fluctuations through the operator r_{f}, $b_{f} \to b_{f} = a_{f} + r_{f}$, where a_{f} is the bare (annihilation) operator. The function Z_{N} (3) has the form [2]

$$Z_{N} = \sum_{...n_{f}...,\ \sum_{f} n_{f}=N} e^{-\beta\,\sum_{f} F(f)\,n_{f}}.$$

CP manifests itself through the critical chemical potential μ_{c} and the critical temperature T_{c}. Let us consider the following power series

$$\frac{P(\bar{\mu})}{\bar{\mu}^{N}} = \sum_{N'=0}^{\infty} \frac{Z_{N'}\,\bar{\mu}^{N'}}{\bar{\mu}^{N}},\ \bar{\mu} = \frac{\mu}{\mu_{c}} \tag{4}$$

on the real axis $0 < \bar{\mu} < R$, where $R \geq 1$ is the convergence radius. Because of positive $Z_{N'}$ the scan function (4) has the only one minimum on $(0, R)$

$$\frac{d^{2}}{d\bar{\mu}^{2}}\left[P(\bar{\mu})\,\bar{\mu}^{-N}\right] = \sum_{N'=0}^{\infty}(N' - N)\,(N' - N - 1)\,Z_{N'}\,\bar{\mu}^{N'-N-2} > 0.$$

The function (4) tends to infinity when $\bar{\mu} \to 0$ and when $\bar{\mu} \to R$. In the interval $(0, R)$ there is a point $\bar{\mu} = \bar{\mu}_{0}$ at which (4) has a single minimum. If one goes alone the vertical axis, (4) has a maximum at

$\bar{\mu}_0$. In case when $\bar{\mu} > \bar{\mu}_0$, the point $\bar{\mu} = \bar{\mu}_0$ is the ground state at given μ. Having in mind that

$$P(\bar{\mu}) = \exp\left\{-\sum_f \ln\left[1 - \bar{\mu}\, e^{-F(f)\beta}\right]\right\},$$

one can find the asymptotic equality $\sum_f \ln\left[1 - \bar{\mu}\, e^{-F(f)\beta}\right] = N\,\Phi(\bar{\mu})$, where $\Phi(\bar{\mu}) = const \cdot v\beta\, K_\chi(\bar{\mu})$, $v = \Omega/N$. $K_\chi(\bar{\mu})$ is the thermochemical potential of the dilaton χ

$$K_\chi(\bar{\mu}) = \beta^{-1} \int \ln\left[1 - \bar{\mu}\, e^{-F(f)\beta}\right] df,$$

which gives the contribution to thermodynamic potential $K = K_\chi + V_\chi + \lambda(f_\chi/2)^4$. Here, V_χ is the potential term in LD of the dilaton $L_\chi = (1/2)\partial_\mu\chi\partial^\mu\chi - V_\chi$, $V_\chi = (\lambda/4)\chi^4[\ln(\chi/f_\chi) - 1/4]$. The thermodynamic potential K does account for dilaton (glueball) and gluon degrees of freedom: $K = \theta(\beta_c - \beta)\, K_\chi(\bar{\mu}) + \theta(\beta - \beta_c)\, K_g$, where K_g is an effective gluon thermodynamic potential with the energy $E_g = \sqrt{|\vec{p}|^2 + m_g^2}$, m_g is an effective gluon mass. The ground state $\bar{\mu}_0$ is defined from the Eq.

$$\sum_f \bar{n}_f = \sum_f \frac{1}{\bar{\mu}_0^{-1}\, e^{F(f)\beta} - 1} = N,$$

where the large number N is correct if the dilatons are light.

3. Critical temperature

Consider the nonrelativistic model where the glueballs are produced in the volume Ω as a cube with the side of the length $L = \Omega^{1/3}$. In the limit $\Omega \to \infty$ and for $v = const$ we consider two cases: high temperature case A), where $\bar{\mu}_0\, e^{\mu\, Q\beta} < 1$, and low T case B), where $\bar{\mu}_0\, e^{\mu\, Q\beta} \sim 1$. In case A) we have the unequality:

$$\frac{2\,\pi^2}{v}\left(\frac{\beta}{2\,m_\chi}\right)^{3/2} < \int_0^\infty \frac{x^2\, dx}{e^{x^2} - 1}.$$

The case A) is realized when $T > T_c$, where

$$T_c = \frac{1}{2\,m_\chi}\left(\frac{2\,\pi^2}{v\,B}\right)^{2/3}, \quad B = \frac{\sqrt{\pi}}{4} \cdot 2{,}612..., \quad m_\chi \neq 0.$$

One can easily find the singular behavior of the correlation length ξ, where divergence of ξ is governed by the ground state $\bar{\mu}_0$:

$$\xi = 2\mu\, Q\left(\frac{v\,B}{2\,\pi^2}\right)^{2/3} \ln^{-1}\left(\frac{1}{\bar{\mu}_0}\right). \tag{5}$$

Actually, $\xi \to \infty$ at $\bar{\mu}_0 \to 1$ that means CP ($\bar{\mu}_0 = \bar{\mu} = 1$).

The case B) takes place at $|p| \leq \delta$ with $T < T_c$ where

$$\lim_{\delta \to 0,\, N \to \infty} \frac{1}{\Omega} \sum_{|p| \leq \delta} \bar{n}_p = \frac{1}{v} - \frac{1}{(2\,\pi)^3} \int_{|p| \geq \delta} \frac{d^3 p}{e^{E_p\beta} - 1} = \frac{1}{v}\left[1 - \left(\frac{\beta_c}{\beta}\right)^{3/2}\right],$$

where the only part of total number of particles proportional $\sim (\beta_c/\beta)^{3/2}$ is distributed on all the spectrum of momenta. The rest one $\sim [1 - (\beta_c/\beta)^{3/2}]$ is the scalar condensate.

The fluctuation of particle density is the function of ξ

$$\langle (n_V - \langle n_V \rangle)^2 \rangle = \langle n_V \rangle \left[1 + \frac{\sqrt{2}\, v}{\pi^2}\left(\frac{T}{\xi}\right)^{3/2} \int_0^\infty \frac{x^2\, dx}{(\bar{\mu}_0^{-1}\, e^{-\mu\, Q\beta}\, e^{x^2} - 1)^2}\right], \langle n_V \rangle = V/\Omega, V < \Omega. \tag{6}$$

The sharp increasing of (6) is expected at CP and it is ξ-independent.

4. Primary direct photons at CP

In the exact scale symmetry, χ couples to SM particles through the trace of $\theta_{\mu\nu}$

$$L = \frac{\chi}{f_\chi}\left(\theta^{\mu}_{\mu_{tree}} + \theta^{\mu}_{\mu_{anom}}\right), \tag{7}$$

where the first term is (contributions from heavy quarks and heavy gauge bosons are neglected)

$$\theta^{\mu}_{\mu_{tree}} = -\sum_q [m_q + \gamma_m(g)]\bar{q}q - \frac{1}{2}m_\chi^2\chi^2 + \partial_\mu\chi\partial^\mu\chi,$$

q is a quark d.o.f. with the mass m_q, γ_m are the corresponding anomalous dimensions. In contrast to SM, the dilaton couples to massless gauge bosons even before running any SM particles in the loop, through the trace anomaly. The latter has the following term in (7) for photons and gluons:

$$\theta^{\mu}_{\mu_{anom}} = -\frac{\alpha}{8\pi}\, b_{EM}\, F_{\mu\nu}F^{\mu\nu} - \frac{\alpha_s}{8\pi}\sum_i b_{0_i}\, G^a_{\mu\nu}G^{\mu\nu\, a},$$

where α is the fine coupling constant, b_{EM} and b_{0_i} are the coefficients of electromagnetic (EM) and QCD β-functions, respectively. If the strong (and EM) interactions are embedded in the conformal sector the following relation for light and heavy particles sectors is established above the scale Λ (in UV): $\sum_{light} b_0 = -\sum_{heavy} b_0$, where the mass of χ splits the light and heavy states. The partial decay width $\chi \to \gamma\gamma$ is

$$\Gamma(\chi \to \gamma\gamma) \simeq \left(\frac{\alpha\, F_{anom}}{4\pi}\right)^2 \frac{m_\chi^3}{16\pi f_\chi^2},$$

where the only CA does contribute through $F_{anom} = -(2\, n_L/3)(b_{EM}/b_0^{light})$, $b_0^{light} = -11 + (2/3)n_L$, $b_{EM} = -4\sum_{q:u,d,s} e_q^2 = -8/3$, e_q is the charge of the light quark. In the vicinity of IRFP there are fluctuations of dilaton field with $m_\chi \simeq \sqrt{1 - N_f/N_f^c}\Lambda$ [3], where N_f^c is the critical value of N_f corresponding to α_s^c at which the chiral symmetry is breaking and the confinement is emerged. In the IR one can estimate the fluctuation rate relevant to primary photons: $r_\chi \sim \Gamma(\pi^0 \to \gamma\gamma)\,(\Lambda/n_L)^2\,\xi^3$. Actually, at CP $r_\chi \to \infty$ when the number of light quarks $n_L \to 0$ as well as the fluctuation length ξ is sharply increasing. The latter is the consequence of $m_\chi \to 0$ as $N_f \to N_f^c$. The measurement of photon fluctuations can be used to determine whether the quantum system is in the vicinity of CP or not.

5. Conclusion

To conclude, the novel approach to an approximate scale symmetry breaking up to the phase transition at the critical point is suggested. We find the CP is achieved at higher μ (see case B) with smaller particle momentum (and, hence, the energy). In the vicinity of CP one has the scalar condensate with the sharp increasing of particle density at CP. The latter can be found as those followed by IRFP where the primary photons are detected. The origin of these photons is CA through the decays of the dilatons. The fluctuations of primary direct photons grow in the IR to become large at CP. An experimental information about the location of CP for the given experimental conditions is obtained by measuring the ratios of γ-quanta yields and compared (fitting) to known model with T and μ.

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