MULTIPLICITY MOMENTS IN QCD AND EXPERIMENT

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Abstract

QCD predictions for moments of *parton* multiplicity distributions are discussed. The next-to-leading terms and conservation law give rise to the peculiar oscillating shape of some ratio of the moments. The similar shape has been found by moment analysis of *hadron* multiplicities. Experimental data, theoretical Monte Carlo models and phenomenological fits have been used for ee, hh, hA, AA reactions at high energies.

In multiparticle production the primary observable is the multiplicity distribution P_n which contains in the integrated form all the correlations of the system. First attempts to derive *parton* multiplicities in the double logarithmic approximation (DLA) of QCD provided extremely wide distributions compared to those found experimentally for *hadrons*. Later, the solution of the QCD equation for the generating function which takes into account higher order terms of the perturbative expansion and conservation law was found [1] and more efficient way of studying the distribution shape based on moment analysis was proposed (see review [2]). When applied to *hadron* multiplicities, it shows peculiar oscillating curves first derived for *partons* in QCD [3]. Their stability for reactions initiated by different projectiles and targets is rather astonishing [4].

Let us describe briefly the theoretical findings. The generating function is defined by

$$G(z) = \sum_{n=0}^{\infty} z^n P_n \tag{1}$$

so that probabilities P_n, factorial (F_q) and cumulant (K_q) moments are

$$P_n = \frac{1}{n!} \frac{d^n G(z)}{dz^n}\bigg|_{z=0}, \quad F_q = \frac{1}{\langle n \rangle^q} \frac{d^q G(z)}{dz^q}\bigg|_{z=1}, \quad K_q = \frac{1}{\langle n \rangle^q} \frac{d^q \ln G(z)}{dz^q}\bigg|_{z=1} \tag{2}$$

with the recurrence relations

$$F_q = \sum_{m=0}^{q-1} C_{q-1}^m K_{q-m} F_m, \tag{3}$$

where $C_n^m = n!/m!(n-m)!$ are the binomial coefficients.

The equation for the generating function in gluodynamics [5] looks like an ordinary birth-death equation but with a typical singular QCD kernel

$$\frac{dG}{dy} = \int_0^1 dx K(x) \gamma_0^2 [G(y + \ln(1-x))G(y + \ln x) - G(y)], \tag{4}$$

where $\gamma_0^2 = 2N_c\alpha_s/\pi$, α_s is the coupling constant, $N_c = 3$ is the number of colours, $K(x) = \frac{1}{x} - (1-x)[2 - x(1-x)]$, $y = \ln Q^2/Q_0^2$, Q^2 is the jet virtuality, Q_0^2=const. The y-dependence enters $G(z, y)$ through the energy dependence of P_n. The variable z is omitted in eq.(4).

For fixed coupling constant, this equation can be reduced to the algebraic one and, therefore, has an exact solution [6, 7]. For running coupling, one can use Taylor series expansion of the terms in the non-linear part of the integral. It is instructive to demonstrate it here because lowest order terms give rise to DLA, and higher orders provide modified leading logarithm and next-to-leading logarithm approximations (see [1, 2]). After the expansion of G at large y in non-linear terms is done and both sides of eq.(4) are divided by $G(y)$, one differentiates both

sides of (4) and the non-linear integro-differential equation (4) is reduced to the differential equation

$$(\ln G)'' = \gamma_0[G - 1 - 2h_1 G' + h_2 G''], \tag{5}$$

where $h_1 = 11/24, h_2 \approx 0.216$ are given by integrals of the kernel $K(x)$ with corresponding weights. The various approximations are obtained by considering in the righthand side of (5) only two (DLA), three (MLLA) or four (NLLA) terms. Higher order derivatives are omitted here.

Using the relation (2) and equating the terms of the same power of z on both sides of the equation, one gets from (5) for the ratio of cumulant to factorial moments

$$H_q \equiv \frac{K_q}{F_q} = \frac{\gamma_0^2[1 - 2h_1 q\gamma + h_2(q^2\gamma^2 + q\gamma')]}{q^2\gamma^2 + q\gamma'} \tag{6}$$

with

$$\gamma \approx \gamma_0 - \frac{1}{2}h_1\gamma_0^2 + \frac{1}{8}(4h_2 - h_1^2)\gamma_0^3. \tag{7}$$

In DLA, one neglects all the terms of higher order in γ_0 and gets from (6)

$$H_q^{(DLA)} = \frac{1}{q^2}. \tag{8}$$

Let us note that such a monotonous behaviour of the ever positive H_q corresponds to the negative binomial distribution with its parameter $k = 2$. The experimental fits show much larger values of k and, consequently, more narrow distributions.

For the realistic values of $\gamma_0=0.48$, the ratio H_q as given by eqs. (5), (6) exhibits the minimum at

$$q_{min} \approx \frac{1}{h_1\gamma_0} \approx 5 \tag{9}$$

and then increases. With higher order terms of Taylor series expansion taken into account the ratio H_q oscillates [3].

These oscillations are due to oscillatory behaviour of cumulants which have the physical meaning of genuine correlations in the q-parton system. The location of the first minimum is determined by the condition $q\gamma \approx 1$ which is related to the well-known quantum-field theory problem of the break-up of the perturbative expansion due to the factorial increase of the number of graphs for n-particle process. In traditional language, it corresponds to the expansion parameter $\alpha_s n$ appearing instead of α_s in that case. At the same time, this location is determined by the singular nature of the kernel at $x \to 0$ (and $x \to 1$ in the symmetrized form). For theories with regular kernels like $\lambda\phi_6^3$ [8] the first minimum is located at larger values of q.
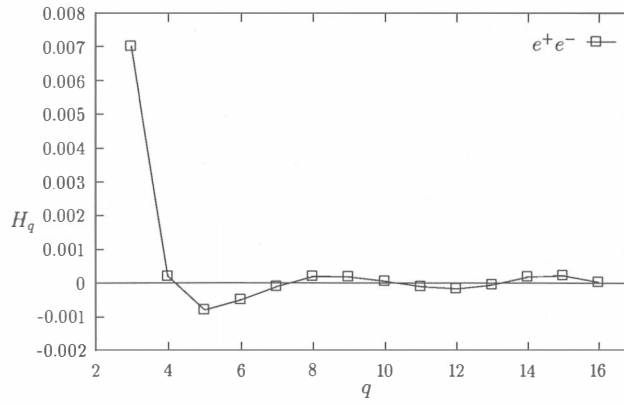
Figure 1:

The KNO-scaling is valid in such an approach, in practice. The energy dependence of H_q should be extremely weak. It is only due to running property of γ_0 and almost cancels in the ratio H_q as seen from (6).

Surely, there is no experimental information on parton distributions. The local parton-hadron duality is often used to relate them to hadron distributions by imposing proportionality assumption. Otherwise, the hadronization scheme should be developed. Anyway, one may not confront directly theoretical predictions about partons to experimental data.

Nevertheless, QCD shows a way to the new method of analysis of experimental data. Cumulants are very sensitive to slight variations of the distributions because of subtractions implied in eq.(3). Therefore, it is worthwhile to calculate factorial moments of experimental multiplicity distributions and then cumulants, using (3), to find out the behaviour of the ratio H_q for hadrons. Its values in e^+e^- collisions at Z^0 peak [9, 10] and in hadronic reactions [4, 9] are shown in Figs.1 and 2, correspondingly.

One concludes that the oscillation pattern is observed in all the cases. The position of the first minimum is in the range $q = 4 - 6$. The value of H_q in it steadily increases (in modulus) from ee to pp, hA, AA. The model calculations according to the dual parton model (DPM) [11] and to the quark-gluon string model (QGSM) [12] were used for hA and AA in Fig.2 but it is shown in [4] that they fit experimental data rather well. On the contrary, no phenomenological fit (ranging from Poisson to modified negative binomial distribution) is able to reproduce such a pattern. They give rise either to monotonous positive ratios or to ones changing the sign at each value of the rank.
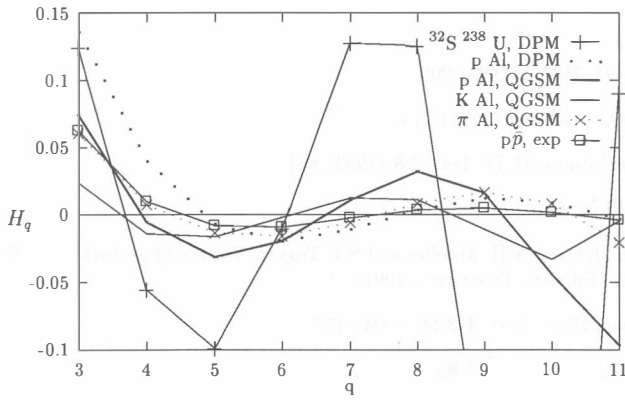
Figure 2:

Thus, the moment analysis is a more powerful method than the direct fits of multiplicity distributions. The cumulants are very sensitive to any uncertainties and require high statistics experiments at high energies. Among different factors influencing the shapes of H_q-curves we mention the cut-off of the high multiplicity tail of the distribution (which is mainly due to limited experimental statistics), difference between the values of moments for charged particles distributions and those for negatives only [4], the similar difference (in model calculations) between clusters and their decay products, and, finally, the experimental selection criteria and error bars of P_n. All of them disappear at asymptotically high energies and with statistics increased but they can be important for any given experiment. No careful analysis of all these factors has been done up to now. We rely on the qualitative similarity of H_q-curves in all the above cases and suppose that it is related rather to the underlying dynamics of the processes than to varying from one experiment to another selection criteria and statistics.

Let us mention at the very end another byproduct of the analysis of the generating functions. When the sum in eq. (1) is cut at some final multiplicity N due to finite experimental statistics and conservation law, the truncated generating function becomes the polynomial of N-th order in z, i.e. it possesses N complex conjugate zeros in z-plane. It happens that those zeros lie near the circle of the unit radius and at large N come very close to the real axis and to the point $z = 1$ (see [4]), where all the moments are calculated according to eq.(2). It explains why the moment analysis is so sensitive to tiny details of the distributions. Some statistical analogies arise also in connection with these zeros [13].

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