Fermi National Accelerator Laboratory

FERMILAB-Pub-80/51-THY
June 1980

Perturbative QCD Corrections to the Hadronic Decay Width of the Higgs Boson

N. SAKAI*
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

(Received

ABSTRACT

Using the operator product expansion I show that perturbative QCD corrections to the hadronic decay width of the Higgs boson can be calculated without encountering mass singularities. The result is given in terms of the "running quark mass" of the renormalization group and calculable corrections in powers of $1/\ln(M^2/\Lambda^2)$. The next-to-leading order correction amounts to about 32% for the Higgs mass M = 50 GeV.

PACS Category Nos.: 13.25.+m, 14.80.Kx

I. INTRODUCTION

The successful SU(2) × U(1) model for the weak and electromagnetic interactions[1] as well as other theoretical possibilities of unification make it urgent to explore Higgs bosons experimentally.[2] Recently Braaten and Leveille calculated gluon radiative corrections to the Higgs decay into a quark-antiquark pair.[3] They found mass singularities which invalidate the perturbative calculation. By summing leading logarithms they obtained the decay rate formula in which the quark mass is replaced by the running quark mass evaluated at the Higgs mass.

The purpose of this paper is to show that perturbative QCD corrections for the hadronic decay of the Higgs boson can be calculated unambiguously without encountering mass singularities. I will give a precise formulation in terms of the operator product expansion and the renormalization group equation, and present the result of the next-to-leading order QCD correction. The correction is about 20% for a 50 GeV Higgs boson.

In the next section I formulate the renormalization group equation for scalar current correlation functions and present the leading and the next-to-leading order perturbative QCD results for the Higgs decay width. Section III is devoted to a brief discussion.

II. RENORMALIZATION GROUP EQUATION

The standard $SU(2) \times U(1)$ model[1] gives the interaction Lagrangian of the decay of the Higgs boson ϕ into a pair of quarks ψ as

$$L = -g_Y \bar{\psi}\psi\phi \equiv -g_Y J \cdot \phi \tag{1}$$

where g_Y and J are the Yukawa coupling and the scalar current. The lowest order decay width for each quark flavor of mass m is given by

$$\Gamma = \frac{3M}{8\pi} g_Y^2 \left(1 - \frac{4m^2}{M^2}\right)^{3/2} \tag{2}$$

where M is the Higgs mass and the color factor 3 is included. This formula can be obtained from the imaginary part of the quark loop diagram in Fig. 1, which is very similar to the well-known process $e^+e^- \to q\bar{q}$. One can easily recognize that perturbative QCD corrections to the hadronic decay of the Higgs boson can be treated almost analogously to the e^+e^- annihilation into hadrons using the operator product expansion.[4-6]

Let us introduce the correlation function of scalar currents with the four momentum q^μ

$$\pi = i\int d^4x e^{iqx} \langle 0 | T(J(x)J(0)) | 0 \rangle \tag{3}$$

whose imaginary part gives the Higgs decay width

$$\Gamma = \frac{g_Y^2}{M} I_m \pi \Big|_{q^2=M^2} \quad . \tag{4}$$

Similarly to the vacuum polarization due to the electromagnetic current,[5-6] π requires subtractive renormalization. It also needs multiplicative renormalization because the scalar current is not conserved in contrast to the electromagnetic current. The renormalized π in $4-\varepsilon$ dimensions is obtained from the bare one π_0

$$\pi(q^2, g, m, \mu) = \frac{Z_Y^2}{Z_2^2}\,\pi_0(q^2, g_0, m_0, \varepsilon) - q^2\mu^{-\varepsilon}K(g, \varepsilon) - m^2\mu^{-\varepsilon}L(g, \varepsilon) \qquad (5)$$

where μ is the renormalization mass scale and the subtractive counter terms are given as a sum of simple poles in ε in the minimal subtraction scheme[7]

$$K(g, \varepsilon) = \sum_{i=1}^{\infty} K^i(g)/\varepsilon^i \quad , \quad L(g, \varepsilon) = \sum_{i=1}^{\infty} L^i(g)/\varepsilon^i \quad . \qquad (6)$$

The renormalized QCD coupling constant g, quark mass m, quark field ψ, and Yukawa coupling g_Y are related to bare ones (with subscript 0) by renormalization constants as

$$\mu^{\varepsilon/2}g = Z_g^{-1}g_0 \quad , \qquad m = Z_m^{-1}m_o \quad , \qquad (7)$$

$$\psi = \sqrt{Z_2}\psi_0 \quad , \quad \mu^{\varepsilon/2}g_Y = g_{Y_0}Z_Y^{-1}Z_2 \quad . \qquad (8)$$

Neglecting nonperturbative effects,[8] π is nothing but the coefficient function of the unit operator in the operator product expansion of the T-product of scalar currents.

Following the standard procedure,[6] I obtain the renormalization group equation for π from the μ independence of π_0 for fixed g_0, m_0, and ε

$$\left[\mu\frac{\partial}{\partial\mu} + \beta\cdot\frac{\partial}{\partial g} + \gamma_m m\frac{\partial}{\partial m} + 2\gamma\right]\pi(q^2, g, m, \mu) = q^2\bar{K}(g) + m^2\bar{L}(g) \qquad (9)$$

where the β-function and anomalous dimensions are given, as usual

$$\beta = -g\mu\frac{\partial}{\partial\mu}\ln Z_g$$

$$\gamma_m = -\mu\frac{\partial}{\partial\mu}\ln Z_m$$

$$\gamma = -\mu\frac{\partial}{\partial\mu}\ln(Z_Y/Z_2) \qquad (10)$$

and inhomogeneous terms are given in terms of $1/\epsilon$ pole terms of the subtractive counter terms in eq. (6)

$$\bar{K}(g) = \frac{\partial g^2 K^1(g)}{\partial g^2} \quad , \quad \bar{L}(g) = \frac{\partial g^2 L^1(g)}{\partial g^2} \quad . \qquad (11)$$

Using the dimensional analysis and the solution of the renormalization group equation I can relate π at q^2 to π at $\hat{q}^2 \equiv q^2 e^{-2t}$

$$\pi(\hat{q}^2 e^{2t}, g, m, \mu) = e^{2t}\pi(\hat{q}^2, g, me^{-t}, \mu e^{-t})$$

$$= e^{2t}\Big\{\pi(\hat{q}^2, \bar{g}(t), \bar{m}(t)e^{-t}, \mu)A(t)$$

$$-\int_0^t dt'\left(\hat{q}^2\bar{K}(\bar{g}(t')) + \bar{m}^2(t')e^{-2t'}\bar{L}(\bar{g}(t'))\right)A(t')\Big\} \qquad (12)$$

where

$$t = \frac{1}{2} \ln (q^2/\hat{q}^2) \tag{13}$$

$$\frac{\partial \bar{g}(t)}{\partial t} = \beta(\bar{g}(t)) \quad , \quad \bar{g}(0) = g \tag{14}$$

$$\frac{\partial \bar{m}(t)}{\partial t} = \bar{m}(t)\gamma_m(\bar{g}(t)) \quad , \quad \bar{m}(0) = m \tag{15}$$

$$A(t) = \exp\left(2\int_0^t \gamma(\bar{g}(t'))dt' \right) \quad . \tag{16}$$

The running quark mass m(M) at M is defined by the solution of eq. (15) for $t = \ln M/\mu$

$$m(M) = \bar{m}(t = \ln M/\mu) = m \bullet \exp\left(\int_0^{\ln M/\mu} \gamma_m(\bar{g}(t'))dt' \right) \quad . \tag{17}$$

Since I am interested in the lowest order in g_Y^2 but to all orders in g, I obtain

$$Z_Y/Z_2 = Z_m \quad , \quad \gamma = \gamma_m \quad . \tag{18}$$

To evaluate the asymptotic behavior of π, I use the perturbative expansion

$$\beta(\bar{g}) = -\frac{\bar{g}^3}{16\pi^2}\beta_0 - \frac{\bar{g}^5}{(16\pi^2)^2}\beta_1 + \dots$$

$$\gamma_m(\bar{g}) = \gamma(\bar{g}) = \frac{\bar{g}^2}{16\pi^2}\gamma_0 + \left(\frac{\bar{g}^2}{16\pi^2}\right)^2\gamma_1 + \dots$$

$$\bar{K}(\bar{g}) = K_0 + \frac{\bar{g}^2}{16\pi^2}K_1 + \dots$$

$$\bar{L}(g) = L_0 + \frac{\bar{g}^2}{16\pi^2}L_1 + \dots \quad . \tag{19}$$

Let us evaluate the asymptotic behavior for $q^2 \to -\infty$ fixing $\hat{q}^2 = -\mu^2$. Leading contributions come from the inhomogeneous terms K_0 and L_0 similarly to the e^+e^- case

$$\pi(q^2, g, m, \mu) \to -q^2\left(\frac{\beta_0 g^2}{16\pi^2}2t\right)^{\gamma_0/\beta_0} \frac{K_0 t}{1+\frac{\gamma_0}{\beta_0}} - m^2\left(\frac{\beta_0 g^2}{16\pi^2}2t\right)^{2\gamma_0/\beta_0} \frac{L_0 t}{1+\frac{2\gamma_0}{\beta_0}} \quad . \tag{20}$$

To obtain the Higgs decay width I take the imaginary part of analytically continued π at $q^2 = M^2$ (Higgs mass)

$$\mathrm{Im}\ \pi(q^2 = M^2, g, m, \mu) = \left(\frac{\beta_0 g^2}{16\pi^2}\ln\frac{M^2}{\mu^2}\right)^{\gamma_0/\beta_0} \frac{\pi}{2}\left(M^2K_0 + m^2\left(\frac{\beta_0 g^2}{16\pi^2}\ln\frac{M^2}{\mu^2}\right)^{\gamma_0/\beta_0} L_0\right) \quad . \tag{21}$$

The γ_0/β_0 power of $\ln M^2/\mu^2$ can be absorbed into the running quark mass (eq. (17)) which becomes in the leading order

$$m(M) \simeq m \cdot \left(\frac{\beta_0 g^2}{16\pi^2}\ln\frac{M^2}{\mu^2}\right)^{\gamma_0/2\beta_0} \quad . \tag{22}$$

In fact the Yukawa coupling in eq. (4) is given by the Fermi constant G_F and the quark mass in the standard SU(2) × U(1) model[1]

$$g_Y = (\sqrt{2}G_F)^{1/2} m \quad . \tag{23}$$

Inserting eqs. (21)-(23) into eq. (4) I obtain the Higgs decay width in the leading order

$$\begin{aligned}\Gamma &= \frac{\pi K_0}{2} M\sqrt{2}G_F[m(M)]^2\left\{1 + \frac{L_0}{K_0}\left(\frac{m(M)}{M}\right)^2\right\} \\ &= \frac{3M}{8\pi}\sqrt{2}G_F[m(M)]^2\left\{1 - 6\left(\frac{m(M)}{M}\right)^2\right\}\end{aligned} \tag{24}$$

where the leading inhomogeneous terms K_0 and L_0 are calculated from the quark loop diagram in Fig. 1 using eqs. (6), (11) and (19)

$$K_0 = \frac{3}{4\pi^2}$$

$$L_0 = -\frac{9}{2\pi^2} \quad . \qquad (25)$$

The leading order result in eq. (24) is the same as the Born term eq. (2) (without gluon corrections) except: (i) the quark mass m (of the Yukawa coupling) in the Born term is replaced by the running quark mass m(M) evaluated at the Higgs mass, and (ii) the quark mass in the phase space factor $(1 - 4m^2/M^2)^{3/2}$ is replaced by the running quark mass m(M) and up to the first term in the $(m(M)/M)^2$ power expansion is retained. Terms of order $(m(M)/M)^{2n}$, $n \geq 2$, are contained in the first term $\pi(\hat{q}^2, \bar{g}(t), \bar{m}(t)e^{-t}, \mu)A(t)$ in eq. (12) and should correspond to contributions from operators $m^{2n} \cdot 1$ in the operator product expansion.

The renormalization group analysis allows precise predictions to any desired order in the running coupling constant $\bar{g}$. As an example I will present the next-to-leading order correction. The β-function[9] and the anomalous dimension[10] γ_m in eq. (19) have been calculated up to the next-to-leading order in the minimal subtraction scheme

$$\beta_0 = 11 - \frac{2}{3}N_f \quad , \quad \beta_1 = 102 - \frac{38}{3}N_f$$

$$\gamma_0 = -8 \quad , \quad \gamma_1 = -108 + \frac{40}{9}N_f \qquad (26)$$

where N_f is the number of quark flavors. I calculate two-loop diagrams in Fig. 2 to obtain the next-to-leading inhomogeneous term K_1 in eq. (19) (for simplicity I neglected the mass in the loop, i.e. $\bar{L}(g) = 0$)

$$K_1 = \frac{5}{\pi^2} \quad . \tag{27}$$

I also need the finite (nonlogarithmic) part of π

$$\pi(q^2 = -\mu^2, g = 0, m = 0, \mu) \equiv (-\mu^2)K_0 \cdot B$$

$$B = \begin{cases} \frac{1}{2}(\ln 4\pi - \gamma_E) + 1 & \text{for MS scheme} \\ 1 & \text{for } \overline{\text{MS}} \text{ scheme} \end{cases} \quad . \tag{28}$$

where the $\ln 4\pi - \gamma_E$ term in the minimal subtraction (MS) scheme is absorbed into the redefinition of the QCD scale parameter Λ in the $\overline{\text{MS}}$ scheme of ref. 11. Other quantities in eqs. (26) and (27) is unchanged in the $\overline{\text{MS}}$ scheme. I can now expand the solution (12) of the renormalization group equation in powers of $(\ln M^2/\Lambda^2)^{-1}$ and obtain the Higgs decay width in the next-to-leading order

$$\Gamma = \frac{3M}{8\pi}\sqrt{2}G_F[m(M)]^2 \left\{ 1 + \frac{1}{\beta_0 \ln\frac{M^2}{\Lambda^2}} \left(\frac{K_1}{K_0} - 2\gamma_0 B \right) \right\} \quad . \tag{29}$$

Taking Higgs mass $M = 50$ GeV, $N_f = 6$, and $\Lambda = 0.5$ GeV for $\overline{\text{MS}}$ scheme, I obtain +35% correction (the second term in the curly bracket). I can also expand the running quark mass m(M) defined in eq. (17) in powers of $(\ln M^2/\Lambda^2)^{-1}$

$$m(M) = \tilde{m} \cdot F(M) \tag{30}$$

$$F(M) = \left(\ln\frac{M^2}{\Lambda^2} \right)^{\gamma_0/2\beta_0} \left\{ 1 + \frac{1}{\beta_0 \ln\frac{M^2}{\Lambda^2}} \left(\frac{\gamma_0\beta_1}{2\beta_0^2} \ln\ln\frac{M^2}{\Lambda^2} + \frac{\gamma_0\beta_1 - \gamma_1\beta_0}{2\beta_0^{\,2}} \right) \right\} \tag{31}$$

$$\tilde{m} = m/F(\mu) \quad . \tag{32}$$

In this expansion I obtain the Higgs decay width

$$\Gamma = \frac{3M}{8\pi}\sqrt{2}G_F \left[\tilde{m} \left(\ln\frac{M^2}{\Lambda^2} \right)^{\gamma_0/2\beta_0} \right]^2 \cdot C$$

$$C = 1 + \frac{1}{\beta_0 \ln\frac{M^2}{\Lambda^2}} \left(\frac{K_1}{K_0} - 2\gamma_0 B + \frac{\gamma_0\beta_1}{\beta_0^2} \ln\ln\frac{M^2}{\Lambda^2} + \frac{\gamma_0\beta_1 - \gamma_1\beta_0}{\beta_0^{\,2}} \right) \quad . \tag{33}$$

The correction factor C becomes 1.32 for the same parameters ($M = 50$ GeV, $N_f = 6$, $\Lambda = 0.5$ GeV, $\overline{MS}$ scheme). The reduction of the magnitude of perturbative corrections in the $(\ln M^2/\Lambda^2)^{-1}$ expansion is also noted in other processes.[12]

III. DISCUSSION

I have shown how to calculate perturbative QCD corrections to the Higgs decay without encountering mass singularities at all. In practical applications, however, there remains two problems: (i) Since Higgs boson decays preferentially to heavy quarks, it may be important to incorporate the phase space kinematical factor $(1 - 4m^2/M^2)^{3/2}$ in eq. (2) and (ii) The magnitude of quark masses is not accurately known.

As for the point (i), I was able to reproduce explicitly the phase space factor for the running quark mass up to the first term in the $(m(M)/M)^2$ power expansion. I expect that higher order terms can also be reproduced by using the operator product expansion. Therefore it seems most reasonable to use the phase space factor $(1 - (2m(M)/M)^2$ for the running quark mass $m(M)$ together with the perturbative QCD correction factors such as eq. (33).

As for the point (ii), one should probably turn the argument around: our formula can be employed to deduce from the Higgs decay width the running quark mass defined in eq. (17) which can be used, e.g. in the discussion of grand unified theories.[10,13]

One can treat Higgs bosons with γ_5 coupling or charged Higgs bosons, analogously. One can calculate perturbative QCD corrections to inclusive hadron distributions from the Higgs decay using similar renormalization group analyses with the cut vertex formalism[14] instead of the operator product expansion.

While this paper was being typed, I received a preprint by T. Inami and T. Kubota[15] which discussed the Higgs decay from a similar point of view and gave the next-to-leading order QCD correction in agreement with my result (33), but did not work out the $(m(M)/M)^2$ term (in my eq. (24)).

ACKNOWLEDGMENTS

I would like to thank W.A. Bardeen and A.J. Buras for valuable discussions and reading of the manuscript. I would also like to thank W. Celmaster, M. Einhorn, and P.Q. Hung for useful discussions.

FOOTNOTES AND REFERENCES

* On leave of absence from Department of Physics, Tohoku University, Sendai 980, Japan.

1 S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in Elementary Particle Physics: Relativistic Groups and Analyticity, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968); S. Glashow, Nucl. Phys. 22, 579 (1961).

2 For a recent review, see G.L. Kane, in Proc. Topical Workshop on the Production of New Particles in Super High Energy Collisions, Wisconsin, Oct. 1979 (preprint UMHE 79-37).

3 E. Braaten and J.P. Leveille, "Higgs boson decay and the running mass," Wisconsin preprint COO-881-127 (1980).

4 T. Appelquist and H. Georgi, Phys. Rev. D8, 4000 (1973).

5 A. Zee, Phys. Rev. D8, 4038 (1973).

6 D.J. Gross, Proc. 1975 Les Houches Summer School, Methods in field theory, ed. R. Balian and J. Zinn-Justin (North-Holland, 1976), p. 141.

7 Since the renormalized π is well-defined even for $m = 0$ in this scheme, one can see that mass singularities obtained in ref. 3 is an artifact of their particular choice of renormalization scheme. They renormalized on the mass shell in order to exploit the Kinoshita's theorem.

[8] M.A. Shifman, A.I. Vainstein and V.I. Zakharov, Nucl. Phys. B147, 385 (1979); A. Patkos and N. Sakai, Nordita preprint 79/49, Nucl. Phys. B, in press. The nonperturbative effects are suppressed at least by a factor $(\Lambda/M)^4$ where Λ is the QCD scale parameter.

[9] D.R.T. Jones, Nucl. Phys. B75, 531 (1974); W. Caswell, Phys. Rev. Lett. 33, 244 (1974).

[10] D.V. Nanopoulos and D.A. Ross, Nucl. Phys. B157, 273 (1979).

[11] W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, Phys. Rev. D18, 3998 (1978). The parameter Λ is given by $\Lambda^2 = \mu^2 \exp\{-16\pi^2/\beta_0 g^2 - \beta_1/\beta_0^2 \ln(\beta_0 g^2/16\pi^2)\}$.

[12] A.J. Buras, "Higher order QCD corrections to quarkonium decays," Fermilab preprint 80/43-THY.

[13] A.J. Buras, J. Ellis, M.K. Gaillard, and D.V. Nanopoulos, Nucl. Phys. B135, 66 (1978); P. Binetruy and T. Schucker, "Gauge and renormalization scheme dependence in GUTS," CERN preprint TH.2802.

[14] A.H. Mueller, Phys. Rev. D18, 3705 (1978).

[15] T. Inami and T. Kubota, "Renormalization group approach to summing QCD corrections in hadronic decays of the Higgs boson," preprint UT-Komaba 80-10, May 1980.

FIGURE CAPTIONS

Fig. 1: The lowest order diagram for the correlation function π of scalar currents.

Fig. 2: The g^2 order diagrams for the correlation function π of scalar currents.

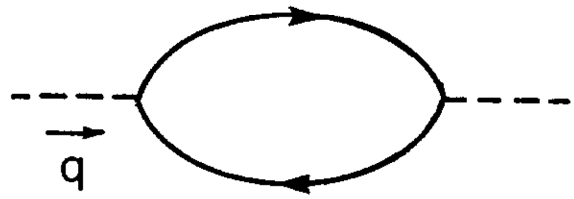

Fig. 1

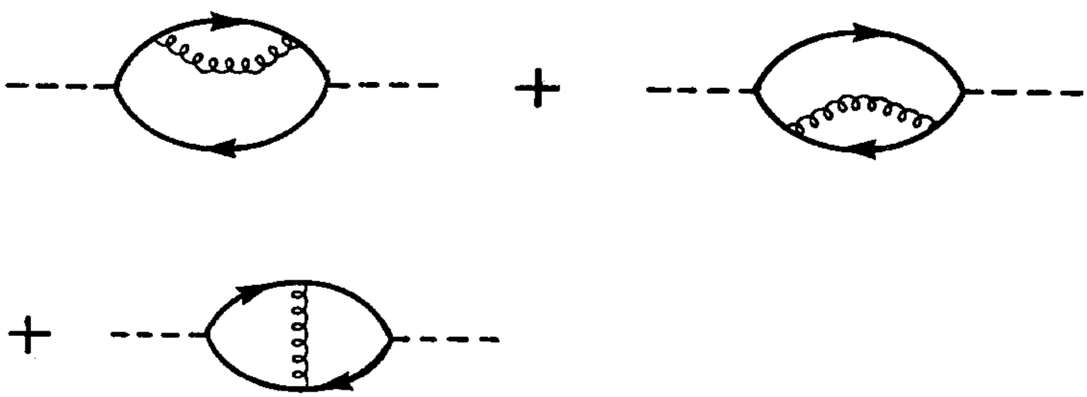

Fig. 2