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Addendum: Aspects of the QCD θ-vacuum

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In the original paper [1], we found that the large-N_c scaling of the fourth cumulant c_4 in chiral perturbation theory is of order $\mathcal{O}\left(N_c^{-3}\right) + \mathcal{O}\left(N_c^{-4}\right)$ (cf. eqs. (2.23) and (2.29)) by calculating this quantity using the full NLO Lagrangian, as well as the Kaiser-Leutwyler δ-expansion. However, while this result was confirmed recently in [2], the authors of this particular paper also recognized that in contrast to our original statements in section 2.4, this scaling behavior does not hold to higher orders in chiral perturbation theory. In fact, the NNLO Lagrangian involving a term $C_4\left(\theta+\Psi\right)^4$ adds a contribution of $\mathcal{O}\left(N_c^{-2}\right)$ to the fourth cumulant c_4 [2], since the LEC C_4 is of the same order, which follows from eq. (2.13).

Indeed, any N^{2n-2}LO Lagrangian $\mathcal{L}_n$ $(n \geqslant 1)$ comes with such a contact term (c.t.)

$$\mathcal{L}_n^{\text{c.t.}} = C_{2n}\left(\theta+\Psi\right)^{2n} ,$$

where the corresponding LEC C_{2n} is of $\mathcal{O}\left(N_c^{2-2n}\right)$ in accordance with eq. (2.13) (for the leading order case, $n=1$, C_2 is just the topological susceptibility of pure gluodynamics $\tau = \mathcal{O}\left(N_c^0\right)$). Clearly, as the minimization problem results in $\Psi \propto \theta$, and also in $\Psi =$

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$\mathcal{O}\left(N_c^{-1}\right)$ (compare eqs. (2.20) and (2.27)), the contribution of the contact term to the vacuum energy density is given by

$$e_n^{\text{vac, c.t.}} = C_{2n}\theta^{2n}\left(1+\mathcal{O}\left(N_c^{-1}\right)\right)^{2n} .$$

With that, one can conclude that the cumulants in general scale as

$$c_{2n} = \left[\frac{\partial^{2n} e_{\text{vac}}}{\partial\theta^{2n}}\right]_{\theta=0} = \mathcal{O}\left(N_c^{2-2n}\right) + \mathcal{O}\left(N_c^{2-2n-1}\right) .$$

Consequently, the previous predictions for the large-N_c scaling of the fourth cumulant should not be corrected from $\mathcal{O}\left(N_c^{-2}\right)+\mathcal{O}\left(N_c^{-4}\right)$ to $\mathcal{O}\left(N_c^{-3}\right)+\mathcal{O}\left(N_c^{-4}\right)$ as we asserted, but to $\mathcal{O}\left(N_c^{-2}\right)+\mathcal{O}\left(N_c^{-3}\right)$ in accordance with the predictions from eq. (2.14).

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JHEP10(2019)028