Parallel Session 15

Dynamical Mass Generation

Organiser

N. P. Chang *(CUNY)*

DYNAMICAL CHIRAL SYMMETRY BREAKING IN QCD AT ZERO AND FINITE TEMPERATURES

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ABSTRACT

A brief review of dynamical chiral symmetry breaking of the flavor group in QCD is given. A discussion of how this phenomenon may be described within the context of the renormalization group and continuum field theory is then presented. The discussion is next extended to finite temperatures, using the real time formalism. It is found that the thermal fermion propagator has a Lorentz invariant massive pole even as $\langle vac|\bar{\psi}\,\psi|vac\rangle_T$ vanishes for temperatures $T > \Lambda_c e^{2/3}$, where Λ_c is the invariant QCD scale parameter. It is shown that indeed chiral symmetry is broken nevertheless, and that there exists a transformation of the Dirac field which manifests this breaking in the conventional way. Comparisons with lattice calculations are carried out.

What I have to say today concerns the question of chiral symmetry in the standard model, its breaking (XSB), and its effects at zero and at finite temperatures. To a large extent, the dynamics of the standard model is controlled by this phenonmenon. The specific XSB which I shall be concerned with in the standard model is the global $SU_L(N_f) \times SU_R(N_f)$ of the quarks. Here N_f is the number of flavors, and the global symmetry is exact in the absence of Yukawa and electroweak gauge couplings. There are no scalar fields in the standard model which carry indices relevant to this group, and the dogma of the standard model is that the color gauge interactions alone are responsible for the spontaneous breaking of this symmetry. The associated Nambu-Goldstone bosons, which acquire masses as a result of the electroweak gauge and Higgs interactions, are identified with the observed pseudo-scalar bosons. The interactions of these are dictated in effect by the spontaneous breaking of this global symmetry.

What is the reason for believing in this dogma? The best substantiation is given by a set of mass inequalities which follow from positivity of the Euclidean action for QCD [1]. These inequalities state that the set of pseudo-scalar boson masses are bounded by a lightest particle, which carries the quantum numbers of a pion, and that the baryon masses are also bounded by this number. The immediate conclusion is that the global chiral symmetry cannot be realized linearly, wherein the baryons are massless, and the pion massive. And then, if one further assumes confinement, and invokes the 't Hooft anomaly matching condition [2], which requires the presence of massless poles in certain weak current amplitudes, the inequalities predict that the pion must be that massless particle. Barring accidental cancellations which would have prevented the pion from coupling to the axial current, this conclusion is tantamount to having XSB.

Lattice calculations, especially within the quenched fermion approximation, generally confirm this this actually takes place [10]. More recently, the Columbia group has announced that the presence of unquenched fermions does not alter this confirmatory evidence in any way [3].

A word about the quenched approximation. One can formally express $\langle\psi\bar{\psi}\rangle$, whose non-trivial value signifies XSB, as an integral over all configurations of the density of states for the eigenfunctions of the Dirac operator in an external field. Now the codimension in gauge configuration space of the zero modes for this operator can be determined, once the gauge group is given [4]. The integral over the gauge configurations can then be performed straightforwardly in the quenched approximation. The result is a finite number, signifying that XSB always occurs [5]. This result holds even in the continuum limit, to which we now turn.

There are several ways of going to this limit. Time permits me only a description of one of these [7]. In this approach, one supposes that QCD in the continuum can be analyzed by using the renormalization group (RG), over the whole range of momentum scales. One now looks at *massive* QCD, and analyzes the chiral flip part of the two point fermion function, $\mathcal{M}(p)$. This is a renormalization group invariant, proportional to the renormalized mass m_r. As $m_r \to 0$, one would have naively expected $\mathcal{M}(p)$ to also vanish. However, RG invariance requires

$$\mathcal{M}(p) = m_r(\lambda_r y_p)^{-6C_f/b}, \tag{1}$$

where $\lambda_r = g_r^2/16\pi^2$, b is the beta function coefficient, and C_f the quadratic Casimir invariant of the quarks. The parameter y_p is an RG invariant. If it vanishes in the limit, XSB will take place. It can be shown [7] that there is a solution to this condition for values of p^2 below a critical value. The effective mass of the fermion therefore exhibits a rather peculiar momentum dependence, which enables many potentially divergent integrals to be carried out [7]. It can therefore be used in calculations of matrix elements of rare decays, where chiral symmetry breaking effects are expected to be important.

A similar analysis can be carried out at finite temperatures. In [6], this extension is actually implemented in the real time formalism. The result turns out to be rather surprising. The same RG invariant mass is now given by

$$\mathcal{M}^2 = \frac{2\pi^2}{3}\frac{T^2}{\ln\frac{T^2}{\Lambda_c}}, \tag{2}$$

for high temperatures. Here Λ_c is the invariant QCD parameter. Notice that $\mathcal{M}$ is independent of momentum, and is unlike a plasmon mass in that respect. Has XSB taken place? A straightforward computation of the order parameter $\langle\psi\bar{\psi}\rangle$, using only the minimal renormalization necessary to get rid of ultraviolet divergences, gives zero. And so apparently chiral symmetry is preserved, even though there is a mass. More explicitly, the effective Lagrangian for the quark field at high T is of the form

$$\begin{aligned}\mathcal{L}_{eff}(T) = \overline{\psi}\vec{\gamma}\cdot\vec{\nabla}(1+\mathcal{A})\psi + \overline{\psi}\gamma_0\frac{\partial}{\partial t}(1+\mathcal{B})\psi \\ - m_r\overline{\psi}(1+\mathcal{C})\psi + \dots\end{aligned} \tag{3}$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are non-polynomail functions of $\vec{\nabla}, T$ and m_r. For $T > \Lambda_c^{2/3}$, the mass term in the effective Lagrangian vanishes in the limit $m_r \to 0$. But, the Dirac equation for this particle remains non-local. It is therefore unclear how to define properly what is meant by chiral transformations.

The issue is put into better focus by expressing chiral transformations in terms of the particle creation and annihilation operators directly. To this end, define the following operators, which satisfy an $SU(2)$ algebra:

$$X_3(\vec{p}) = -\frac{1}{2}\sum_s s\left(a^\dagger_{p,s}a_{p,s} + b^\dagger_{-p,s}b_{-p,s}\right) \tag{4}$$

$$X_2(\vec{p}) = \frac{i}{2}\sum_s s\left(a^\dagger_{p,s}b^\dagger_{-p,s} - b_{-p,s}a_{p,s}\right) \tag{5}$$

$$X_1(\vec{p}) = \frac{1}{2}\sum_s \left(a^\dagger_{p,s}b^\dagger_{-p,s} + b_{-p,s}a_{p,s}\right) \tag{6}$$

The operators a, b are the annihilation operators for the Dirac fermion and anti-fermion with the appropriate momentum and spin projection. In this notation, the usual chiral rotation for a *massless* Dirac fermion is generated by X_3. As a simple example of what is going on here, let us now imagine that the massless fermions pick up a mass via the Nambu-Jona-Lasinio mechanism. The new BCS-like ground state in this model is a transform of the ground state for the massless fermions, and is generated by X_2 through an angle $\theta = \arctan m/2p$, summed over all momenta [8]. The resultant fermion field operator, $\tilde{\Psi}$, however, now satisfies a non-local equation, similar to the one above, and without a mass term. Its behavior under chiral transformations is unclear. In this instance, though, what is clear is that some form of XSB has indeed occured, even if the equation does not apparently have a mass-like term. And we can actually demonstrate that this is so by performing a Cini-Touschek transformation on this field operator [9],

$$\tilde{\Psi}(\vec{p}, t) \equiv \exp -i\theta_p\vec{\gamma}\cdot\hat{p}\Psi(\vec{p}, t). \tag{7}$$

The operator Ψ now satisfies an ordinary local Dirac equation; its transformation under chiral rotations is again generated by X_3, and the ground state will have a non-zero expectation value relative to $\Psi\bar{\Psi}$.

A further transformation is therefore necessary in order that the effective Lagrangian above be brought into local form. This is effected by a transformation of the Cini-Touschek form, similar to what has been described above. With respect to the field expressed in this basis, the order parameter $\langle\Psi\bar{\Psi}\rangle$ is directly proportional to the RG invariant mass [6], and so persists to arbitrarily high temperatures. Therefore, chiral symmetry is not restored at high temperatures. Lattice calculations generally do not perform this final transformation, and so give the apparent restoration of chiral symmetry at high temperatures [10]. The reason is that chiral transformations depend upon the wave functions used in expanding the fermi field, and are therefore sensitive to the choice of the zero of energy. For finite temperatures, this choice acquires a dependence on this temperature. The Cini-Touschek transformation is necessary to put in this dependence on the wave-functions in the expansion of the fermi field. Relative to this basis, where chiral transformations are well-defined, XSB is not restored [6].

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DISCUSSION

Q. **D. Soper** *(Univ. Oregon)*: Your approach used the renormalization group evaluated at one loop order. Is this a good approximation?

A. **L. N. Chang**: In the *critical* limit, when $m_r \to 0$, $y_p \to 0$ our result that there is a bifurcation in M_p is unchanged by inclusion of higher loops. This question has been examined by Chang and Li (Ngee Pong Chang, Da-Xi Li, *Phys. Rev.* **D30**, 790 (1984).

Q. **M. D. Scadron** *(Univ. Arizona)*: Concerning your last point on the chiral symmetry restoration temperature T_c: there are alternative schemes where the dynamical quark mass m_{dyn} also vanishes at $T = T_c$. It gives the same result as when the scalar mass vanishes at T_c, namely $T_c = 2f_\pi$.

A. **L. N. Chang**: I am not familiar with these alternative schemes. Certainly, within our context, the dynamical mass does not vanish at high temperatures.

DYNAMICAL ELECTROWEAK SYMMETRY BREAKING WITH COLOR-SEXTET QUARKS

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ABSTRACT

Under the assumption of the existence of heavy quarks belonging to a sextet representation of the color SU(3) of quantum chromodynamics we seek a possibility that the condensation of these quarks gives rise to the dynamical breaking of the electroweak SU(2)×U(1) gauge symmetry. The quantum numbers are assigned to the color-sextet quarks starting from some natural requirements. The mass of the color-sextet quarks is estimated by using the formula for the weak-boson masses. Phenomenological implications of the model are briefly discussed.

In electroweak theory an elementary scalar field, the Higgs field, is introduced to trigger the spontaneous breaking of the SU(2)×U(1) gauge symmetry. As a result we have proliferation of free parameters in the basic Lagrangian.

To resolve this unsatisfactory situation possible dynamical mechanisms of the electroweak symmetry breaking have been investigated in which the Higgs field is replaced by a bound state of some fundamental entities. A typical example of such attempts is the technicolor model proposed a decade ago by Weinberg[1] and Susskind.[2] Another attractive model of this type is the top-quark condensation model proposed recently.[3]

Here we consider yet another model of the dynamical electroweak symmetry breaking, i. e. the color-sextet-quark condensation model first proposed by Marciano.[4] In this model it is assumed that there exist heavy quarks belonging to the higher-dimensional representation of the color SU(3). The lightest of such heavy quarks is assumed to be the one in the 6-dimensional representation which we call the color-sextet quark. Their condensates play the role of the Higgs field and are responsible for the electroweak symmetry breaking.

We first determine quantum numbers of the color-sextet quarks. To do so we need some basic requirements fulfilled by these quarks Q. We require that

1. Q belongs to 6 (or 6^*) of the color SU(3),
2. $\overline{Q}Q$ condenses and behaves like a Higgs field in the standard theory, i.e. $<\overline{Q}Q>\neq 0$ and $\overline{Q}Q$ ~SU(3) color-singlet, SU(2) doublet and $Y=1$ where Y represents the hypercharge associated with U(1),
3. Q is not stable and decays into ordinary quarks and leptons. This requirement is necessary for our model to conform with present cosmological observations,
4. possible anomalies should be cancelled by adding a suitable number of extra fermions.

Let us denote the quantum numbers of fermions participating in the electroweak theory by the symbol

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(N,n,Y) where N and n represent the dimensions of the color SU(3) and weak SU(2) representation respectively. The color-sextet quark will be denoted by Q_h where index h represents the handedness.

Due to the requirement 1, we have $Q_h \sim$(6,n,Y) or (6*,n,Y). Here for later convenience we consider the case (6*,n,Y). The requirement 2 imposes the condition $\overline{Q}_k Q_h \sim$(1,2,1) where k designates the handedness opposite to h so that, if $h = L$ (left-handed), then $k = R$ (right-handed) and vice versa. According to the above condition we find that

$$Q_k = (6^*, n, Y), \quad Q_h = (6^*, n-1, Y \pm 1).$$

Although n($\geqq$ 2) is arbitrary, we fix the dimension n to be 2, the simplest possible choice. Hence

$$Q_k = (6^*, 2, Y), \quad Q_h = (6^*, 1, Y \pm 1).$$

An information on Y may be obtained through the requirement 3. We assume that the decays of Q_k and Q_h take place through one of the following effective interactions (1) $\overline{Q}^c q\bar{q}q$, (2) $\bar{q}Q\bar{q}l^c$, (3) $\bar{q}Q\bar{q}l$ and (4) $\overline{Q}^c qG$. After some arguments we find that the simplest possible choice of the quantum numbers for Q_h with $h = L$ reads

$$\begin{aligned} &Q_L \sim (6^*, 2, 1/3) && \text{for case (1) and (2),} \\ &Q_L \sim (6^*, 2, 5/3) && \text{for case (3),} \\ &Q_L \sim (6^*, 2, -1/3) && \text{for case (4).} \end{aligned}$$

The requirement 4 puts a condition on the number of extra fermions needed to cancel anomalies. In case (1) and (2) we have two extra leptons, in case (3) ten leptons, and in case (4) two extra quarks. Hence the case (3) is unreal istic with too many extra leptons and the case (4) is unreasonable since the asymptotic freedom is spoiled. We shall consider either the case (1) or (2) in the following. The decision regarding which of these two cases is acceptable will be made by experimental observations. We write

$$Q_L = (U, D)_L.$$

Our basic Lagrangian L is invariant under the local color SU(3) and electroweak SU(2)×U(1) gauge transformations. In this Lagrangian terms consisting of the color-sextet quarks and extra heavy leptons are included in addition to those of the ordinary quarks and leptons, and the Higgs field is of course absent in the Lagrangian.

The role of the Higgs field is assumed to be played by a dynamically generated bound state $\overline{Q}Q$. We, however, do not have a precise knowledge of the mechanism to form the bound state. Possibly the strong color force due to the large quadratic Casimir invariant for color-sextet quarks may be responsible for this mechanism.[4] Instead of directly dealing with the dynamics in QCD we introduce effective four-fermion interaction terms[3] including the color-sextet quarks to trigger the dynamical symmetry breaking. These four-fermion terms are constructed to be invariant under SU(3)×SU(2)×U(1) transformations. Our basic Lagrangian now reads

$$L = L_{QCD} + L_{EW} + L_4$$

where L_{QCD} is the QCD Lagrangian in which the terms consisting of color-sextet quarks are included, L_{EW} is the electroweak Lagrangian without the Higgs field, and L_4 is the four-fermion interaction Lagrangian.

We assume that the four-fermion term consisting of the color-sextet quarks is the dominant term among others triggering the condensation of color-sextet quarks. To see whether the condensation of color-sextet quarks takes place we examine the Schwinger-Dyson equation for the self-energy part of the color-sextet quark. If the Schwinger-Dyson equation allows a nontrivial solution for the self-energy part, there occurs the condensation of color-sextet quarks that signals the dynamical breaking of the electroweak symmetry.

An explicit solution may be obtained in the linearized version of the Schwinger-Dyson equation in the quenched planar approximation with the Higashijima trick[5] for incorporating the QCD running coupling constant α_s. The solution for the self-energy part Σ is given by

$$\Sigma_a(x) = m_a(\alpha_s(x)/\alpha_s(m_a^2))^{A/2}, \quad a = U, \ D,$$

where x is the Euclidean momentum squared of the color-sextet quark, m_U and m_D are the mass of the color-sextet quark U and D respectively and the exponent A is a numerical constant given by the quadratic Casimir invariant and the number of species of the triplet and sextet quarks.

Once Σ is known, the vacuum expectation value of the composite operator $\overline{Q}Q$ is calculated. Accordingly all the ferminon masses are given by this vacuum expectation value through the four-fermion terms. At the same time the gauge boson masses m_W and m_Z are given by

$$m_W = (1/2)\, g\, f_{\pi^+}, \quad m_Z = (1/2)\sqrt{g^2 + g'^2}\, f_{\pi^0},$$

where f_{π^+} and f_{π^0} are charged and neutral "pion" decay constants and g and g' the SU(2) and U(1) coupling constant respectively. Here f_{π^+} and f_{π^0} may be calculated in terms of the self-energy part of the color-sextet quarks.[6] Hence the gauge boson masses are calculated once the color-sextet quark masses are known. Or inverting the above mass formulae one may calculate the color- sextet quark masses in terms of the gauge boson masses. Thanks to the recent precise experimental data the color-sextet quark masses are thus calculable. The predicted masses are

$$m_U = 340 - 400 \text{ GeV},$$
$$m_D = 300 - 360 \text{ GeV},$$
$$m_t = 77 - 150 \text{ GeV}.$$

In the above estimate we assumed that the top quark can condense to give a minor contribution to the dynamical mass generation. We took into account the experimental constraint on the ρ parameter and the one-loop QCD running coupling constant is employed where the scale parameter in the color-sextet mass region is obtained by using the Georgi-Politzer beta function.

Since the large Casimir invariant is assosiated with the color-sextet quarks, the e^+e^- cross section may show a big rise at the color-sextet production threshold$\sqrt{s} \sim 700$ GeV. We may observe baryons of the type Qqq with mass around 350 GeV which is color-singlet where q is the ordinary quark. The lightest of the mesons has the configuration of $Q\overline{q}\overline{q}q$. The extraordinary hadrons of the configuration $QG\overline{q}$ may also be observed at around 350 GeV where G represents the gluon. The phenomenology with the color-sextet quarks with mass around 350 GeV seems to be an exciting new field.

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DISCUSSION

Q. **P. O'Donnell** *(Univ. Toronto)*: Could you tell us about the lepton problem?

A. **T. Muta**: According to the requirement of the anomaly cancellation we need two extra lepton families. Their masses cannot be very small due to the recent experimental analysis in LEP. If their masses are large (of order 100 GeV), then they contribute to the dynamical mass generation through our four-fermion terms. Since their self-energy part is constant in our approximation, the contribution from the high energy region to the mass formula seems to be large. We are now in the process of reanalysing our estimation of masses.

LINEAR SIGMA MODEL IN ONE-LOOP ORDER

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ABSTRACT

Exploiting quark and meson loop tadpole graphs, it is shown that the linear σ model in one-loop order naturally recovers the well-known chiral-limiting meson masses $m_\pi = 0$ and $m_\sigma = 2m_{qk}$.

Gell-Mann and Levy's SU(2) linear σ model [1] (LσM) originally treated nucleons, pions, and the σ meson as elementary particles in the spontaneous chiral symmetry-broken lagrangian. In the chiral limit (CL) their conclusion was that 0^- pions are massless $m_\pi = 0$, while the 0^+ σ mass could take on arbitrary values - at least in tree order. This LσM lagrangian involving nucleons was taken as the phenomenological final step - not to be reiterated via field theory into (unphysical) nucleon loop diagrams.

By way of contrast, the competing four-fermion chiral lagrangian studied a year later by Nambu and Jona-Lasinio [2] (NJL) treated fermions (now understood as nonstrange quarks) alone as elementary. Then $\bar{q}q$ pions and σ mesons were realized as dynamical bound states with CL masses

$$m_\pi = 0 \quad , \quad m_\sigma = 2m_{qk} \quad , \tag{1}$$

(where m_{qk} is the CL constituent quark mass) as computed in one-loop order in Hartree approximation.

Since quarks are now assumed to be real objects (even if confined), it is natural to try to reformulate the LσM at the quark level. In this talk we follow the recent Ref. [3] and show that at the one-loop quark level the NJL meson masses (1) are also a consequence of the LσM. But to begin, we start with the usual LσM spontaneous symmetry-broken lagrangian with vacuum expectation of the (old) scalar field $\langle\sigma_{old}\rangle = -f_\pi$. Shifting this field to $\sigma = \sigma_{old} + f_\pi$, the new minimum with $\langle\sigma\rangle = 0$ means that the interaction part of the LσM lagrangian density

$$\mathscr{L}_{int} = g'\sigma(\sigma^2+\vec{\pi}^2) - (g'/4f_\pi)(\sigma^2+\vec{\pi}^2)^2 + g\,\bar{\psi}(\sigma+i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi - gf_\pi\bar{\psi}\psi \tag{2}$$

now breaks chiral symmetry dynamically. Here ψ represents a quark field. While the second and third terms in (2) preserve chiral symmetry, the last $\bar{\psi}\psi$ term in (2) breaks chiral symmetry via the quark mass occurring in the Goldberger-Treiman (GT) relation at the quark level

$$f_\pi g = m_{qk} \quad . \tag{3a}$$

We may treat (3a) as scaling the dimensionless meson-quark coupling constant in (2) to $g \approx 3.5$ for $m_{qk} \approx M_N/3 \approx 313$ MeV and $f_\pi \approx 90$ MeV in the CL. Also the first term in (2) breaks chiral symmetry when σ decays to 2π with amplitude $2g'$ and meson-meson coupling [1]

$$g' = m_\sigma^2/2f_\pi \approx 2.2 \text{ GeV} \tag{3b}$$

in order that $m_\pi = 0$ in tree order in the CL. Here we have used the NJL value $m_\sigma = 2m_{qk} \approx 630$ MeV to find a numerical estimate of g'.

Proceeding to compute one-loop order graphs based on the LσM lagrangian (2) with coupling constants (3), we first study the CL pion decay constant obtained from the quark loop of Fig. 1 when $q_\pi \to 0$ and with $đ^4p = (2\pi)^{-4}d^4p$,

$$f_\pi = -i4N_c g m_{qk} \int \frac{đ^4p}{(p^2-m_{qk}^2)^2} \quad , \tag{4}$$

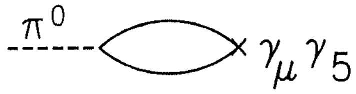

Fig. 1. Quark loop representation of f_π.

which, using (3a) is a log-divergent "gap equation" [3]. Then computing the two quark loops for $\sigma \to \pi^+\pi^-$ in Fig. 2, we encounter the same log-divergent integral as in (4) with CL scale

$$g_{\sigma\pi\pi} = g' = 2g^2 f_\pi \approx 2.2 \text{ GeV} \quad . \tag{5}$$

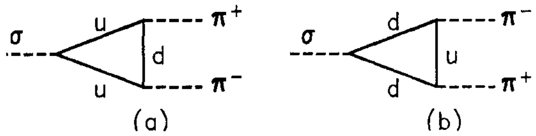

Fig. 2. Quark loop representation of $g_{\sigma\pi\pi}$.

It is not an accident that the two numerical values for g′ in (3b) and (5) are the same. In fact, equating their analytical forms together with (3a) we obtain $m_\sigma = 2m_{qk}$. This is the LσM one-loop level analog of the NJL result in (1).

Recovery of the CL value $m_\pi = 0$ in one-loop order in the LσM follows from analyzing the pion self-energy graphs of Figs. 3 and 4. The quark loop graphs of Fig. 3 proportional to color number N_c are of the vacuum polarization (VP) and quark tadpole (qktad) type. In the CL with $q_\pi \to 0$ these pion amplitudes as generated by the LσM lagrangian (2) are

$$M^o_{VP} = -i4N_cN_f g^2 \int \frac{d^4p}{p^2-m^2_{qk}} \quad , \qquad (6a)$$

$$M^o_{qktad} = \frac{i4N_cN_f 2g'g}{m^2_\sigma} \int \frac{d^4p \; m_{qk}}{p^2-m^2_{qk}} \quad . \qquad (6b)$$

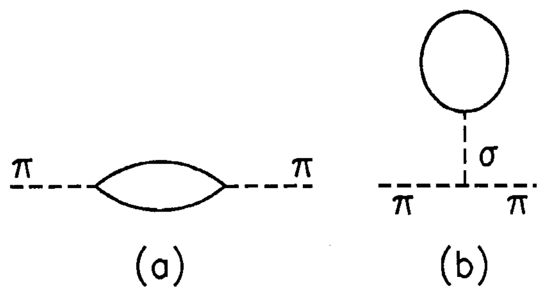

Fig. 3. Quark loop contributions to pion self energy.

While both amplitudes in (6) are quadratically divergent, their coefficients are of equal but opposite sign due to the chiral symmetry conditions of Eqs. (3). Thus we learn that the pion self-energy generated by the quark loops of Fig. 3 vanishes in the CL:

$$M^o_{VP} + M^o_{qktad} = 0 \quad . \qquad (7)$$

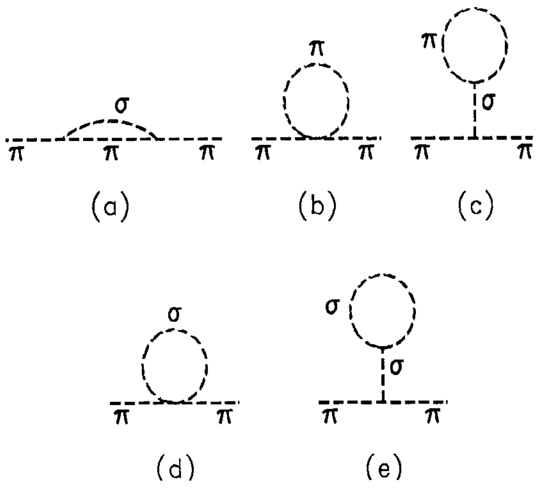

Fig. 4. Meson loop contributions to pion self energy.

Likewise the meson loops generated in Figs. 4 by the LσM lagrangian (2) give the CL pion self-energy part

$$M^o_{\substack{meson\\loops}} = 4g'^2 i \int \frac{d^4p}{p^2(p^2-m^2_\sigma)}$$

$$+ \left[\frac{5g'}{f_\pi} - \frac{6g'^2}{m^2_\sigma}\right] i \int \frac{d^4p}{p^2} \qquad (8a)$$

$$+ \left[\frac{g'}{f_\pi} - \frac{6g'^2}{m^2_\sigma}\right] i \int \frac{d^4p}{p^2-m^2_\sigma} \quad .$$

Using (3) and the partial fraction identity $m^2_\sigma p^{-2}(p^2-m^2_\sigma)^{-2} = (p^2-m^2_\sigma)^{-2} - p^{-2}$, the first log-divergent integral in (8a) precisely cancels the difference of the two remaining quadratically-divergent integrals in (8a). Thus we find in the CL

$$M^o_{meson\ loops} = 0 \quad . \qquad (8b)$$

The sum of the (vanishing) pion self energies (7) and (8b) means that the entire pion mass vanishes in the CL to one-loop order in the LσM. Of course $m_\pi = 0$ vanishes through all loop orders by virtue of the Goldstone theorem, but Eqs. (6)-(8) just give a LσM realization of this theorem in one-loop order.

We conclude that the one-loop CL Nambu relations $m_\pi = 0$, $m_\sigma = 2m_{qk}$ in (1) not only follow in the NJL four-fermion model but also hold for the LσM when the elementary fermions are taken as quarks. Central to our approach are three one-loop level tadpole diagrams in Figs. 3 and 4. Although these tadpole graphs are quadratically divergent, they are also intrinsically negative (for σ propagator $-m_\sigma^{-2}$ with zero momentum transfer to the tadpole). As such, these three tadpole graphs act as natural CL counter-term renormalizations.

In passing we note that the one-loop equivalence of the LσM and the NJL model can be extended away from the CL $m_\pi \neq 0$. In this case the VP amplitude of Fig. 3a becomes in the LσM

$$M_{VP} = -i4N_c N_f g^2 \int \frac{d^4p[p^2-\hat{m}^2-m_\pi^2/4]}{\left[\left(p+\frac{1}{2}q\right)^2-\hat{m}^2\right]\left[\left(p-\frac{1}{2}q\right)^2-\hat{m}^2\right]} . \quad (9)$$

Here $\hat{m} = m_{qk} + \hat{m}_{cur}$ is the nonstrange constituent quark mass and $\hat{m}_{cur}$ is the nonstrange current quark mass which is nonvanishing when $m_\pi \neq 0$. Subtracting the CL M_{VP}^o amplitude (6a) from the chiral-broken M_{VP} amplitude (9) and assuming $\hat{m}_{cur} << \hat{m}$ then leads to the incremental self-energy shift [3]

$$\delta M_{VP} = M_{VP} - M_{VP}^o \approx -(5/4)m_\pi^2 + 4\hat{m}_{cur}m_{qk} \quad . \quad (10)$$

Furthermore $\delta M_{qktad} + \delta M_{meson\ loops} = 0$ must be the case if the LσM matches the NJL model (it is numerically approximately valid). But because the net mass shift must be the entire (pion) chiral-broken mass, $\delta M_{VP} = m_\pi^2$, Eq. (10) then predicts [3]

$$\hat{m}_{cur} \approx (9/16)\ m_\pi^2/m_{qk} \approx 34 \text{ MeV} \quad . \quad (11)$$

This same current mass also follows in the NJL model. For our CL quark mass $m_{qk} \approx 313$ MeV, the meaning of (11) is that the total nonstrange constituent quark mass is

$$\hat{m}_{con} = m_{qk} + \hat{m}_{cur} \approx (313+34)\text{ MeV} \approx 347 \text{ MeV} \quad . \quad (12)$$

Such a constituent quark mass as (12) has long been obtained from magnetic moment [4] and from hyperfine splitting [5] quark models.

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Nucleon Mass in a Skyrmion Lagrangian

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Abstract

The soliton description of the nucleon in nonlinear σ-model with gauge bosons predicts too large a mass for the nucleon. If the model is supplemented by a quark sector, where left and right quarks interact via a scalar field, a condensed ground state can form. The system can then reduce its total energy or mass by a phase transition. This mechanism also implies a composite structure of the nucleon that is in accord with analyses of high energy elastic scattering.

The nonlinear σ-model with vector mesons ω, ρ, A_1 introduced as gauge bosons describes the nucleon as a topological soliton or skyrmion.[1] Baryonic charge in this model turns out to be topological and the vector meson ω is identified as the gauge boson coupled to the baryonic charge. Studies by several groups have shown that the model works very well in describing the low-energy properties of the nucleon, but consistently leads to a large mass of the soliton ($\sim$1500 MeV) compared with the nucleon mass of 939 MeV.[2] This suggests that something basic is missing in the nonlinear σ-model. We address this question. We are further motivated by phenomenological analyses of pp and $\bar{p}p$ high energy elastic scattering[3] at the CERN ISR and SPS Collider, which appear to support the nonlinear σ-model. The analyses have shown that the vector meson ω behaves as a spin-1 elementary boson and probes a matter density distribution inside the proton that can be interpreted as a quark number distribution.

Our strategy is to start with the more general linear σ-model, which has besides a pseudoscalar meson sector, a fermion or quark sector, a scalar field and an interaction between the fermions via the scalar field.[4] Using a path integral formulation, we relate the fermion measure to an invariant measure and show that the Jacobian between the two measures can be identified as $\exp[i\Gamma_{WZ}]$, where Γ_{WZ} is the Wess-Zumino action. When in the pseudoscalar meson sector the scalar field is replaced by its vacuum value f_π, the model breaks up into two parts. The pseudoscalar part of the model becomes the nonlinear σ-model. The other part involves chiral fermions, the scalar field and their interaction. We find that if the scalar field vanishes at small r, but rises sharply to its vacuum value at some R, the ground state energy of the interacting quark-scalar field system can be lower than the ground state energy of the non-interacting system. The interaction between quarks and the scalar field can, therefore, lead to a condensed ground state or vacuum, and the system can reduce its total energy or mass by making a phase transition similar to superconductivity.

We conclude: 1) Conventional nonlinear σ-model needs to be supplemented by a quark sector, where left and right quarks interact via a scalar field. 2) The scalar field should have a critical behavior, so that a condensed vacuum can form and the system can reduce its energy by a phase transition. 3) The nucleon has a core of topological baryonic charge, surrounded by a condensed vacuum of zero baryonic charge. Precisely the same picture emerges from high-energy elastic scattering.

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VACUUM QUANTIZATION AND LEPTON GENERATION

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Abstract

A theory of lepton generation is suggested. The theory consider the manifold of gauge group as a quantized physical field. This and the other principles result in the generation structure of lepton. We get the unified formulae for mass and life time of lepton generations.

The fourth generation are proved to be forbidden by uncertainty principle.

Semiclassically, the vacuum of gauge group has been studied for many years by many peoples. It led to the conclusion that vacuum can be classified according to the topological number n (winding number). As a quantum field theory we consider the group element U(x) as the wave field of group manifold and quantize it through the following equation (for SU_2 case)

$$\hat{n} = \frac{1}{24\pi^2}\int d^3x tr\epsilon_{ijk}(\hat{U}^{-1}\partial_i\hat{U})(\hat{U}^{-1}\partial_j\hat{U})(\hat{U}\partial_k\hat{U}) \quad (1)$$

where $\hat{n} = a^+a$, $\hat{U}(a^+$, a, U(x)) is the operator form of U(x). Equation (1) sets a correspondence between the winding number of homotopy theory and the particle number of quantum field theory. We get the general form of $\hat{U}$

$$\hat{U} = I^{\hat{n}}U_1^{\hat{n}} \quad (2)$$

where I is a c number, U_1 is the group element of SU(2) with winding number $n = 1$.

Now the Higgs field will be represented as

$$\hat{\Phi} = \hat{U}\begin{pmatrix} 0 \\ \varphi \end{pmatrix} \quad (3)$$

the Yukawa coupling term between Higgs field and fermion in standard model gives a mass term

$$\frac{Ga}{\sqrt{2}}I^n\int_V \overline{\psi}\psi d^3x \quad (4)$$

For a lepton with finite life time τ_n, the integral domain V in (4) is different from the normalization volume (Ω) and the former can be estimated by causality principle in plane wave representation. We get

$$m_n = I^n\frac{Ga\,V}{\sqrt{2}\,\Omega} = I^n\frac{Ga\,\tau_n}{\sqrt{2}\,T_0} \quad (5)$$

where $I = \frac{\alpha}{32\pi^2}$ ($\alpha = 3\times10^{10}$), $T_0 = 1$ sec. If we assume $\tau_0 = T_0$, then equation (5) gives the mass spectrum of lepton generation for which $n = 0$ corresponds to (e, ν_e), $n = 1$ corresponds to (μ, ν_μ) and so on.

It has been proved that (5) can satisfy the uncertainty principle $\Delta m_n \cdot \tau_n \geq 1$ only when $n < 2.85$. Thus we get the answer that why there is no fourth generation in nature[1].

The lepton number conservation is a topological deduction of our theory. Let us attach the winding number n (which is equivalent to lepton number) to each massive particle of corresponding generation and -n to its antiparticle. For the massless lepton ν, we give the inverse assignment. Then the following processes keep the total topology number n conservation.

$$\mu^- \longrightarrow e^- + \nu_\mu + \overline{\nu}_e$$
$$n \longrightarrow p + e^- + \overline{\nu}_e$$

Thus we give a topological interpretation for the lepton number conservation.

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[1] Measurement of Z^0 Decay to Hadrons and Precise Determination of the Number of Neutrino Species. The L3 Collaboration L3 preprint 004 Dec, 24, 1989.

Q. **R. R. Volkas** *(Univ. Melbourne)*: If a 4th generation charged lepton were to exist, can you predict its mass and lifetime?

A. **J. Tang**: The mass and lifetime of the 4th generation charged lepton can be predicted as

$$m_0(n = 3) \sim 100 \text{ GeV}$$
$$\tau_{n=3} \sim 10^{-20} \text{ sec} \; .$$

But for further reason of our theory it actually cannot exist in nature.

Comments on the Renormalization of a Supersymmetric Nonlinear σ-Model in 2+1 Dimensions

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In $d = 2+1$ the O(N) supersymmetric nonlinear σ-model (SSNSM) is defined by the action[1, 2]

$$S = \int d^3x\, d^2\theta \left[\frac{1}{2}\Phi_j D^2 \Phi_j + \frac{1}{2}\Sigma\Big(\Phi_j\Phi_j - \frac{N}{g^2}\Big)\right], \quad (1)$$

with the sum over the flavor index j running from 1 to N. The superfield $\Phi_j = A_j + \bar{\theta}\psi_j + \frac{1}{2}\bar{\theta}\theta F_j$ and the lagrange multiplier superfield $\Sigma = \sigma + \bar{\theta}\xi + \frac{1}{2}\bar{\theta}\theta\,\alpha$. In component form and upon elimination of the non-propagating F_j, the lagrangian from (1) is

$$\begin{aligned}\mathcal{L} = \; & -\tfrac{1}{2}A_j\partial^2 A_j + \tfrac{i}{2}\bar{\psi}_j\partial\!\!\!/\psi_j - \tfrac{1}{2}\sigma^2 A_j^2 - \tfrac{1}{2}\alpha A_j^2 \\ & + \tfrac{1}{2}\sigma\bar{\psi}_j\psi_j + \bar{\xi}\psi_j A_j + \tfrac{N}{2g^2}\alpha\,. \qquad (2)\end{aligned}$$

It is now easy to see that α, ξ, and σ are the respective lagrange multipliers for the constraints $A_jA_j = N/g^2$, $A_j\psi_j = 0$, and $A_jF_j = \frac{1}{2}\bar{\psi}_j\psi_j$ and α accounts for the ordinary nonlinear σ-model (NSM) sector, σ accounts for the four-fermi sector, and ξ accounts for the mixed sector[1, 2].

Integrating over the fields A_j and ψ_j we obtain an effective action for the fields α, σ, and ξ:

$$\begin{aligned}S_{eff}\;[\alpha,\sigma,\xi] = \; & \tfrac{N}{2} tr\; ln\Big(\partial_E^2 + \alpha + \sigma^2 + \bar{\xi}\tfrac{1}{i\partial\!\!\!/_E+\sigma}\xi\Big) \\ & -\tfrac{N}{2}Tr\; ln(i\partial\!\!\!/_E + \sigma) - \tfrac{N}{2g^2}\textstyle\int d^3x_E\,\alpha \qquad (3)\end{aligned}$$

This model exhibits two supersymmetric phases[2] but if we demonstrate 1/N renormalizability in one phase, renormalizability in the other phase immediately follows since phase transitions are infra-red effects. We thus discuss renormalization in the O(N)-symmetric phase which is obtained by setting $\frac{\delta S_{eff}}{\delta\alpha}$ to zero at $\langle\alpha\rangle = 0$, $\langle\xi\rangle = 0$, $\langle\sigma\rangle \neq 0$[1] and this yields a gap equation. This gap equation comes from the variation of S_{eff} w.r.t. α but the variation w.r.t. σ is identically zero because of cancellations between fermion and boson loops. So there is no fine-tuning of the coupling in the four-fermi sector, in contrast to the four-fermi model by itself[4]. Also, because of SUSY, the fields α, σ, and ξ, and their induced propagators do not need any renormalization and the fine-tuning in the gap equation does not shift at higher orders [2]. These results are in contrast to the nonsupersymmetric NSM[3] and four-fermi model[4]. Moreover, the ordinary NSM is not BPHZ renormalizable but the SSNSM is[2], and we now briefly discuss why this is so.

In the symmetric phase, all N bosons $(A_1,\cdots,A_N)$ and fermions $(\psi_1,\cdots,\psi_N)$ acquire the same dynamical mass $|\langle\sigma\rangle|$ so that SUSY is preserved. We perform the shifts $\sigma = \sigma' + |\langle\sigma\rangle|$, and $\alpha' = \alpha + 2m\sigma'$, $m \equiv |\langle\sigma\rangle|$, with $\langle\sigma'\rangle = \langle\alpha\rangle = 0$ and expand S_{eff} in (3) about $\alpha' = 0$ and $\sigma' = 0$ to obtain the Feynman rules.

It turns out[2] that cancellations between fermion and boson loops eliminate the linear and quadratic divergences for the next-to-leading order corrections to the boson propagator so that no A_j^2 counterterms are induced and a wavefunction renormalization will eliminate all of the infinities. Likewise, at even higher orders, the divergences between the four-point functions completely cancel as do the divergences between the six-point functions , and no A_j^4 or A_j^6 counterterms are induced. All other higher order terms are either finite or they may be handled by standard BPHZ techniques which preserve SUSY [2]; this is not the case in the NSM. The reason for this is the presence of $\frac{1}{2}\sigma^2A_j^2$ and $\bar{\xi}\psi_jA_j$ in the lagrangian (2) lead to miraculous cancellations. In case of NSM, the only way to eliminate the A^2, A^4, and A^6 counterterms so that they do not spoil the nonlinear constraint and make the model inconsistent is to leave all graphs with external α lines unrenormalized[3]. Thus the NSM is not BPHZ renormalizable whereas the SSNSM is. This result is especially interesting since these models are not renormalizable in weak coupling pertubation theory. Details are contained in [2].

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