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The Flavour of Seesaw

Memoria de Tesis Doctoral realizada por

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A mis padres

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Motivations and Goals **or why do I write a thesis**

Arguably, the main issue of fundamental physics today is that we still don't know how to do the matching between our two paradigm theories - General Relativity and the Standard Model (SM) of Particle Physics - that explain with incredible precision phenomena at the two opposite ends of the energy scale. Moreover, each of these theories has problems of its own. Indeed, cosmology has shown that we don't understand as much as 95% of the energy content of the universe. About 75% of this is a Dark Energy fluid which drives the accelerated expansion of the universe and of which very little is known except for its thermodynamic properties. The rest of that 95% is matter, dubbed Dark Matter, and the lack of a candidate particle to account for it is one of a few experimental failures of the SM. Among the latter it should also be mentioned the lack of an explanation for the observed baryon asymmetry of the universe.

However, the SM as a theoretical construction leaves many questions unanswered. We can mention among the most relevant the smallness of the θ-parameter of QCD which appears to be fine-tuned up to 11 orders of magnitude. There is also the so called *Hierarchy Problem* or why the Higgs particle is so light, taking into account that the presence of New Physics (NP) seems to imply that the Higgs mass should receive corrections proportional to the scale at which NP appears. Even more, we don't have any explanation for the large set of parameters that must be introduced by hand in connection with the Higgs sector nor for its seemingly arbitrary values. This is known as the *Flavour Puzzle*. Unfortunately, most attempts to solve one of these problems, such as Supersymmetry or Technicolor, do not help and often worsen the others.

In this thesis we deal with flavour and also with still another experimental fact of particle physics that is not accounted for in the SM, that is, the fact that *neutrinos are massive particles*. On the one hand, I will discuss our recent work in the area of neutrino physics. The uncertainties in this field come first and foremost from the question: what are the tiny neutrino masses telling us about fundamental physics? Secondly, we will also discuss some ideas inspired by the Flavour Puzzle. Let me explain both in a bit more detail.

When the SM was conceived, neutrinos were postulated massless. This was not an arbitrary decision. There was no experimental indication of neutrino masses and setting them to zero implied that $B - L$ was a global symmetry of the Lagrangian. In the formalism of the SM, no neutrino masses meant that no right-handed singlet fermion was introduced as a counterpart to the ν_L. This wasn't regarded as a problem, although it was in clear contrast with the rest of the fermions present in the theory,

Conclusive evidence for very small neutrino masses, but different from zero nonetheless, arrived at the turn of the century. Although, in principle, this could make the quark and lepton sector symmetrical, new questions were introduced. For one, we already think that there is a need to explain the strongly hierarchical pattern of the masses of the particles. Neutrinos tiny masses only make the problem much worse. To put a worldly perspective, we know that if the top quark had the mass of the Giza pyramid then the electron mass would amount roughly to that of one of the blocks with which it was built. Neutrino masses in that picture can't be greater than that of a grain of desert sand. Why should there be such a large difference among the masses of the fundamental particles?

The Majorana paradigm of neutrino mass generation provided an answer to that question. In its most general formulation, it simply postulates the appearance of the small mass for the neutrinos as a consequence of the extension of the SM with new, heavy particles involved in $B - L$ violating interactions with the neutrinos. At low energies, the effects of these particles are parametrized by an effective Lagrangian, whose first term is the famous $B - L$ violating Weinberg operator that leads to neutrino masses after EW symmetry breaking. With Majorana neutrinos, the problem of the smallness of neutrino masses is solved and we are left with the question of determining which is the correct model that produces them. The simplest examples of this paradigm are the so called *Seesaw models* which implement the Majorana idea at tree level. We will assume in this thesis that the mechanism providing mass for the neutrinos is of the Majorana type.

Because it must provide masses for all three neutrinos, the mechanism in question is inevitably connected to flavour physics and the Flavour Puzzle and, in general, models of neutrino mass also predict other kinds of exotic flavour processes. The new effects are readily described by the remaining terms in the effective Lagrangian. It is not hard to show that exotic flavour processes that do not violate $B - L$ are represented by operators of even dimension, $d = 6$, 8, etc, while $B - L$ violating processes are given by operators of odd dimensionality. A recurrent subject in this thesis will be to ascertain whether we can associate consistently different energy scales, Λ_{FL} and Λ_{LN} to be precise, to these two types of processes.

Given the little success met by ideas to understand the flavour structure of the SM, a more modest alternative has appeared, namely, that of *Minimal Flavour Violation*. Rather than solving the Flavour Puzzle straight away, certain reasonable assumptions are made about the structure of its solution. It is well known that if all matter fermions were massless, a large global *flavour* symmetry would be present in the SM. The hypothesis of Minimal Flavour Violation is that the Yukawas of the SM are in fact the couplings that break the flavour symmetry in the fundamental theory of the universe; and not just the manisfestation of that breaking at low energies. The Yukawa couplings manifest at low energies as spurions and the coefficients of any flavour charged operator in the low energy theory must be formed by a combination of them.

In the case of the lepton sector, this applies in particular for the coefficient of the $d = 5$ operator, giving rise to neutrino masses, and for those of the $d = 6$ operators linked with rare flavour processes. While we already have much information about the $d = 5$ coefficient via our knowledge about neutrino masses and mixings, little is known about the exotic flavour $d = 6$ processes. It is natural to ask what is the relation between neutrino masses and flavour processes imposed by MFV and how do Majorana neutrinos fit in this picture. Are Seesaw models compatible with MFV? And if that is the case, can the scales Λ_{LN} and Λ_{FL} of $B-L$ violation and MFV respectively be separated? We develop a positive answer to the latter questions in this thesis and show how the MFV hypothesis can guide us to very atractive possibilities for models of neutrino mass.

The scale Λ_{LN} must be very large in order to give natural neutrino masses. If Λ_{FL} were comparable then all rare lepton processes would be suppressed and we could never hope to see a hint of them. It is plausible on the other hand that Λ_{LN} is much larger than Λ_{FL} and that flavour effects are at reach of present or near future experiments. In this work a neat example of this is developed in a very simple MFV Seesaw model where all the flavour physics is fixed by the knowledge of the lepton masses and mixings.

Another possibility to suppress neutrino masses and still have sizable flavour processes is for some additional symmetry to forbid the $d=5$ Weinberg operator of neutrino masses. If $B-L$ is still violated, neutrino masses would be given by $d=7$ operators with a $1/M^3$ suppression or higher, presumably allowing for the couplings to be $\mathcal{O}(1)$ without having the energy scale M much higher than the TeV. It is therefore pertinent to determine whether it is possible to build a model that gives rise to neutrino masses only by means of effective operators of $d>5$. We will show how this can be done both at the tree and loop level in SM extensions.

For the last part of this thesis we consider four-lepton exotic interactions involving neutrinos. These are interesting since the constraints on them are not very strict. *Non Standard Neutrino Interactions* (NSNIs) are a general prediction of models beyond the SM. A very exciting possibility emerges if we are able to lower the typical scale at which they appear to the accesible energies of present experiments.

The drawback is that NSNIs are related through gauge invariance with four charged-lepton processes which are very constrained by present precision data. The relevant question here is, how strong are the contraints imposed on the NSNIs by charged-lepton precision experiments? Can we evade those constraints and obtain sizeable NSNIs while keeping the charged-lepton processes suppressed? By carrying a careful model-independent analysis we are able to show what general conditions are required to do so at the tree-level. We will further show with examples how our methods can be used in practice.

The document is organized as follows. In Chapter 1 we review the status of neutrino masses. We go on to introduce the Seesaw paradigm and the reasons for its appeal as an explanation for the origin of neutrino masses. We present the most well known examples. Chapter 2 is an eclectic mix of relevant information related with flavour in the lepton sector. Minimal Flavour Violation is explained in detail. Non-Standard Neutrino Interactions are presented and the bounds are summarized. From Chapter 3 to Chapter 5 our original work is presented. In Chapter 3 the relation between Minimal Flavour Violation and the Seesaw hypothesis is established and several models are shown that exemplify this connection. Chapter 4 is devoted to neutrino mass generation by means of $d \geq 7$ operators. In Chapter 5 we analyze Non-Standard Neutrino Interactions in the light of the gauge principle and determine what are the constraints that it imposes on working models.

Chapter 1

Massive neutrinos

We have come a long way since Pauli postulated the neutrino "as a desperate remedy to save the principle of energy conservation in beta decay". From that inspired suggestion to four-Fermi interactions; from the establishment of the Standard Model as a successful theory of fundamental interactions, with neutrinos embedded in it as fundamental particles, to the definitive proof that neutrinos are massive particles at the very end of the last century, much water has flowed. We present here some of the main ideas that have been brought up through all these years and that form part of what could be called the "standard lore" of neutrino theory. Though given the amount of effort put in the subject this presentation is by no means exhaustive, it does aim to introduce concisely the important subject of neutrino mass which plays a central role in this thesis.

1.1 The role of neutrinos in the SM

The Standard Model (SM) [1] is a gauge theory with an $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge group. The gauge principle is a powerful one and predicts precisely the quantum numbers of the gauge bosons. On the contrary, matter content is selected to accomodate that observed in nature with the only constraint that the gauge symmetries must remain conserved at the quantum level(anomaly cancellation). In the SM the quark and lepton matter fields are chosen as

$$Q_{L\alpha} \sim (\mathbf{3},\, \mathbf{2})_{\frac{1}{6}}\,, \quad U_{R\alpha} \sim (\mathbf{3},\, \mathbf{1})_{\frac{2}{3}}\,, \quad D_{R\alpha}(\mathbf{3},\, \mathbf{1})_{-\frac{1}{3}}\,, \tag{1.1}$$

$$L_{L\alpha} \sim (\mathbf{1},\, \mathbf{2})_{-\frac{1}{2}}\,, \quad E_{R\alpha} \sim (\mathbf{1},\, \mathbf{1})_{-1}\,, \tag{1.2}$$

where the notation $(C, W)_Y$ stands for the field's color, weak and hypercharge representations. The index α runs through the three generations.

The left-handed (LH) quarks are components of a weak doublet comprising the U and D type quarks

$$Q_{L\,\alpha} = \begin{pmatrix} U_{L\alpha} \\ D_{L\alpha} \end{pmatrix}\,. \tag{1.3}$$

and they have non trivial electroweak interactions through weak boson exchange

$$i\bar{Q}_L \not{D} Q_L = i\bar{U}_L \not{\partial} U_L + i\bar{D}_L \not{\partial} D_L - \frac{g}{2\sqrt{2}} \bar{U}_L \not{W}_+ D_L - \frac{g}{2\sqrt{2}} \bar{D}_L \not{W}_- U_L - \frac{2}{3} g \cos\theta_W \bar{U}_L \not{A} U_L +$$
$$+ \frac{1}{3} g \cos\theta_W \bar{D}_L \not{A} D_L - \frac{g}{2\cos\theta_W} \bar{U}_L \not{Z} U_L + \frac{g}{2\cos\theta_W} \bar{D}_L \not{Z} D_L \,, \tag{1.4}$$

where g is the weak coupling constant and θ_W is the Weinberg angle. Flavour indices have been omitted.

The lepton sector is colorless and left-handed neutrinos are components of a weak doublet, also including the left-handed charged leptons

$$L_{L\alpha} = \begin{pmatrix} \nu_{L\alpha} \\ \ell_{L\alpha} \end{pmatrix} . \tag{1.5}$$

Both charged leptons and neutrinos have non-trivial interactions with the weak bosons via the kinetic term

$$\bar{L}_L i \not{D} L_L = \bar{\nu}_L i \not{\partial} \nu_L + i \bar{\ell}_L \not{\partial} \ell_L - \frac{g}{2\sqrt{2}} \bar{\nu}_L \not{W}_+ \ell_L - \frac{g}{2\sqrt{2}} \bar{\ell}_L \not{W}_- \nu_L$$
$$+ g \cos\theta_W \bar{\ell}_L \not{A} \ell_L - \frac{g}{2\cos\theta_W} \bar{\nu}_L \not{Z} \nu_L + \frac{g}{2\cos\theta_W} \bar{\ell}_L \not{Z} \ell_L \dots \,, \tag{1.6}$$

Neutrinos are neutral particles that do not interact electromagnetically.

Dirac mass terms of the form $\bar{\Psi}_L \Psi_R$ are forbidden in the SM by the $SU(2)_W$ gauge symmetry. As a consequence all matter fields acquire their masses following the spontaneous breaking of the electroweak symmetry. The mechanism requires the presence of the Higgs scalar

$$H \sim (\mathbf{1},\, \mathbf{2})_{\frac{1}{2}} \tag{1.7}$$

which couples to the matter fields through the Yukawa couplings

$$\mathscr{L}_Y = -Y_U^{\alpha\beta} \bar{Q}_{L\alpha} \tilde{H} U_{R\beta} - Y_D^{\alpha\beta} \bar{Q}_{L\alpha} H D_{R\beta} - Y_E^{\alpha\beta} \bar{L}_{L\alpha} H E_{R\beta} \tag{1.8}$$

where $\tilde{H} = i\tau_2 H^*$. Spontaneous symmetry breaking occurs when the Higgs field acquires a vacuum expectation value (VEV)

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} . \tag{1.9}$$

For the case of leptons, substituting in Eq.(1.8) and diagonalizing with the double unitary transformation

$$L_L \to U_L L_L \,, \quad E_R \to U_E E_R \tag{1.10}$$

such that

$$\frac{Y_\ell^{\alpha\beta} v}{\sqrt{2}} \to U_L \frac{Y_\ell^{\alpha\beta} v}{\sqrt{2}} U_E^\dagger = \mathrm{diag}\{m_e,\, m_\mu,\, m_\tau\} \tag{1.11}$$

one obtains the mass terms for the charged leptons

$$\mathscr{L}_{m_\ell} = -m_e \bar{e}_L e_R - m_e \bar{\mu}_L \mu_R - m_e \bar{\tau}_L \tau_R + \text{h.c.} \tag{1.12}$$

while the interaction terms with the weak bosons in Eq.(1.6) are unchanged. Because no right-handed partners have been introduced it is not possible to write Yukawa interactions for the ν_Ls. The SM thus predicts massless neutrinos.

Furthermore, we can redefine the lepton fields so as to diagonalize the Yukawa couplings

$$\ell_L \to U_\ell L_L\,, \quad E_R \to U_E E_R\,, \quad U_\nu = U_\ell \tag{1.13}$$

After diagonalization, the only parameters remaining in the leptonic Lagrangian are the three lepton masses while the weak interaction terms in Eq.(1.6) are unchanged. It is instructive to see how this can be understood by counting the number of parameters in the Lagrangian before applying the diagonalization procedure and comparing it with the number of them we can absorb through the change of basis in Eq.(1.10). Notice that

- **i)** The only parameters relevant for this analysis that appear in the Lagrangian are those in Y_E. This amounts to one complex 3×3 matrix with 9 real parameters and 9 phases.

- **ii)** Parameters are absorbed through the matrices U_ℓ and U_E. These are unitary matrices with 3 real parameters and 6 phases each. Overall, 6 real parameters and 12 phases can be absorbed by means of Eq.(1.10).

- **iii)** However, among the transformations in Eq.(1.10) there is one symmetry. Indeed, the Lagrangian remains unchanged for $U_\ell = U_E = \text{diag}\{e^{i\alpha},\, e^{i\alpha},\, e^{i\alpha}\}$. This is the *Lepton Number* (LN) symmetry. It is a phase transformation that *does not* absorb any parameter and therefore must be substracted from ii). So, in fact, only 11 phases can be absorbed .

On comparing we see that there should remain 3 real parameters in the Lagrangian, these are the 3 lepton masses. We can also absorb all 9 phases and we still even have 2 phases to spare which we identify with two $U(1)$ symmetries that were hidden in the original Lagrangian and become self-evident only after diagonalization. The latter add to the LN symmetry to give the conservation of all three, electron, muon and tau lepton numbers.

Carrying the same analysis above for the quark sector turns out to be rewarding since it will leads us to flavour mixing. There, two Yukawas, Y_U and Y_D, amount to 18 real parameters and 18 phases. We can absorb parameters with three unitary matrices if we want to leave the kinetic term unchanged: U_Q, U_U and U_D

$$Q_L \to U_Q Q_L\,, \quad U_R \to U_U U_R\,, \quad D_R \to U_D D_R\,. \tag{1.14}$$

Thus we can absorb 9 real parameters and 18 phases minus one, because of Baryon Number Symmetry (BN). Upon substracting we find that 9 real parameters and 1 phase should remain in the Lagrangian as physical.

The BN and LN symmetries we have found are in fact anomalous and do not remain as symmetries at the quantum level. However, the linear combination of the two, $B-L$ does remain a good symmetry of the quantum theory. This symmetry is *accidental* since it wasn't imposed to begin with but rather emerged as a consequence of the field content of the SM.

The 9 real parameters we found for the quarks are 6 quark masses and 3 mixing angles while the phase is responsible for CP violation in the quark sector. If we write the Lagrangian with the mass term diagonal, the three angles and the phase appear in the interaction term of the left-handed quarks with the charged weak bosons

$$g\bar{U}_{L\alpha}\not{W}_{+}D_{L\alpha} \to gU^{\alpha\beta}_{CKM}\bar{U}_{L\alpha}\not{W}_{+}D_{L\beta}\,, \quad \text{with } U_{CKM} = U_Q^{\dagger}U_D\,, \tag{1.15}$$

while the neutral current term is unperturbed. Mixing appears in charged currents because the mass and weak interaction eigenstates do not coincide. At one loop, mixing also shows in the neutral currents thus giving rise to Flavour Changing Neutral Currents. Their smallness is crucial for binding SM extensions.

The *mixing matrix* U_{CKM} [2,3] is usually parametrized as [4]

$$U_{CKM} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \tag{1.16}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{1.17}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. Some emphasis should be made on the CP phase δ. If the parameter counting were to be repeated for the SM with two generations one would find that no phase remains physical. In fact, it can be shown that an imaginary component in the mixing matrix only appears in the case of three or more non-degenerate families The number of families in the SM is minimal such that it allows for CP violation.

That CP was not a symmetry of nature was indeed a fundamental discovery [3], more so taking into account that CP violation is an essential ingredient for explaining the matter-antimatter asymmetry of the universe. There was the hope that CP violation in the quark sector could account for that puzzle. The amount of CP violation can be parametrized by the Jarlskog invariant [5] that appears in all CP violating processes

$$J = \mathrm{Im}(U_{ij}U_{kl}U^{*}_{kj}U^{*}_{il}) = s_{12}s_{13}s_{23}c_{12}c^{2}_{13}c_{23}\sin\delta\,. \tag{1.18}$$

CP is conserved if any of the conditions $\theta_{ij} = 0,\ \pi/2,\ \delta = 0\,, \pi/2$ are satisfied. With the present bounds, $J = (3.05^{+0.19}_{-0.20}) \times 10^{-5}$, which is not enough to generate the observed baryon asymmetry.

None of this rich phenomenology could possibly happen in the lepton sector due to the neutrinos being massless.

1.2 ...but neutrinos were massive

The fact that neutrinos were massive was established at the turn of the last century as the only consistent explanation for the wealth of data gathered by many experiments. All of these experiments studied neutrinos arriving at detectors from a number of different sources: nuclear reactors, the sun, neutrinos produced in the atmosphere due to cosmic ray interactions. The remarkable result of this great experimental effort was the establishment that the SM prediction for the expected fluxes *for specific neutrino types* was wrong.

It is remarkable that already in the 60s, the Homestake experiment, measuring electron neutrinos coming from the sun through the reaction

$$\nu_e + \,^{37}\mathrm{Cl} \to e^- + \,^{37}\mathrm{Ar}\,, \tag{1.19}$$

identified a neutrino flux smaller than theoretical predictions [6, 7]. Subsequent experiments such as Kamiokande [8, 9], SAGE [11] and GALLEX [12] confirmed the deficit and, more importantly, they ruled out any astrophysical explanation to the solar neutrino problem. Several solutions based on particle physics were proposed by the 80s, among them the one that would turn out to be correct, i.e., the possibility that neutrinos were massive particles and thus capable of oscillating between flavours. In that case, an experiment designed to measure electron neutrinos far away from their source should find a deficit.

At the same time a similar puzzle was revealed in the measurement of neutrinos generated by cosmic rays hitting the atmosphere at experiments such as NUSSEX or Soudan [13], apart from Kamiokande [9]. There, the relevant reaction was

$$\pi^{\pm} \to \mu^{\pm} + \nu_\mu \;\; (\text{or } \; \bar{\nu}_\mu) \tag{1.20}$$

$$\mu^{\pm} \to e^{\pm}\,\nu_e\,\nu_\mu \;\; (\text{or } \bar{\nu}_e\,\bar{\nu}_\mu)\,, \tag{1.21}$$

and from this, the expected ratio of muon to electron neutrinos should be roughly 2:1. The measured ratio however consistently resulted about 40 percent smaller. To throw even more wood into the fire, Kamiokande found an up-down asymmetry on the already depleted muon neutrino flux reaching its detector [10]. This made sense if neutrino oscillations were oscillating on their way through Earth.

SuperKamiokande, the largest Cherenkov detector ever built, proved to be the decisive machine. It confirmed the dissapearance of muon neutrinos [14] and, more importantly, it established for good the dependence of the atmospheric neutrino flux on the angle with which it reached the detector. The up-down ratio of muon neutrino fluxes measured at SuperK differed from 1 in more than 6 sigma! Though uncertainties in the flux or cross section predictions as well as experimental biases were safely ruled out, the data gathered by SuperKamiokande was easily fit by a two flavour $\nu_\mu \to \nu_\tau$ oscillation. After a couple of years SuperK's results became widely accepted and their 98 paper [15], the most cited experimental particle physics paper ever.

The rest of the story is downhill. The Sudbury Neutrino Observatory was able to measure the total $\nu_e + \nu_\mu + \nu_\tau$ neutrino flux coming from the sun and showed it to agree with the total neutrino flux expected thus confirming that solar neutrinos weren't actually dissappearing but merely transforming into one another [16,17]. Totally Earth-based experiments followed. They proved that we could reproduce neutrino oscillations in a controlled way. The atmospheric or solar oscillations could be probed by choosing wisely the quotient between the baseline and the energy of the neutrino beam. Thus Kamland confirmed the solar sector [18] while MINOS [19] and K2K [20] did the same for the atmospheric one. Ultimately, in 2004, SuperKamiokande and KamLAND presented evidence of neutrino dissapearance *and* reappearance at another detector thus eliminating mercilessly whatever little hope there was for any non-oscillating solution for the neutrino puzzles.

1.2.1 Direct and cosmological bounds to neutrino mass

The requirement for neutrinos to oscillate is not merely that they have masses, but rather that their masses are diferent. Henceforth, oscillation experiments are useful for determining *mass differences*, not absolute values. From decay processes that involve neutrinos in the final state very precisely one can obtain bounds for neutrino masses or, to be more specific, for the effective flavour mass defined as

$$m_\alpha^2 = \sum_i |V_{i\alpha}|^2 m_i^2 \,. \tag{1.22}$$

So far, the following direct bounds on neutrino masses have been achieved

- From the measurement of the end-point of the β-decay spectrum of tritium

  $$^3H \to ^3 He + e + \bar{\nu}_e \tag{1.23}$$

  the bound $m_{\nu_e}^2 < 5\,\mathrm{eV}^2$ was obtained by the Mainz collaboration although others have performed the same experiments with similar results. [21, 22]. Since we already know the mass differences from oscillation data, this is the only direct neutrino mass determination type of experiment that is being pursued at present. The forthcoming KATRIN experiment has a proposed sensitivity of 0.35 eV [23].

- From the measurement of the energy and momentum of the muon produced in pion decay at rest

  $$\pi^+ \to \mu^+ + \nu_\nu \tag{1.24}$$

the bound $m_{\nu_\mu} < 190\,\text{keV}$ was obtained by a team at the accelerator laboratory at the Paul Scherrer Institute [24].

- From the measurement of the total energy and momentum of pions produced by hadronic tau-decays
$$\tau \to N\pi + \nu_\tau \,, \quad N > 3 \tag{1.25}$$
the bound $m_{\nu_\tau} < 18.2\,\text{MeV}$ was obtained by the ALEPH collaboration [25].

Less robust but much stronger bounds can be obtained from cosmological arguments. Generally speaking, neutrino masses influence in a number of ways the history of the universe so we can use the measured cosmological parameters to put upper bounds on them. For instance:

- The number density of neutrinos n_ν can be obtained from the assumption of thermal equilibrium at the time of decoupling and from the knowledge of the photon energy density from the CMB. Knowing the present n_ν we can calculate their energy density and imposing that the latter does not exceed the measured energy density of matter we obtain the naive cosmological bound on neutrino masses
$$\sum m_\nu < 94\Omega_m h^2 \text{ eV} \simeq 11.5 \text{ eV} \tag{1.26}$$
for $\Omega_m \leq 0.25$ and $h = 0.7$. This bound can be further improved by noting that neutrinos are relativistic and not given to form gravitationally bound systems. If neutrinos were to form too large a fraction of the energy density of the universe, Large Scale Structure (LSS) formation would be inhibited. To correct this, neutrinos shouldn't exceed $\sim 30\%$ of the total matter density or [CITE]
$$\sum m_\nu < 3.6 \text{ eV} \,. \tag{1.27}$$

- Massive neutrinos are reflected on the CMB power spectrum. Two effects can be seen in the angular fluctuations of the CMB: peaks in the power spectrum shift to the left as neutrino masses increase and there is a decrease of the first peak height. Analysis of WMAP data allows to restrict the neutrino mass from above to [26]
$$\sum m_\nu < 0.61 \text{ eV} \,. \tag{1.28}$$

- A more careful analysis of LSS theory shows that, in a universe dominated by neutrinos, very large structures of order a few hundred Mpc should be formed first; after that, smaller structures are formed by fragmentation. Therefore smaller size structures are younger. The larger the neutrino mass (but still small so that they remain relativistic) the longer the delay in the formation of small structures. In particular, the number of Lyman-α absorption lines - indicative of young structures

- at high red-shifts $z \geq 1$ is very sensitive to light massive neutrinos. Combined data analysis with WMAP [27] restricts neutrino mass to

$$\sum m_\nu < 0.17\ \text{eV}\,. \tag{1.29}$$

1.3 Neutrinos as Majorana particles

Neutrino masses can be introduced in the SM straightforwardly by mirroring the Dirac mechanism which gives masses to the other fermions. We shall see in a moment that there are several drawbacks in these Dirac neutrinos and more importantly, there is an appealing alternative. However, the Dirac Lagrangian for neutrino masses is a good starting point and we present it first.

Masses of the Dirac type require the SM field content to be enlarged by two or more right-handed neutrinos, singlets under all gauge groups

$$N_{R\alpha} \sim (\mathbf{1},\, \mathbf{1})_0 \tag{1.30}$$

For the case of three flavours the relevant Lagrangian is given by

$$\mathscr{L}_{\nu-int} = -\frac{g}{2\sqrt{2}}\bar{\ell}_L \not{W}_+ \nu_L - \frac{g}{2\cos\theta_W}\bar{\nu}_L \not{Z}\nu_L - Y_E^{\alpha\beta}\bar{L}_{L\alpha} H E_{R\beta} - Y_N \bar{L}_{L\alpha}\tilde{H} N_{R\beta} + \text{h.c.} \tag{1.31}$$

After EW symmetry breaking, the mass terms for charged leptons and neutrinos can be diagonalized as in Sec.1.1. The procedure mirrors that of the quarks and, in particular, we have mixing and a CKM-like matrix - Eq.(1.17) - appears accompanying the charged weak interactions.

However, at least two things are worrisome about the preceding picture of neutrino masses:

- Right-handed neutrinos are singlets of all gauge groups. Therefore, a Majorana mass term
$$\frac{M}{2}\bar{N^c}_R N_R \tag{1.32}$$
is compatible with all the symmetries of the Lagrangian. It does violate LN by two units but LN conservation was never a requirement *a priori*. It is actually an accidental symmetry that appeared only as a consequence of the particular field content of the theory. If the field content varies, there is no reason why it should be preserved.[1]

[1] Strictly speaking, Lepton Number symmetry is anomalous and therefore, it is already violated at the quantum level. A Majorana mass term as the one in Eq.(1.32) also violates $B-L$, a good symmetry of the Lagrangian. But since LN or BN violation are not seen at the perturbative level, we will use LN and $B-L$ interchangeably in this thesis.

- In order to satisfy the present bound on neutrino masses, the Yukawa couplings must be tiny. From the equation

$$\frac{Y_N v}{\sqrt{2}} < 1\text{eV} \tag{1.33}$$

we get $Y_N \sim 10^{-11}$. This is very unnatural. Furthermore, given the electron Yukawa $Y_e \sim 10^{-6}$, we are actually introducing a new question by implementing neutrinos this way. Namely, why should the Yukawa couplings for the neutrinos be so tiny when compared to the rest? In particular, why is there such a huge hierarchy *within the same family*?

At the relatively small price of dropping LN, both issues are addressed beautifully. From the modern point of view, the SM is only an effective theory where operators of dimension higher than four ($d > 4$) are suppressed by some energy scale of new physics.

$$\mathscr{L} = \mathscr{L}_{SM} + c^{\alpha\beta}_{d=5}\mathcal{O}_{d=5} + \dots = \mathscr{L}_{SM} + \frac{\hat{c}^{\alpha\beta}_{d=5}}{\Lambda_{LN}}\mathcal{O}_{d=5} + \dots \tag{1.34}$$

where Λ_{LN} is the scale of new physics and $\hat{c}_{d=5}$ is dimensionless coefficient. If LN is not a fundamental symmetry of nature then Λ_{LN}-suppressed, LN violating couplings should appear among the non-renormalizable operators. As a matter of fact, there is only one $d = 5$ operator that can be built out of the fields of the SM and respecting all its gauge symmetries. It already violates LN. This operator was first identified by Weinberg in a seminal paper [28] and it can be written as

$$\mathcal{O}_{d=5} = (\bar{L}^c{}_{L\alpha}\tilde{H}^*)(\tilde{H}^\dagger L_{L\beta})\,. \tag{1.35}$$

After EWSB this operator leads to a Majorana mass term for the left-handed neutrinos proportional to v^2/Λ_{LN}. The mass matrix defined as

$$m_\nu^{\alpha\beta} = \frac{\hat{c}^{\alpha\beta}_{d=5} v^2}{2\Lambda_{LN}} \tag{1.36}$$

is symmetric and can be diagonalized by the transformation

$$\nu_{L\alpha} \to U_\nu \nu_{L\alpha}\,, \quad m_\nu \to U_\nu^* m_\nu U_\nu^\dagger = \text{diag}\{m_1,\, m_2,\, m_3\} \tag{1.37}$$

We see that the mass eigenvalues m_i can be made very small if Λ_{LN} is large enough. That is, if $\hat{c}_{d=5} \sim O(1)$ then $\Lambda_{LN} \sim M_{GUT}$ would fit the experimental value of neutrino masses, a tantalizing possibility. If $\hat{c}_{d=5} \sim Y_E^2$ then $\Lambda_{LN} \sim O(\text{TeV})$ could be appropriate. All in all, no extra fine-tuning with respect to the Standard Model is needed for TeV $\lesssim \Lambda_{LN} \lesssim M_{GUT}$.

We can also reduce the charged lepton mass terms to diagonal form by means of the transformations

$$\ell_{L\alpha} \to U_\ell \ell_{L\alpha}\,, \quad E_{R\alpha} \to U_E E_R \tag{1.38}$$

$$Y_E \to U_\ell Y_E U_E^\dagger = \mathrm{diag}\{m_e,\ m_\mu,\ m_\tau\} \tag{1.39}$$

and analogous to what happens for the quarks, the charged current interaction is not invariant under this transformation

$$\bar{\ell}_L \not{W}_+ \nu_L \to \bar{\ell}_L U_\ell U_\nu^\dagger \not{W}_+ \nu_L \tag{1.40}$$

with the new coefficients given by the mixing matrix

$$U_{PMNS} = U_\ell U_\nu^\dagger \quad . \tag{1.41}$$

Notice that for the particular case in which all three neutrinos are degenerate $M \propto \mathbf{1}$, Eq.(1.37), the matrix U_ν is arbitrary and we can make $U_\nu = U_\ell$. Therefore, from Eq.(1.41), there is no flavour violation if the neutrinos are degenerate in mass.

As it was previously done with the CKM matrix, we can do the parameter counting for the PMNS matrix as follows:

i) In the "raw" Lagrangian we have on one hand Y_E with 9 real parameters and 9 phases. On the other hand, m_ν is symmetric so there are only 6 independent real parameters and 6 phases. Overall, we have 15 real parameters and 15 phases in the lepton sector of the Lagrangian before diagonalizing.

ii) The Lagrangian can absorb the U_E and U_ℓ rotations with $U_\nu = U_\ell$, Eq.(1.40). Both U_ℓ and U_E are unitary matrices with 3 real parameters and 6 phases. In total, 6 real parameters and 12 phases can be absorbed by transformations of the fields.

iii) There are no accidental symmetries.

Therefore, 9 real physical parameters and 3 physical phases remain in the Lagrangian. The real parameters are the six charged lepton masses and the three mixing angles as before. However, there are more physical phases in the PMNS matrix than in the CKM. It is customary to parametrize the PMNS as

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_{ph} \tag{1.42}$$

with

$$U_{ph} = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & 1 \end{pmatrix} . \tag{1.43}$$

1.4 Seesaw Models

Unless some other symmetry forbids it[2], any fundamental theory that violates $B - L$ will give rise to the Weinberg operator at low energies. However, there are just three ways of generating this operator at tree level. These are the so called Seesaw Models.

We can find all three types of seesaw by a procedure we call *opening* the Weinberg vertex, Fig.1.1. Diagramatically, the idea consists in grouping the four external legs of the operator in Eq.(1.35) in two sets of two. Each of these bilinears should form part of a renormalizable coupling in the fundamental theory. The particle missing from the vertex or *mediator* can be determined uniquely by imposing Lorentz and gauge invariance.

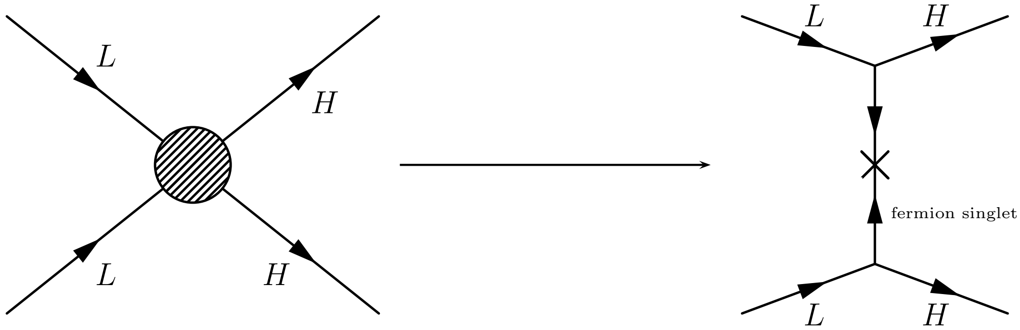

Figure 1.1: Opening the Weinberg vertex to give Type I Seesaw

Seesaw Types I and III refer to fermionic particles mediating: an electroweak singlet and triplet respectively. Type II refers to a scalar triplet mediator. We consider these basic models in what follows.

1.4.1 Tree-level Seesaw: Mediating fermions

The historical way to implement the seesaw mechanism was adding a Majorana mass term for the right-handed singlets to the Dirac neutrino Lagrangian in Eq.(1.31) [29]. For an illustration of the mechanism consider the SM with just one generation of leptons, plus both Dirac (Yukawa) and Majorana couplings,

$$\mathscr{L}_N = i\bar{N}_R \partial\!\!\!/ N_R - Y_N \bar{L}_L \tilde{H} N_R - \frac{M}{2} \bar{N^c}_R N_R + \text{h.c.} \,, \tag{1.44}$$

[2]See Ch.4 for work along that line.

where flavour indices have been omitted. After EWSB the mass terms in Eq.(1.44) can be rewritten as

$$\mathscr{L}_{m_\nu} = \begin{pmatrix} \bar{\nu} & \bar{N}^c \end{pmatrix} \begin{pmatrix} 0 & \frac{Y_\nu v}{\sqrt{2}} \\ \frac{Y_\nu v}{\sqrt{2}} & M \end{pmatrix} \begin{pmatrix} \nu^c \\ N \end{pmatrix} , \tag{1.45}$$

with eigencvalues

$$m_{1,2} = \frac{M}{2} \mp \sqrt{\left(\frac{M}{2}\right)^2 + \left(\frac{Y_\nu v}{\sqrt{2}}\right)^2} . \tag{1.46}$$

It is natural to assume that the mass M is large, $M \gg v$, since it is not protected by any of the SM symmetries. In this case one eigenvalue is very light while the other is very heavy. This is the *seesaw* mechanism at work

$$m_1 \simeq \frac{Y_\nu^2 v^2}{2M} \tag{1.47}$$

$$m_2 \simeq M \tag{1.48}$$

It thus provides an explanation for the lightness of the neutrino mass if the mass of the right-handed field is located at some large energy scale. One can estimate the order of magnitude of the mass M, such that $Y_\nu \sim 1$ and m_1 is compatible with the present upper bounds, to be of the order ot a typical GUT theory

$$M \sim 10^{13}\ \text{GeV} . \tag{1.49}$$

In the language of effective field theory what we are doing is to *integrate out* the heavy right-handed field. This procedure leads indeed to the $d = 5$ Weinberg operator, Eqs.(1.34, 1.35), with the identification

$$M \equiv \Lambda_{LN} , \quad \hat{c}_{d=5} \equiv Y_\nu^2 \tag{1.50}$$

After EWSB, we are left with the tiny Majorana mass for the SM neutrinos,

$$\mathscr{L}_{d=5} = \frac{\hat{c}_{d=5}}{M}\mathcal{O}_{d=5} = \frac{Y_\nu^2}{M}(\bar{L}^c \tilde{H}^*)(\tilde{H}^\dagger L) + \text{h.c.} \to \frac{Y_\nu^2 v^2}{2M}\bar{\nu^c}\nu + \text{h.c.} \tag{1.51}$$

where $\hat{c}_{d=5}$ encondes the flavour structure as before. It is possible to generalize the Seesaw mechanism for three families of leptons and n heavy right-handed neutrinos. Both Y_ν and M are upgraded to matrices with flavour indices and after integrating out the heavy fields, one is left with a Majorana mass matrix for the three light neutrinos of the form

$$m_\nu = \frac{v^2}{2} Y_\nu \frac{1}{M} Y_\nu^T . \tag{1.52}$$

$m_\nu^{\alpha\beta}$ is a symmetric complex matrix and can be diagonalized by a unitary transformation on the neutrino fields as in Eq.(1.37). Diagonalization of the fermion mass terms give rise to U_{PMNS}.

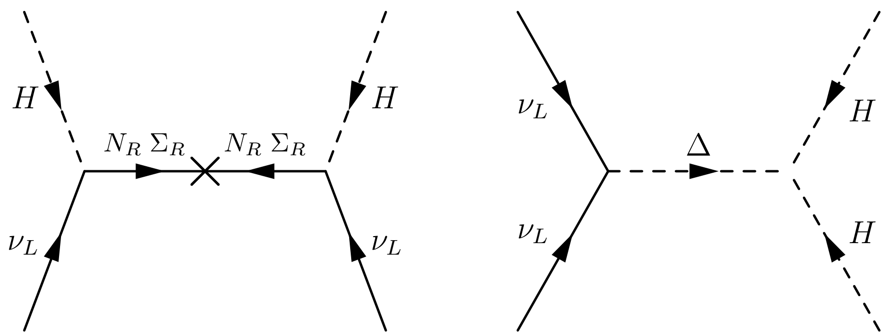

Figure 1.2: Tree-level diagrams giving rise to the Weinberg operator. *Left*: Types I and III Seesaws. *Right*: Type II Seesaw.

As we said above, in the case of a fermion mediator there is also the possibility of adding to the SM zero hypercharge triplets of $SU(2)$ [30], Fig.1.2. We denote this fields by $\vec{\Sigma}$ with

$$\vec{\Sigma} = (\Sigma_1,\, \Sigma_2,\, \Sigma_3) \tag{1.53}$$

where the flavour indexes are omitted in what follows. The interactions of the new field are described by the Lagrangian

$$\mathscr{L}_\Sigma = i\overline{\vec{\Sigma}_R}\not{D}\vec{\Sigma}_R - \left[\frac{1}{2}\overline{\vec{\Sigma}^c_R}M_\Sigma\vec{\Sigma}_R + \overline{\vec{\Sigma}_R}Y_\Sigma(\tilde{\phi})^\dagger L_L) + \text{h.c.}\right] \tag{1.54}$$

Notice that a Majorana mass is now a possible gauge invariant term since the $\vec{\Sigma}$ field is in the adjoint representation. Here, the covariant derivative is given by

$$D_\mu = \partial_\mu - igT^aW^a_\mu \tag{1.55}$$

with

$$T^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}\ ,\quad T^2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}\ ,\quad T^3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{1.56}$$

The $\vec{\Sigma}$ fields are not eigenstates of electric charge, the latter given by the combinations

$$\Sigma_\pm = \frac{\Sigma_1 \mp i\Sigma_2}{\sqrt{2}}\ ,\quad \Sigma_0 = \Sigma_3 \tag{1.57}$$

After EWSB the Yukawa term Y_Σ induces Majorana masses for the LH neutrino fields of the SM through the exchange of $\vec{\Sigma}$ particles. The coefficient of the $d = 5$ operator turns out to be given as before by

$$c_{d=5} = Y_\Sigma^T\frac{1}{M_\Sigma}Y_\Sigma \tag{1.58}$$

At low energies we get again the neutrino mass matrix

$$m_\nu = -\frac{v^2}{2} Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma \,. \tag{1.59}$$

being completely equivalent to Type I Seesaw in what refers to neutrino masses.

1.4.2 Tree-level Seesaw: Mediating scalars

The $d = 5$ Weinberg operator, Eq.(1.35), can also be obtained as a low energy effect of a theory in which the SM field content is enlarged with a scalar field Δ triplet of $SU(2)$ [31]. There are a number of new terms now in the Lagrangian which can be written as

$$\mathscr{L}_\Delta = (D_\mu \Delta)^\dagger (D^\mu \Delta) - \left(Y_\Delta^{\alpha\beta} \bar{\tilde{L}}_\alpha (\tau \cdot \Delta) L_\beta + \mu_\Delta \tilde{H}^\dagger (\tau \cdot \Delta)^\dagger H + \text{h.c.} \right) - V(H, \Delta) \tag{1.60}$$

where $\tilde{L} = i\tau_2 L^c$ and $V(H, \Delta)$ includes all couplings in the scalar sector that are compatible with the symmetry. Notice μ_Δ is a dimensionful parameter with mass dimension ~ 1 and that Y_Δ is a symmetric matrix.

For the seesaw mechanism to operate the mass of the triplet M_Δ is assumed very large. It can be integrated following the procedure described and leading to the Weinberg operator as promised with coefficient

$$c_{d=5}^{\alpha\beta} = 2 Y_\Delta^{\alpha\beta} \frac{\mu_\Delta}{M_\Delta^2} \,. \tag{1.61}$$

Hence, after EWSB, the mass matrix for the neutrinos has the form

$$m_\nu^{\alpha\beta} = Y_\Delta^{\alpha\beta} v^2 \frac{\mu_\Delta}{M_\Delta^2} \tag{1.62}$$

In the scalar seesaw, the breaking of the $B - L$ symmetry is given by the simultaneous presence of the Yukawa Y_Δ and the μ_Δ coupling. Hence, setting any one of them to zero leads to massless neutrinos. The neutrino mass matrix can again be diagonalized following the procedure described above. The number of parameters to be determined in the experiment is the same as in the leptonic seesaw.

Notice the fact that in this seesaw version, the neutrino mass matrix m_ν is linear in the Yukawa couplings instead of quadratic as seen in Types I and III. This implies that a future measurement of all elements of the low energy neutrino mass matrix would directly provide the full flavour structure of the high-energy theory. Another key difference with respect to leptonic seesaws is the fact that Type II is born with two different mass scales, μ_Δ and M_Δ, embedded on it. If $Y_\Delta \sim \mathcal{O}(1)$ we must have

$$\Lambda_{LN} \equiv \frac{M_\Delta^2}{\mu_\Delta} \sim 10^{13} \text{ GeV} \,, \tag{1.63}$$

and we can lower M up to the TeV if $\mu_\Delta \sim 10^{-6}$ GeV. This fact will turn out to be essential when we discuss the relation between Minimal Flavour Violation and neutrino mass generation in Ch.2.3.

1.4.3 Seesaws at the low scales

Enlarging the right-handed sector in the case of fermionic seesaw allows one to ontain natural neutrino masses with a low new physics scale, maybe even as low as the TeV. We exemplify the argument for the case of Type I Seesaw. We introduce two sets of right-handed neutrino singlets $N_{R\alpha}$ and $N'_{R\alpha}$ coupling to the left-handed neutrinos of the SM through the mass matrix

$$\mathcal{M} = \begin{array}{c} \\ \\ \end{array} \begin{array}{cc} \begin{array}{ccc} \bar{\nu}_L & \overline{N_R^c} & \overline{N_R'^c} \end{array} & \\ \begin{pmatrix} 0 & Y_N & 0 \\ Y_N & 0 & m_N \\ 0 & m_N & \mu \end{pmatrix} & \begin{array}{c} \nu_L^c \\ N_{R\alpha} \\ N'_{R\alpha} \end{array} \end{array} , \tag{1.64}$$

where all entries of the mass matrix may or may not be flavour-charged. For m_N larger than the EW scale, we can integrate out the two right-handed species to obtain neutrino masses at tree level. Since for μ very small, the right-handed fields have mass eigenvalues of order m_N, one would guess naively a suppression for the mass of the SM neutrinos of $O(1/m_N)$. However, this is not the case. The contributions of N_R and N'_R to the Weinberg operator cancel each other and leave a stronger suppression

$$c_{d=5} = Y_N^T \frac{\mu}{m_N^2} Y_N \,. \tag{1.65}$$

As a matter of fact, this could have been inferred from the beginning. Lepton Number violation dissappears from this Lagrangian if we make either $\mu \to 0$ or $m_N \to \infty$ [3] and therefore, the coefficient of the Weinberg operator must go to zero in these limits. The rest is dimensional analysis. In this case as in Type II, it is the interplay of two different energy scales what gives the lepton number violation. This mechanism is known as *inverse seesaw* [32].

Finally it should be stated there are other ways to generate neutrino masses, other than Seesaw models, in order to lower the scale of new physics. We mention two other general possibilities

1. The neutrino mass is generated radiatively. Λ_{LN} doesn't need to be very high since additional suppression is guaranteed by the loop integrals [121–133].

2. The Weinberg operator is forbidden by a symmetry but neutrino masses still appear from effective operators of dimension higher than 5 [145–150].

There is much literature on loop-generated neutrino masses. A less studied subject is the viability of the second possibility, an issue to which we will come back in Ch. 4

[3] Notice that the same reasoning *does not apply* for the limit $m_N \to 0$. That is because if we take this limit leaving the rest of the parameters constant, at some point m_N becomes smaller than μ. That is, the right-handed neutrinos become active and they can no longer be integrated out!

1.4.4 Cosmological baryon asymmetry

Apart from leading to naturally small neutrino masses, the Seesaw Models have another appealing implication. Namely, they are among the simplest and most popular ways to explain the puzzle of baryon and lepton asymmetry.

It has been known for some time that, even when the SM fulfills all three Sakharov conditions for successful baryogenesis [33] - i.e., BN Violation, $C$and CP violation and Departure from Thermal Equilibrium, the amount of CP violation, provided by the CP phase δ in the U_{CKM}, does not suffice to generate the observed baryon asymmetry. Therefore, the puzzle of cosmological baryon asymmetry is another evidence of BSM physics.

The fact that the simple extension of the SM with two or more right-handed neutrino singlets provided the necessary ingredients to solve the puzzle was realized in [34]. The BN violation is provided, first by the LN violating decay of the heavy N_Rs and then by the $B - L$ conserving but $B + L$ violating sphaleron processes. These turn some of the lepton asymmetry into baryon asymmetry after the universe has cooled down to a temperature below the typical masses of the heavy neutrinos. On the other hand, a sufficient amount of CP asymmetry is generated through interference of the tree and one-loop level diagrams of neutrino decay. Finally, the out of equilibrium epoch appears at the time the decay process of the heavy neutrinos freezes due to the cooling of the universe. In order for baryogenesis to work in this scenario, it is necessary that the freezing occurs at $T > 10^{10-12}$ GeV which puts a bound on the masses of heavy neutrinos

$$M \gtrsim 10^{12}\text{GeV} \tag{1.66}$$

1.5 Neutrino oscillations: Theory

As we briefly reviewed in Sec.1.2, historically, non-zero neutrino masses were inferred from neutrino oscillation experiments. It is pertinent to recall the theory as well as the knowledge we have gained so far of the oscillation parameters.

The field basis in which the mass terms of the Lagrangian are diagonal is called the *mass basis* and it is associated with the eigenstates of the SM Hamiltonian. This is opposed to the *flavour basis* in which the interaction terms have the canonical form. The two field basis are related by the PMNS matrix. Neutrino states are related by the same matrix U_{PMNS} that operates on the fields

$$|\nu_i\rangle = U|\nu_\alpha\rangle\,. \tag{1.67}$$

From now on we refer to the PMNS matrix as U.

The states that take part in the production and measurement processes are the neutrino flavour eigenstates. Neutrinos coming from typical sources - solar, atmospheric or reactor neutrinos - have energies of the order of the MeV or higher and, given the upper bounds on neutrino masses, can be considered ultrarelativistic.

Neutrino oscillations is a process of quantum interference between components a neutrino state with different four-momenta. However, the detection process in a neutrino experiment consists in the interaction of a neutrino with a nucleon in the detector that generates a transition between two nucleon states with well defined energy. On the other hand, the wavefunction of both nucleon states must consist of a superposition of momentum eigenstates such that the probability of finding the nucleon outside the detector vanishes. A transition between the two is produced by components of the neutrino state with well defined energy and different momenta. Because it is impossible to determine which momentum component of the neutrino state is responsible for the nucleon transition, there is interference. This is in other words, a double-slit experiment in momentum space [35].

It is therefore correct to treat the incident neutrino as a superposition of states with different momenta that peak around a common energy

$$p_i \simeq E - \frac{m_i^2}{2E} \,. \tag{1.68}$$

We can write the evolution equation as

$$i\frac{d}{dL}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} \simeq U \begin{pmatrix} 0 & & \\ & \frac{\Delta m_{12}^2}{2E} & \\ & & \frac{\Delta m_{13}^2}{2E} \end{pmatrix} U^\dagger \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} , \tag{1.69}$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$. The transition amplitude between states for a produced $|\nu_\alpha\rangle$ to be detected as a ν_β is given by

$$\mathcal{A} = \langle \nu_\beta(L,t)|\nu_\alpha(0,0)\rangle = \sum_{i,j} \langle \nu_i | U_{\beta i}^* e^{-i(p_i L - Et)} U_{\alpha j} |\nu_j\rangle \,, \tag{1.70}$$

where L and t are the distance travelled and the time ellapsed since the neutrino production and the p_i are the average neutrino momenta. For the transition probability we have

$$P(\nu_\alpha \to \nu_\beta) = |\mathcal{A}_{\alpha\beta}|^2 = \sum_{i,j} U_{\alpha j} U_{\beta j}^* U_{\alpha i}^* U_{\beta i} e^{-i(p_j - p_i)L} \,. \tag{1.71}$$

Equation (1.71) is completely general and depends on the full PMNS matrix U with three real parameters. However, depending on the initial state and the baseline L the oscillation formula for a specific experiment can be well approximated by two-flavour vacuum oscillations where just one out of the three angles is relevant. Taking the $\nu_e \to \nu_\mu$ oscillation as an example we have in this approximation

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle - \sin\theta|\nu_2\rangle \,, \\ |\nu_\mu\rangle &= \sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle \,, \end{aligned} \tag{1.72}$$

and the assumptions above we can write the evolution equation for the two-flavour case as

$$i\frac{d}{dL}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = U\begin{pmatrix} 0 & \\ & \frac{\Delta m_{21}^2}{2E} \end{pmatrix}U^\dagger \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} . \tag{1.73}$$

Solving for the two-flavour $\nu_e \to \nu_\mu$ oscillation it follows for the vacuum oscillation probability

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) . \tag{1.74}$$

Notice that the probability of oscillation has a maximum for

$$\frac{\Delta m^2 L}{2E} \sim \pi . \tag{1.75}$$

Naturally, Eq.(1.74) is valid for the other two cases as well just replacing θ and Δm^2 with the proper values. The formula is useful because most experiments are designed to better probe one of the mixing angles and the oscillation probabilities can be approximated by it reasonably well.

For neutrinos traversing matter things change a little bit. The four-Fermi interaction of neutrinos with matter adds terms to the potential that depend on the fermionic constituents of matter. This wouldn't have any effects if all neutrino types were influenced in the same way but as it happens only electron neutrinos can interact coherently through charged currents with electrons

$$\mathscr{L}_{Fermi} = -2\sqrt{2}G_F(\bar{\nu}_{eL}\gamma^\mu \nu_{eL})(\bar{e_L}\gamma_\mu e_L) . \tag{1.76}$$

Averaging the electronic factor

$$\langle \bar{e}_L \gamma^\mu e_L \rangle = \delta_{\mu 0}\frac{N_e}{2} , \tag{1.77}$$

where N_e is the average electron number density. The evolution equation, Eq.(1.69), gets a new term

$$i\frac{d}{dL}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \left[U\begin{pmatrix} 0 & & \\ & \frac{\Delta m_{12}^2}{2E} & \\ & & \frac{\Delta m_{13}^2}{2E} \end{pmatrix}U^\dagger + \begin{pmatrix} \sqrt{2}G_F N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} . \tag{1.78}$$

This is one of the most important equations in neutrino oscillations. In fact, it provides the final theoretical piece needed to solve the solar neutrino puzzle. A complete analysis of the solutions of this equation is beyond the scope of this chapter though we will come back to it when we examine neutrino non-standard interactions in Ch.2. Nevertheless, we can provide a taste of what's going on with the two flavour case.

With two flavours we can write Eq.(1.78) more explicitly by substituting U in terms of the mixing angle. We have

$$i\frac{d}{dL}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}=\begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta+\sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \tag{1.79}$$

Upon diagonalizing the characteristic angle of the transformation is no longer θ but some effective θ_{eff} characteristic of matter. That is, matter effects change the mixing angle between the mass and flavour basis!

The expression for the angle θ_{eff} as a function of θ and N_e can be found by diagonalizing the matrix in Eq.(1.79). We have

$$\tan 2\theta_{eff}=\frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\frac{\Delta m^2}{2E}\cos 2\theta-\sqrt{2}G_F N_e}\,. \tag{1.80}$$

The value zero in the denominator corresponds to $\theta_{eff}=\pi/4$ or maximal mixing. This is the so called MSW *resonance condition*[4] as

$$\sqrt{2}G_F N_e=\frac{\Delta m_2}{2E}\cos 2\theta\,. \tag{1.81}$$

The presence of a resonance has some striking consequences. It implies that, for a medium of constant N_e, the oscillation probability can be quite large - even maximal - no matter how small the vacuum mixing angle may be. The analysis can be extended for a medium such as the Sun, where the electron decreases notably as neutrinos proceed in their way out, provided the neutrino evolution takes place adiabatically. That is, if the matter density varies slow enough. In that case the oscillation probability measured in a far away detector comes out to be

$$P(\nu_e\to\nu_\mu)=\cos^2\theta \tag{1.82}$$

implying that conversion is greatest for small mixing angles. This can be understood as follows.

Only electron neutrinos are created inside the sun. A small θ means that electron neutrinos in the vacuum are chiefly composed of the eigenstate 1. But, according to Eq.(1.80), θ_{eff} - which replaces θ in matter - increases as N_e does, it passes through $\pi/4$ giving the resonance condition, and keeps increasing until $\frac{\Delta m^2}{4E}\ll N_e$. In this limit the evolution matrix is almost diagonal and we must have $\theta_{eff}\sim\pi/2$. But, from Eq.(1.72), we see that this means that electron neutrinos created inside the sun are almost entirely made of energy eigenstate 2! Adiabaticity implies that the transitions between energy eigenstates are exponentially suppressed so the neutrinos that come out of the sun are mostly in eigenstate 2 which, due to the smallness of the mixing angle is chiefly composed of the muon flavour. Therefore, the smaller the mixing angle, the higher the oscillation probability.

[4]For Mikheyev, Smirnov and Wolfenstein who first described the effect [36].

1.6 Neutrino oscillations: Experiments

Oscillations experiments have been by far the most successful ones in constraining the parameters in the SM pertaining to neutrino physics. First and foremost, they have allowed us to gain precise information about the angles in the mixing matrix, Eq.(1.42). Secondly we have been able to determine the two independent mass squared differences that can be defined for three light neutrinos. Oscillation experiments are not able to determine the overall scale of neutrino mass which is therefore left undetermined.

It is customary to define the mass states ν_1 and ν_2 as the ones that give the smallest mass-squared of all and to further impose $m_1^2 < m_2^2$. This leaves ν_3 as a state than can be heavier or lighter than the couple ν_1 and ν_2. These two scenarios are called the *normal* and *inverted* hierarchy respectively.

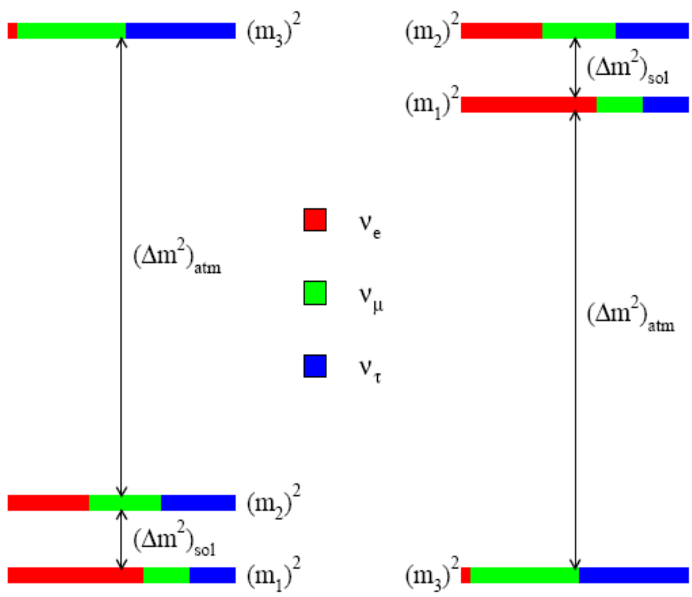

Figure 1.3: Neutrino mass hierarchies: Left, normal hierarchy, Right: inverted hierarchy.

From data it follows [**?**, 37–39]

$$\Delta m^2_{21} = \big(7.59 \pm 0.20\big) \times 10^{-5}\ \mathrm{eV}^2\,, \tag{1.83}$$

$$|\Delta m^2_{31}| = \big(2.40^{+0.12}_{-0.11}\big) \times 10^{-3}\ \mathrm{eV}^2\,. \tag{1.84}$$

The most relevant mass-related unconstrained parameter is undoubtedly the sign of Δm^2_{31} which selects the type of hierarchy. This is important because the two hierarchies might suggest quite different sets of theories of neutrino mass.

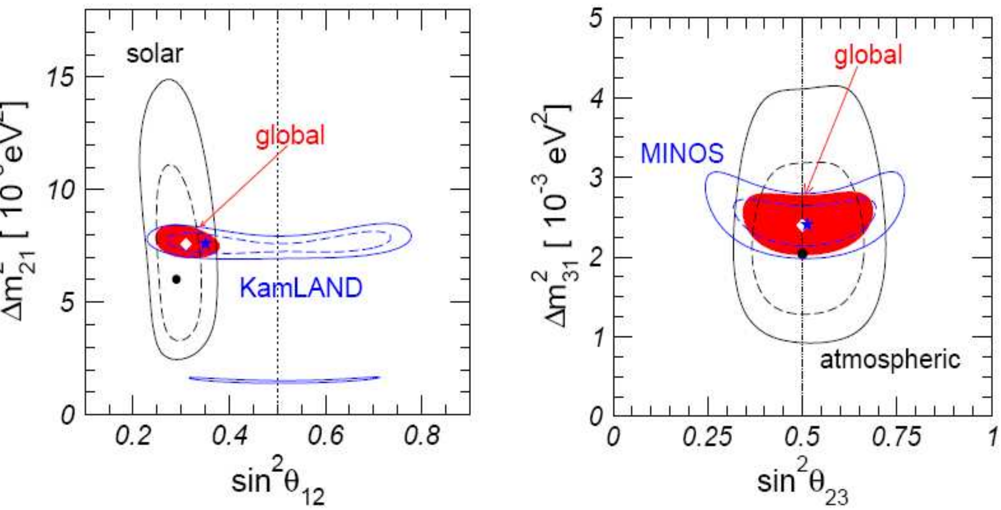

Figure 1.4: Global fit for the ranges of the solar (left) and atmospheric (right) oscillation parameters. Notice the orthogonal constraints put by reactor experiments.

For the mixing angles $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ have been measured to be fairly big and are well determined while accurate data on $\sin^2\theta_{13}$ is lacking. The following list summarizes the most recent data [37]:

$$\sin^2\theta_{12} = 0.304^{0.022}_{-0.016}\,, \tag{1.85}$$

$$\sin^2\theta_{23} = 0.50^{+0.07}_{-0.06}\,, \tag{1.86}$$

$$\sin^2\theta_{13} = 0.01^{+0.016}_{-0.011}\,. \tag{1.87}$$

There are a number of unknowns remaining that call our attention within these values. First, $\sin^2\theta_{13}$ is virtually unconstrained. This is important since it is the remaining key to be found to measure the Dirac-like CP phase δ and thus it is a majora planetary goal at present. All we know is that it is small compared to the other two. Furthermore, a glance at U_{PMNS} shows that $\sin^2\theta_{13}$ has a straightforward interpretation: it is the amount of ν_e in the ν_3 state. Our present data allows θ_{13} to be zero and therefore we can not tell whether there is some electronic component in the third eigenstate or not.

Secondly, in a very good approximation $\sin^2\theta_{23} \sim 1/2$. In other words, the sign of $1 - 2\sin^2\theta_{23} = \cos 2\theta_{23}$ is unconstrained. Along with θ_{13}, the fact that at least two of the mixing angles for the neutrinos seem to have "special" values ($\theta_{13} \simeq 0$, $\theta_{23} \simeq \sqrt{2}/2$) remains puzzling and seems to be calling for a theoretical explanation. Ideas that have been explored include discrete symmetries among others [40].

We know nothing about the CP-phase either. There is at present no experiment capable of measuring δ though several proposals under discussion are commented below.

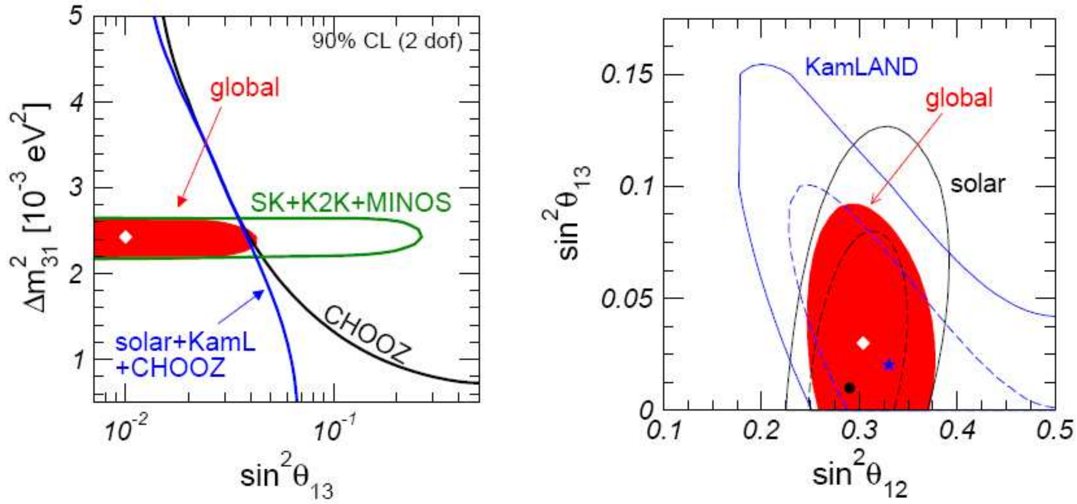

Figure 1.5: Upper bounds for $\sin^2\theta_{13}$. Left: $\sin^2\theta_{13}$ vs Δm^2_{31}. Right: $\sin^2\theta_{12}$ vs $\sin^2\theta_{13}$

Notice that in any case, the amount of CP violation is proportional to the leptonic Jarlskog invariant, analogous to that of the quarks, Eq.(1.18). The latter is in turn proportional to all three mixing angles and in particular to $\sin\theta_{13}$. Therefore, measuring CP violation in the leptonic sector is only possible if $\sin\theta_{13}$ is not too small.

The experimental future for all these parameters looks nevertheless promising, the precise determination of them being the goal of the next generation of experiments.

1.6.1 Future prospects

New experiments are being built with the express purpose of improving the bounds or measuring the value of θ_{13} in case it is significantly different from zero [41]. In order to probe this sector one needs to match the baseline L to the energy of the neutrino beam E_ν optimally for the $1 \to 3$ oscillation, Eq.(1.74). For a reactor anti-neutrino beam energy a few MeV, a baseline $L \sim 1$km is needed.

The Chooz experiment put the best present bounds to θ_{13} [42]. It used a detector located at 1.05 km from the reactor core. Several upgrades to the Chooz approach are being carried out in experiments that expect to be sensible to a value of $\sin^2 2\theta_{13}$ as small as 10^{-2}. Two already approved experiments seem to be the main contenders in the coming race of measuring or improving the bounds on θ_{13}. These are Double Chooz [43], located at the same site of the original Chooz experiment, and the Daya Bay Reactor Neutrino Experiment, located near the homonimous body of water in China, which will deploy three identical detectors at various distances ($\tilde{1}$ km) from three nuclear reactors [44].

The other possibility to measure θ_{13} is by looking for $\nu_\mu \to \nu_e$ oscillations at values of L/E matched to Δm^2_{32}. This is a clever design! Since most muon neutrinos will oscillate into taus, detecting electron neutrinos probes in fact θ_{13}. Actually, by looking at a process used to probe the 12 sector with an experimental setup appropriate for probing the 23 sector one makes the oscillation probability dependent on all three angles *and* the CP phase δ. Also in this case two experiments compete: T2K in Japan [45], that will produce an intense beam of muon neutrinos shooted from the J-PARC facility and use the existing SuperKamiokande detector 295 km away. The other experiment that might is NoνA [46] which will use the beam currently used by MINOS at two detectors, far and near. NoνA hasn't been approved yet though.

The next generation of experiments brings in new designs that will improve the accuracy of the measurements of θ_{13} and the CP phase δ. Two proposals of new machines have been extensively explored in the recent literature and deserve a comment here. These are the Neutrino Factories (ν-Factories) and the Beta Beams (β-Beams).

The β-Beam is a future neutrino facility [47] which would produce pure and intense (anti) electron neutrino beams, by accelerating radioactive ions and storing them in a decay ring. The resulting beam is virtually background free and the fluxes can be easily computed by the properties of the beta decay of the parent ion and by its Lorentz boost factor γ. The major goal of β-Beam experiments is to measure CP violation in the lepton sector. This requires a comparison of neutrino versus antineutrino oscillations in an appearance experiment [48]. The scenario considered is to produce the neutrino beams at CERN and fire them into a water Cerenkov detector located at the Fréjus Underground Laboratory, at about 130 km from CERN. This distance fits greatly the $\nu_e \to \nu_\mu$ oscillation maximum at the atmospheric scale.

The main competitor for the next generation neutrino money is the Neutrino Factory [49]. This is a machine which would exploit muon decays ($\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ and $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ with $E_\mu \sim 50\,GeV$) as source to obtain a neutrino beam The NF beam is produced by circulating μ^- or μ^+ beams in accumulators with big straight sections (the muon life time is quite larger than the pion one), but still smaller than the β-beam rings. The energy and flavour spectra of a ν-factory beam is easily and accurately computed. Also, conventional neutrino beams from π^- (or π^+) decays are dominantly composed by $\bar{\nu}_\mu$ (or ν_μ). On the contrary, a neutrino beam produced by decays of μ^- (or μ^+) consists of $\nu_\mu + \bar{\nu}_e$ (or $\bar{\nu}_\mu + \nu_e$). Finally, the energies handled at the ν-Factory allows to extend the baseline to several thousand kilometers of distance.

We conclude this section by mentioning that as for the Majorana phases we have absolutely no clue about what they might be. Typically they are much more hard to measure since they tend to cancel in the amplitudes for most processes. In any case, the one experiment that depends crucially on the Majorana phases and is expected to shed light on their values in the near future is *neutrinoless double beta decay*.

1.7 Neutrinoless double-beta decay

Nuclear double beta decay occurs whenever the ordinary beta decay is forbidden due to energy conservation or very suppressed due to angular momentum conservation. *Neutrinoless* double beta decay ($0\nu\beta\beta$) [50] is particularly interesting since it violates $B-L$ by two units and it can decide whether neutrinos are Majorana particles or not. The nuclear reaction is

$$(A, Z) \to (A, Z+2) + e^- + e^- \,. \tag{1.88}$$

with the typical mechanism being via the exchange of a Majorana neutrino between the two decaying neutrons. The process is depicted in Fig.1.6.

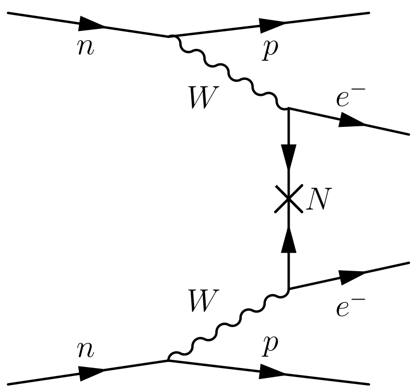

Figure 1.6: Neutrinoless double beta decay.

The effective beta decay Hamiltonian has the form

$$\mathscr{H} = \sqrt{2} G_F (\bar{\ell}_{Le} \gamma_\mu \nu_{Le}) J_L^{\mu\dagger} + \text{h.c.} \tag{1.89}$$

where J_L^μ represents the hadronic current. Neutrino mixing is given as usual by

$$\nu_{Le} = U^{ek} \nu_{Lk} \,. \tag{1.90}$$

Under the assumptions that massive neutrinos ν_j are Majorana particles and that $\beta\beta_{0\nu}$-decay is generated *only* by the exchange of Majorana neutrinos via the $V-A$ charged current weak interaction, the amplitude of neutrinoless double beta decay is proportional to an effective Majorana mass defined in terms of the mixing parameters

$$\langle m_{ee} \rangle = m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{2i\alpha_1} + m_3 |U_{e3}|^2 e^{2i\alpha_2} \,. \tag{1.91}$$

It is possible to constrain the values of $\langle m_{ee} \rangle$ using the experimental values for the known entries of the U matrix. This is particularly useful for discriminating between normal and inverted hierarchy. The fact that it is so can be understood by noticing that the effective mass in Eq.(1.91) is proportional to the mass eigenvalues. In the case of the inverted hierarchy the mass eigenstate that couples to the electron neutrino - which is the only flavour that participates in the process - is the heaviest one and thus we expect the decay ratio for the case of inverted hierarchy to be considerably larger.

We can confirm the intuitive idea above with a quick computation. In the case of the normal hierarchy we can neglect m_1 and approximate the mass differences by

$$m_2^2 \simeq \Delta m_{12}^2\,, \quad m_3^2 \simeq \Delta m_{23}^2 \tag{1.92}$$

The phase ambiguity is replaced by a sign ambiguity and the effective mass is approximately given by

$$\langle m_{ee} \rangle \simeq |c_{13}^2 s_{12}^2 \Delta m_{12}^2 \pm s_{13}^2 \Delta m_{23}^2| \tag{1.93}$$

In this case, a small factor, Δm_{12}^2 and $\sin\theta_{13}^2$ respectively, appears in both terms. On the other hand, for the case of the inverse hierarchy we can neglect the combination $m_3 \sin^2\theta_{13}$ and approximate

$$m_1 \simeq m_2 \simeq \Delta m_{23}^2, \cos^2\theta_{13} \simeq 1 \tag{1.94}$$

and the effective mass is written as

$$\langle m_{ee} \rangle \simeq \sqrt{\Delta m_{23}^2} |\cos^2\theta_{12} \pm \sin^2\theta_{12}|\,. \tag{1.95}$$

One can see that now the large mass difference Δm_{23}^2 weighs both terms. As a consequence $\langle m_{ee} \rangle$ is significantly different in both cases. This implies that not only a positive signal would be enough to determine which is he correct hierarchy but even a negative signal, with sensitivity at reach in the next generation of experiments, would exclude the inverse hierarchy scenario. For $\theta_{13} = 0$ and $|\Delta m_{23}^2| = 2 \times 10^{-3}\mathrm{eV}^2$ one has

$$0.0 \leq \langle m_{ee}^{NH} \rangle (\mathrm{eV}) \leq 2.6 \tag{1.96}$$

$$19.9 \leq \langle m_{ee}^{NH} \rangle (\mathrm{eV}) \leq 50.5 \tag{1.97}$$

Many experiments have put direct bounds on the $\langle m_{ee} \rangle$ combinationThe strongest bounds on the half-live of the $0\nu\beta\beta$-decay of different nuclei were obtained by the Heidelberg-Moscow collaboration and the IGEX experiment and more recently by the CUORICINO and NEMO experiments.

In both the Heidelberg-Moscow and the IGEX collaboration the decay ${}^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se} + e^- + e^-$. Both experiments measured a half-life for germanium of around 2×10^{25}y thus the bound

$$\langle m_{ee} \rangle < 0.35\,\mathrm{eV} \tag{1.98}$$

was inferred by the Heidelberg-Moscow team while the IGEX collaboration, using different nucleon matrix elements (NMEs) found

$$\langle m_{ee} \rangle < (0.33 - 1.35)\,\mathrm{eV} \tag{1.99}$$

In the CUORICINO experiment, the search for $0\nu\beta\beta$-decay of ^{130}Te was performed. No evidence was found so the upper limit

$$\langle m_{ee} \rangle < (0.19 - 0.68)\,\mathrm{eV} \tag{1.100}$$

was inferred.

For the future, there are a number of proposed designs which will be able to lower the upper bound of $\langle m_{ee} \rangle$ down something of order 10^{-2}eV. These include the Italian based GERDA and CUORE as well as the innovative EXO experiment in New Mexico using Xenon. All these experiments should be bale to probe fully the inverse hierarchy region.

Neutrinoless double beta decay experiments remain the only practical way to discern the fundamental question, are neutrinos really Majorana particles? As such they are of utmost importance. A positive signal would be a strong support for the seesaw classes of models as theories of neutrino masses with every bit of exciting physics they bring, including new sources of flavour violatio, $B - L$ violation and a viable candidate mechanism for baryogenesis through leptogenesis.

Chapter 2

Elements of flavour physics in the lepton sector

It has been remarked that the SM, with the addition of a mechanism to provide masses for the neutrinos, is experimentally a very successful theory. Still, we do not expect the SM to be valid all the way up to the Planck scale. That is because we don't have a quantum theory of all four interactions and extreme gravity phenomena, such as black holes, require a consistent quantum theory of gravity which we do not possess.

On the other hand, if we assume that there is physics Beyond the Standard Model (BSM), then the *Hierarchy Problem* arises because the Higgs mass is not protected by any symmetry from acquiring large quantum corrections coming from the scale at which such new physics lives. A handful of solutions have been proposed and have managed to atract enough attention. The most popular is supersymmetry which relates particles with spins that differ in 1/2. If supersymmetry is a symmetry of nature then quantum corrections to the Higgs mass would come in pairs with the fermionic and bosonic contributions cancelling among themselves. That is, the supersymmetric solution to the Hierarchy Problem amounts to protecting the Higgs mass from quadratic corrections using the enlarged symmetry of the Lagragian. Other solutions share this rationale. In models with extra-dimensions, the Higgs can be identified with the extradimensional component of gauge fields and the gauge symmetry prevents it from acquiring large quantum corrections. In Technicolor and the more recent Little Higgs models, the Higgs is identified with the Goldstone boson of a global symmetry which again prevents it from becoming too massive. Solutions that do not rely in a symmetry to fix the Higgs mass scale include lowering the Planck scale as in models with Large Extra-dimensions.

Many of these models fail though and the reason lies in the physics of flavour. The flavour structure of the SM cannot be easily tweaked without generating signals that should have been measured long ago. The naive versions of BSM models typically introduce too large Electric Dipole Moments (EDMs) and/or Flavour Changing Neutral Currents (FCNCs) that are not allowed in the SM. And unfortunately, our theoretical understanding of flavour - The Flavour Puzzle - has been stalled for decades even when neutrino masses added precious bits of information.

In this chapter we deal with the physics of flavour in the lepton sector. In particular we touch aspects that are central to this thesis. We examine FCNCs and explain why they are so suppressed in the SM. Then we introduce an ansatz, Minimal Flavour Violation, that mimics the flavour behaviour of the SM. We proceed to examine two other signatures of exotic flavour structure in models BSM, namely, non-unitarity of the PMNS matrix and Non-Standard Neutrino Interactions.

2.1 Scales of flavour physics

We have argued in Ch.1 that the smallness of neutrino masses is more easily accomodated if $U(1)_{LN}$ symmetry is only violated at some very high energy scale Λ_{LN}. Still it is possible for lepton flavour violation to be induced at a different scale Λ_{FL} - which could be as low as the TeV - while LN is still an approximate symmetry. As long as those new scales are larger than the electroweak one, a model-independent representation is given by an effective theory of the type

$$\mathscr{L} = \mathscr{L}_{SM} + \frac{\hat{c}^{d=5}}{\Lambda_{LN}}\mathcal{O}^{d=5} + \sum_i \frac{\hat{c}_i^{d=6}}{\Lambda_{FL}^2}\mathcal{O}_i^{d=6} + ... \tag{2.1}$$

where the operators $\mathcal{O}$ are composed of SM fields. As usual, the only $d = 5$ operator is Weinberg's . The dimensionless couplings $\hat{c}^{d=5}$, $\hat{c}_i^{d=6}$, ... may be assumed to be of $\mathcal{O}(1)$, while the effective scales Λ_{FL}, Λ_{LN}, take care of the suppressions of each type of contribution.

At $d = 6$ there is a plethora of operators [28] contributing to the sum in Eq.(2.1). Among them, there are four-fermion operators such as those typical of the Fermi interaction and also operators with two fermions and two Higgses that modify the fermionic kinetic terms. As it is shown below, all these operators may be responsible for exotic flavour physics that would be a confirmation of BSM physics. No operator at $d = 6$ violates LN.

As it is discussed below, exotic flavour processes are a good probe of new physics. It is clear that in order to have them sizable, Λ_{FL} can not be very large. This is not so if, for instance, $\Lambda_{LN} \equiv \Lambda_{FL}$ as it happens when the same new physics is responsible for giving rise to both the Weinberg operator and the exotic flavour processes. In that case, the LN scale suppresses the $d = 6$ operators strongly and their effects are not observable. A question to which we will return is whether we can impose a relation $\Lambda_{LN} \gg \Lambda_{FL}$ consistently. Another possibility we consider is forbidding the Weinberg operator in Eq.(2.1) by means of an additional symmetry. In that case, the first LN violating operators appear in the series above at $d = 7$, suppressed by a factor Λ_{LN}^3. Then a huge Λ_{LN} is not necessary to generate neutrino masses and sizable flavour processes might be at reach even if $\Lambda_{LN} \sim \Lambda_{FL}$. Finally, if the neutrino mass appears at the loop level, the loop suppression factors can help to reduce the scale of new physics.

2.2 Flavour changing neutral processes in leptons

From the point of view of the experimentalist, in order to tell new physics apart, one would want a clean signal that can be easily discriminated from the SM backgrounds. From the point of view of the theorist it is important to determine what kind of new physics appears in a majority of the SM extensions, or putting it the other way around, which of the SM predictions makes it special when compared to other possibilities. With that in hand, we could point to the eager experimentalist in the right direction and tell her: 'Expect the SM to breakdown here and there since most of our models say it should'.

EDMs experiments are ideal in this sense since they only appear at the two-loop level in the SM while typically less suppressed in SM extensions. Processes involving FCNCs are almost as good, especially in the lepton sector. That is because they emerge in the SM at the loop level and even then they appear very suppressed, by means of the *GIM mechanism* which is particularly efficient for leptons. These are salient features that can be easily disrupted. For instance, many models of neutrino mass predict relatively large FCNC processes that could be detected in near future experiments. Since we don't think that the SM is the final theory of nature, we expect EDMs and FCNCs to be there.

Since we are mainly interested in the physics of flavour, we will leave aside the EDMs and proceed to illustrate FCNCs in this section. In order to do so, it is convenient to examine the decay $\mu \to e\,\gamma$, a typical example useful to carry the more general discussion [51]. Incidentally, we recall that it was shown in Chap.1 that in the limit in which the masses of all three charged leptons are degenerate there is no flavour violation. Thus, the FCNC processes that we study in this section only happen in theories with at least two neutrinos with non-degenerate masses.

The diagram for $\mu \to e\gamma$ is pictured in Fig.2.2. If p and p' are the incoming and outcoming momenta respectively then, using Lorentz invariance and the Ward identities from charge conservation, one can infer the general form for the amplitude of the process to be

$$\bar{u}'(p')\left[F(q^2) + F_5(q^2)\gamma_5\right]\sigma_{\lambda\rho}q^\rho u(p)\epsilon^\lambda \tag{2.2}$$

where $q = p' - p$ and $\bar{u}'(p)$, $u(p)$ and ϵ^λ are the muon, electron and photon polarizations respectively. From here, putting in the kinematics, we have that the decay rate in the rest frame of the muon is given by

$$\Gamma = \frac{(m_\mu^2 - m_e^2)^3}{8\pi m_\mu^3}\left(|F|^2 + |F_5|^2\right). \tag{2.3}$$

This is completely general. All the model dependency of the process is coded in the functions F and F_5.

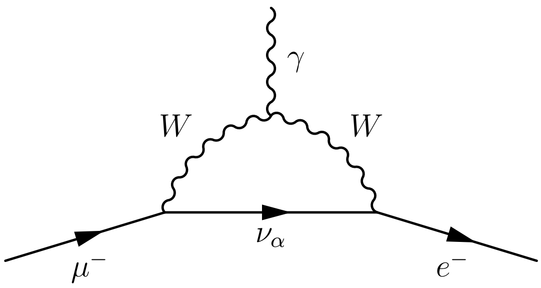

Figure 2.1: Diagram for $\mu \to e\,\gamma$.

For the Standard Model, with $m_\mu \gg m_e$ it follows from the diagram

$$F = F_5 = \frac{eG_F m_\mu}{8\sqrt{2}\pi^2} \sum_\alpha U^*_{\mu\alpha} U_{e\alpha} f(r_\alpha) \tag{2.4}$$

with $r_\alpha = (m_{\nu_\alpha}/M_W)^2$. The function f can be expanded around $r_\alpha = 0$ and the constant term doesn't contribute due to the unitarity of the mixing matrix. This is the GIM mechanism in action. The first contribution to the amplitude of the process is proportional to the small quotient r_α.

We shall see in a moment that as a consequence all that is needed for this treatment is an order of magnitude estimate of the decay. In that case, the first order term in the expansion of the function $f(r_\alpha)$ can be approximated by r_α. Using Eqs.(2.3) and (2.4) we obtain for the decay rate

$$\Gamma \simeq \frac{1}{2}\alpha G_F^2 \left(\frac{1}{32\pi^2}\right)^2 m_\ell^5 \left|\sum_\alpha U^*_{\mu\alpha} U_{e\alpha} r_\alpha\right|^2 \tag{2.5}$$

Customarily this result is rather expressed as the branching ratio (BR)

$$B_{\mu\to e\,\gamma} = \frac{\Gamma(\mu \to e\,\gamma)}{\Gamma(\mu \to e\bar{\nu}_e \nu_\mu)} \simeq \frac{3\alpha}{32\pi} \left|\sum_\alpha U^*_{\mu\alpha} U_{e\alpha} r_\alpha\right|^2 . \tag{2.6}$$

We see that the last factor is proportional to the fourth power of the ratio m_{ν_α}/M_W. This is a huge suppression! Taking the reasonable upper bound for $m_{\nu_\alpha} \sim 5$ eV (see Sec. 1.2.1) we obtain

$$B_{\mu\to e\,\gamma} < 10^{-48} \tag{2.7}$$

Notice that the BR doesn't depend on the mass of the decaying charged lepton. Since the entries of the PMNS matrix are $\mathcal{O}(1)$ this limit is in fact also valid for the other two decays: $\tau \to \mu\,\gamma$ and $\tau \to e\,\gamma$. The present experimental bounds for these three processes are summarized as follows

$$B_{\mu\to e\,\gamma} < 1.2 \times 10^{-11} \tag{2.8}$$

$$B_{\tau\to\mu\,\gamma} < 4.5 \times 10^{-8} \tag{2.9}$$

$$B_{\tau\to e\,\gamma} < 1.1 \times 10^{-7} \tag{2.10}$$

It is clear that the SM decays $\ell_\alpha \to \ell_\beta\,\gamma$ are absolutely inaccessible for any foreseeable experiment. This justifies our order of magnitude calculation from Eq.(2.5) onwards.

Another FCNC process that has been extensively studied is the decay $\mu \to e^- e^- e^+$. It is not hard to see diagramatically that this one is equally suppressed since in the SM, it can only happen through the $\mu \to e\gamma$ vertex we have already seen;

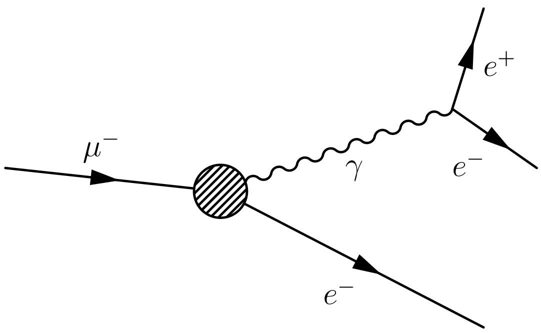

Figure 2.2: Diagram for $\mu \to e^- e^- e^+$.

The detailed calculation reveals that the bound on the corresponding branching ratio is weaker by a couple of orders of magnitude, it is clear that it is again unreachable.

Why do we get such extraordinary suppression? In the SM context, in order for a flavour violating neutral process to occur, a lepton of unspecified flavour must always run inside a loop. This implies in turn that the product $U_{i\alpha} U_{\alpha j}$ will appear in the amplitude. There is no way out of this and thus the GIM mechanism must operate in every FCNC process.

A carbon-copy analysis is valid for the case of the quarks as well although the suppression there is milder because the neutrino masses in r_α are substituted by the u-type quark masses and in particular for the top quark mass. An analogous process to $\mu \to e\gamma$ would be the radiative quark transition $b \to s\gamma$. This process has been found to agree with the SM prediction at the B-factories.

We might also notice that this is a very delicate feature of the SM. New bosons in extended theories can contribute easily to flavour changing processes and new fermions yield non-unitarity of the mixing matrix which forces us to consider the constant term in Eq.(2.4), see Sec.2.4. This makes of FCNCs excellent probes for BSM physics. If the SM model is only the low energy limit of a more fundamental theory we should expect violations of the GIM mechanism to occur, maybe even as big as to produce signals in present or near future experiments.

Interestingly though, it has been known for some time that there is a class of models that extend the SM in which FCNCs remain under control. They obey an ansatz, Minimal Flavour Violation, that we discuss in what follows. This is useful in particular if one wants to mimic the SM flavour behaviour.

2.3 Minimal Flavour Violation

In the SM all flavour is realized through the Yukawa couplings. If the Yukawas are set to zero the Lagrangian gains invariance under a *global flavour symmetry* $U(3)^5_f = SU(3)^3_q \otimes SU(3)^2_l \otimes U(1)^5$. The non-abelian part of this symmetry is particularly important with

$$SU(3)^3_q \equiv SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R}\,, \tag{2.11}$$
$$SU(3)^2_l \equiv SU(3)_{L_L} \otimes SU(3)_{E_R}\,, \tag{2.12}$$

under which the quarks and leptons transform as triplets

$$Q_L \sim (3,1,1)_q\,, \quad U_R \sim (1,3,1)_q\,, \quad D_R \sim (1,1,3)_q\,, \tag{2.13}$$
$$L_L \sim (3,1)_l\,, \quad E_R \sim (1,3)_l\,. \tag{2.14}$$

Then, back in the massive theory we find that any flavour violating process must be proportional to the fermion masses so that the limit holds. These processes are therefore suppressed by the EW scale by dimensional analysis as in Eq.(2.5).

It is never a bad thing to learn a lesson too well. *Minimal Flavour Violation* (MFV), as first proposed in [52] for technicolor models, assumes that "the global symmetry is broken explicitly by terms proportional to the three mass matrices (of the Standard Model)...". Faced with the difficulty of large FCNCs the idea was simply to take advantage of the one symmetry we already know that works, namely that of the SM. If the SM Yukawas are all there is to flavour violation and if in particular, all the amplitudes of all flavour violating couplings imaginable must be proportional to some product of them, then all FCNC effects induced are automatically suppressed by powers of the quark masses and by the scale of new physics. This is necessary to fit quark data coming from B-factories.

It is interesting to notice that many if not most of the modern treatment of MFV was already contained in the original paper by Chivukula and Georgi. In technicolor one introduces some new fermions, the preons, charged under some new strong gauge interaction. The strong dynamics breaks the chiral symmetry of these *preons* in a way similar to the breaking of chiral symmetry in massless QCD. The low energy quasi-Goldstone bosons, analogous to pions, can be used to fulfill the role of the Higgs in giving masses to the SM fields. Minimal Flavour Violation guarantees that these masses will have their SM values. For instance, if Λ_C is the compositeness scale, the operators giving rise to lepton and quark masses would occur at $d = 6$ as[1]

$$\frac{1}{\Lambda_C^2}(\bar{L}_{\alpha L} Y_E^{\alpha\beta} E_{\beta R})(\bar{\psi}_R \Psi_L) + \text{h.c.} \tag{2.15}$$

$$\frac{1}{\Lambda_C^2}(\bar{Q}_{\alpha L} Y_U^{\alpha\beta} U_{\beta R})(\bar{\phi}_R \Psi_L) + \text{h.c.} \tag{2.16}$$

$$\frac{1}{\Lambda_C^2}(\bar{Q}_{\alpha L} Y_D^{\alpha\beta} D_{\beta R})(\bar{\psi}_R \Psi_L) + \text{h.c.} \tag{2.17}$$

where the new fields are the preons and $\Psi_L = (\phi_L \ \psi_L)$ is a doublet of $SU(2)_W$. The flavour structure in these equations is fixed by MFV - that is, it must consist of the low energy Yukawas and not some new flavour couplings. Below the compositeness scale we are left with the mass terms for the SM fermions. Therefore, MFV predicts the SM masses and mixings for these class of models.

The beauty of it all is that we get more than we asked for. We not only achieve the desired GIM-like suppression of FCNCs but we also obtain strong predictions for the flavour physics since now all flavour processes are related among themselves. Furthermore, because the exact amplitudes depend on the details of the underlying model, we can actually have significant deviations from the SM predictions that we could probe in future experiments.

2.3.1 Effective theory of MFV

MFV can be analyzed in a very general framework based on effective field theory [53]. The crucial point is to note that *once we have identified* the couplings that are the source of $SU(3)^5$ flavour symetry breaking, the low energy theory follows simply by making use of the group properties. The real issue lies in this "identification" which has to be done only with the knowledge we have of low energy couplings. Indeed, MFV is "minimal" as a prescription to perform this identification: we assume that Nature works the simplest way possible, that is, we identify the flavour violating couplings we measure at low energies with the fundamental ones.

Assuming MFV we can forget about the fundamental theory and focus instead on the relations it implies between the effective flavour coefficients. This will in turn yield relations between the amplitudes of flavour processes Working for the moment in the

[1] It is to note that in the original paper flavour structures such as $\bar{Q}_{\alpha L} Y_E^{\alpha\beta} U_{\beta R}$) were not considered. Although not emphasized, the original authors were already treating MFV as it is implemented today after [CITE STRUMIA], Sec.2.3.1

quark sector, the key observation is that one can formally recover the flavour symmetry, Eq.(2.11), in the flavour violating Lagrangian

$$\mathscr{L} = -Y_D^{\alpha\beta}\bar{Q}_{L\alpha}HD_{R\beta} - Y_U^{\alpha\beta}\bar{Q}_{L\alpha}\tilde{H}U_{R\beta} + \text{h.c.} \tag{2.18}$$

by having the Yukawas transform under it as

$$Y_U \sim (3,\, \bar{3},\, 1)\,, \quad Y_D \sim (3,\, 1,\, \bar{3})\,, \tag{2.19}$$

that is, by considering the Yukawas as spurion fields so that the theory is invariant under flavour. Hence we define an effective field theory as Minimal Flavour Violating if all higher dimensional operators constructed with the quarks and the Yukawas of the SM *are formally invariant under* $SU(3)_q^3$. In a MFV effective theory, all flavour violating operators are prescribed by group theory arguments.

We can push forward a bit and simplify things by noticing that all other Yukawa couplings can be neglected against that of the top quark. Rotating the spurions by using the $SU(3)_q^3$ symmetry, we have

$$Y_D \to \lambda_d\,, \quad Y_U \to V^\dagger \lambda_u \tag{2.20}$$

where the λ are diagonal matrices and V here is the CKM matrix. Because any combination of Yukawas will be dominated by the mass of the top quark, it makes sense to define

$$(\lambda)_{\alpha\beta} = \begin{cases} (Y_U Y_U^\dagger)_{\alpha\beta} \sim \frac{\sqrt{2}m_t}{v} V_{t\alpha}^* V_{t\beta} & i \neq j \\ 0 & i = j \end{cases} \tag{2.21}$$

This coupling relates down-type quarks so we see that in this scheme it is more likely to find new physics in processes with down-type quarks as final states. Neglecting coefficients of order λ_d^2 we can construct two basic bilinears for processes with external down quarks

$$\bar{Q}_L Y_U Y_U^\dagger Q_L \sim \bar{Q}_L \lambda Q_L\,, \quad \bar{D}_R Y_D^\dagger Y_U Y_U^\dagger Q_L \sim \bar{D}_R \lambda_d \lambda Q_L \tag{2.22}$$

and with these bilinears one can build a full set of independent dimension six ($d = 6$) operators $\mathcal{O}_i$ invariant under the flavour group. In [53] the operators were conveniently divided in $\Delta F = 2$ operators, which necessarily involve four quarks and of which only the following is linearly independent

$$\mathcal{O}_0 = \frac{1}{2}\left(\bar{Q}_L \lambda \gamma_\mu Q_L\right)^2; \tag{2.23}$$

and $\Delta F = 1$ operators, mixing a quark bilinear with a pair of either Higgses, gauge bosons or leptons. There are many of these. In particular they include the operators responsible for the rare decays $B \to X_s\gamma$, involving the B meson plus some other meson charged under strangeness.

$$H^\dagger(\bar{D}_R\lambda_d\lambda\sigma_{\mu\nu}Q_L)F^{\mu\nu}\,, \quad (\bar{Q}_L\lambda\gamma_\mu Q_L)D_\mu F_{\mu\nu} \tag{2.24}$$

This process is analogous to the $\ell_\alpha \to \ell_\beta\gamma$ analyzed in the elast section. We refer to [CITE] for the complete list of operators.

The program is now clear. The determination of the CKM matrix is not strongly affected by the MFV new physics. For one, the bounds on the rare flavour processes can be translated into lower bounds on the effective scale of new physics Λ. For the effective operators described, these are situated around the TeV with a somewhat larger bound for the operators involving photons. More importantly it is possible to relate predictions for FCNC processes for instance in the meson systems. This is clearly a prediction in itself to be probed in case FCNCs are measured with high accuracy.

Finally, one can tackle MFV in the context of particular extensions of the Standard Model. Things will certainly change if an enlarged Higgs sector with two scalar doublets is assumed. This is because the Yukawa coupling of the bottom quark in that case can be of order unity provided $\tan\beta$ is very large. In any case, the effective field theory approach may be pertinent, even with some more exotic possibilities, like extra-dimensional theories or supersymmetry.

2.3.2 MFV in the lepton sector

Neutrino masses and neutrino oscillations in particular established clearly the non-conservation of flavour in the lepton sector. This opens up the possibility of implementing the MFV hypothesis for leptons in a way analogous to how it is done for the quarks. Moreover, the smallness of neutrino masses is already a strong indication for new particles becoming active at energies greater than the EW scale and participating in flavour violating couplings. This is the natural scenario for MFV.

Nevertheless, opposed to what happens with quarks, there is no natural way to define the spurions, that is, there is no unique MFV effective Lagrangian of leptons. Since the correct model of neutrino mass generation is not known, MFV can take different forms. Each case will predict different amplitudes for flavour processes, in particular for the decay $\mu \to e\gamma$, depending not only on the energy scales but also on the flavour parameters. Minimal Flavor Violation is not as restrictive in the lepton sector as it is in the quarks since different models of neutrino masses will lead to different scenarios.

The first paper addressing these issues was [54] where two different, broad scenarios of MFV in the lepton sector, differing only on the low energy field content, were identified:

- The *minimal* case. The low energy field content is that of the SM, namely, three left-handed lepton doublets $L_{L\alpha}$ and three right-handed charged lepton siglets $E_{R\alpha}$. Here the flavour symmetry group is simply

$$\mathscr{G}_{LF} = SU(3)_L \times SU(3)_E\,. \tag{2.25}$$

- The *extended* case. Three right-handed neutrinos $N_{R\alpha}$ are added to the SM field content. The symmetry group is $\mathscr{G}_{LF} \times SU(3)_N$.

The next step in the MFV rationale is to identify the irreducible flavour spurions in each case and use the relations imposed by MFV on the higher dimensional coefficients to predict the rates and cross-sections of the exotic flavour processes. In the minimal case the irreducible coefficients are by assumption the charged Yukawa coupling and the coefficient of the Weinberg operator

$$\mathscr{L}_{min} = -Y_E^{\alpha\beta} \bar{L}_{L\alpha} H E_{R\beta} - \frac{1}{2\Lambda_{LN}} g_\nu^{\alpha\beta} (\bar{L}^c{}_{L\alpha} \tilde{H}^*)(\tilde{H}^\dagger L_{L\beta}) + \text{h.c.} \tag{2.26}$$

while in the extended case we have both the charged lepton and neutrino Yukawas

$$\mathscr{L}_{ext} = -Y_E^{\alpha\beta} \bar{L}_{L\alpha} H E_{R\beta} - Y_N^{\alpha\beta} \bar{L}_{L\alpha} \tilde{H} N_{R\beta} - \frac{\Lambda_{LN}}{2} \bar{N}^c{}_{R\alpha} N_{R\alpha} + \text{h.c.} \tag{2.27}$$

In this second case, the Majorana mass matrix for the right-handed neutrino is assumed flavourless. In each case, the coefficients of the rare flavour processes must now be combinations of these irreducible spurions fixed by the flavour symmetry. For instance, the coefficient$\Delta^{\alpha\beta}$ of the operator

$$\mathcal{O}^{(1)} = \bar{L}_{L\alpha} \gamma^\mu L_{L\beta} H^\dagger i D_\mu H \tag{2.28}$$

contributing, after EW symmetry breaking, to the radiative lepton flavor changing decays $\ell_\alpha \to \ell_\beta \gamma$ must be given, for the minimal and extended case by

$$\Delta_{min}^{\alpha\beta} \propto g_\nu^{\dagger\,\alpha\gamma} g_\nu^{\gamma\beta}\,, \quad \Delta_{ext}^{\alpha\beta} \propto Y_\nu^{\dagger\,\alpha\gamma} Y_\nu^{\gamma\beta} \tag{2.29}$$

respectively, plus terms of order $Y_e \Delta^{\alpha\beta}$ which can be safely neglected. Thus, within these two rather broad realizations in the lepton sector, MFV not only asserts that there must exist a relation between the flavour couplings but specifically predicts the coefficients of the radiative decays in terms of the flavour spurions. Notice that the latter can be expressed roughly as functions of the neutrino mass matrix: in the minimal case $\Delta_{min}^{\alpha\beta} \sim m_\nu^{\alpha\gamma} m_\nu^{\gamma\beta}$ while in the extended case $\Delta_{ext}^{\alpha\beta} \sim m_\nu^{\alpha\beta}$. The coincidence is exact if there is CP conservation[2].

Following a line of reasoning similar to that of the previous section we can write now the whole list of flavour operators. One would be interested in those $d = 5$ or $d = 6$ operators that could lead to lepton flavour violating processes. Nevertheless, these operators must conserve lepton number; if they don't, they would be suppressed

[2]Recall that, with three right-handed neutrinos added to the SM field content, the mass matrix for the light neutrinos comes proportional to $Y_N^T Y_N$ which is equal to $Y_N^\dagger Y_N$ if it can be made real, that is, if there is no CP violation.

by the LN breaking scale which is assumed very large. Therefore, no $d = 5$ operator appears.

The $d = 6$ operators must include at least one of the following leptonic bilinears

$$\bar{L}_{L\,\alpha}\Gamma L_{L\,\beta}\,, \quad \bar{E}_{R\,\alpha}\Gamma L_{L\,\beta}\,, \quad \bar{E}_{R\,\alpha}\Gamma E_{R\,\beta} \tag{2.30}$$

where Γ represents a generic matrix from the Dirac algebra. We can divide them in four-lepton operators and semileptonic operators, the latter involving other fields of the SM. Among the four-lepton operators we have

$$\bar{L}_L\gamma^\mu\Delta L_L\bar{L}_L\gamma_\mu L_L\,, \quad \bar{E}_R\gamma^\mu\Delta E_R\bar{L}_L\gamma_\mu L_L\,, \quad \text{etc}\,. \tag{2.31}$$

which contribute for instance to the four lepton process $\mu \to e^+e^-e^-$. On the other hand, a process like $\mu \to e\gamma$, analized in Sec.2.2, is produced through operators such as the one in Eq.(2.28) and others

$$g'(Y_E\Delta)^{\alpha\beta}H^\dagger\bar{e}_{R\,\alpha}\sigma^{\mu\nu}L_{L\,\beta}B_{\mu\nu}\,, \quad g(Y_E\Delta)^{\alpha\beta}H^\dagger\tau^a(\bar{e}_{R\,\alpha}\sigma^{\mu\nu}L_{L\,\beta})W^a_{\mu\nu}\,, \quad \text{etc}\,. \tag{2.32}$$

All of the latter operators have in common that they include a factor of Δ. It is clear then that in order to say something more about $\mu \to e\gamma$ we would need the values for that coefficient. In turn, using Eq.(2.29), Δ can be rewritten in terms of the parameters measured in neutrino oscillations. So it follows, for the minimal case:

$$\Delta_{\mu e} = \frac{\Lambda^2_{LN}}{v^4}\frac{1}{\sqrt{2}}(s_{12}c_{12}\Delta m^2_{12} + s_{13}e^{i\delta}\Delta m^2_{23})\,, \tag{2.33}$$

and for the extended case:

$$\Delta_{\mu e} \simeq \frac{\Lambda_{LN}}{v^2}\frac{1}{\sqrt{2}}\left[s_{12}c_{12}\sqrt{\Delta m^2_{12}} + s_{13}\sqrt{\Delta m^2_{23}}\right]\,. \tag{2.34}$$

In both cases normal hierarchy and no CP violation has been assumed.

A number of questions remained unanswered nonetheless regarding MFV among the leptons. How general is the division between minimal and extended cases? Are these the only two possibilities? Moreover, the fact that we attribute the tiny neutrino masses to the physics of $B - L$ violation has no parallel in the quark sector. This implies in particular, that the $B - L$ violating scale Λ_{LN} and flavour violating scale Λ_{FL} must be decoupled if sizable amplitudes are to be generated for the exotic flavour processes. We will tackle this issue in Ch.3.

2.4 Non-unitarity

There's another kind of new physics effects intimately related with FCNCs and often appearing along with it. Many times, models of neutrino mass will also yield Λ_{fl}-suppressed operators, Eq.(2.1) that, after EW symmetry breaking contribute to a mixing matrix in the low energy Lagrangian that is non-unitary. In particular all Seesaw theories in which the heavy mediators are fermions induce a non-unitary PMNS matrix [96, 98, 99, 102–104, 111, 163, 167, 168]. This is because whenever new fermions are included in a SM extension, we expect them to mix with the known leptons after EW symmetry breaking. The total mixing matrix of the theory will still be unitary but the submatrix U_{PMNS} corresponding to the mixing of low energy fields will not. This is relevant, for instance, to discriminate between different seesaw models in case we have a positive signal of Majorana neutrinos, such as $0\nu\beta\beta$ decay.

We will consider unitarity violations in the lepton sector of the SM Lagrangian with three light neutrinos. As such, we follow closely in this presentation the scheme dubbed as Minimal Unitarity Violation in [56].

Non-unitarity is parametrized in the low energy Lagrangian with a matrix N that appears in the charged and neutral currents involving neutrinos. Concretely, the neutrino interaction terms in the Lagrangian in Eq.(1.6) are modified to

$$\mathscr{L}_{int} = \dots - \frac{g}{2\sqrt{2}}(\bar{\ell}_{L\alpha}\not{W}^{+}N_{\alpha i}\nu_{Li} + \text{h.c.}) - \frac{g}{2\cos\theta_W}(\bar{\nu}_{Li}\not{Z}(N^{\dagger}N)_{ij}\nu_{Lj} + \text{h.c.}) + \dots \quad (2.35)$$

with the matrix N that replaces U_{PMNS} being a non-unitary matrix.

The origin of non-unitarity effects can be conveniently illustrated by considering the interaction term of the charged lepton current in the SM with the W^{+} along with a perturbation from a $d = 6$ effective operator involving two neutrinos and two Higgs fields. We have

$$\mathscr{L} = i\bar{\nu}_{L\alpha}\not{\partial}\nu_{L\alpha} - c^{\alpha\beta}(\bar{L}_{L\alpha}\tilde{H})i\not{\partial}(\tilde{H}L_{L\beta}) - \frac{g}{2\sqrt{2}}\bar{\ell}_{L\alpha}\not{W}^{+}_{\mu}\nu_{L\alpha} + \dots \quad (2.36)$$

The second term is a $d = 6$ gauge-invariant effective operator that appears in typical SM extensions such as the Seesaw Models. Notice the matrix $c^{\alpha\beta}$ must be hermitian if the Lagrangian is to be real. After EW symmetry breaking this term leads to a correction of the kinetic term for the neutrinos

$$\mathscr{L}_{SB} = iK^{\alpha\beta}\bar{\nu}_{L\alpha}\not{\partial}\nu_{L\beta} - \frac{g}{2\sqrt{2}}\bar{\ell}_{L\alpha}\not{W}^{+}_{\mu}\nu_{L\alpha} + \dots, \quad (2.37)$$

with

$$K^{\alpha\beta} = \left(1 + \frac{v^2 c^{\alpha\beta}}{2}\right). \quad (2.38)$$

In order to make physical sense of this theory we need to take the kinetic term to the canonical form. The coefficient of the kinetic term is hermitian and can be diagonalized by a unitary rotation U. Furthermore, if κ_i, $i = 1, 2, 3$ are the real eigenvalues of the matrix K, we can rescale the neutrino fields $\nu_i = \nu'_i/\kappa_i$ to obtain the Lagrangian

$$\mathscr{L}_{SB} = i\bar{\nu'}_{Li}\not{\partial}\nu'_{Li} - \frac{g}{2\sqrt{2}}\bar{\ell}_{L\alpha}\not{W}^{+}_{\mu}U^{\dagger}\sqrt{K_D}\nu_{Li} + \dots \quad (2.39)$$

with $K_D = \text{diag}\{\kappa_1, \kappa_2, \kappa_3\}$. Notice that in the limit of $SU(3)_L$ flavour symmetry - that is, in the limit in which the lepton mass matrix is proportional to the identity and neutrinos are massless - the unitary matrix U can be absorbed by the ℓ fields. Thus, in this limit, non-unitarity simply expresses itself as a non-universal coupling constant of the weak bosons to the charged lepton currents. Massless neutrinos can now be differentiated by the strength of their interaction to their charged partners! This is reflected for instance in the Fermi constant G_F^M which now will depend on the flavours involved in the particular process. Notice that non-unitarity remains even in the limit in which the particles involved interact at very high energies so that all leptons can be assumed massless.

It is not difficult to address the general case in Eq.(2.35). In particular we can find the amplitudes and branching ratios for the flavour violating processes. For the case-study leptonic process $\ell_\alpha \to \ell\gamma$ the branching ratio is given exactly by the same expression as in the unitary case with the matrix N replacing U_{PMNS}

$$B_{\mu\to e\gamma} = \frac{3\alpha}{32\pi}\frac{\sum_k N_{\mu\alpha}N^\dagger_{\alpha e}f(x_\alpha)}{(NN^\dagger)_{\mu\mu}(NN^\dagger)_{ee}}\,. \tag{2.40}$$

Following Sec.2.2 we expand Eq.(2.40) in powers of $x_\alpha = m_\alpha^2/M_W^2$. However, now it is not possible to neglect the constant term due to the non-unitarity of N. The GIM mechanism is destroyed. Using $f(x) \simeq 10/3$ it follows that

$$B_{\mu\to e\gamma} \simeq \frac{100\alpha}{96\pi}\frac{\sum_k N_{\mu\alpha}N^\dagger_{\alpha e}f(x_\alpha)}{(NN^\dagger)_{\mu\mu}(NN^\dagger)_{ee}}\,. \tag{2.41}$$

A similar expression is obtained for the other two flavour changing decays. $\ell_\alpha \to \ell_\beta\gamma$. This allows one to constrain the elements of the matrix $NN^\dagger$ [56, 81]

$$|(NN^\dagger)_{\alpha\beta} - \delta_{\alpha\beta}| < \begin{pmatrix} 4.0\times 10^{-3} & 1.2\times 10^{-4} \times 3.2\times 10^{-3} \\ 1.2\times 10^{-4} & 1.6\times 10^{-3} \times 2.1\times 10^{-3} \\ 3.2\times 10^{-3} & 2.1\times 10^{-4} \times 5.3\times 10^{-3} \end{pmatrix}. \tag{2.42}$$

The $\mu \to e\gamma$ case is particularly relevant because of the strong experimental bound which translates in a bound for the matrix component. Other bounds on the non-unitarity of the mixing matrix can be obtained by analyzing neutrino oscillation experiments as well as weak boson decays.

Of course, non-unitarity is a non-issue in the context of MFV. This is because MFV is particularly designed not to contribute to FCNCs. Specifically, in a typical MFV model, the same unitary matrix that we use to diagonalize the neutrino mass matrix U_{PMNS} would be used for diagonalization of the coefficient $K_{\alpha\beta}$. $K_{\alpha\beta}$ would in turn be made of spurions so the diagonal matric K_D would only differ from the identity by factors of order m_α^2/Λ^2 - with Λ the scale of new physics - which implies a non-unitarity correction not bigger than that allowed by the GIM mechanism.

2.5 Non-Standard Neutrino Interactions

Non standard neutrino interactions (NSNI) is the generic name given to exotic couplings involving neutrinos excluding the simple Dirac or Majorana extensions to the SM. A simple example of such a couplings is provided by the toy non-unitary Lagrangian in Eq.(2.39). It was pointed out above that, in the limit of global $SU(3)_L$ flavour symmetry, one effect provoked in the Lagrangian by non-unitarity was the breaking of universality in the coupling of the W to the charged currents. Such a breaking could have implications in the detection and production process of neutrinos as well as in their passing through matter.

With respect to NSNIs, the trademark of non-unitarity is that the coefficients of the NSI operators induced by it and contributing to neutrino production, detection, and matter effects are not independent but related. Barring fine-tuned cancellations, the stringent bounds and future signals on non-unitarity [169–171] apply as well to NSI, except for those NSI operators affecting exclusively the propagation in matter. Recently, the value of the elements of the PMNS matrix have been extracted from data without assuming a unitary mixing matrix [172], and new related CP-odd signals have been proposed as well [112, 114, 116]. For a detailed discussion of the NSI-non-unitarity relationship, see [81]. For the following discussion and for the developments in Ch.5 we will leave non-unitary NSIs aside, mantioning only its qualitative implications.

It should be stated at this point that the most appealing models of neutrino mass, i.e., the Seesaw models, have not been shown to be linked to NSNIs other than non-unitarity. As such, NSNIs are at the moment not central in the picture of the lepton sector. NSNIs is nonetheless an enveloping concept worth to explore. Neutrino exotic couplings can affect the production process, the time evolution of neutrinos, the detection process or any combination of them. In the most general treatment one can identify four relevant bases [165]

- The mass basis $|\nu_i^m\rangle$ where the neutrino mass matrix is diagonal.
- The weak basis $|\nu_\alpha^W\rangle$ in which the leptonic couplings to the W are diagonal.
- The source basis $|\nu_\alpha^s\rangle$ where the interactions of the production process are diagonal.
- The detector basis $|\nu_\alpha^d\rangle$ where the interactions at the detector are diagonal.

If neutrino interactions are described fully by the minimal Dirac or Majorana extensions of the SM, the last three definitions coincide and we call that basis the flavour basis. It is when we consider new physics effects that those three bases can be different.

All other bases are related to the mass basis by linear transformations that differ in the general case and that are not necessarily unitary. We can write

$$|\nu_\alpha^s\rangle = V_{\alpha i}^s|\nu_i^m\rangle\,, \quad |\nu_\alpha^d\rangle = V_{\alpha i}^d|\nu_i^m\rangle\,, \quad |\nu_\alpha^W\rangle = V_{\alpha i}^W|\nu_i^m\rangle\,. \tag{2.43}$$

These formulae are all we need to treat, for instance, neutrino oscillations in the vacuum which evolve after the production process as mass eigenstates until they are detected. The amplitude for finding a ν_β^d in the original ν_α^s at time t is given by [165]

$$\langle \nu_\beta^d | \nu_\alpha^s \rangle = \sum_{i,j} \langle \nu_i^m | V_{i\beta}^{d\dagger} e^{-iE_j t} V_{\alpha j}^s | \nu_j^m \rangle = \sum_i e^{-iE_i t} V_{\alpha i}^s V_{\beta i}^{d*}. \tag{2.44}$$

In any case, NSNIs affecting the production or the detection process are zero-distance effects and are extremely constrained by rare decays. Constraints on matter interactions in the other hand are weaker and may be worth investigating. In this work we will focus mainly in the effect of Non-Standard Interactions in the case of neutrinos passing through matter. There NSNIs are conveniently described in terms of an effective propagation Hamiltonian where off-diagonal terms may be present. We define the $\epsilon_{\alpha\beta}^f$ parameters as the coefficients of the terms

$$\mathscr{L}_{NSI} = 2\sqrt{2} G_F \sum_f \epsilon_{\alpha\beta}^f (\bar{\nu}_L^\alpha \gamma^\mu \nu_L^\beta)(\bar{f}\gamma_\mu f) \tag{2.45}$$

where f can be any right- or left-handed fermion constituent of matter. For ordinary, neutral matter, the total coefficient is given by

$$\epsilon_{\alpha\beta}^m = \epsilon_{\alpha\beta}^e + 2\epsilon_{\alpha\beta}^u + \epsilon_{\alpha\beta}^d + \frac{n_n}{n_e}(\epsilon_{\alpha\beta}^u + 2\epsilon_{\alpha\beta}^d) \tag{2.46}$$

where n_n and n_e are the number density of neutrons and electrons respectively. The effective Hamiltonian is now given by

$$\mathscr{H} = U_{PMNS} \begin{pmatrix} 0 & & \\ & \frac{\Delta m_{21}^2}{2E} & \\ & & \frac{\Delta m_{31}^2}{2E} \end{pmatrix} U_{PMNS}^\dagger + \sqrt{2} G_F N_e \begin{pmatrix} 1+\epsilon_{ee}^m & \epsilon_{e\mu}^m & \epsilon_{e\tau}^m \\ \epsilon_{\mu e}^m & \epsilon_{\mu\mu}^m & \epsilon_{\mu\tau}^m \\ \epsilon_{\tau e}^m & \epsilon_{\tau\mu}^m & \epsilon_{\tau\tau}^m \end{pmatrix} \tag{2.47}$$

which is to be compared with Eq.(1.78).

NSNIs could hence strongly affect neutrino propagation in matter. This would be very relevant for experiments with long baselines such as the neutrino factory. Opposed to this, non standard interactions can be constrained by looking at short baselines experiments and determining the parameters of the Hamiltonian before they travel long. The present direct bounds for the ϵ parameters are not very strict and are summarized in the following table

$$\begin{pmatrix} -4 < \epsilon_{ee}^m < 2.6 & |\epsilon_{e\mu}^m| < 1.4 \cdot 10^{-4} & |\epsilon_{e\tau}^m| < 1.9 \\ & -0.05 < \epsilon_{\mu\mu}^m < 0.08 & |\epsilon_{\mu\tau}^m| < 0.25 \\ & & |\epsilon_{\tau\tau}^m| < 19 \end{pmatrix},$$

at the 90% CL. Notice that the bounds are significantly weaker for the τ type neutrinos.

It can be argued however that these bounds can not be justified as such. Neutrinos are part of weak doublet also involving the corresponding charged lepton. If a NSNI like

$$(\bar{\nu}_\mu \gamma^\mu \nu_e)(\bar{\ell}_e \gamma_\mu \ell_e) \tag{2.48}$$

is allowed then, by $SU(2)$ gauge invariance, the operator

$$(\bar{\mu} \gamma^\mu \ell_e)(\bar{\ell}_e \gamma_\mu \ell_e) \tag{2.49}$$

may be also allowed. It follows a contribution to the FCNC $\mu^- \to e^+ e^- e^-$ of Sec.2.2, which as we have seen in Sec.2.2, is very well constrained. This should forbid a big coefficient for the operator in Eq.(2.48). The NSNIs need to be addressed in a gauge invariant framework to see what windows remain.

2.5.1 Imposing gauge invariance

NSNIs are exotic flavour processes that conserve lepton number. In the Lagrangian in Eq.(2.1) they are represented by operators of even dimension, $d = 6, 8, \ldots$. In order to set off the discussion about gauge invariant NSIs, let us consider four-fermion operators such as the one in Eq.(2.48). If we want to implement them in a gauge invariant framework we require effective operators of $d \geq 6$ with four fermion fields plus Higgses in the case of operators with $d > 6$ [80, 173, 174]. There is a plethora of $d = 6$ [78] and $d = 8$ [79] operators[3], with different classes of models resulting in different operators and operator coefficients.

In order to get a taste of the complications involved let $\{\mathcal{O}^{d=6}_{C}\}$ be the set of $d = 6$ gauge-invariant operators that induce both NSNIs *and* four charged lepton processes and $\{\mathcal{O}^{d=6}_{\not{C}}\}$ the set of those operators that *only* induce non-standard neutrino interactions. As an example one can check that

$$(\bar{L}_\alpha i\tau^2 L^c_\beta)(\bar{L^c}_\gamma i\tau^2 L_\delta) \in \{\mathcal{O}^{d=6}_{\not{C}}\} \tag{2.50}$$

where τ^2 is the Pauli matrix and we have explicited the flavour indices. Operators belonging to $\{\mathcal{O}^{d=6}_{\not{C}}\}$ are not constrained in principle by the bounds on the charged lepton processes. In fact, the operator in Eq.(2.50) is the only operator belonging to $\{\mathcal{O}^{d=6}_{\not{C}}\}$. In any case, from the model building perspective one can go further and ask what type of models will lead to such operators.

Using the *mediator decomposition method* described in Sec.1.4 we see that, for instance, the operator in Eq.(2.50) can be generated at tree level by the exchange of a singlet massive scalar with hypercharge $Y = 1$. In this simple case the amplitudes

[3] $d = 5, 7, \ldots$ operators [175] are odd under lepton number and not relevant for the present discussion.

for several NSNI processes turn out to be linked, something that might pose some phenomenological problems or not but interesting by its own right. The case will be studied in detail in Ch.5.

In the other hand, the exchange of a singlet vector particle could generate the following gauge-invariant operator

$$(\bar{L}_\alpha \gamma_\mu L_\beta)(\bar{L}_\gamma \gamma^\mu L_\delta) \in \{\mathcal{O}_C^{d=6}\}\,. \tag{2.51}$$

The amplitude for NSNIs generated through Eq.(2.51) is suppressed because the coefficient of this operator also appears in the amplitude for the well constrained charged lepton process $\mu \to e^+e^-e^-$, Sec.2.2. Not much hope seems to be in this kind of operators unless some cancellations with other operators take place in the Lagrangian.

If we want some more freedom to generate NSNIs than Eq.(2.50) we can turn to $d = 8$ operators by adding two Higgses to the fermion content. The drawback is that these are suppressed with respect to $d = 6$ operators by a factor v^2/Λ_{FL}^2, Eq.(2.1). Therefore, in order for these to be relevant, the scale of new physics should be close enough to the EW scale. The advantage is that at $d = 8$ there are a number of operators that only contribute to NSNIs. An example that has been studied[CITE] is

$$\mathcal{O}_{\mathrm{NSI}} = (\bar{L}^i H_i)\gamma^\rho (H^{\dagger i} L_i)(\bar{E}\gamma_\rho E)\,, \tag{2.52}$$

Nevertheless, it seems interesting to explore how can this operator be realized in practice. What models could generate it, at least at tree level? Ch.5 is devoted to develop a systematic approach to these questions.

Chapter 3

Minimal Flavour Seesaw Models

Let us come back to the question of the separation of the lepton number violating and flavour scales, Λ_{LN} and Λ_{fl} respectively. The MFV setup developed in Sec. 2.3.2 [57], assumes two fundamental -a priori unrelated- conditions to hold:

a) Hierarchy between the operators that break and preserve lepton number or, in other words, a large hierarchy between the corresponding scales, $\Lambda_{FL} \ll \Lambda_{LN}$.

b) Flavour structure of the $d = 6$ operator coefficients fixed by that of the $d = 5$ one.

In particular, condition a) guarantees: first, that neutrino masses appear naturally in the effective theory with $c^{d=5} \sim 1$; second, that flavour violating processes are not as suppressed. Thus, rare processes such as $\mu \to e\gamma$ can be generally quite large if the scale Λ_{FL} is of $\mathcal{O}$(TeV). On the other hand, condition b) guarantees the predictability that falls within the logic of MFV.

This setup rises however several fundamental questions. In both extended and minimal MFV models, flavour spurions are introduced which are coupled to the physical fields responsible for the LN scale. How exactly can these spurions remain coupled, for example in the $d = 6$ operator coefficients, after the large scale Λ_{LN} is integrated out? In order to fulfill conditions a) and b), is it necessary to have two distinct scales, Λ_{LN} and Λ_{FL}. Do these scales correspond to physical particle masses? Would this imply a naturalness problem [59,62]? In Sec.2.3 we defined two scenarios of MFV, *minimal* and *extended*, which differred in the fundamental spurions. Consequently, each of them led to different relations between $d = 5$ and $d = 6$ coefficients but, are these two the only cases possible? In this chapter we address these questions by considering simple explicit seesaw models that satisfy criteria a) and b).

Given that we consider explicit models and not just some generic effective theory, we can distinguish two situations. Either condition b) is satisfied by the intrinsic structure of the model, or it is a consequence of a restrictive MFV hypothesis. Obviously the former case is more interesting and we will show a couple of examples of this type (in sections 3.1 and 3.3), where the whole lepton flavour structure of the model can be extracted from the light neutrino mass matrix. Furthermore, we will present a very simple model in Sec. 3.3 that satisfies conditions a) and b), but in which the relation between $d = 5$ and $d = 6$ operators is none of the kind considered in Ref. [57].

3.1 MFV in scalar mediated (Type-II) seesaw models

We are interested in explicit models fulfilling the two criteria a) and b) above. In this section we stress that the Type-II seesaw model is nothing but a MFV model of the minimal type (that is, where the basic flavour spurion is the coefficient of Weinberg's operator). It is the simplest example of such minimal MFV model.

The type-II seesaw model [31] in its basic form only adds to the SM fields one scalar hypercharge 2 scalar triplet field $\Delta \equiv (\Delta^{++}, \Delta^{+}, \Delta^{0})$. Writing this triplet as $(\frac{1}{\sqrt{2}}(\tilde\Delta_1 - i\tilde\Delta_2), \tilde\Delta_3, \frac{1}{\sqrt{2}}(\tilde\Delta_1 + i\tilde\Delta_2))$, the Lagrangian in Eq.(1.60) can be easily written in terms of $\tilde\Delta \equiv (\tilde\Delta_1, \tilde\Delta_2, \tilde\Delta_3)$:

$$\begin{aligned}\mathscr{L}_\Delta = &\left(D_\mu\tilde\Delta\right)^\dagger\left(D^\mu\tilde\Delta\right) + \left(\overline{\widetilde{L}_L} Y_\Delta (\tau\cdot\tilde\Delta) L_L + \mu_\Delta \widetilde{H}^\dagger (\tau\cdot\tilde\Delta)^\dagger H + \text{h.c.}\right) - \tilde\Delta^\dagger {M_\Delta}^2 \tilde\Delta \\ &- \frac{\lambda_2}{2}\left(\tilde\Delta^\dagger\tilde\Delta\right)^2 - \lambda_3\left(H^\dagger H\right)\left(\tilde\Delta^\dagger\tilde\Delta\right) - \frac{\lambda_4}{2}\left(\tilde\Delta^\dagger T^i\tilde\Delta\right)^2 - \lambda_5\left(\tilde\Delta^\dagger T^i \tilde\Delta\right) H^\dagger\tau^i H\,, \quad (3.1)\end{aligned}$$

with $H \equiv (H^+\, H^0)^T$, T_i being the three-dimensional representation of the $SU(2)$ generators (as defined in Ref. [59]) and τ_i the Pauli matrices. In the absence of charged-lepton Yukawa couplings and Y_Δ, the leptonic Lagrangian exhibits a global flavour symmetry group $SU(3)_L \otimes SU(3)_E$. The coexistence of Y_Δ and μ_Δ explicitly breaks lepton number, inducing at low energies the Weinberg operator with coefficient given in Eq.(1.61). It is proportional to Y_Δ, which is the only flavour spurion of the model. As for the generated $d = 6$ operators, there is only one at tree level which involves four leptons [1]:

$$\delta\mathscr{L}^{d=6} = c^{d=6}_{\alpha\beta\gamma\delta}\left(\overline{\ell_{L\beta}}\gamma_\mu \ell_{L\delta}\right)\left(\overline{\ell_{L\alpha}}\gamma_\mu \ell_{L\gamma}\right)\,, \quad (3.2)$$

with

$$c^{d=6}_{\alpha\beta\gamma\delta} = -\frac{1}{M^2_\Delta} {Y_\Delta}^\dagger_{\alpha\beta} Y_{\Delta\delta\gamma}\,. \quad (3.3)$$

As a matter of fact the structure of $c^{d=6}$ is also generic for $d = 6$ leptonic operator coefficients in all seesaw models, $c^{d=6} \sim (M^{-1}Y)^\dagger M^{-1} Y$, where Y and M denote new Yukawas and scales, respectively The comparison of the $d = 5$ coefficient, Eq.(1.61), with Eq.(3.3) shows that, in addition, the flavour structure of the type II seesaw $d = 6$

[1] As shown in Ref. [59], this model generates also two other $d = 6$ operators involving scalar Higgs doublets and gauge bosons and no fermions, hence less interesting for our purpose since they do not carry any flavour structure.

leptonic coupling goes basically like the square of that of the $d = 5$ coupling, as in the minimal MFV described in Sec.2.3 In other words, in the type-II seesaw model if we know the flavour structure of the $d = 5$ coefficient we know that of the $d = 6$ ones. This is a well-known fact.

In this framework, while the $d = 5$ operator coefficient is proportional to μ_Δ, the $d = 6$ coefficient is not. Therefore the decoupling in size of $d = 5$ and $d = 6$ couplings is automatic. With small enough μ_Δ, a tiny neutrino mass doesn't require large M_Δ and/or small Yukawa couplings Y_Δ, hence the $d = 6$ couplings can be sizeable. The only limit to this pattern is given by the rare decay constraints. For example if $M_\Delta \sim 1$ TeV, $Y_\Delta \sim 10^{-1}$, $\mu_\Delta \sim 10^{-13}$ TeV, one gets neutrino masses of order 10^{-1} eV and saturates the experimental upper bound on the $\mu \to eee$ rate. The latter gives the most stringent constraint as $l \to 3l'$ decays are induced at tree level by the $d = 6$ operator.

The flavour breaking scale Λ_{FL} is well defined in this case: it is the mass of the triplet. The lepton number violating scale Λ_{LN} is more subtly defined: a large lepton number scale has been traded by a small μ_Δ one, which does not correspond to the mass of any new physical particle. The effective Λ_{LN} scale in eq. (2.1) would rather correspond now to the combination $\Lambda_{LN} \sim M_\Delta^2/\mu_\Delta$. Since the μ_Δ term explicitly breaks lepton number (in conjunction with the dimensionless Yukawa coupling Y_Δ), its small value is stable because $\mu_\Delta = 0$ restores the lepton number symmetry. Therefore μ_Δ does not necessarily require any large new physics scale to generate it.

Alternatively, μ_Δ could come from the spontaneous breaking of lepton number, i.e. from the vev v_S of an extra scalar field. It could then be small owing to a seesaw-type mechanism i.e. $\mu_\Delta \sim v_S^2/\Lambda'$ (in which case the scale of the new physics responsible for the small value of μ_Δ could effectively be a large scale $\Lambda_{LN} = \Lambda'$), or because v_S is small and $\mu_\Delta = c \cdot v_S$ (with c a dimensionless coefficient). Problems of stability of the scale v_S are nevertheless to be expected in this framework with spontaneous breaking of lepton number, as discussed in Appendix A, unless the smallness of μ_Δ is due to the smallness of the dimensionless coefficient c rather than to the smallness of v_S.

In summary, the type-II seesaw model satisfies both criteria a) and b) above and to our knowledge there is no simpler model which satisfies them in a minimal–content minimal–flavour way.

3.2 Two-scale fermionic mediated seesaw models (type-I and type-III)

In general all type I seesaw models are described by the following Lagrangian:

$$\mathscr{L} \;=\; \mathscr{L}_{SM} + i\bar{N}_\alpha \partial\!\!\!/ N_\alpha - \left[\lambda_N^{\alpha\beta} \bar{N}^\alpha \tilde{H}^\dagger L_L^\beta + \frac{M_{\alpha\beta}}{2} \bar{N}^\alpha N^{c\beta} + h.c.\right], \qquad (3.4)$$

giving rise to a neutrino mass matrix with the following block structure:

$$M_\nu = \begin{pmatrix} 0 & \lambda_N^T v/\sqrt{2} \\ \lambda_N v/\sqrt{2} & M \end{pmatrix}, \tag{3.5}$$

where λ_N is in general a $N \times 3$ matrix and M is $N \times N$, with N the number of sterile Weyl species. The lepton symmetry can be ensured for particular choices of the λ_N and M matrices.

In the simplest Type I Seesaw [29] there is only one new scale encoded within the heavy right-handed neutrino mass matrix M. Since lepton number is violated by the simultaneous presence of M and λ_N, we can identify this scale with Λ_{LN}. The flavour spurions, which in this case are the leptonic Yukawa couplings λ_N, would decouple when the heavy LN scale goes to infinity. Type I Seesaw thus fails in satisfying condition a) above and it is not a valid model of MFV.

In order to achieve a successful MFV fermionic-mediated seesaw theory, some extra flavour dynamics at a lower scale, Λ_{FL}, is needed [2]. Moreover, it is also necessary to identify the basic flavour spurions –if there is more than one possible choice – and to guarantee that in the limit $\Lambda_{LN} \to \infty$ they remain coupled to the degrees of freedom active at the lower scale Λ_{FL}.

Type-I seesaw models with two scales built in do exist, an example having been mentioned in Sec.1.4.3. Through the assumption of an approximately conserved lepton number $U(1)_{LN}$ symmetry [32, 64] models with suppressed $d = 5$ coefficients but large $d = 6$ interactions can be implemented. [3]. The basic mechanism is to have a number of chiral fermions such that some with opposite $U(1)_{LN}$ charges pair up into Dirac fermions. One or several charged species remain unpaired and therefore massless. These massless neutrinos could only get masses if LN symmetry breaking interactions at a new scale are included. The two scales are therefore related to the typical Dirac masses (Λ_{FL}) and the typical lepton number breaking scale (Λ_{LN}).

At least two generic types of flavour structures which do not decouple in the limit of LN conservation, $\Lambda_{LN} \to \infty$, can be identified:

- Type A: λ_N and M have the following block structures:

$$\lambda_N^T = \begin{pmatrix} Y_N^T & 0 \end{pmatrix}, \quad M = \begin{pmatrix} 0 & \Lambda^T \\ \Lambda & 0 \end{pmatrix}, \tag{3.6}$$

  In this case the $N = 2n$ sterile species divide in two groups with opposite lepton number charges, which we will denote by N and N'. The corresponding Lagrangian

[2]For instance, this happens in type-I seesaw models with two scales built in. Recall as well that the scalar mediated type-II seesaw model in the previous section naturally encoded two distinct scales.

[3]Seesaw models of type III [30] with unsuppressed $d = 6$ operators can be constructed analogously [59]. Since the phenomenology of flavour violating decays will be very similar, we restrict the explicit analysis to models with singlet fermions.

would read:

$$\mathscr{L}_A = \mathscr{L}_{SM} + i\bar{N}^\alpha \not{\partial} N^\alpha + i\bar{N'}^\alpha \not{\partial} N'^\alpha - \left[Y_N^{\alpha\beta} \bar{N}^\alpha \tilde{H}^\dagger L_L^\beta + \frac{\Lambda_{\alpha\beta}}{2} \left(\bar{N'}^\alpha N^{\beta c} + \bar{N}^\beta N'^{\alpha c} \right) + h.c. \right] . \tag{3.7}$$

Models of this type include those in Refs. [32, 65, 66], often denominated *inverse* or *multiple* seesaw models. The lepton number assignments are $L_N = -L_{N'} = L_{\ell_L} = 1$. The pairs (N^α, N'^α) combine into n massive Dirac fermions, while the 3 neutrinos remain massless for any n.

- Type B: λ_N and M have the following block structures:

$$\lambda_N = \begin{pmatrix} Y_N^T & 0 & 0 \end{pmatrix}, \quad M = \begin{pmatrix} 0 & \Lambda^T & 0 \\ \Lambda & 0 & 0 \\ 0 & 0 & \Lambda' \end{pmatrix} , \tag{3.8}$$

in which M includes two distinct scales Λ and Λ' even in the lepton number conserving limit under discussion. The Lagrangian is then

$$\mathscr{L}_B = \mathscr{L}_{SM} + i\bar{N}^\alpha \not{\partial} N^\alpha + i\bar{N'}^\alpha \not{\partial} N'^\alpha + i\bar{N''}^\alpha \not{\partial} N''^\alpha - \left[Y_N^{\alpha\beta} \bar{N}^\alpha \tilde{H}^\dagger L_L^\beta + \frac{\Lambda_{\alpha\beta}}{2} \left(\bar{N'}^\alpha N^{c\beta} + \bar{N}^\beta N'^{c\alpha} \right) + \frac{\Lambda'_{\alpha\beta}}{2} \bar{N''}^\alpha N''^{\beta c} + \text{h.c.} \right] , \tag{3.9}$$

with the lepton number assignments are L, N', $N \sim 1_{LN}$ and $N'' \sim 0_{LN}$.

In this case therefore $N = 3n$, where $2n$ of the sterile species have opposite charges combining into n massive Dirac fermions, as in model of Type A. The third group of n massive Majorana singlets, N'', is decoupled again in the lepton number conserving limit, leaving behind 3 massless neutrinos. It should be noted that the simplest example of type B model in eq. (3.8) corresponds to $n = 1$. In this case, Y_N^T is a three-dimensional vector and Λ and Λ' are just numbers. This model has been recently discussed in Refs. [59, 68], and it also corresponds to the structure of the models considered earlier in Refs. [64, 67].

Obviously there could be generalizations of the above to more species, but we will discuss MFV in the context of these two possibilities.

The Lagrangian in eq. (3.7) leads (for all n) to n quasi Dirac fermions of masses $\sim \Lambda \gg v$ and three massless neutrinos that can get masses only if lepton number breaking entries are switched on. Let us next consider how it can be implemented.

3.3 The simplest MFV Type-I Seesaw model

We will now present the simplest possibility of leptonic seesaw satisfying conditions a) and b), which will turn out to be a model of type A with $n = 1$.

Consider type A models above for general n. In order to obtain neutrino masses, it is necessary to break the $U(1)_{LN}$ symmetry, lifting the zeros in eq. (3.6). By naturalness arguments we should therefore lift all zeros at once. Let us then consider the matrix

$$M_\nu = \begin{pmatrix} 0 & Y_N^T v & \epsilon Y_N'^T v \\ Y_N v & \mu' & \Lambda^T \\ \epsilon Y_N' v & \Lambda & \mu \end{pmatrix}, \tag{3.10}$$

where ϵ is a flavour-blind constant. ϵ, μ and μ' are "small parameters", that is, the scales in μ, μ' are much smaller than those in Λ and v, and $\epsilon \ll 1$, to ensure an approximate $U(1)_{LN}$ symmetry.

The entry in the 22 element in eq. (3.10) does not modify $c^{d=5}$ *at tree level*, and we will obviate it in what follows, while entries in either the 13 or 33 elements do. When the n quasi Dirac neutrinos are integrated out, they give rise to both $d = 5$ and $d = 6$ effective operators (as expected in all type I seesaw models [55, 59]):

$$\delta\mathscr{L}^{d=5} = c^{d=5}_{\alpha\beta} \left(\overline{L^c_{L\alpha}} \tilde{H}^*\right) \left(\tilde{H}^\dagger \mathrm{L}_{L\beta}\right), \tag{3.11}$$

$$\delta\mathscr{L}^{d=6} = c^{d=6}_{\alpha\beta} \; (\bar{L_L}^\alpha \tilde{H} i\partial\!\!\!/) \left(\tilde{H}^\dagger L_L^\beta\right), \tag{3.12}$$

with coefficients [4]

$$c^{d=5}_{\alpha\beta} \equiv \epsilon \left({Y'_N}^T \frac{1}{\Lambda^T} Y_N + Y_N^T \frac{1}{\Lambda} Y'_N \right)_{\alpha\beta} - \left(Y_N^T \frac{1}{\Lambda} \mu \frac{1}{\Lambda^T} Y_N \right)_{\alpha\beta}, \tag{3.13}$$

$$c^{d=6}_{\alpha\beta} \equiv \left(Y_N^\dagger \frac{1}{\Lambda^\dagger \Lambda} Y_N \right)_{\alpha\beta} + \mathcal{O}(\epsilon). \tag{3.14}$$

Note that in general there is no apparent relation between $c^{d=5}$ and $c^{d=6}$. However, we will see below that a direct connection does exists in the case $n = 1$. In this case, Y_N and Y'_N are three dimensional complex column vectors, while Λ, μ and μ' are in general complex numbers. This model gives rise to just one massless neutrino, which is a viable possibility.

In order to prove the connection between $c^{d=5}$ and $c^{d=6}$, we will start by showing that in the case $\mu = \mu' = 0$, we can reconstruct the Yukawa vectors Y_N and Y'_N (up to a

[4] As recalled in the previous section, the leptonic $c^{d=6}$ coefficients are expected to depend on $(\Lambda^{-1}Y_N)^\dagger(\Lambda^{-1}Y_N)$, $(\Lambda^{-1}\epsilon Y'_N)^\dagger(\Lambda^{-1}Y_N)$ and $(\Lambda^{-1}\epsilon Y'_N)^\dagger(\Lambda^{-1}\epsilon Y'_N)$, and the last two contributions can thus be neglected at leading order in ϵ.

global normalization) from $c^{d=5}$, and therefore we can fully predict the flavour structure of $c^{d=6}$. We will then show that the general case, eq. (3.10) for $n=1$, can be treated similarly.

Let us then first consider the mass matrix

$$M_\nu = \begin{pmatrix} 0 & Y_N^T v & \epsilon Y_N'^T v \\ Y_N v & 0 & \Lambda^T \\ \epsilon Y_N' v & \Lambda & 0 \end{pmatrix} . \tag{3.15}$$

The $d=5$ and $d=6$ operator coefficients are then given by

$$c_{\alpha\beta}^{d=5} \equiv \epsilon \left(Y_N'^{\,T} \frac{1}{\Lambda^T} Y_N + Y_N^T \frac{1}{\Lambda} Y_N' \right)_{\alpha\beta} , \; c_{\alpha\beta}^{d=6} \equiv \left(Y_N^\dagger \frac{1}{\Lambda^\dagger \Lambda} Y_N \right)_{\alpha\beta} + \mathcal{O}(\epsilon) . \tag{3.16}$$

The texture in eq. (3.15) has been considered previously in Ref. [69] for $n=3$. In that texture, lepton number is broken due to the simultaneous presence of all three types of terms, and light neutrino masses are then expected to depend on Y_N, Y_N' and Λ. The flavour breaking in this model stems from both Y_N, Y_N', and in consequence there is flavour violation even in the lepton-number conserving $\epsilon \to 0$ limit, as Y_N remains active in that limit: non-trivial leptonic flavour physics can thus affect processes other than neutrino masses.

The structure of the effective Lagrangian in eq. (2.1) is therefore recovered if one identifies $\Lambda_{FL} \to \Lambda$ and $\Lambda_{LN} \to \Lambda/\sqrt{\epsilon}$. The separation of scales is achieved by having a small ϵ, which is technically natural since $\epsilon = 0$ restores the lepton number symmetry. The Λ_{LN} scale does not correspond to any particle mass at this level, while Λ_{FL} corresponds to the Dirac heavy right-handed neutrino mass scale, as expected.

We will show that in this case the coefficient $c_{\alpha\beta}^{d=5}$ contains sufficient information to reconstruct *both* Yukawa vectors, up to a global normalization, and therefore also the flavour structure of $c_{\alpha\beta}^{d=6}$ up to a global normalization. Furthermore, this statement is valid even in the presence of CP violation, up to discrete degeneracies in the Majorana phases.

It is only slightly more difficult in this case to perform the counting of the number of real and imaginary parameters in the complete model as we did in Ch.1 for the SM and the Seesaws. We have

i) We may assume that Y_E is diagonalized in the raw Lagrangian so it contributes with 3 real parameters. Y_N and Y_N' contribute with 3 real parameters and 3 phases and on top of that we have Λ which adds one of each. Overall we have 10 real parameters and 7 phases in the raw Lagrangian.

ii) In order to keep Y_E diagonal the matrices U_E and U_L must be equal and diagonal. Since they are unitary we must have

$$U_\ell = U_E = \text{diag}\{e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}\} \tag{3.17}$$

So we can absorb 3 phases by rotating the lepton fields and 2 more by rotating the N and N' fields. Overall we can absorb 5 phases.

iii) There are no symmetries in this Lagrangian.

Upon substraction we see we end up with 10 real parameters and 2 physical phases. The former correspond in the to 3 lepton masses, 3 neutrino masses, 1 Dirac mass for the heavy neutrinos and 3 mixing angles in U_{PMNS}. Except for the mass of the heavy neutrinos all of these are low energy parameters. Therefore we predict that, except for an overall scale factor, this model can be completely fixed by the low energy physics! The two phases correspond to one CP violating phase δ of the CKM type and one Majorana phase α. Furthermore, there is then a certain freedom in the choice of basis for the complete theory, for instance it is possible to take real Λ and Y_N and also get rid of one of the 3 phases in Y'_N.

In what follows we will work in a basis in which Λ is real while both Y_N and Y'_N may be taken as complex.

Let us explicitly reconstruct the Yukawa couplings from the neutrino mass matrix. It is useful to introduce the notations:

$$Y_N^T \equiv y\mathbf{u} \quad {Y'_N}^T \equiv y'\mathbf{v}, \tag{3.18}$$

where y and y' are real numbers and $\mathbf{u}$ and $\mathbf{v}$ are three complex vectors with unit norm. That is

$$\langle \mathbf{u}, \mathbf{u} \rangle = \langle \mathbf{v}, \mathbf{v} \rangle = 1, \tag{3.19}$$

where the scalar product is between complex vectors $\langle \mathbf{u}, \mathbf{v} \rangle \equiv \mathbf{u}^\dagger \cdot \mathbf{v}$.

The coefficient $c^{d=5}$ in eq. (3.11) can be rewritten as

$$c^{d=5} = \frac{\epsilon y y'}{\Lambda} \left(\mathbf{u}\mathbf{v}^T + \mathbf{v}\mathbf{u}^T \right) \equiv \frac{\epsilon y y'}{\Lambda} \hat{O}, \tag{3.20}$$

$$c^{d=6} = \frac{y^2}{\Lambda^2} \left(\mathbf{u}\mathbf{u}^\dagger \right) + \mathcal{O}(\epsilon^2) \, . \tag{3.21}$$

Note that $c^{d=5}$ is symmetric in the exchange $\mathbf{u} \leftrightarrow \mathbf{v}$. This will result in discrete degeneracies of the Majorana phase α, which cannot be resolved by the measurement of neutrino masses and mixing parameters.

$\hat{O}$ is a symmetric complex matrix and can therefore be diagonalized by a transformation of the form:

$$\frac{\epsilon y y' v^2}{\Lambda} \, U^T \hat{O} U = \frac{\epsilon y y' v^2}{\Lambda} \, \hat{O}_d \equiv - \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{3.22}$$

where m_i denote the mass eigenvalues, which are taken real, and U is the unitary PMNS matrix.

We can determine the mass eigenvalues and the entries of the U matrix diagonalizing the hermitian matrix $\hat{O}^\dagger\hat{O}$, since

$$U^\dagger\hat{O}^\dagger\hat{O}U = \hat{O}_d^2. \tag{3.23}$$

The three eigenvalues and eigenvectors of the matrix $\hat{O}^\dagger\hat{O}$ read:

$$\mu_0 = 0\,, \quad \mathbf{e}_0 = \frac{\mathbf{u}\times\mathbf{v}}{\sqrt{1-|\mathbf{u}\cdot\mathbf{v}|^2}}\,, \tag{3.24}$$

$$\mu_\pm = (1\pm\rho)^2 \quad \mathbf{e}_\pm = \frac{1}{\sqrt{2(1\pm\rho)}}\left(e^{-i\theta/2}\mathbf{u}^* \pm e^{i\theta/2}\mathbf{v}^*\right)\,, \tag{3.25}$$

where

$$\langle\mathbf{u},\mathbf{v}\rangle = \langle\mathbf{v},\mathbf{u}\rangle^* = \rho e^{i\theta}. \tag{3.26}$$

The PMNS matrix U is now given by the matrix whose columns are precisely these eigenvectors [5]. Aside from discrete degeneracies in α, the measurement of the neutrino masses and mixing parameters fully fixes then the eigenvectors and allows to reconstruct the vectors $\mathbf{u}$ and $\mathbf{v}$ since:

$$\mathbf{u}^* = \frac{e^{i\theta/2}}{\sqrt{2}}\left(\sqrt{1+\rho}\,\mathbf{e}_+ + \sqrt{1-\rho}\,\mathbf{e}_-\right), \tag{3.27}$$

$$\mathbf{v}^* = \frac{e^{-i\theta/2}}{\sqrt{2}}\left(\sqrt{1+\rho}\,\mathbf{e}_+ - \sqrt{1-\rho}\,\mathbf{e}_-\right), \tag{3.28}$$

while the ratio of the two mass splittings fixes ρ (it quantitatively depends on the neutrino hierarchy). The phase θ is not physical since it can be reabsorbed by rephasing the N field by $e^{i\frac{\theta}{2}}$ and N' by $e^{-i\frac{\theta}{2}}$ (leaving Λ real) and therefore we set it to zero for simplicity.

In order to do the matching precisely, we have to distinguish the cases of the two possible neutrino hierarchies.

Normal hierarchy

In this case the ordering of the neutrino mass eigenstates is:

$$m_1 = 0\,, \quad |m_2| = \frac{\epsilon y y' v^2}{\Lambda}\,(1-\rho)\,, \quad |m_3| = \frac{\epsilon y y' v^2}{\Lambda}\,(1+\rho)\,, \tag{3.29}$$

and therefore the columns of U are ordered as $(\mathbf{e_0}, \mathbf{e}_-, \mathbf{e}_+)$. From the ratio of the two neutrino splittings we can fix ρ:

$$r \equiv \frac{|\Delta m^2_{solar}|}{|\Delta m^2_{atmos}|} = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{23}|}\,, \quad \rho = \frac{\sqrt{1+r}-\sqrt{r}}{\sqrt{1+r}+\sqrt{r}}\,. \tag{3.30}$$

[5] Note that one mass is negative in our convention. That sign can be reabsorbed in a shift of the Majorana phase.

Reading the columns of the PMNS matrix, one obtains

$$Y_{Ni} = \frac{y}{\sqrt{2}} \left(\sqrt{1+\rho}\, U_{i3}^* + \sqrt{1-\rho}\, U_{i2}^* \right) , \tag{3.31}$$

$$Y'_{Ni} = \frac{y'}{\sqrt{2}} \left(\sqrt{1+\rho}\, U_{i3}^* - \sqrt{1-\rho}\, U_{i2}^* \right) . \tag{3.32}$$

We will use the standard angular parametrization of the PMNS matrix:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_{ph} \tag{3.33}$$

where U_{ph} contains the Majorana phases and can be parametrized in our case as:

$$U_{ph} = \begin{pmatrix} e^{-i\alpha} & & \\ & e^{i\alpha} & \\ & & 1 \end{pmatrix} . \tag{3.34}$$

Up to terms of $\mathcal{O}(\sqrt{r}, s_{13})$, we find

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta}s_{13} + e^{-i\alpha}s_{12}r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha}r^{1/4}c_{12}c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha}r^{1/4}c_{12}s_{23} \end{pmatrix} . \tag{3.35}$$

Since the lightest neutrino is massless, from the central values of the atmospheric and solar parameters [70], we can also fix the combination

$$\left|\frac{\epsilon y y' v^2}{\Lambda}\right| \sim 0.029\,\text{eV} \to \left|\frac{\epsilon y y'}{\Lambda}\right| \sim 4.9 \times 10^{-13}\,\text{TeV}^{-1}. \tag{3.36}$$

Inverted hierarchy

In this case the ordering of the neutrino mass eigenstates is:

$$m_3 = 0\,, \quad |m_1| = \frac{\epsilon y y' v^2}{\Lambda}\,(1-\rho)\,, \quad |m_2| = \frac{\epsilon y y' v^2}{\Lambda}\,(1+\rho), \tag{3.37}$$

and therefore the columns of U are ordered as $(\mathbf{e}_-, \mathbf{e}_+, \mathbf{e_0})$. We find:

$$r = \frac{|\Delta m_{12}^2|}{|\Delta m_{13}^2|}\,, \quad \rho = \frac{\sqrt{1+r} - 1}{\sqrt{1+r} + 1}\,. \tag{3.38}$$

and

$$Y_{Ni} = \frac{y}{\sqrt{2}}\left(\sqrt{1+\rho}\, U^*_{i2} + \sqrt{1-\rho}\, U^*_{i1}\right) , \tag{3.39}$$

$$Y'_{Ni} = \frac{y'}{\sqrt{2}}\left(\sqrt{1+\rho}\, U^*_{i2} - \sqrt{1-\rho}\, U^*_{i1}\right) . \tag{3.40}$$

For the explicit parametrization of the PMNS matrix U, we will use that in eq. (3.33). Again, up to terms of $\mathcal{O}(\sqrt{r}, s_{13})$ we find

$$Y_N^T \simeq \frac{y}{\sqrt{2}} \begin{pmatrix} c_{12}e^{i\alpha} + s_{12}e^{-i\alpha} \\ c_{12}\left(c_{23}e^{-i\alpha} - s_{23}s_{13}e^{i(\alpha-\delta)}\right) - s_{12}\left(c_{23}e^{i\alpha} + s_{23}s_{13}e^{-i(\alpha+\delta)}\right) \\ -c_{12}\left(s_{23}e^{-i\alpha} + c_{23}s_{13}e^{i(\alpha-\delta)}\right) + s_{12}\left(s_{23}e^{i\alpha} - c_{23}s_{13}e^{-i(\alpha+\delta)}\right) \end{pmatrix} . \tag{3.41}$$

From the central values of the atmospheric and solar parameters [70], for the inverted hierarchy under study it follows that

$$\left|\frac{\epsilon y y' v^2}{\Lambda}\right| \sim 0.049\,\mathrm{eV} \to \left|\frac{\epsilon y y'}{\Lambda}\right| \sim 8.1\times 10^{-13}\,\mathrm{TeV}^{-1}. \tag{3.42}$$

Having reconstructed the full Yukawa vectors, it is now possible to make predictions for other lepton flavour violating processes. It is interesting to estimate the rate for $l_i \to l_j\gamma$ processes and establish how do they depend on the unique free real parameter, θ_{13}, and on the neutrino mass hierarchy. We will analyze the ratios

$$B_{ji} \equiv \frac{\Gamma(l_i \to l_j\gamma)}{\Gamma(l_i \to l_j \nu_i \bar{\nu}_j)} \sim |u_i^* u_j|^2 \;=\; \frac{1}{y^2}\,|Y_{N_i} Y_{N_j}|^2 \,. \tag{3.43}$$

In Figs. 3.1 and 3.2 we show the results for the ratios $B_{e\mu}/B_{e\tau}$ and $B_{e\mu}/B_{\mu\tau}$ as a function of θ_{13}, for the normal and inverted hierarchies. The most striking feature is the strong dependence on the Majorana phase α of one of these ratios for both hierarchies: $B_{e\mu}/B_{e\tau}$ in the case of normal hierarchy, and $B_{e\mu}/B_{\mu\tau}$ for inverted hierarchy. In fact, within the ranges of δ and θ_{13} studied, the following prediction holds for the normal hierarchy:

$$\begin{aligned} B_{e\mu} &\simeq \frac{9}{2}B_{e\tau} \qquad && \alpha = 0, \\ B_{e\mu} &\simeq \frac{5}{2}B_{e\tau} && \alpha = \pi/4, \\ B_{e\mu} &\simeq B_{e\tau} && \alpha = \pi/2\,. \end{aligned} \tag{3.44}$$

while $B_{\mu\tau} > B_{e\mu}$. In contrast, a mild dependence on the δ phase holds for any θ_{13} value within the allowed range.

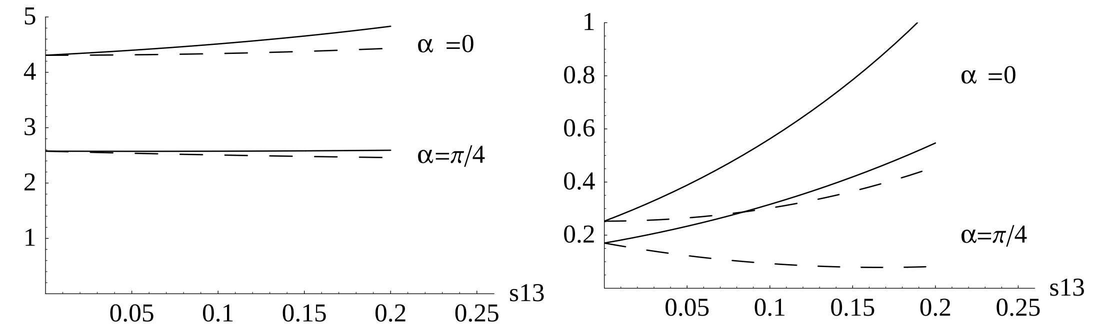

Figure 3.1: Normal hierarchy. Left: Ratio $B_{e\mu}/B_{e\tau}$ for different values of the CP phase $\delta = 0$ (solid) and $\delta = \pi/2$ (dashed), with the two pairs of curves corresponding to $\alpha = 0$ and $\alpha = \pi/4$ as denoted. Right: the same for the ratio $B_{e\mu}/B_{\mu\tau}$.

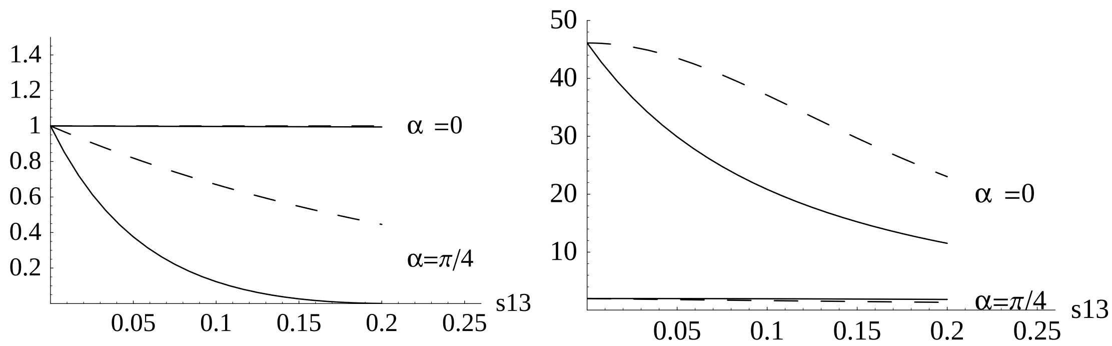

Figure 3.2: Inverted hierarchy. Left: Ratio $B_{e\mu}/B_{e\tau}$ for different values of the CP phase $\delta = 0$ (solid) and $\delta = \pi/2$ (dashed), with the two pairs of curves corresponding to $\alpha = 0$ and $\alpha = \pi/4$ as denoted. Right: the same for the ratio $B_{e\mu}/B_{\mu\tau}$.

A different situation is found for the inverse hierarchy where, i.e. for vanishing $\theta_{13} = 0$,

$$\begin{aligned} B_{e\mu} &\gg B_{\mu\tau} \qquad \alpha = 0\,, \\ B_{e\mu} &\simeq 2B_{\mu\tau} \qquad \alpha = \pi/4, \\ B_{e\mu} &\ll B_{\mu\tau} \qquad \alpha = \pi/2\,, \end{aligned} \tag{3.45}$$

while $B_{e\mu} = B_{e\tau}$ holds. A significant dependence on δ may also develop for $\theta_{13} \neq 0$ for the two ratios considered depending on the value of the Majorana phase α

The α-dependence of the ratios considered has been plotted in Fig. 3.3 for both hierarchies, for $\delta = 0, s_{13} = 0.2$.

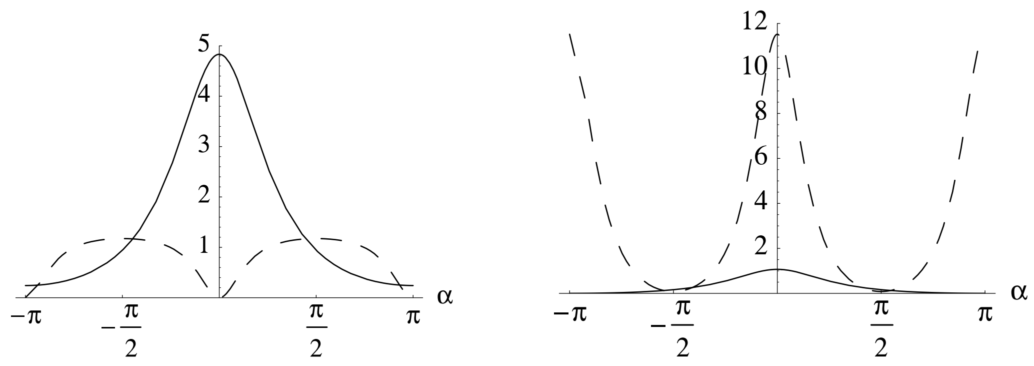

Figure 3.3: Left: Ratio $B_{e\mu}/B_{e\tau}$ for the normal hierarchy (solid) and the inverse hierarchy (dashed) as a function of α for $(\delta, s_{13}) = (0, 0.2)$. Right: the same for the ratio $B_{e\mu}/B_{\mu\tau}$.

Note that the absolute normalization of the branching ratios is unconstrained, since neutrino masses only fix the combination $yy'v^2/\Lambda$, while the branching ratios depend on y^2v^2/Λ^2. Λ not far from the TeV scale is thus a viable possibility, and these branching ratios could therefore be measurable, provided y' is small enough to account for the tiny neutrino masses.

In Figs. 3.4 and 3.5 we show the expected value of $|m_{ee}|$ to be measured in neutrinoless beta decay, for the normal and inverse hierarchies and for the central experimental values of the known parameters as a function of s_{13} and α. Note that these figures show degeneracies in the value of α that can be resolved from the measurement of the $c^{d=6}$ couplings, i.e. from the radiative decays discussed above. As expected, the value of $|m_{ee}|$ is of $\mathcal{O}(10^{-3}eV)$ for the normal hierarchy and one order of magnitude above for the inverse one. Expanding in the small parameters s_{13} and $r^{1/2}$, the following approximate

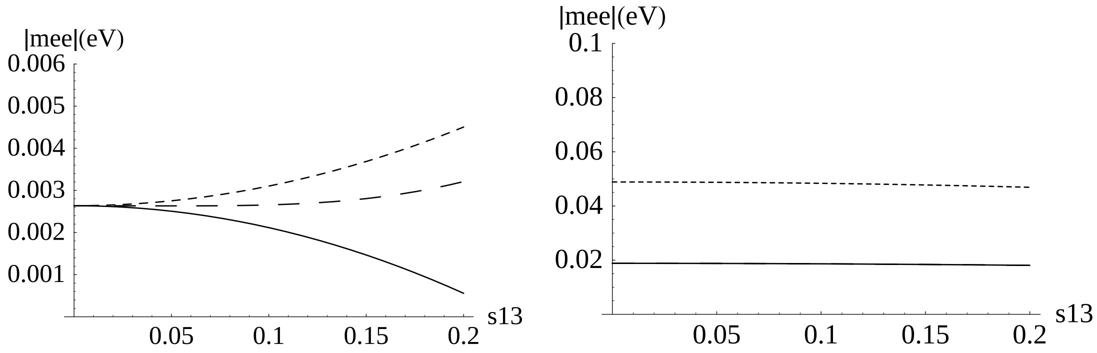

Figure 3.4: Left: $|m_{ee}|(eV)$ for the normal hierarchy as a function of $\sin\theta_{13}$ and for $(\delta, \alpha) = (0, 0)$ (solid), $(0, \pi/4)$ (dotted) and $(\pi/2, 0)$ (dashed). Right: the same for the inverse hierarchy.

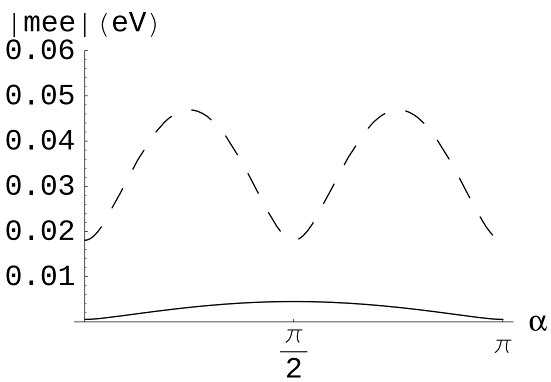

Figure 3.5: m_{ee} as a function of α for the normal (solid) and inverted (dashed) hierarchies, for $(\delta, s_{13}) = (0, 0.2)$.

expressions result (taking the central values for s_{23} and $s_{23} \simeq c_{23}$):

$$
\begin{aligned}
|m_{ee}|_{NH} &\simeq 0.058 \; eV \;\; \left| s_{13}^2 e^{2i\delta} - s_{12}^2 e^{-2i\alpha} \sqrt{r}(1-\sqrt{r}) \right| \\
|m_{ee}|_{IH} &\simeq 0.049 \; eV \;\; \left| s_{12}^2 e^{-2i\alpha} - c_{12}^2 e^{2i\alpha} \right| + \mathcal{O}(r, s_{13}^2) \, .
\end{aligned}
\tag{3.46}
$$

The inverse hierarchy case is in consequence approximately independent of s_{13} and therefore of the CKM-like phase δ, but very sensitive to the Majorana phase α. In the normal hierarchy case, the dependence on all the parameters is significant. In both cases, it is important to stress that the measurement of $|m_{ee}|$, together with that of the neutrino mixing parameters in future neutrino oscillation experiments can in principle fix *all* the parameters of the model, except the absolute normalization of the $d = 6$ operator [6].

Let us now turn to the more general case when $\mu, \mu' \neq 0$ in eq. (3.10). It turns out that *all* the results previously derived in this section hold as well for this general case. This can be easily seen by noting that, for the corresponding $c^{d=5}$ coefficient in eq. (3.13),

$$
\begin{aligned}
c^{d=5}_{\alpha\beta} &= \epsilon \left(Y_N'^T \frac{1}{\Lambda^T} Y_N + Y_N^T \frac{1}{\Lambda} Y_N' \right)_{\alpha\beta} - \left(Y_N^T \frac{1}{\Lambda} \mu \frac{1}{\Lambda^T} Y_N \right)_{\alpha\beta} \\
&= \epsilon \left[\left(Y_N' - \frac{k}{2} Y_N \right)^T \frac{1}{\Lambda^T} Y_N + Y_N^T \frac{1}{\Lambda} \left(Y_N' - \frac{k}{2} Y_N \right) \right]_{\alpha\beta} ,
\end{aligned}
\tag{3.47}
$$

with

$$
k \equiv \frac{1}{\epsilon} \mu \frac{1}{\Lambda^T} . \tag{3.48}
$$

Therefore $c^{d=5}$ has the same structure of that in eq. (3.16) with the substitution

$$
Y_N' \longrightarrow Y_N' - \frac{k}{2} Y_N . \tag{3.49}
$$

We can consequently reconstruct Y_N and the combination in eq. (3.49) from the neutrino mass matrix, that is from $c^{d=5}$, exactly as we did before. From these two combinations, we cannot determine Y_N' in eq. (3.10) , because the factor k is a new free parameter. Nevertheless, all the flavour violating processes induced by $c^{d=6}$ depend only on Y_N, at leading order in the lepton-number violation parameters, and are therefore the same. In

[6] Note also that the relation between the $d = 6$ and $d = 5$ flavour structures obtained above is not of the "minimal" or "extended" MFV types and is not based on the assumption of an underlying flavour symmetry (such as a $O(n)$ symmetry enforced in Ref. [57] to have a right-handed neutrino mass matrix proportional to the identity).

other words, the structure in eq. (3.10) is as predictive as that in eq. (3.15). The low-energy physics (i.e. the relation between flavour violation transitions and the neutrino mass matrix) is the same in both models.

A nice feature of the model considered in this section, eq. (3.10), is its naturalness characteristics. It does not contribute significantly to the electroweak hierarchy problem for Λ values near the TeV scale, as all loop corrections relevant to Higgs physics are proportional to small parameters.

Finally, given the predictivity of the model, it is interesting to explore whether it leads to successful leptogenesis. This has been done in [74]. At low scale, a small mass splitting between the right-handed neutrinos is necessary in order to have a large resonant enhancement of the CP-asymmetry. This indeed happens in the model discussed here, eq. (3.10), which induces a tiny mass difference of order of the size of the $U(1)_{LN}$ breaking, and hence leads to a large resonant enhancement (with however e.g. large washout effects from inverse decays and $\Delta L = 2$ scatterings for large values of the Y_N couplings). Even more, the small mass splitting implies that washout is suppressed. The conclusion is that indeed this model provides all necessary ingredients for successful leptogenesis, coexisting with large lepton flavour effects as discussed above.

We will consider next an alternative class of candidate MFV models: those in which lepton number violation results from lifting the zeros in the diagonal entries of the M_ν matrix, with no 13 entry and $n > 1$. These are the well known inverse seesaw models [32].

3.4 MFV in type-I inverse seesaw models

This section deals, as did the previous one, with models of type A, see eq. (3.6). We consider now the case in which light neutrino masses result from lifting the zeros in the diagonal entries of M_ν. In contrast to the case with only off-diagonal lepton-violating entries, eq. (3.15), the diagonal entries are soft-breaking terms and therefore would not induce by themselves off-diagonal terms. The fundamental neutrino mass matrix is of the form:

$$M_\nu = \begin{pmatrix} 0 & Y_N^T v/\sqrt{2} & 0 \\ Y_N v/\sqrt{2} & \mu' & \Lambda^T \\ 0 & \Lambda & \mu \end{pmatrix} . \tag{3.50}$$

For $n = 1$ however it leads to two massless neutrinos and in consequence is of no physical interest. $n \geq 2$ is needed to get at least 2 massive neutrinos [32]. The simultaneous presence of Y_N, Λ and the Majorana couplings μ and/or μ' breaks lepton number. As explained before the μ' scale does not play any role at low-energies at tree level.

The tree-level exchange of the heavy species gives rise to the same $d = 5$ and $d = 6$

effective operators in eqs. (3.11)-(3.12) with coefficients

$$c_{\alpha\beta}^{d=5} \equiv -\left(Y_N^T \frac{1}{\Lambda} \mu \frac{1}{\Lambda^T} Y_N\right)_{\alpha\beta}, \qquad c_{\alpha\beta}^{d=6} \equiv \left(Y_N^\dagger \frac{1}{\Lambda^\dagger \Lambda} Y_N\right)_{\alpha\beta}. \tag{3.51}$$

The structure of the effective Lagrangian in eq. (2.1) is therefore recovered if one identifies $\Lambda_{FL} \to \Lambda$ and $\Lambda_{LN} \to \Lambda^2/\mu$. The separation of scales is achieved by having a small μ, which is technically natural since $\mu = 0$ restores the lepton number symmetry.

Concerning the flavour structure of the $d = 5$ and $d = 6$ operators in eq. (3.51), they are, in general, unrelated. That is, unless $\mu \sim I_{n\times n}$, which amounts to saying that the term preserves an additional $O(n)$ symmetry. Obviously this symmetry is broken by the Y_N and Λ couplings, and in consequence it can be argued that there is a priori no justification for this choice, which will not be stable under radiative corrections. Nevertheless, this choice is equivalent to the MFV hypothesis: that the only sources of flavour violation are encoded in the charged lepton Yukawa coupling, Y_e, in Y_N and maybe also in Λ. If these three couplings were zero, then the lepton sector would have a symmetry group:

$$SU(3)_{\ell_L} \times SU(3)_E \times SU(n)_N \times O(n)_{N'}. \tag{3.52}$$

Alternatively, the option $\lambda_E = Y_N = 0$ with Λ proportional to the identity would imply that the flavour symmetry group is

$$SU(3)_{\ell_L} \times SU(3)_E \times O(n)_{N,N'}. \tag{3.53}$$

In the former case the neutrino sector spurions are $Y_N \sim (\bar{3}, 1, n, 1)$ and $\Lambda \sim (1, 1, n, n)$, while in the latter $Y_N \sim (\bar{3}, 1, n)$. In both cases, the exact connection of $d = 5$ and $d = 6$ couplings *only holds up to CP phases*. Indeed, in the absence of CP violation it follows that

$$c_{\alpha\beta}^{d=5} = -\mu\, c_{\alpha\beta}^{d=6}, \tag{3.54}$$

and the flavour processes induced by the $d = 6$ operator are fixed, up to a global normalization, by the neutrino mass matrix. This model with diagonal μ is therefore the simplest example of the extended class of models defined in Ref. [57].

In Refs. [57, 61], the implications for flavour-violating processes $l_i \to l_j\gamma$ as well as μe conversion in extended models of MFV have been discussed and should apply as well to the model discussed here. However, it turns out that the $d = 6$ Lagrangian at tree level contains just one operator, eq. (3.12), which is none of those appearing in the basis considered in Ref. [57]. It can be rewritten in terms of operators in that list:

$$\begin{aligned} \delta\mathscr{L}^{d=6} &= c_{\alpha\beta}^{d=6}\, \bar{L}_L^\alpha \tilde{H} i\partial\!\!\!/ \left(\tilde{H}^\dagger L_L^\beta\right) \\ &= \frac{c_{\alpha\beta}^{d=6}}{2} \left(\bar{L}_L^\alpha \gamma_\mu L_L^\beta\ H^\dagger i D_\mu H - \bar{L}_L^\alpha \tau \gamma_\mu L_L^\beta\ H^\dagger \tau i D_\mu H\right). \end{aligned} \tag{3.55}$$

The combination is however a *blind* direction: $l_i \to l_j\gamma$ and $\mu \to e$ do not take place at tree level, as it happens separately for any of the two operators on the right -hand side of eq. (3.55), but only at one loop. In consequence, the bounds derived from these processes in Refs. [57, 61] are further suppressed by an additional loop factor, roughly $1/(4\pi)^2 \sim 10^{-2}$. The flavour structure is however the same. Similar plots to those shown in Figs. 3.1, 3.2 can be found in Ref. [57], which should be strictly applicable to our case. They found the pattern $B_{\mu\tau} \gg B_{e\mu} \sim B_{e\tau}$, which is to be contrasted with the findings in the previous section.

Also in this case it is necessary to justify the presence of the μ, μ' terms and no other $U(1)_{LN}$ breaking term, such as for instance a 13 entry in eq. (3.50) as in the model in previous section. The symmetry pattern shown in eq. (3.52) could justify it. Alternatively, such a choice could be justified if the $U(1)_{LN}$ symmetry is spontaneously broken by the vacuum expectation value (vev) of a scalar singlet S with charge -2, leading to a Lagrangian of the form:

$$\begin{aligned}\mathscr{L}_A = \mathscr{L}_{SM} + i\bar{N}\not{\partial}N + i\bar{N}'\not{\partial}N' - \Big[Y_N \bar{N}\tilde{H}^\dagger L_L + \frac{\Lambda}{2}\left(\bar{N}'N^c + \bar{N}N'^c\right) + \\ + \frac{gS}{2}\bar{N}'N'^c + \frac{g'S^\dagger}{2}\bar{N}N^c + \text{h.c.}\Big] + V(S,H).\end{aligned} \tag{3.56}$$

A vev of the singlet would induce the μ and μ' couplings $\mu = g\langle S\rangle, \mu' = g'\langle S^\dagger\rangle$. Nevertheless, this possibility results in a naturalness problem, that is, of the stability of the separation of scales at the quantum level.

3.5 MFV in type-I seesaw models of type B

The models of type B, e.g. with $3n$ sterile species, also satisfy an exact global $U(1)_{LN}$ symmetry, which ensures the presence of three massless neutrinos for any value of n. In order to lift their masses it is necessary to have some entries in the mass matrix that violate the symmetry. There are several possibilities with different implications in what respects MFV. One possibility is to include some small entries in the zeros of M. The modification of only the diagonal entries in M reduces the model to one of type A, since the N'' fields would remain decoupled in this case. The modification instead of only the off-diagonal entries induces a neutrino mass matrix of the form:

$$M_\nu = \begin{pmatrix} 0 & Y_N v/\sqrt{2} & 0 & 0 \\ Y_N^T v/\sqrt{2} & 0 & \Lambda & \mu_2 \\ 0 & \Lambda^T & 0 & \mu_1 \\ 0 & \mu_2 & \mu_1 & \Lambda' \end{pmatrix}. \tag{3.57}$$

The main interest of these models, in comparison with models of type A, is that it is no longer necessary to assume that μ_1 and μ_2 are very small scales. Even more, in the limit in which Λ' is much larger than all the other scales present, it reduces to a Type A model. In other words, type B models can be seen as an ultraviolet completion of type A scenarios, whose small scales are then explained in terms of large ones in the fundamental theory. Let us discuss this point in detail.

The separation of scales, that is, the implementation of criterium a) in the Introduction, can be achieved through a hierarchy of scales: $\Lambda' \gg \Lambda, \mu_1, \mu_2$. In principle μ_1 and μ_2 could be roughly $\sim \Lambda$, because the $U(1)_{LN}$ symmetry is recovered when the scale Λ' decouples, no matter how large are the other scales. Indeed, integrating out the scale Λ', the effective theory at energies below Λ' is:

$$\begin{aligned}\mathscr{L}_B \simeq \mathscr{L}_{SM} + i\bar{N}\partial\!\!\!/ N + i\bar{N}'\partial\!\!\!/ N' - \Bigg[Y_N \bar{N}\tilde{H}^\dagger L_L + \frac{1}{2}\left(\Lambda + \mu_2 \frac{1}{\Lambda'}\mu_1^T\right)\left(\bar{N}' N^c + \bar{N} N'^c\right) \\ + \frac{1}{2}\mu_2 \frac{1}{\Lambda'}\mu_2^T \bar{N} N^c + \frac{1}{2}\mu_1 \frac{1}{\Lambda'}\mu_1^T \bar{N}' N'^c + \text{h.c.}\Bigg] . \end{aligned} \tag{3.58}$$

This is nothing but a model of type A, with symmetry-breaking entries of the μ, μ' type, in eq. (3.50) suppressed by the large scale Λ'. The scale of lepton number violation can be simply identified with $\Lambda_{LN} \sim \Lambda'$, which corresponds to the mass of the heavy Majorana neutrinos, while the scale of lepton flavour violation would be $\Lambda_{FL} \sim \Lambda$. This pattern is close to that of the extended models of Ref. [57].

When the scale Λ is sufficiently above the electroweak scale, it can be integrated out, resulting in the same d=5 and d=6 operators than in eq. (3.51), with μ given now by $\mu_1 \frac{1}{\Lambda'}\mu_1^T$. The effective theory at scales much lower than Λ is therefore:

$$\begin{aligned}\mathscr{L}_B \simeq \mathscr{L}_{SM} - \left(Y_N^T \frac{1}{\Lambda}\mu_1 \frac{1}{\Lambda'}\mu_1^T \frac{1}{\Lambda^T} Y_N\right)_{\alpha\beta}\left(\overline{L^c_{L\alpha}}\tilde{H}^*\right)\left(\tilde{H}^\dagger L_{L\beta}\right) \\ + \left(Y_N^\dagger \frac{1}{\Lambda^\dagger}\frac{1}{\Lambda} Y_N\right)_{\alpha\beta} \bar{L_L}^\alpha \tilde{H} i\partial\!\!\!/ \left(\tilde{H}^\dagger L_L^\beta\right) + \mathcal{O}\left(\frac{1}{\Lambda'^2}, \frac{1}{\Lambda^2 \Lambda'}\right) , \end{aligned} \tag{3.59}$$

to be compared with the typical structure of inverse seesaw models, eq. (3.51). The $c^{d=6} \propto c^{d=5}$ relation between the flavour structures of d=5 and d=6 operators discussed in section 4 holds (up to CP phases), provided we assume that the flavour symmetry group is

$$SU(3)_{\ell_L} \times SU(3)_E \times SU(n)_N \times O(n)_{N',N''}, \tag{3.60}$$

and is only broken by the spurions $Y_N \sim (\bar{3}, 1, n, 1)$ and $\Lambda \sim (1, 1, n, n)$, while both Λ' and μ_1 are invariant under $O(n)$ rotations of the N'' and N' fields. In this situation, $\mu_2 \sim (1, 1, n, n) \sim \Lambda$. Would Λ be instead proportional to the identity and Y_N the only spurion, then the symmetry group would be

$$SU(3)_{\ell_L} \times SU(3)_E \times O(n)_{N,N',N''}, \tag{3.61}$$

and μ_2 would also be proportional to the identity.

Concerning the justification of the zeros in eq. (3.57), we note that the flavour symmetries just described are not enough to forbid, for example, a 33 entry in the case of eq. (3.60), or 13 and 14 entries (proportional to Y_N) in the case of eq. (3.61). However , it is easy to justify a breaking of the $U(1)_{LN}$ symmetry only through the μ_1 and μ_2 terms, if we assume that the symmetry has been spontaneously broken through the vev of a singlet scalar S with lepton number $L_S = +1$. The only possible renormalizable couplings of the scalar to fermions would then be precisely those giving rise to the μ_1 and μ_2 terms, see eq. (A.10) in Appendix A. As in the type A models with spontaneous symmetry breaking, questions of naturalness may arise though, as we briefly discuss in that appendix.

As in the case of type A models, an alternative to break the global symmetry is to lift the zeros in λ_N, that is the 13 or 14 entries in the neutrino matrix in eq. (3.57). A 13 entry would reduce the model at low energies to that discussed in section 3. On the contrary, a 14 entry would be qualitatively different:

$$M_\nu = \begin{pmatrix} 0 & Y_N v/\sqrt{2} & 0 & Y'_N v/\sqrt{2} \\ Y_N^T v/\sqrt{2} & 0 & \Lambda & 0 \\ 0 & \Lambda^T & 0 & 0 \\ Y'^T_N v/\sqrt{2} & 0 & 0 & \Lambda' \end{pmatrix}, \tag{3.62}$$

with Y'_N and Y_N being distinct spurions, since the quantum numbers of N_α and N''_α are different. The approximate $U(1)_{LN}$ symmetry is ensured in this case not by a suppressed Y'_N, but rather by a large hierarchy $\Lambda' \gg \Lambda$. The integration of the scale Λ' and Λ in this case gives now rise to the d=5 and d=6 operators with coefficient matrices given by:

$$c^{d=5}_{\alpha\beta} \equiv \epsilon \left(Y'^T_N \frac{1}{\Lambda'} Y'_N \right)_{\alpha\beta}, \; c^{d=6}_{\alpha\beta} \equiv \left(Y^\dagger_N \frac{1}{\Lambda^\dagger \Lambda} Y_N \right)_{\alpha\beta} + \mathcal{O}\left(\frac{1}{\Lambda'}\right). \tag{3.63}$$

Therefore, their flavour structures are completely unrelated and condition b) is not satisfied for these models. Also, in contrast with type A models, the simplest case with $n = 1$ does not lead here to a phenomenologically viable model since there is only one massive neutrino, and at least $n = 2$ should have to be explored.

A possibility to enforce MFV in this case would be to have both Λ and Λ' proportional to the identity matrix, and $Y_N \propto Y'_N$. This might be justified assuming for instance the flavour symmetry in eq. (3.61). This would not forbid however a 13 entry in eq. (3.62) proportional to the Y_N spurion, and additional small parameters would thus be required to ensure suppressed neutrino masses in this case. Note also that a spontaneously broken symmetry pattern cannot generate any 14 entry in eq. (3.62) at the renormalizable level. Finally, note that leptogenesis has been studied in some models of type B in Ref. [71], and in the "extended MFV" framework in Ref. [72].

In summary, Type B models involve two physical scales, associated to the masses of extra heavy fermions - SM singlets or triplets. The approximate $U(1)_{LN}$ symmetry is recovered in the limit of large Λ_{LN}, characteristic of some heavy fermion mass, and not by introducing very small mass terms or couplings. Although physically more appealing, the presence of two distinct mass scales is not stable under radiative corrections (unless some couplings are small), which is nothing but the standard naturalness problem. Models of type B are interesting in particular as a possible ultraviolet completion of MFV neutrino mass models of type A. Another ultraviolet completion of Type A models involving scalars instead of fermions is analyzed next.

Chapter 4

Neutrino masses from higher than d=5 effective operators

In this chapter we explore the possibility, already mentioned in Sec. 1.4, that the Weinberg operator is forbidden in a theory that violates LN symmetry. In that case, Majorana neutrino masses would be induced by $d > 5$ operators. What kind of symmetries could forbid the $d = 5$ neutrino mass operator and allow those with $d \geq 7$ and what is their relation with lepton number symmetry?. We begin in the next section with some considerations that will prove useful to tackle this problem.

4.1 Rationale

The general form of effective SM Lagrangian was presented in Eq.(2.1). It was remarked that in the SM effective Lagrangian we find, among the even dimension operators, some that contribute to exotic flavour processes and non-unitarity. Both are typical signals of new physics. In the other hand, at odd dimensions we find operators that violate LN symmetry. In this chapter it will be assumed that there is just one scale of new physics, $\Lambda_{fl} \equiv \Lambda_{LN}$. Then, disregarding flavor, spinor, and gauge indices, the lepton number violating $d = 5$, $d = 7$, *etc.*, operators in Eq.(2.1) that contribute to Majorana neutrino masses are of the form

$$\begin{aligned} \mathcal{O}^5 &= \mathcal{O}_W = LLHH && (4.1)\\ \mathcal{O}^7 &= (LLHH)(H^\dagger H) && (4.2)\\ \mathcal{O}^9 &= (LLHH)(H^\dagger H)(H^\dagger H) && (4.3)\\ &\vdots \end{aligned}$$

If we assume that the neutrino Yukawa coupling is naturally $\mathcal{O}(1)$, then the neutrinos masses generated by these operators are roughly given by

$$m_\nu \sim v \left(\frac{v}{\Lambda_{\rm LN}} \right)^{d-4} . \tag{4.4}$$

For a typical neutrino mass of $O(\text{eV})$, this relationship gives the energy scale of new physics as a function of the dimension d of the operator responsible for neutrino masses. If we want to lower the scale of new physics down to that of present or near future experiments, $\Lambda_{\rm LN} \sim 1 - 10$ TeV, then $d \geq 9$ suffices in case no additional suppression mechanism is provided. On the other hand, if Yukawas of the order $m_e/v \simeq 10^{-6}$ are considered natural, then $d \geq 7$ is enough.

Our goal is to generate neutrino masses through an effective operator with $d \leq 7$. It is worth to take a closer look at the complications involved. For that, let D be the dimension of the operator that gives the dominant contribution to neutrino masses. In order to claim $D > 5$, we need all relevant operators of dimension $d < D$ to be strictly forbidden. For if we take for instance the operator in Eq. (4.2), it is clear that the $(H^\dagger H)$ component can be closed in a loop, Fig.4.1. This leads to the $d = 5$ Weinberg operator with the additional suppression factor $1/(16\pi^2)$ – unless the loop contributions cancel:

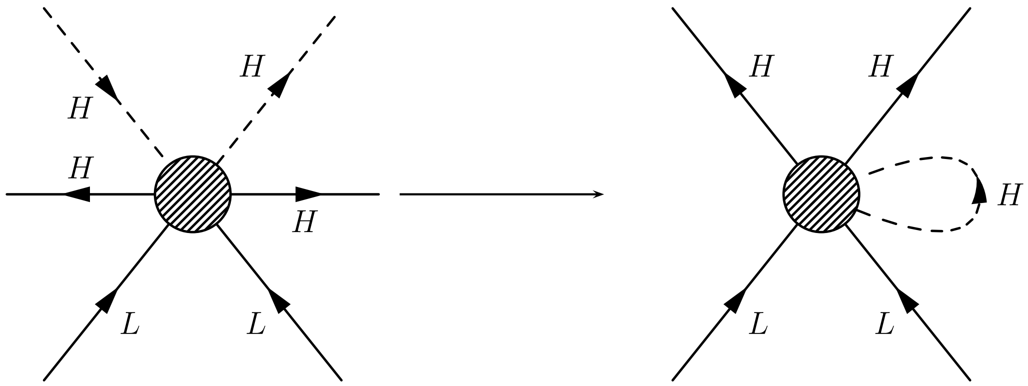

Figure 4.1: $d = 7$ neutrino mass operators made of SM fields inevitably generate the Weinberg operator radiatively.

$$\frac{1}{\Lambda_{\rm LN}^3} (LLHH)(H^\dagger H) \to \frac{1}{16\pi^2} \frac{1}{\Lambda_{\rm LN}} (LLHH) . \tag{4.5}$$

The latter will be the leading contribution to neutrino masses if $1/(16\pi^2) \gtrsim (v/\Lambda_{\rm LN})^2$, that is, if $\Lambda_{\rm LN} \gtrsim 3\,\text{TeV}$. Note that in both cases $\Lambda_{\rm LN} \lesssim 3$ TeV and $\Lambda_{\rm LN} \gtrsim 3$ TeV, the new physics might have implications at the LHC. For a robust model to be valid in the entire LHC-testable range, one should therefore have a symmetry to justify the absence of the $d = 5$ neutrino mass operator.

We call a dimension $d \geq 7$ operator *genuine* if it is impossible to generate some other neutrino mass operator of lower dimension by closing loops. In this work, we seek for a genuine operator, which means that we need a symmetry that forbids the appearance of neutrino masses at dimension $d < D$. One can easily see from Eqs. (4.1) to (4.3) that the symmetry cannot be implemented with SM fields only. This is because the combination $(H^\dagger H)$ is a singlet under any symmetry and therefore, if one operator is allowed, then the whole tower must be so. On the contrary, one can slightly enlarge the Higgs sector and charge the fields under a new U(1) or discrete symmetry (a so-called "matter parity" [151]) that allows a dimension D operator while forbidding all others with lower dimension.

In this context, the simplest possibilities to enhance the field content of the SM are the addition of a Higgs singlet [146, 147]

$$\mathscr{L}_{\text{eff}}^{d=n+5} = \frac{1}{\Lambda_{\text{LN}}^{d-4}}(LLHH)(S)^n\,, \quad n = 1, 2, 3, \ldots \tag{4.6}$$

or the addition of a Higgs doublet, leading to the Two Higgs Doublet Model (THDM) [145, 148, 152]

$$\mathscr{L}_{\text{eff}}^{d=2n+5} = \frac{1}{\Lambda_{\text{LN}}^{d-4}}(LLH_uH_u)(H_dH_u)^n\,, \quad n = 1, 2, 3, \ldots\,. \tag{4.7}$$

where the H_u couples to the U-type quarks and the H_d couples to the D-type quarks and the charged leptons. More complicated options include, for instance, the next-to-minimal SUSY standard model (NMSSM) using two Higgs doublets and a scalar, see Ref. [147]. In this study, we only consider Eq. (4.7) within the THDM. However, as we shall discuss elsewhere [153], our mechanism can be applied to SUSY models as well.

We discuss in Sec. 4.2 the conditions to obtain neutrino masses from genuine effective operators of dimension $d \geq 7$. Then we show in Sec. 4.3 several tree level decompositions of the only $d = 7$ operator allowed in both SUSY and the THDM, which describe the smallness of the lepton number violating terms naturally. Furthermore, we discuss generic extensions of the standard see-saw scenarios in Sec. 4.4, and we illustrate additional suppression mechanisms, such as from even higher dimensional operators or loop suppression factors, in Sec. 4.5.

4.2 Neutrino mass from higher dimensional operators

In order to have a *genuine* dimension D operator to be the leading contribution to neutrino mass, we forbid all $d < D$ operators by means of a new U(1) or $\mathbb{Z}_n$ symmetry. We assign matter charges q (see, *e.g.*, Refs. [151, 154]) to the new fields H_u (q_{H_u}), H_d (q_{H_d}), and the SM fields, *i.e.*, the lepton doublets L (q_L), right-handed charged leptons

E (q_E), quark doublets Q (q_Q), right-handed up-type quarks U (q_U), and right-handed down-type quarks D (q_D).

For the following discussion, we show the charge assignments assuming a discrete $\mathbb{Z}_n$ symmetry. Note, however, that the effective operators can be controlled as well by a new U(1) symmetry. If that is the case, additional (unwanted) Goldstone bosons may appear after the spontaneous breaking of the electroweak and U(1) symmetry. As we will discuss later, this can be avoided by breaking the U(1) explicitly, either by an enhanced scalar sector, or by a soft breaking term. Since the actual implementation of this U(1) breaking depends on the model, we will not touch it in this section, and focus on the discrete symmetries for the moment.

We list the possible $d = 5$, $d = 7$, and $d = 9$ effective operators that generate neutrino mass together with the charge of the effective interaction in Table 4.1. In SUSY models, only the operators with the column "SUSY" checked are allowed because of the holomorphy of super-potential. In the last two columns, we show the charge of the effective interaction with respect to our discrete symmetry, and we number the independent conditions.

Obviously, not all of the charges are independent, which we illustrate by giving each independent condition a number (second-last column). Genuine operators are precisely the ones whose charge is independent from all those of lower dimension. For instance, at order $d = 7$, the only possible genuine operators are #4 and #11. At $d = 9$, there are again only two possibilities, operators #12 and #26. In the following, we will use operator #4 as an example, since it is the simplest realization of our mechanism which is allowed in both the THDM and SUSY. Note that in SUSY models, only the operators with the column "SUSY" checked are allowed because of the holomorphy of super-potential.

In order to have operator #4 as leading contribution, we need to allow this operator by the condition on the $\mathbb{Z}_n$ charges

$$(2q_L + q_{H_d} + 3q_{H_u}) \mod n = 0 \tag{4.8}$$

and suppress all lower dimensional operators and all other $d = 7$ operators by charging them as (*cf.*, Table 4.1):

$$\begin{aligned}
(2q_L + 2q_{H_u}) &\mod n \neq 0\,, &(4.9)\\
(2q_L - q_{H_d} + q_{H_u}) &\mod n \neq 0\,, &(4.10)\\
(2q_L - 2q_{H_d}) &\mod n \neq 0\,, &(4.11)\\
(2q_L - 3q_{H_d} - q_{H_u}) &\mod n \neq 0\,. &(4.12)
\end{aligned}$$

	SUSY	Op.#	Effective interaction	Cond.#	Charge of effective int.
dim.5	✓	1	LLH_uH_u	1	$2q_L + 2q_{H_u}$
		2	$LLH_d^*H_u$	2	$2q_L - q_{H_d} + q_{H_u}$
		3	$LLH_d^*H_d^*$	3	$2q_L - 2q_{H_d}$
dim.7	✓	4	$LLH_uH_uH_dH_u$	4	$2q_L + q_{H_d} + 3q_{H_u}$
		5	$LLH_uH_uH_d^*H_d$	1	$2q_L + 2q_{H_u}$
		6	$LLH_uH_uH_u^*H_u$	1	$2q_L + 2q_{H_u}$
		7	$LLH_d^*H_uH_d^*H_d$	2	$2q_L - q_{H_d} + q_{H_u}$
		8	$LLH_d^*H_uH_u^*H_u$	2	$2q_L - q_{H_d} + q_{H_u}$
		9	$LLH_d^*H_d^*H_d^*H_d$	3	$2q_L - 2q_{H_d}$
		10	$LLH_d^*H_d^*H_u^*H_u$	3	$2q_L - 2q_{H_d}$
		11	$LLH_d^*H_d^*H_u^*H_d^*$	5	$2q_L - 3q_{H_d} - q_{H_u}$
dim.9	✓	12	$LLH_uH_uH_dH_uH_dH_u$	6	$2q_L + 2q_{H_d} + 4q_{H_u}$
		13	$LLH_uH_uH_dH_uH_d^*H_d$	4	$2q_L + q_{H_d} + 3q_{H_u}$
		14	$LLH_uH_uH_dH_uH_u^*H_u$	4	$2q_L + q_{H_d} + 3q_{H_u}$
		15	$LLH_uH_uH_d^*H_dH_d^*H_d$	1	$2q_L + 2q_{H_u}$
		16	$LLH_uH_uH_d^*H_dH_u^*H_u$	1	$2q_L + 2q_{H_u}$
		17	$LLH_uH_uH_u^*H_uH_u^*H_u$	1	$2q_L + 2q_{H_u}$
		18	$LLH_d^*H_uH_d^*H_dH_d^*H_d$	2	$2q_L - q_{H_d} + q_{H_u}$
		19	$LLH_d^*H_uH_d^*H_dH_u^*H_u$	2	$2q_L - q_{H_d} + q_{H_u}$
		20	$LLH_d^*H_uH_u^*H_uH_u^*H_u$	2	$2q_L - q_{H_d} + q_{H_u}$
		21	$LLH_d^*H_d^*H_d^*H_dH_d^*H_d$	3	$2q_L - 2q_{H_d}$
		22	$LLH_d^*H_d^*H_d^*H_dH_u^*H_u$	3	$2q_L - 2q_{H_d}$
		23	$LLH_d^*H_d^*H_u^*H_uH_u^*H_u$	3	$2q_L - 2q_{H_d}$
		24	$LLH_d^*H_d^*H_d^*H_u^*H_d^*H_d$	5	$2q_L - 3q_{H_d} - q_{H_u}$
		25	$LLH_d^*H_d^*H_d^*H_u^*H_u^*H_u$	5	$2q_L - 3q_{H_d} - q_{H_u}$
		26	$LLH_d^*H_d^*H_u^*H_d^*H_u^*H_d^*$	7	$2q_L - 4q_{H_d} - 2q_{H_u}$
dim.11			$\ldots$		

Table 4.1: Effective operators generating neutrino mass in the THDM.

In addition, we have to allow the ordinary Yukawa interactions which requires[1]

$$(q_E + q_L + q_{H_d}) \mod n = 0\,, \tag{4.13}$$

$$(q_D + q_Q + q_{H_d}) \mod n = 0\,, \tag{4.14}$$

$$(q_U + q_Q + q_{H_u}) \mod n = 0\,. \tag{4.15}$$

Without loss of generality, we fix the charge of the quark doublet to be $q_Q = 0$.

[1]Note that $E = (e_R)^c$, $U = (u_R)^c$, and $D = (d_R)^c$. We assume Yukawa interactions of the THDM type II (and MSSM) in which Higgs-mediated flavour changing neutral current processes are suppressed [152].

We have tested all possibilities for charge assignments and discrete symmetries systematically in order to identify the simplest possibility in terms of group order (we do not consider group products). It has turned out that a $\mathbb{Z}_5$ symmetry is the simplest one, with, for instance, the following charge assignments

$$q_{H_u} = 0\,,\ q_{H_d} = 3\,,\ q_L = 1\,,\ q_E = 1\,,\ q_Q = 0\,,\ q_U = 0\,,\ q_D = 2\,. \tag{4.16}$$

For operator #11, we also obtain $\mathbb{Z}_5$ as option with the lowest group order. For the $d = 9$ operators #12 and #26, we need at least a $\mathbb{Z}_7$. If SUSY is implemented, both operators #4 and #12 can be realized within a $\mathbb{Z}_3$. Note that the charge assignments are not unique.[2]

From the discussion above, it should be clear that these operators can be generated at tree level, which we consider in the following two sections. The discrete symmetry (matter parity), which we have introduced, must be broken by the Higgses taking their VEVs, because the effective Majorana mass terms obviously violate the $\mathbb{Z}_5$. Note, however, that this symmetry is not the same as lepton number. This can easily seen by the effective operator $(\#1)^5$ made from operator #1 in Table 4.1. This operator is obviously invariant under the $\mathbb{Z}_5$, but it violates lepton number.

4.3 Inverse see-saw mechanisms with naturally suppressed lepton number violation

So far we have only discussed the effective operators and the necessary conditions to have a genuine $d > 5$ operators as leading contribution for neutrino mass. We show in this section several examples to illustrate the completions of the theory at high energies.

We consider see-saw-like models of the fermionic type. It is easy to convince oneself that the simplest cases, such as the type I see-saw, can not produce a genuine $D \geq 7$ operator.[3] Hence, we focus on tree level decompositions of $d = 7$ operators which require the addition of two extra fermion singlet fields N_{Ra} and N'_{La}. This leads to an inverse see-saw-like structure [135–137] of the neutral fermion mass matrix of the form of Eq.(3.10) (in the basis $\begin{pmatrix}\nu_L^c & N_R & N_L'^{\,c}\end{pmatrix}$):

$$M_\nu = \begin{pmatrix} 0 & (Y_\nu^{\mathsf{T}})v & \epsilon(Y_\nu'^{\mathsf{T}}) \\ (Y_\nu)v & \mu' & \Lambda_{\mathrm{LN}} \\ \epsilon(Y_\nu') & \Lambda_{\mathrm{LN}}^T & \mu \end{pmatrix} . \tag{4.17}$$

[2] There are $3 \times 2 = 6$ possibilities for $\mathbb{Z}_3$, $5 \times 4 = 20$ possibilities for $\mathbb{Z}_5$, $42 = 7 \times 6$ possibilities for $\mathbb{Z}_7$, *etc.*, because the first assignment is always arbitrary, the second is also arbitrary but cannot be hypercharge (one possibility subtracted), and the rest is determined by these two.

[3] The type I see-saw implies the introduction of the right-handed heavy Majorana mass term and the Yukawa interaction with the lepton doublet. If the Yukawa interaction is present in the theory, the right-handed Majorana mass term has to be obviously forbidden, because otherwise the usual $d = 5$ operator is generated. Without the Majorana mass, however, no suppression is obtained. Therefore, at least one more fermionic field is required, and the fermionic fields need to form a mass term.

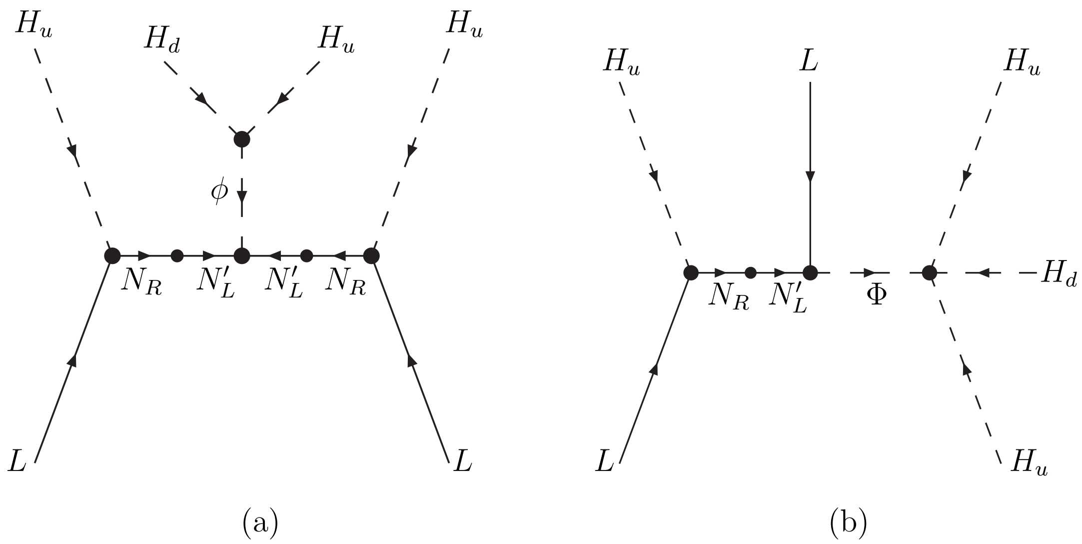

Figure 4.2: Tree level decompositions of the dimension seven operator $LLH_uH_uH_dH_u$ (#4 in Table 4.1) for neutrino masses.

With only N_R and N'_L added to the SM, the interactions leading to the mass matrix in Eq. (4.17) can only be obtained via non-renormalizable operators. Indeed, assuming the charge assignments in Eq. (4.16), the only renormalizable term that can be written is

$$\overline{N_R} Y_\nu H_u \mathrm{i}\tau^2 L + \text{H.c.} \tag{4.18}$$

with $q_{N_R} = q_{N'_L} = 1$ in order to conserve the $\mathbb{Z}_5$ symmetry. Now constructing, with the SM fields plus N_R and N'_L, the possible effective operators that respect the $\mathbb{Z}_5$ symmetry, one obtains the $d = 5$ operators

$$\frac{\lambda_1}{\Lambda_{\text{LN}}}(H_d \mathrm{i}\tau^2 H_u)\overline{N_R}N_R^c + \frac{\lambda'_1}{\Lambda_{\text{LN}}}(H_d \mathrm{i}\tau^2 H_u)\overline{N_L'^c}N'_L + \text{H.c.}, \tag{4.19}$$

and the $d = 6$ operator

$$\frac{\lambda_2}{\Lambda_{\text{LN}}^2}(H_d \mathrm{i}\tau^2 H_u)\overline{N_L'^c}Y'_\nu H_u \mathrm{i}\tau^2 L + \text{H.c.}\,. \tag{4.20}$$

Matching these with Eq. (4.17) leads to

$$\mu = \frac{\lambda_1}{\Lambda_{\text{LN}}}\langle H_d^0 H_u^0\rangle, \quad \mu' = \frac{\lambda'_1}{\Lambda_{\text{LN}}}\langle H_d^0 H_u^0\rangle, \quad \epsilon = \frac{\lambda_2}{\Lambda_{\text{LN}}^2}\langle H_d^0\rangle\langle H_u^0\rangle^2\,. \tag{4.21}$$

In order to generate those coefficients through renormalizable interactions, extra scalar fields need to be added, whose masses will define Λ_{LN}. From Eqs. (4.19) and (4.20), it is clear that, at tree-level, the same field that generates the operators in Eq. (4.19) cannot generate the operator in Eq. (4.20). In other words, depending on the decomposition, the μ-term or the ϵ-term will be generated, but not both simultaneously. This justifies a small μ or ϵ resulting from high energy scales plus discrete symmetries.

In the following, we will show fundamental theories which predict a small lepton number violating (LNV) μ- or ϵ-term suppressed by the new physics scale. The diagrams generating neutrino mass are shown in Fig. 4.2 where N'_L and N_R refer to SU(2) singlet fermions, ϕ to a singlet scalar, and Φ to a doublet scalar. They lead to inverse see-saw scenarios with entries in the (3,3) element (a) and (1,3) element (b).

4.3.1 Decomposition (a): The μ-term

For the decomposition (a) in Fig. 4.2, we introduce two chiral fermions, singlets of the SM: N_R (right-handed) and N'_L (left-handed), and a SM singlet scalar ϕ. The relevant interactions are then given by

$$\begin{aligned}\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \big[& Y_\nu \, \overline{N_R} H_u \mathrm{i}\tau^2 L + \kappa (\overline{N_L'^c})(N'_L)\phi + \mu \phi^* H_d \mathrm{i}\tau^2 H_u + \\ & + \overline{N_R}\, M \,(N'_L) + \mathrm{H.c.}\big] + M_\phi^2 \phi^* \phi + \cdots \,. \end{aligned} \tag{4.22}$$

The mass matrix for the neutral fermion fields can be summarized as

$$\mathscr{L} = \frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{N_R} & \overline{N_L'^c} \end{pmatrix} \begin{pmatrix} 0 & Y_\nu^\mathsf{T} \langle H_u^0 \rangle & 0 \\ Y_\nu \langle H_u^0 \rangle & 0 & M \\ 0 & M^\mathsf{T} & (\Lambda^{-1}) \langle H_d^0 H_u^0 \rangle \end{pmatrix} \begin{pmatrix} \nu_{L\beta} \\ N_R^c \\ N'_L \end{pmatrix} + \mathrm{H.c.}, \tag{4.23}$$

similar to the inverse see-saw one in Eq. (4.17) with the μ-term as source of LNV. Here the Majorana mass term for N'_L arises after the spontaneous breaking of the electroweak symmetry (and the matter parity) with the coefficient

$$(\Lambda^{-1})_{\alpha\beta} = 2\kappa_{\alpha\beta} \frac{\mu}{M_\phi^2} \sim O\left(\frac{1}{\Lambda_{\mathrm{LN}}} \right) \tag{4.24}$$

suppressed by the new physics scale.

The effective neutrino masses are then given by

$$m_\nu = \frac{v_u^3 v_d}{4} Y_\nu^\mathsf{T} (M^{-1})^\mathsf{T} \Lambda^{-1} M^{-1} Y_\nu \sim O\left(\frac{v^4}{\Lambda_{\mathrm{LN}}^3} \right) , \tag{4.25}$$

where $v_u = \sqrt{2}\langle H_u^0 \rangle$ and $v_d = \sqrt{2}\langle H_d^0 \rangle$. If we assume $\Lambda_{\mathrm{LN}} \sim 1\,\mathrm{TeV}$ and $m_\nu \sim 1$ eV, then we have the Dirac mass term for the $\overline{N_R}$-ν_L interaction with $Y_\nu v_u \sim 10$ MeV, which is smaller than in the ordinary see-saw scenario but of the same order as the charged lepton masses, *i.e.*, Y_ν is not *extremely* small in comparison with the other fermion Yukawa couplings.

In order to have the interactions in Eq (4.22) and to forbid the Majorana mass term for the SM singlet fermion N_R and the Yukawa interaction with N'_L, we assign the following charges[4] under a $\mathbb{Z}_5$:

$$q_{H_u} = 0\,, \quad q_{H_d} = 3\,, \quad q_L = 1\,, \quad q_{N_R} = q_{N'_L} = 1, \quad q_\phi = 3\,. \tag{4.26}$$

Note that we cannot forbid the interaction $\overline{N_R^c} N_R \phi$ by any charge assignment, which means that the (2,2) element (μ'-term) in Eq. (4.23) is actually non-zero after EW symmetry breaking, but suppressed with respect to the Dirac masses of N_R and N'_L. Nevertheless, such a Majorana mass term

$$\frac{M_R}{2} \overline{N_R^c} N_R + \text{h.c.}\,, \tag{4.27}$$

gives a contribution to neutrino masses proportional to $(v_u^4 v_d^2 M_R)/(M^4 \Lambda_{\text{LN}}^2)$, which is of second order, and can thus be omitted from this discussion.

It is interesting to compare our approach to the original inverse see-saw model. In the original model, the texture of the mass matrix is justified by the lepton number symmetry with the charge assignment $L(\nu_L) = 1$, $L(N_R) = 1$ and $L(N'_L) = 1$. Then, the Majorana mass term of the N'_L field (the μ-term in Eq. (4.17)) is introduced by hand and its smallness is justified by the fact that it is the only lepton number violating quantity. In our model, the texture of the mass matrix is determined by the $\mathbb{Z}_5$ symmetry. Moreover, the Majorana mass for the N'_L field is generated after electroweak symmetry (and matter parity) breaking and is naturally small since it is suppressed by the scale of new physics Λ_{LN}. We thus implement what is sometimes called "double see-saw" rather than inverse see-saw. Indeed we have one see-saw mechanism which generates a small Majorana mass for the new fermion singlet N'_L suppressed by Λ_{LN}, and another one which generates small neutrino masses suppressed by M.

In fact, this model, defined by the SM Lagrangian plus the interactions displayed in Eq. (4.22), has more than a $\mathbb{Z}_5$ symmetry: it is also invariant under a new U(1) symmetry.[5] This is potentially dangerous since the breaking of the electroweak symmetry also breaks this U(1) symmetry leading to a massless Goldstone boson. However, this can be avoided by an enhanced scalar sector, provided the term

$$\frac{\lambda}{\Lambda_\phi} \phi^5\,, \tag{4.28}$$

appears in the effective Lagrangian after integration of the degrees of freedom of some high energy theory (here Λ_ϕ denotes the typical scale). We do not provide such a theory

[4]Note that allowing the Yukawa interaction in Eq. (4.22), together with Eq. (4.9) automatically gives the necessary conditions to forbid these two terms.

[5]The new symmetry neither corresponds to lepton number nor hypercharge. It contains $\mathbb{Z}_5$ which is often called "matter parity". With respect to the Higgs potential, it plays the same role as the Peccei-Quinn symmetry [156–159].

here explicitly since it is not directly relevant for the generation of neutrino mass in this context. Neverthless, we have checked that one can have an enhanced scalar sector to produce Eq. (4.28) in this model, without having massless Goldstone bosons and unwanted tadpole of additional scalars at the same time. Instead, we refer to Sec. 4.5 for an explicit model where neutrino masses depend crucially on the breaking of some U(1) symmetry down to $\mathbb{Z}_5$.

As an alternative, one can introduce a soft violation of the U(1) symmetry (and also $\mathbb{Z}_5$)

$$\mathscr{V}_{\rm soft} = m_3^2 H_d \mathrm{i}\tau^2 H_u + \text{H.c.}\,, \tag{4.29}$$

where m_3 is assumed to be the electroweak scale. This term is generally introduced in the THDM as the soft breaking term of $\mathbb{Z}_2$ to forbid FCNC (Flavor Changing Neutral Current) processes [160, 161]. The introduction of this soft term makes the Higgs phenomenology MSSM-like. With this term, the Goldstone boson obtains the mass proportional to m_3, which is identified with the CP odd Higgs boson, A^0, in the THDM and the MSSM. This soft term also affects neutrino masses since it explicitly violates $\mathbb{Z}_5$, which implies that the dimension five Weinberg operator must appear at the loop level by insertion of that term inside a loop. Note that the loop diagram is constructed by closing the Higgs propagators of the dimension seven operator $LLH_uH_uH_dH_u$. Therefore, it has to be proportional to the dimension seven contribution, Fig. 4.3. The size of the contribution is estimated as

$$\frac{1}{16\pi^2}\frac{m_3^2}{\Lambda_{\rm LN}^3}(LLH_uH_u), \tag{4.30}$$

which is still suppressed with respect to the tree level dimension seven contribution by the loop suppression factor. Therefore, the introduction of a soft term would not disturb our main line of argumentation.[6]

4.3.2 Decomposition (b): The ϵ-term

For the decomposition (b) in Fig. 4.2, we introduce two chiral fermions, singlets of the SM model N_R (right-handed) and N'_L (left-handed), and a $SU(2)_L$ doublet scalar Φ. We then need the following relevant interactions:

$$\begin{aligned}\mathscr{L} =& \mathscr{L}_{\rm SM} + \Big[Y_\nu\, \overline{N_R} H_u \mathrm{i}\tau^2 L + Y'_\nu\, \overline{N_L'^c} \Phi^\dagger L + \zeta (H_d \mathrm{i}\tau^2 H_u)(\Phi \mathrm{i}\tau^2 H_u) \\ &+ \overline{N_R} M N'_L + \text{H.c.}\Big] + M_\Phi^2 \Phi^\dagger \Phi + \cdots\,. \end{aligned} \tag{4.31}$$

[6]Note that compared to the loop contributions from closing the $H^\dagger H$ loop, there is factor of m_3^2 in the numerator compared to $\Lambda_{\rm NP}^2$, which means that this contribution is effectively suppressed as strong as the one loop $d = 7$ operator instead of the one loop $d = 5$ operator. Therefore, our usual argumentation (in the introduction) on closing the loops does not apply.

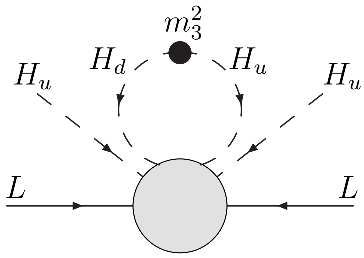

Figure 4.3: One loop contribution with the soft breaking term of Eq. (4.29).

These lead to the mass matrix

$$\mathscr{L} = \frac{1}{2}\begin{pmatrix}\overline{\nu_L^c} & \overline{N_R} & \overline{N_L'^c}\end{pmatrix}\begin{pmatrix} 0 & Y_\nu^\mathsf{T}\langle H_u^0\rangle & \frac{\zeta Y_\nu'^\mathsf{T}}{M_\Phi^2}\langle H_d^0\rangle\langle H_u^0\rangle^2 \\ Y_\nu\langle H_u^0\rangle & 0 & M \\ \frac{\zeta Y_\nu'}{M_\Phi^2}\langle H_d^0\rangle\langle H_u^0\rangle^2 & M^\mathsf{T} & 0\end{pmatrix}\begin{pmatrix}\nu_L \\ N_R^c \\ N_L'\end{pmatrix} + \text{H.c.}. \tag{4.32}$$

with a non-trivial $(1,3)$ element (ϵ-term in Eq. (4.17)) again suppressed by the new physics scale. The joint presence of the three entries violates lepton number and yields the neutrino mass

$$m_\nu = \frac{\zeta v_u^3 v_d}{4M_\Phi^2}\left(Y_\nu^\mathsf{T} M^{-1} Y_\nu' + Y_\nu'^\mathsf{T}(M^{-1})^\mathsf{T} Y_\nu\right) \sim \mathcal{O}\left(\frac{v^4}{\Lambda_{\text{LN}}^3}\right). \tag{4.33}$$

A contribution of the same order of that of the case we considered previously.

The conditions on the charges imposed by the fundamental interactions can be implemented by the following assignments under a $\mathbb{Z}_5$:

$$q_{H_u} = 0\,, \quad q_{H_d} = 3, \quad q_L = 1\,, \quad q_{N_R} = q_{N_L'} = 1, \quad q_\Phi = 2\,. \tag{4.34}$$

This model is also a double see-saw but involves the product of two Dirac masses, contrary to the previous case where Majorana masses were involved. Again, the U(1) has to be broken explicitly.

4.4 Generalization of standard see-saws

Here we show all possible decompositions of the dimension seven operator $LLH_uH_uH_dH_u$ (#4 in Table 4.1) at tree level. We do not go into the details of the models, such as the Lagrangian and the matter parity conditions to implement it. Therefore, these results should be interpreted as the necessary conditions to find tree level neutrino mass models from this dimension seven operator. For the new fields, we follow the notation in Ref. [82]. They are denoted by $\mathbf{X}_Y^{\mathcal{L}}$, where

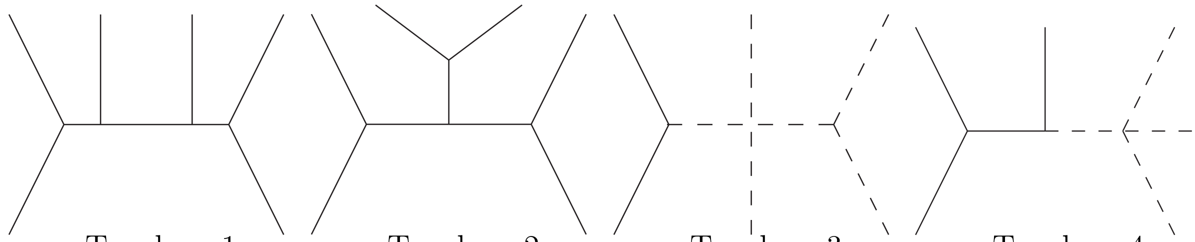

Figure 4.4: Possible topologies for the tree level decomposition of the dimension seven operator $LLH_uH_uH_dH_u$.

- **X** denotes the SU(2) nature, *i.e.*, singlet **1**, doublet **2**, or triplet **3**.
- $\mathcal{L}$ refers to the Lorentz nature, *i.e.*, scalar (s), vector (v), left-handed (L) or right-handed (R) chiral fermion.
- Y refers to the hypercharge $Y = Q - I_3^W$.

The possible topologies of the Feynman diagrams are shown in Fig. 4.4 where the dashed lines denote always scalars (scalar mediators or the Higgs doublet). The solid lines in Topology 1, 2, and 4 should be interpreted as fermions or scalars depending on the decomposition.

We present our results in Table 4.2. In this table, we show the decompositions of all possible combinations leading to the effective operator $LLH_uH_uH_dH_u$. The brackets in the operators show the fundamental interactions, *i.e.*, each operator corresponds to a Feynman diagram with the topology listed in the 3rd column (*cf.*, Fig. 4.4). The fourth column shows the SM quantum numbers of the required mediators, where each symbol represents a separate new field. The abbreviation "X/Y" means that either X or Y or both are different possibilities, depending on the topology, whereas the abbreviation "(X)" means that X is optional, depending on the topology. The last columns indicate the phenomenology one may expect in this model, where "NU" stands for non-unitarity of the lepton mixing matrix, "δg_L" for a shift of the neutral current coupling with charged leptons, and "4ℓ" for charged lepton flavor violation or non-standard neutrino interactions.

The decompositions which we have discussed in Sec. 4.3 are #1 and #13 in the table. The table is useful to read off the potential models for any possible set of mediators. For example, one can read off from Table 4.2 the generic realizations[7] of the standard

[7]By generic realization we mean a decomposition which includes one or two copies of the original mediator ($\mathbf{1}_0^{R/L}$ for the Type I see-saw, $\mathbf{3}_{-1}^{s}$ for the Type II see-saw and $\mathbf{3}_0^{R/L}$ for the Type III see-saw) plus extra scalar mediators.

#	Operator	Top.	Mediators	Phenom. NU	δg_L	4ℓ
1	$(H_u \mathrm{i}\tau^2 \overline{L^c})(H_u \mathrm{i}\tau^2 L)(H_d \mathrm{i}\tau^2 H_u)$	2	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{1}_0^s$	✓		
2	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(H_u \mathrm{i}\tau^2 L)(H_d \mathrm{i}\tau^2 \vec{\tau} H_u)$	2	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$ $\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{3}_0^s$	✓	✓	
3	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(H_u \mathrm{i}\tau^2 \vec{\tau} L)(H_d \mathrm{i}\tau^2 H_u)$	2	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{1}_0^s$	✓	✓	
4	$(-\mathrm{i}\epsilon^{abc})(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(H_u \mathrm{i}\tau^2 \tau^b L)(H_d \mathrm{i}\tau^2 \tau^c H_u)$	2	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{3}_0^s$	✓	✓	
5	$(\overline{L^c} \mathrm{i}\tau^2 \vec{\tau} L)(H_d \mathrm{i}\tau^2 H_u)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	2/3	$\mathbf{3}_{-1}^s$, $\mathbf{3}_{-1}^s/\mathbf{1}_0^s$			✓
6	$(-\mathrm{i}\epsilon^{abc})(\overline{L^c} \mathrm{i}\tau^2 \tau^a L)(H_d \mathrm{i}\tau^2 \tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^c H_u)$	2/3	$\mathbf{3}_{-1}^s$, $\mathbf{3}_{-1}^s/\mathbf{3}_0^s$			✓
7	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L \mathrm{i}\tau^2 \vec{\tau} H_d)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	2	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{3}_{-1}^s$	✓	✓	
8	$(-\mathrm{i}\epsilon^{abc})(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(L \mathrm{i}\tau^2 \tau^b H_d)(H_u \mathrm{i}\tau^2 \tau^c H_u)$	2	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{3}_{-1}^s$	✓	✓	
9	$(H_u \mathrm{i}\tau^2 \overline{L^c})(\mathrm{i}\tau^2 H_u)(L)(H_d \mathrm{i}\tau^2 H_u)$	1	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{2}_{-1/2}^R$, $\mathbf{2}_{-1/2}^L$, $\mathbf{1}_0^s$	✓		
10	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(\mathrm{i}\tau^2 \vec{\tau} H_u)(L)(H_d \mathrm{i}\tau^2 H_u)$	1	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{2}_{-1/2}^R$, $\mathbf{2}_{-1/2}^L$, $\mathbf{1}_0^s$	✓	✓	
11	$(H_u \mathrm{i}\tau^2 \overline{L^c})(\mathrm{i}\tau^2 H_u)(\vec{\tau} L)(H_d \mathrm{i}\tau^2 \vec{\tau} H_u)$	1	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{2}_{-1/2}^R$, $\mathbf{2}_{-1/2}^L$, $\mathbf{3}_0^s$	✓		
12	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\mathrm{i}\tau^2 \tau^a H_u)(\tau^b L)(H_d \mathrm{i}\tau^2 \tau^b H_u)$	1	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{2}_{-1/2}^R$, $\mathbf{2}_{-1/2}^L$, $\mathbf{3}_0^s$	✓	✓	
13	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L)(\mathrm{i}\tau^2 H_u)(H_d \mathrm{i}\tau^2 H_u)$	1/4	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{2}_{-1/2}^s$, $(\mathbf{1}_0^s)$	✓		
14	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(\vec{\tau} L)(\mathrm{i}\tau^2 H_u)(H_d \mathrm{i}\tau^2 H_u)$	1/4	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{2}_{-1/2}^s$, $(\mathbf{1}_0^s)$	✓	✓	
15	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L)(\mathrm{i}\tau^2 \vec{\tau} H_u)(H_d \mathrm{i}\tau^2 \vec{\tau} H_u)$	1/4	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{2}_{-1/2}^s$, $(\mathbf{3}_0^s)$	✓		
16	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\tau^a L)(\mathrm{i}\tau^2 \tau^b H_u)(H_d \mathrm{i}\tau^2 \tau^b H_u)$	1/4	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{2}_{-1/2}^s$, $(\mathbf{3}_0^s)$	✓	✓	
17	$(H_u \mathrm{i}\tau^2 \overline{L^c})(H_d)(\mathrm{i}\tau^2 H_u)(H_u \mathrm{i}\tau^2 L)$	1	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{2}_{-1/2}^R$, $\mathbf{2}_{-1/2}^L$	✓		
18	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(\vec{\tau} H_d)(\mathrm{i}\tau^2 H_u)(H_u \mathrm{i}\tau^2 L)$	1	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{2}_{-1/2}^R$, $\mathbf{2}_{-1/2}^L$, $\mathbf{1}_0^R$, $\mathbf{1}_0^L$	✓	✓	
19	$(H_u \mathrm{i}\tau^2 \overline{L^c})(H_d)(\mathrm{i}\tau^2 \vec{\tau} H_u)(H_u \mathrm{i}\tau^2 \vec{\tau} L)$	1	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{2}_{-1/2}^R$, $\mathbf{2}_{-1/2}^L$, $\mathbf{3}_0^R$, $\mathbf{3}_0^L$	✓	✓	
20	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\tau^a H_d)(\mathrm{i}\tau^2 \tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b L)$	1	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{2}_{-1/2}^R$, $\mathbf{2}_{-1/2}^L$,	✓	✓	
21	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a L)(H_u \mathrm{i}\tau^2 \tau^a)(\tau^b H_d)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	1/4	$\mathbf{3}_{-1}^s$, $\mathbf{2}_{+1/2}^s$, $(\mathbf{3}_{-1}^s)$			✓
22	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a L)(H_d \mathrm{i}\tau^2 \tau^a)(\tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	1/4	$\mathbf{3}_{-1}^s$, $\mathbf{2}_{+3/2}^s$, $(\mathbf{3}_{-1}^s)$			✓
23	$(\overline{L^c} \mathrm{i}\tau^2 \vec{\tau} L)(H_u \mathrm{i}\tau^2 \vec{\tau})(H_u)(H_d \mathrm{i}\tau^2 H_u)$	1/4	$\mathbf{3}_{-1}^s$, $\mathbf{2}_{+1/2}^s$, $(\mathbf{1}_0^s)$			✓
24	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a L)(H_u \mathrm{i}\tau^2 \tau^a)(\tau^b H_u)(H_d \mathrm{i}\tau^2 \tau^b H_u)$	1/4	$\mathbf{3}_{-1}^s$, $\mathbf{2}_{+1/2}^s$, $(\mathbf{3}_0^s)$			✓
25	$(H_d \mathrm{i}\tau^2 H_u)(\overline{L^c} \mathrm{i}\tau^2)(\vec{\tau} L)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	1	$\mathbf{1}_0^s$, $\mathbf{2}_{+1/2}^L$, $\mathbf{2}_{+1/2}^R$, $\mathbf{3}_{-1}^s$			
26	$(H_d \mathrm{i}\tau^2 \tau^a H_u)(\overline{L^c} \mathrm{i}\tau^2 \tau^a)(\tau^b L)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	1	$\mathbf{3}_0^s$, $\mathbf{2}_{+1/2}^L$, $\mathbf{2}_{+1/2}^R$, $\mathbf{3}_{-1}^s$			
27	$(H_u \mathrm{i}\tau^2 \overline{L^c})(\mathrm{i}\tau^2 H_d)(\vec{\tau} L)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	1	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{2}_{+1/2}^R$, $\mathbf{2}_{+1/2}^L$, $\mathbf{3}_{-1}^s$	✓		
28	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\mathrm{i}\tau^2 \tau^a H_d)(\tau^b L)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	1	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{2}_{+1/2}^R$, $\mathbf{2}_{+1/2}^L$, $\mathbf{3}_{-1}^s$	✓	✓	
29	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L)(\mathrm{i}\tau^2 \vec{\tau} H_d)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	1/4	$\mathbf{1}_0^R$, $\mathbf{1}_0^L$, $\mathbf{2}_{+1/2}^s$, $(\mathbf{3}_{-1}^s)$	✓		
30	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\tau^a L)(\mathrm{i}\tau^2 \tau^b H_d)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	1/4	$\mathbf{3}_0^R$, $\mathbf{3}_0^L$, $\mathbf{2}_{+1/2}^s$, $(\mathbf{3}_{-1}^s)$	✓	✓	
31	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a H_d)(\mathrm{i}\tau^2 \tau^a H_u)(\tau^b L)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	1	$\mathbf{3}_{+1}^L$, $\mathbf{3}_{+1}^R$, $\mathbf{2}_{+1/2}^L$, $\mathbf{2}_{+1/2}^R$, $\mathbf{3}_{-1}^s$	✓	✓	
32	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a H_d)(\tau^a L)(\mathrm{i}\tau^2 \tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	1/4	$\mathbf{3}_{+1}^L$, $\mathbf{3}_{+1}^R$, $\mathbf{2}_{-3/2}^s$, $(\mathbf{3}_{-1}^s)$	✓	✓	
33	$(\overline{L^c} \mathrm{i}\tau^2 \vec{\tau} H_d)(\mathrm{i}\tau^2 \vec{\tau} H_u)(H_u)(H_u \mathrm{i}\tau^2 L)$	1	$\mathbf{3}_{+1}^L$, $\mathbf{3}_{+1}^R$, $\mathbf{2}_{-3/2}^L$, $\mathbf{2}_{-3/2}^R$, $\mathbf{1}_0^L$, $\mathbf{1}_0^R$	✓	✓	
34	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a H_d)(\mathrm{i}\tau^2 \tau^a H_u)(\tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b L)$	1	$\mathbf{3}_{+1}^L$, $\mathbf{3}_{+1}^R$, $\mathbf{2}_{-3/2}^L$, $\mathbf{2}_{-3/2}^R$, $\mathbf{3}_0^L$, $\mathbf{3}_0^R$	✓	✓	

Table 4.2: Decompositions of the effective dimension seven operator $LLH_uH_uH_dH_u$.

type I, II, and III see-saw mechanisms by using their field content and additional mediators:

Type I (fermionic singlet mediator) Operators #1, #13, #15, and #29 are simple generalizations. In fact, our decompositions in Sec. 4.3 represent some of the simplest possible generalizations of the type I see-saw mechanism, which require only three types of new fields in total.

Type II (scalar triplet mediator) Operators #5, #6, #21, #22, #23, and #24 are the simplest possibilities. For example, #5 requires an additional triplet scalar and/or singlet scalar.

Type III (fermionic triplet mediator) Operators #3, #4, #14, #16, #30 are possible options. For example, operator #3 is the natural type III counterpart of the inverse see-saw mechanism in the previous section.

One can also reads from Table 4.2 that some decompositions are combinations of different types of see-saws: for example operator #2 can be viewed as a Type I + Type III see-saw. Note that the decomposition shown in Ref. [149] does not appear in Table 4.2 because we concentrate on the decomposition with $\mathrm{SU}(2)_L$ singlet, doublet, and triplet mediators.

4.5 Additional suppression mechanisms

In this section, we qualitatively sketch options for additional suppression mechanisms compared to the simplest possibility, the tree level decompositions of the $d = 7$ operators. For this discussion, it is useful to consider the following expansion of the effective operators, where $\mathscr{L}^{(k)}_{d=D}$ stands for the dimension D contribution to the k-loop correction:

$$\begin{aligned}\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{SM}} \quad & + \quad \frac{1}{\Lambda_{\text{LN}}}\left[\mathscr{L}^{(0)}_{d=5} + \delta\mathscr{L}^{(1)}_{d=5} + \delta\mathscr{L}^{(2)}_{d=5} + \cdots\right] \\ & + \quad \frac{1}{\Lambda^3_{\text{LN}}}\left[\mathscr{L}^{(0)}_{d=7} + \delta\mathscr{L}^{(1)}_{d=7} + \delta\mathscr{L}^{(2)}_{d=7} + \cdots\right] \\ & + \quad \frac{1}{\Lambda^5_{\text{LN}}}\left[\mathscr{L}^{(0)}_{d=9} + \delta\mathscr{L}^{(1)}_{d=9} + \delta\mathscr{L}^{(2)}_{d=9} + \cdots\right] \\ & + \quad \frac{1}{\Lambda^7_{\text{LN}}}\left[\mathscr{L}^{(0)}_{d=11} + \delta\mathscr{L}^{(1)}_{d=11} + \delta\mathscr{L}^{(2)}_{d=11} + \cdots\right] \\ & + \quad \cdots \end{aligned} \tag{4.35}$$

In general, the vertical expansion is controlled by the new symmetry, whereas the horizontal expansion is suppressed by the loop suppression factor. If, for instance, we go to $d = 7$, we can switch off the first row in Eq. (4.35) by imposing a new U(1) symmetry,

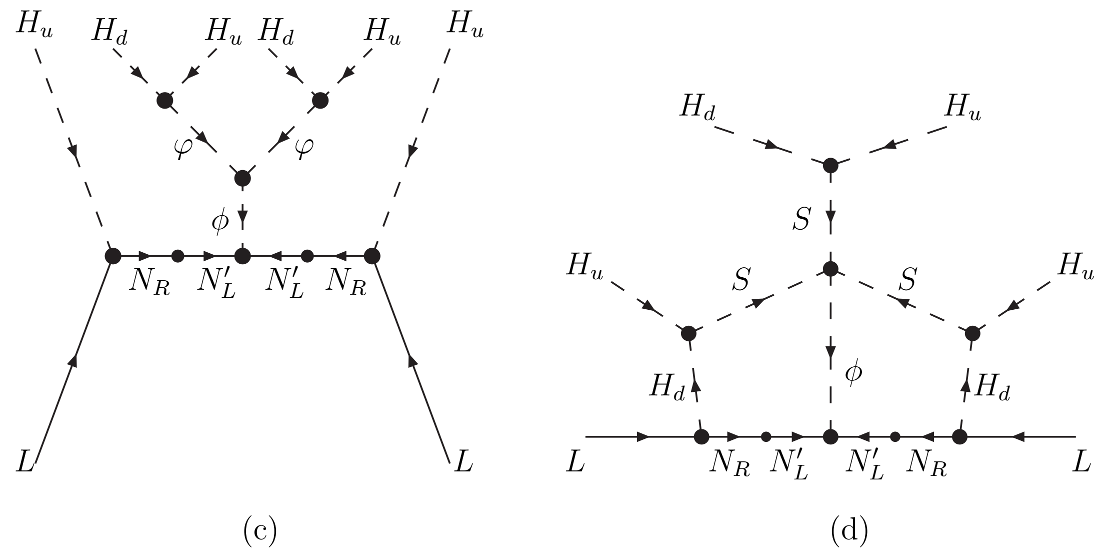

Figure 4.5: Left panel: A possible tree level decomposition of the dimension nine operator $LLH_uH_uH_dH_uH_dH_u$ for the generation of neutrino mass. It can also be interpreted within the inverse see-saw framework. Right panel: A possible two loop decomposition of the dimension seven operator $LLH_uH_uH_dH_u$.

and there is no need to worry about loop contributions at $d = 5$. In this case, $\mathscr{L}^{(0)}_{d=7}$ gives the leading contribution for neutrino mass generation, as it was used the previous two sections.

However, if one wants to implement additional suppression from higher dimensional operators either in the horizontal (loops) or vertical (higher d) direction, it is necessary to ensure that the discussed contribution is the leading order effect. For instance, it is *a priori* not clear which of the operators $\delta\mathscr{L}^{(1)}_{d=7}$ and $\mathscr{L}^{(0)}_{d=9}$ gives a larger contribution if both are allowed. As already discussed in the introduction, if $1/(16\pi^2) \gtrsim (v/\Lambda_{\text{LN}})^2$, or $\Lambda_{\text{LN}} \gtrsim 3\,\text{TeV}$, one would generically expect that the loop contributions are larger than the ones from the higher dimensional operators. However, a too low new physics scale $\sim$ TeV may be potentially harmful for a loop model if there are higher dimensional tree level contributions. We will discuss a two loop model from $\delta\mathscr{L}^{(2)}_{d=7}$ in Sec. 4.5.2. Since there is no contribution from $\mathscr{L}^{(0)}_{d=7}$, $\delta\mathscr{L}^{(1)}_{d=7}$, and $\mathscr{L}^{(0)}_{d=9}$ in this model, it gives the leading contribution to neutrino mass for $\Lambda_{\text{LN}} \gtrsim 3\,\text{TeV}$. If $\Lambda_{\text{LN}} \lesssim 3\,\text{TeV}$, $\mathscr{L}^{(0)}_{d=11}$ and $\delta\mathscr{L}^{(1)}_{d=9}$ have to be avoided as well. Furthermore, we show an example for $d = 9$ from $\mathscr{L}^{(0)}_{d=9}$ in Sec. 4.5.1.

4.5.1 Higher than $d = 7$ at tree level

Here we qualitatively sketch an example of neutrino mass generation from $\mathscr{L}^{(0)}_{d=9}$ ($d = 9$, tree level). The relevant diagram, corresponding to #12 in Table 4.1, is shown in Fig. 4.5 (c). We introduce two SM singlet chiral fermions N_R (right-handed) and N'_L (left-handed), and two SM singlet scalars ϕ and φ. The interaction Lagrangian is given by

$$\begin{aligned}\mathscr{L} =& \mathscr{L}_{\mathrm{SM}} + \Big[Y_\nu \overline{N_R} H_u \mathrm{i}\tau^2 L + \kappa \overline{N'^c_L}) N'_L \phi + \mu \varphi^* H_d \mathrm{i}\tau^2 H_u + \omega \varphi\varphi\phi^* \\ &+ \overline{N_R} M N'_L + \mathrm{H.c.}\Big] + M^2_\phi \phi^*\phi + M^2_\varphi \varphi^*\varphi + \cdots . \end{aligned} \tag{4.36}$$

It can be implemented by the following charge assignments under a $\mathbb{Z}_7$ (*cf.*, Sec. 4.2):

$$q_{H_u} = 0\,, \quad q_{H_d} = 6\,, \quad q_L = 1, \quad q_{N_R} = q_{N'_L} = 1\,, \quad q_\varphi = 6\,, \quad q_\phi = 5\,. \tag{4.37}$$

If the scalars are integrated out, we obtain the inverse see-saw mass matrix with a μ-term

$$\mathscr{L} = \frac{1}{2} \begin{pmatrix} \overline{\nu^c_L} & \overline{N_R} & \overline{N'^c_L} \end{pmatrix} \begin{pmatrix} 0 & Y^\mathsf{T}_\nu \langle H^0_u \rangle & 0 \\ Y_\nu \langle H^0_u \rangle & 0 & M \\ 0 & M^\mathsf{T} & \Lambda^{-3} \langle H^0_d H^0_u \rangle^2 \end{pmatrix} \begin{pmatrix} \nu_L \\ N^c_R \\ N'_L \end{pmatrix} + \mathrm{H.c.} \tag{4.38}$$

with

$$(\Lambda^{-3})_{\alpha\beta} = 2\kappa_{\alpha\beta} \frac{\mu^2 \omega}{M^2_\phi M^4_\varphi} \sim \mathcal{O}\left(\frac{1}{\Lambda^3_{\mathrm{LN}}}\right) . \tag{4.39}$$

Now the μ-term is suppressed by $\Lambda^{-3}_{\mathrm{LN}}$ and the LNV parameter κ, *i.e.*, extremely small. Neutrino mass, of course, acquires additional suppression from M.

4.5.2 Two loop generated $d = 7$ operator

Here we show an example for neutrino mass generation from $\delta\mathscr{L}^{(2)}_{d=7}$.[8] This possibility is a very neat example, because all the different ideas mentioned in Sec. 2.1 to reduce the new physics scale are implemented at once: radiative generation of neutrino mass, small LNV parameter, and neutrino mass generation from a higher dimensional operator. In all the previous examples, the resulting Lagrangian had a full (new) U(1) symmetry instead of a $\mathbb{Z}_n$, and the U(1) had to be broken by a sector which is independent of neutrino mass. In this example, we will demonstrate that the neutrino mass emerges from the breaking of the U(1) to the $\mathbb{Z}_n$.

[8] Another realization of the loop suppressed inverse see-saw with an extended Higgs sector is shown in Ref. [162].

We introduce two SM singlet chiral fermions N_R and N'_L, and two SM singlet scalars ϕ and S:

$$\begin{aligned}\mathscr{L} =&\mathscr{L}_{\rm SM} + \Big[Y_N \overline{N_R} H_d^\dagger L + \alpha_1 \phi \overline{N_R^c} N_R + \alpha_2 \phi \overline{N_L'^c} N'_L + \\ &+ \mu S^* H_d \mathrm{i}\tau^2 H_u + \overline{N_R} M N'_L + \text{H.c.}\Big] - \mathscr{V}(H_u,\, H_d,\, \phi,\, S)\,. \end{aligned} \tag{4.40}$$

The relevant part of the scalar potential is given by

$$\mathscr{V}(H_u,\, H_d,\, \phi,\, S) = \big[\lambda_1 S\phi^3 + \mu_1 S^* \phi^2 + \lambda_2 S^3 \phi^* + \text{H.c.}\big] + M_S^2 S^* S + M_\phi^2 \phi^* \phi + \cdots\,. \tag{4.41}$$

Let us focus on the terms in the bracket in Eq. (4.40): These terms respect three independent U(1) symmetries, which can be identified with hypercharge, lepton number and an additional (new) U(1) symmetry. Since lepton number is conserved in this sector of the Lagrangian, no operator can be written for neutrino masses. On the other hand, the scalar potential in Eq. (4.41) violates all continuous symmetries but hypercharge, while respecting $\mathbb{Z}_5$. Neutrino masses are therefore only allowed in the presence of the scalar potential, which violates lepton number and the new U(1). In fact, the scalar potential in Eq. (4.41) just generates the effective U(1) breaking term in Eq. (4.28) after integrating out the S field.

In the following, we choose the $\mathbb{Z}_5$ charges

$$q_{H_u} = 0\,,\quad q_{H_d} = 1\,,\quad q_L = 2,\quad q_{N_R} = q_{N'_L} = 1\,,\quad q_\phi = 3\,,\quad q_S = 1 \tag{4.42}$$

to implement the Lagrangian in Eq. (4.40) leading to neutrino mass from operator #4 in Table 4.1, while operators #1 to #3 are forbidden.

Now the mass matrix for three types of neutral fermions can be written as

$$\mathscr{L} = \frac{1}{2}\begin{pmatrix}\overline{\nu_L^c} & \overline{N_R} & \overline{N_L'^c}\end{pmatrix}\begin{pmatrix} m_\nu^{(\text{2-loop})} & Y_N^\mathsf{T}\langle H_d^{0*}\rangle & \epsilon Y_\nu'^{(\text{1-loop})\mathsf{T}} \\ Y_N \langle H_d^{0*}\rangle & \mu'^{(\text{tree})} & M \\ \epsilon Y_\nu'^{(\text{1-loop})} & M^\mathsf{T} & \mu^{(\text{tree})}\end{pmatrix}\begin{pmatrix}\nu_L \\ N_R^c \\ N'_L\end{pmatrix} + \text{H.c.}. \tag{4.43}$$

Assuming that

$$M_S \sim M_\phi \sim \mu \sim M \equiv \Lambda_{\rm NP} \tag{4.44}$$

we can estimate the elements as

$$m_{\nu\alpha\beta}^{(\text{2-loop})} \sim \frac{1}{(16\pi^2)^2}\frac{v_d v_u^3}{\Lambda_{\rm LN}}\lambda_2 [Y_N^\mathsf{T} (M^\mathsf{T})^{-1} \alpha_2 M^{-1} Y_N]_{\alpha\beta}, \tag{4.45}$$

$$(\epsilon Y'_\nu)_{\alpha\beta}^{(\text{1-loop})} \sim \frac{1}{16\pi^2}\frac{v_d^2 v_u^3}{\Lambda_{\rm LN}^3}\lambda_2[\alpha_2 M^{-1} Y_N]_{\alpha\beta}, \tag{4.46}$$

$$\mu_{\alpha\beta}^{(\text{tree})} \sim \frac{v_d^3 v_u^3}{\Lambda_{\rm LN}^5}\lambda_2 (\alpha_2)_{\alpha\beta}, \tag{4.47}$$

$$\mu_{\alpha\beta}'^{(\text{tree})} \sim \frac{v_d^3 v_u^3}{\Lambda_{\rm LN}^5}\lambda_2^* (\alpha_1^*)_{\alpha\beta}. \tag{4.48}$$

The two-loop contribution to neutrino masses comes from the diagram shown in Fig. 4.5 (d). The one-loop contribution to ϵ-term can be obtain by cutting a propagator of H_d in Fig. 4.5 (d) and giving the VEVs to the end of the cut propagator. Assuming the parameters α_2 and λ_2 are $O(1)$, we find that in order to obtain neutrino masses of the order of the eV if $\Lambda_{\rm LN} \sim 10\,\mathrm{TeV}$.

Notice that the order of magnitude result of Eqs. (4.47) and (4.48) takes into account the fact that the scalars S and ϕ acquire VEVs, after electroweak symmetry breaking, due to the mixed terms in the Lagrangian. These VEVs can be estimated at tree level by minimizing the potential. In particular, the VEV of the ϕ, which contributes to the Majorana masses of the heavy neutrinos, is consistent with the formulas above.

Two things are different in this scenario from those previously considered and are worth stressing. First of all, postulating that neutrino mass is generated from the breaking of the (new) $U(1)$ symmetry down to $\mathbb{Z}_5$ has forced us to consider loop diagrams in order to generate neutrino masses. These introduce an extra suppression by about two orders of magnitude. Secondly, the neutrino masses must come proportional to the couplings in the scalar sector α_1 and λ_2 since they are responsible for LNV and the $U(1)$ breaking. If these couplings are perturbative, they can easily account for some more suppression while still being natural. That is, this model predicts neutrino masses at a scale of new physics that is naturally the TeV scale with large Yukawa couplings. Since flavor violating processes appear at tree level as $d = 6$ operators, we expect new physics within the reach of near future experiments.

Chapter 5

Large gauge invariant non-standard neutrino interactions

Up to now we have been focused in models of neutrino mass. In the road, we have always kept in mind near future experimental capabilities. In this last chapter we shift focus from the origin of neutrino masses to explore the possibility of Non-Standard Neutrino Interactions.

We saw in Sec.2.5 that exotic couplings involving neutrinos must still respect the SM gauge symmetry. Electroweak symmetry relates these couplings to processes among charged leptons that might be strongly suppressed. We examine in what follows what kind of constraints impose these relations to non-standard interactions.

This work is partly focused on the model building aspect of gauge invariant NSNIs. In order to determine possible models leading to NSIs we will make use of the tree-level mediator decomposition technique that was illustrated in Sec.1.4 for the seesaw models. We will classify all $d = 6$ and $d = 8$, $SU(3) \times SU(2) \times U(1)$ invariant, leptonic NSI operators in terms of the heavy mediators that induce them.

We will first emphasize the case in which the mediators exchanged only couple to SM bilinear field combinations. In this study, we refer to "SM bilinears" as fundamental interactions of *exactly two* SM fields with one or two exotic fields, where the latter possibility amounts to couplings between two exotic bosons and two Higgs doublets. Other than that, there can be in addition new exotic couplings involving only *one* SM field, which will be addressed in a later stage.

In the general analysis, after determining all possible mediators, the resulting correlations between the possible $d = 6$ and $d = 8$ operators will be systematically studied. We will then establish which mediators or combinations of mediators can lead to large NSI, without inducing experimentally excluded leptonic charged flavour-changing transitions, and/or other undesired phenomenological consequences.

Our main motivation in this study is to determine the minimum level of complexity needed for a viable model of NSI. As an illustration for model building, a particular simple toy model will be developed in which the operator $\mathcal{O}_{\rm NSI}$ of Eq.(2.52) is induced unaccompanied by any leptonic $d = 6$ operator. The aim is to show the generic prize to pay at the theoretical level for allowing observable NSI effects at future experiments.

Note that we focus in this study on the necessary conditions to build a model with large NSI, while for any given model additional limitations may arise. Supplementary constraints which could arise from a phenomenological analysis at one-loop are also not considered here and should be addressed when considering a particular model. We will not make any explicit statement how likely it is to observe large NSI. We leave the interpretation of this likeliness by judging the necessary conditions for a viable model to the reader. Finally, possible NSI involving quark fields are neither included in this study.

5.1 Effective operator formalism

The SM Lagrangian is extended to accommodate the tower of effective even dimension operators

$$\delta\mathscr{L}_{\rm eff} = \frac{1}{\Lambda^2}\sum_i^{d=6} \mathcal{C}_i\,\mathcal{O}_i^{d=6} + \frac{1}{\Lambda^4}\sum_k^{d=8} \mathcal{C}_k\,\mathcal{O}_k^{d=8}\,, \tag{5.1}$$

related with exotic flavour processes. Here the two terms run over all possible $d = 6$ and $d = 8$ operators relevant for purely leptonic NSI. The flavour composition will be made explicit in each coefficient and operator, *i.e.*, $(\mathcal{C}_i)^{\alpha\gamma}_{\beta\delta}(\mathcal{O}_i)^{\beta\delta}_{\alpha\gamma}$. All distinct flavour combinations for the same operator structure will be taken into account, as they correspond in fact to independent operators.

Effective operator basis

In order to find all possible $d = 6$ and $d = 8$ effective operators leading to purely leptonic NSI, we will use the following bases:

- $\boldsymbol{d = 6}$ **operators**. A complete basis of $d = 6$ operators invariant under the SM gauge group and made out of the SM light fields was proposed by Buchmüller and Wyler (BW) [78]. The four fermion operator structures relevant to our problem are:

$$(\mathcal{O}_{LE})^{\beta\delta}_{\alpha\gamma} =(\bar{L}^\beta E_\gamma)(\bar{E}^\delta L_\alpha)\,, \tag{5.2}$$
$$(\mathcal{O}^{\mathbf{1}}_{LL})^{\beta\delta}_{\alpha\gamma} =(\bar{L}^\beta\gamma^\rho L_\alpha)(\bar{L}^\delta\gamma_\rho L_\gamma)\,, \tag{5.3}$$
$$(\mathcal{O}^{\mathbf{3}}_{LL})^{\beta\delta}_{\alpha\gamma} =(\bar{L}^\beta\gamma^\rho\vec{\tau} L_\alpha)(\bar{L}^\delta\gamma_\rho\vec{\tau} L_\gamma)\,, \tag{5.4}$$
$$(\mathcal{O}_{EE})^{\beta\delta}_{\alpha\gamma} =(\bar{E}^\beta\gamma^\rho E_\alpha)(\bar{E}^\delta\gamma_\rho E_\gamma)\,, \tag{5.5}$$

  where L (E) refers to the $SU(2)$ leptonic doublet (singlet). We will refer to the coefficient matrices for these operators by $(\mathcal{C}_{LE})^{\alpha\gamma}_{\beta\delta}$, $(\mathcal{C}^{\mathbf{1}}_{LL})^{\alpha\gamma}_{\beta\delta}$, $(\mathcal{C}^{\mathbf{3}}_{LL})^{\alpha\gamma}_{\beta\delta}$, and $(\mathcal{C}_{EE})^{\alpha\gamma}_{\beta\delta}$, respectively. The operators $\mathcal{O}_{EE}$ do not produce NSI directly, but will play a role when considering charged lepton flavor violation (since they share some mediators with the operators in Eq. (5.2)-(5.4)).

On top of the above, there are two $d = 6$ operator structures including two lepton doublets L and two Higgs doublets H,

$$(\mathcal{O}^{\mathbf{1}}_{LH})^{\beta}_{\alpha} = \left(\bar{L}^{\beta} H\right) \mathrm{i}(H^{\dagger} L_{\alpha}), \tag{5.6}$$

$$(\mathcal{O}^{\mathbf{3}}_{LH})^{\beta}_{\alpha} = \left(\bar{L}^{\beta} \vec{\tau} H\right) \mathrm{i}(H^{\dagger} \vec{\tau} L_{\alpha}), \tag{5.7}$$

and a operator with two E's and two H's

$$(\mathcal{O}_{EH})^{\beta}_{\alpha} = \left(H^{\dagger} \mathrm{i} D^{\rho} H\right) \left(\bar{E}^{\beta} \gamma_{\rho} E_{\alpha}\right) , \tag{5.8}$$

where D denotes the SM covariant derivative. These three operators belong to the class which, after EWSB, correct the parameters of the SM Lagrangian. In particular, they renormalize the kinetic energy of neutrinos and/or charged leptons [96,103,104]. As previously mentioned, they result in non-unitary corrections to the the leptonic mixing matrix and/or correct the charged and neutral electroweak currents [78], and will not be further developed in this work. We include them above only for the sake of completeness (see also Sec.5.3.2).

- $\boldsymbol{d = 8}$ **operators**. A basis was discussed by Berezhiani and Rossi (BR) [79], with the relevant operators given by

$$\begin{aligned}
(\mathcal{O}^{\mathbf{1}}_{LEH})^{\beta\delta}_{\alpha\gamma} &= (\bar{L}^{\beta} \gamma^{\rho} L_{\alpha})(\bar{E}^{\delta} \gamma_{\rho} E_{\gamma}) \left(H^{\dagger} H\right), && (5.9)\\
(\mathcal{O}^{\mathbf{3}}_{LEH})^{\beta\delta}_{\alpha\gamma} &= (\bar{L}^{\beta} \gamma^{\rho} \vec{\tau} L_{\alpha})(\bar{E}^{\delta} \gamma_{\rho} E_{\gamma}) \left(H^{\dagger} \vec{\tau} H\right), && (5.10)\\
(\mathcal{O}^{\mathbf{111}}_{LLH})^{\beta\delta}_{\alpha\gamma} &= (\bar{L}^{\beta} \gamma^{\rho} L_{\alpha})(\bar{L}^{\delta} \gamma_{\rho} L_{\gamma})(H^{\dagger} H), && (5.11)\\
(\mathcal{O}^{\mathbf{331}}_{LLH})^{\beta\delta}_{\alpha\gamma} &= (\bar{L}^{\beta} \gamma^{\rho} \vec{\tau} L_{\alpha})(\bar{L}^{\delta} \gamma_{\rho} \vec{\tau} L_{\gamma})(H^{\dagger} H), && (5.12)\\
(\mathcal{O}^{\mathbf{133}}_{LLH})^{\beta\delta}_{\alpha\gamma} &= (\bar{L}^{\beta} \gamma^{\rho} L_{\alpha})(\bar{L}^{\delta} \gamma_{\rho} \vec{\tau} L_{\gamma})(H^{\dagger} \vec{\tau} H), && (5.13)\\
(\mathcal{O}^{\mathbf{313}}_{LLH})^{\beta\delta}_{\alpha\gamma} &= (\bar{L}^{\beta} \gamma^{\rho} \vec{\tau} L_{\alpha})(\bar{L}^{\delta} \gamma_{\rho} L_{\gamma})(H^{\dagger} \vec{\tau} H), && (5.14)\\
(\mathcal{O}^{\mathbf{333}}_{LLH})^{\beta\delta}_{\alpha\gamma} &= (-\mathrm{i}\epsilon^{abc})(\bar{L}^{\beta} \gamma^{\rho} \tau^{a} L_{\alpha})(\bar{L}^{\delta} \gamma_{\rho} \tau^{b} L_{\gamma})(H^{\dagger} \tau^{c} H), && (5.15)\\
(\mathcal{O}_{EEH})^{\beta\delta}_{\alpha\gamma} &= (\bar{E} \gamma^{\rho} E)(\bar{E} \gamma_{\rho} E)(H^{\dagger} H) \,. && (5.16)
\end{aligned}$$

In these operators, subscripts correspond to a shortcut notation for their SM field composition, whereas superscripts denote the corresponding $SU(2)$ charges of the field combinations. Once again, although the operators $\mathcal{O}_{EEH}$ cannot induce NSI by themselves, they will come to play a related role, as they induce charged lepton flavour violating transitions.

Strictly speaking, not all of the above operators are independent when the full flavor structure is taken into account, as

$$(\mathcal{O}^{\mathbf{313}}_{LLH})^{\beta\delta}_{\alpha\gamma} = (\mathcal{O}^{\mathbf{133}}_{LLH})^{\delta\beta}_{\gamma\alpha} \,. \tag{5.17}$$

However, the expressions below will look much simpler if both operators are used. Notice that the phenomenologically interesting $\mathcal{O}_{\rm NSI}$ operator in Eq.(2.52) can be expressed as a combination of the two first operators in the list above,

$$\mathcal{O}_{\rm NSI} = \frac{1}{2}\,(\mathcal{O}^{\mathbf{1}}_{LEH} \,+\, \mathcal{O}^{\mathbf{3}}_{LEH})\,. \tag{5.18}$$

This means for instance that if a model only induces at $d=8$ the operators $\mathcal{O}^{\mathbf{1}}_{LEH}$ and $\mathcal{O}^{\mathbf{3}}_{LEH}$ with similar weights and no $d=6$ NSI operator, it could be an optimal candidate for viable large NSI. We will explore later some examples of this kind.

Decomposition in terms of $SU(2)$ field components

After EWSB, the contributions from the $d=6$ and $d=8$ gauge invariant operators result in two very simple sets of operators. From the $\bar{L}L\bar{E}E$-type operators, Eqs. (5.2), (5.9) and (5.10), we find:

$$\begin{aligned}\delta\mathscr{L}_{\rm eff} =&\frac{1}{\Lambda^2}\left(-\frac{1}{2}\mathcal{C}_{LE} + \frac{v^2}{2\Lambda^2}(\mathcal{C}^{\mathbf{1}}_{LEH} + \mathcal{C}^{\mathbf{3}}_{LEH})\right)^{\alpha\gamma}_{\beta\delta}\left(\bar{\nu}^{\beta}\gamma^{\rho}\mathrm{P}_L\nu_{\alpha}\right)\left(\bar{\ell}^{\delta}\gamma_{\rho}\mathrm{P}_R\ell_{\gamma}\right)\\ &+\frac{1}{\Lambda^2}\left(-\frac{1}{2}\mathcal{C}_{LE} + \frac{v^2}{2\Lambda^2}(\mathcal{C}^{\mathbf{1}}_{LEH} - \mathcal{C}^{\mathbf{3}}_{LEH})\right)^{\alpha\gamma}_{\beta\delta}\left(\bar{\ell}^{\beta}\gamma^{\rho}\mathrm{P}_L\ell_{\alpha}\right)\left(\bar{\ell}^{\delta}\gamma_{\rho}\mathrm{P}_R\ell_{\gamma}\right) + \text{h.c.}\,,\end{aligned} \tag{5.19}$$

The first line in this equation produces the relevant NSI, whereas the second line leads to the (unwanted) four charged lepton contributions. The NSI in the first line involve only right-handed charged leptons. In consequence, their effect at the neutrino source will be chirally suppressed[1].

From the operators involving four lepton doublets, Eqs. (5.3), (5.4), (5.11)-(5.15), it results that [2]

$$\begin{aligned}\delta\mathscr{L}_{\rm eff} =& \frac{1}{\Lambda^2}\left(\mathcal{C}^{\bar{L}L\bar{L}L}_{\rm NSI}\right)^{\alpha\gamma}_{\beta\delta}\left(\bar{\nu}^{\beta}\gamma^{\rho}\mathrm{P}_L\nu_{\alpha}\right)\left(\bar{\ell}^{\delta}\gamma^{\rho}\mathrm{P}_L\ell_{\gamma}\right)\\ &+\frac{1}{\Lambda^2}\left(\mathcal{C}^{\mathbf{1}}_{LL} + \mathcal{C}^{\mathbf{3}}_{LL} + \frac{v^2}{2\Lambda^2}\left(\mathcal{C}^{\mathbf{111}}_{LLH} + \mathcal{C}^{\mathbf{331}}_{LLH} - \mathcal{C}^{\mathbf{133}}_{LLH} - \mathcal{C}^{\mathbf{313}}_{LLH}\right)\right)^{\alpha\gamma}_{\beta\delta}\left(\bar{\ell}^{\beta}\gamma^{\rho}\mathrm{P}_L\ell_{\alpha}\right)\left(\bar{\ell}^{\delta}\gamma^{\rho}\mathrm{P}_L\ell_{\gamma}\right)\\ &+\text{h.c.}\,,\end{aligned} \tag{5.20}$$

[1]At detection, the effect of these NSI is subdominant because of the dominance of the neutrino-nucleon cross section.

[2]Here we do not show the interactions among four neutrinos which these operators also induce. See the Appendix B for a discussion of these interactions.

where

$$
\begin{aligned}
\left(\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L}\right)_{\beta\delta}^{\alpha\gamma} = &\left(\mathcal{C}_{LL}^{\mathbf{1}} - \mathcal{C}_{LL}^{\mathbf{3}} + \frac{v^2}{2\Lambda^2}\left(\mathcal{C}_{LLH}^{\mathbf{111}} - \mathcal{C}_{LLH}^{\mathbf{331}} - \mathcal{C}_{LLH}^{\mathbf{133}} + \mathcal{C}_{LLH}^{\mathbf{313}}\right)\right)_{\beta\delta}^{\alpha\gamma} \\
&+ \left(\mathcal{C}_{LL}^{\mathbf{1}} - \mathcal{C}_{LL}^{\mathbf{3}} + \frac{v^2}{2\Lambda^2}\left(\mathcal{C}_{LLH}^{\mathbf{111}} - \mathcal{C}_{LLH}^{\mathbf{331}} + \mathcal{C}_{LLH}^{\mathbf{133}} - \mathcal{C}_{LLH}^{\mathbf{313}}\right)\right)_{\delta\beta}^{\gamma\alpha} \\
&+ \left(2\,\mathcal{C}_{LL}^{\mathbf{3}} + \frac{v^2}{\Lambda^2}\left(\mathcal{C}_{LLH}^{\mathbf{331}} - \mathcal{C}_{LLH}^{\mathbf{333}}\right)\right)_{\delta\beta}^{\alpha\gamma} \\
&+ \left(2\,\mathcal{C}_{LL}^{\mathbf{3}} + \frac{v^2}{\Lambda^2}(\mathcal{C}_{LLH}^{\mathbf{331}} + \mathcal{C}_{LLH}^{\mathbf{333}})\right)_{\beta\delta}^{\gamma\alpha} . \qquad (5.21)
\end{aligned}
$$

Note the different flavor structure in the four lines in Eq. (5.21). In addition, note that the term relevant for the NSI, *i.e.*, the first line in Eq. (5.20), couples to left-handed charged leptons, which means that source NSI can be generated as well. In resume, matter NSI are (not) correlated with source and production NSI for $\bar{L}L\bar{L}L$ ($\bar{L}L\bar{E}E$)-type operators.

Connection to NSI and phenomenology

Let us first consider NSI in matter. The Hamiltonian describing neutrino propagation in matter under these conditions is that of Eq.(2.47). where, a_{CC} is the usual matter effect term defined as $a_{\text{CC}} \equiv 2\sqrt{2}EG_F N_e$ (with N_e the electron number density in Earth matter).

From Eqs. (5.19) and (5.20) it follows that

$$
\epsilon_{\beta\alpha}^{m,L} = \frac{v^2}{2\Lambda^2}\left(\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L}\right)_{\beta e}^{\alpha e}, \quad \epsilon_{\beta\alpha}^{m,R} = \frac{v^2}{2\Lambda^2}\left(-\frac{1}{2}\mathcal{C}_{LE} + \frac{v^2}{2\Lambda^2}(\mathcal{C}_{LEH}^{\mathbf{1}} + \mathcal{C}_{LEH}^{\mathbf{3}})\right)_{\beta e}^{\alpha e}, \qquad (5.22)
$$

with $\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L}$ as defined in Eq. (5.21). These two parameters in matter lead to a total

$$
\epsilon_{\beta\alpha}^{m} = \epsilon_{\beta\alpha}^{m,L} + \epsilon_{\beta\alpha}^{m,R}, \qquad (5.23)
$$

because matter effects are only sensitive to the vector component.

In addition to the propagation in matter, the production or detection processes can be affected by NSI. For the specific case of a neutrino factory and considering just the purely leptonic NSI under discussion, only effects at the source are relevant, since the detection interactions involve quarks [3]. They are customarily parametrized in terms of $\epsilon_{\alpha\beta}^{s}$, which describes an effective source state $|\nu_\alpha^s\rangle$ as [165, 193, 220]

$$
|\nu_\alpha^s\rangle = |\nu_\alpha\rangle + \sum_{\gamma=e,\mu,\tau} \epsilon_{\alpha\gamma}^{s}|\nu_\gamma\rangle . \qquad (5.24)
$$

[3] Superbeams, for instance, use hadronic interactions for neutrino production, which are not affected by purely leptonic NSI to first order.

In this case, the muon decay rate could be modified by the NSI interaction in Eq. (5.20), with the largest effect resulting from the coherent contribution to the state at the source [193,198]. It appears as an admixture of a given flavour ν_α with all other flavours, encoded by ν_γ in Eq. (5.24). Two types of contributions are possible,

$$\epsilon^s_{\mu\beta} = \frac{v^2}{2\Lambda^2}(\mathcal{C}^{\bar{L}L\bar{L}L}_{\rm NSI})^{e\mu}_{\beta e} \quad \text{or} \quad \epsilon^s_{e\beta} = \frac{v^2}{2\Lambda^2}(\mathcal{C}^{\bar{L}L\bar{L}L}_{\rm NSI})^{\mu e}_{\beta\mu} \, . \tag{5.25}$$

The second possibility will affect the golden $\nu_e \to \nu_\mu$ appearance channel, where the effect might be easiest to observe. If the coefficients in Eq. (5.21) are known for a specific model, one can easily calculate the connection between source and propagation effects via Eqs. (5.22) and (5.25), a connection which does not hold for $\bar{L}L\bar{E}E$-type operators above, as explained earlier.

Conditions to suppress charged lepton processes

Let us discuss now potentially dangerous contributions to charged lepton flavour violation processes, possible modifications of G_F and the constraints on lepton universality. The focus is set on pure charged lepton processes at tree level. These interactions can result from the second terms in Eqs. (5.19) and (5.20). They should be very suppressed in any phenomenologically viable model. In order to cancel those terms, the putative beyond the SM theory has to satisfy, to a high degree of accuracy, the following constraints:

$$\left(-\frac{1}{2}\mathcal{C}_{LE} + \frac{v^2}{2\Lambda^2}(\mathcal{C}^{\mathbf{1}}_{LEH} - \mathcal{C}^{\mathbf{3}}_{LEH})\right)^{\alpha\gamma}_{\beta\delta} = 0 \, , \tag{5.26}$$

$$\left(\mathcal{C}^{\mathbf{1}}_{LL} + \mathcal{C}^{\mathbf{3}}_{LL} + \frac{v^2}{2\Lambda^2}\left(\mathcal{C}^{\mathbf{111}}_{LLH} + \mathcal{C}^{\mathbf{331}}_{LLH} - \mathcal{C}^{\mathbf{133}}_{LLH} - \mathcal{C}^{\mathbf{313}}_{LLH}\right)\right)^{\alpha\gamma}_{\beta\delta} = 0 \, , \tag{5.27}$$

for all possible values of the flavour indices (Greek letters). A possibility suggested by these equations is that there could be cancellations among $d = 6$ and $d = 8$ operator coefficients. However, we will not discuss such a possibility in this study, as it would correspond to fine-tune the scale Λ. We will therefore require that the $d = 6$ and $d = 8$ operator coefficients in Eq. (5.26) and Eq. (5.27) cancel independently.

For the $d = 6$ operator coefficients, it reads (omitting flavor indices)

$$\mathcal{C}_{LE} = 0 \, , \quad \mathcal{C}^{\mathbf{1}}_{LL} = -\mathcal{C}^{\mathbf{3}}_{LL}, \, , \quad \mathcal{C}_{EE} = 0 \, , \tag{5.28}$$

which implies that only $\bar{L}L\bar{L}L$-type operators can induce large NSI. One possibility for its implementation is the antisymmetric operator mediated by a $SU(2)$ singlet scalar in Ref. [174], which turns out to be the only $d = 6$ possibility requiring just one tree-level mediator, as we shall explicitely demonstrate.

For the $d=8$ operator coefficients, the cancellation conditions read

$$\mathcal{C}^{\mathbf{1}}_{LEH} = \mathcal{C}^{\mathbf{3}}_{LEH}\,, \quad \mathcal{C}^{\mathbf{111}}_{LLH} + \mathcal{C}^{\mathbf{331}}_{LLH} - \mathcal{C}^{\mathbf{133}}_{LLH} - \mathcal{C}^{\mathbf{313}}_{LLH} = 0\,, \quad \mathcal{C}^{\mathbf{333}}_{LLH}\ \text{arbitr.}\,, \quad \mathcal{C}_{EEH} = 0\,, \tag{5.29}$$

where the first condition corresponds to operators of the type $\mathcal{O}_{\text{NSI}}$ in Eq.(2.52) which we repeat here for reference

$$\mathcal{O}_{\text{NSI}} = (\bar{L}^i H_i)\gamma^\rho (H^{\dagger i} L_i)(\bar{E}\gamma_\rho E)\,, \tag{5.30}$$

see Eq. (5.18). In the following, we will refer to operators satisfying Eq. (5.29) as $\mathcal{O}_{\text{NSI}}$, *i.e.*, we define the class of potential non-standard neutrino interaction operators in mass dimension eight as the one which does not introduce any harmful $d=8$ processes with four charged leptons. Eq. (5.30) is (apart from Fierz rearrangements) the only such possibility with two right-handed charged leptons involved. When considering leptonic NSI involving four left-handed fields, several new operators of this kind will be determined later on.

As far as the possible NSI in terms of $SU(2)$ field components are concerned, not all flavour structures can be generated from the $d=6$ effective gauge invariant operators if charged lepton processes are suppressed. Applying the $d=6$ cancellation conditions in Eq. (5.28) to Eq. (5.21), it results that the $d=6$ contribution to the coefficient $\left(\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L}\right)^{\alpha\gamma}_{\beta\delta}$ is antisymmetric in the flavor index exchanges $(\alpha,\gamma)\to(\gamma,\alpha)$ and $(\beta,\delta)\to(\delta,\beta)$, which means that $\alpha\neq\gamma$ and $\beta\neq\delta$ for viable NSI. As regards matter effects, this implies that only $\epsilon^m_{\mu\mu}$, $\epsilon^m_{\mu\tau}$, and $\epsilon^m_{\tau\tau}$ – defined in Eq. (5.22) – can be generated from $d=6$ operators and the connection with the source effects is given by

$$\epsilon^m_{\mu\mu} = -\epsilon^s_{ee} = -\epsilon^s_{\mu\mu}\,, \tag{5.31}$$

$$\epsilon^m_{\mu\tau} = -(\epsilon^s_{\mu\tau})^*\,. \tag{5.32}$$

In contrast, $\epsilon^m_{\tau\tau}$ is not connected to the source effects at the effective operator level [4]. Notice that, for instance, the NSI in Eq. (5.31) contribute to the G_F measurement coherently (*i.e.*, the interference with SM couplings contributes linearly to the rates), for which quite stringent bounds exist. These results hold in general for any purely leptonic NSI $d=6$ operator with suppressed interactions among four charged leptons, *i.e.*, Eq. (5.28). Furthermore, for the particular case of a neutrino factory, the antisymmetry conditions and constraints described above imply that the only possible non-negligible NSI source terms induced by $d=6$ operators are $\epsilon^s_{e\tau}$ and $\epsilon^s_{\mu\tau}$.

ΔL	Lorentz	Mediator	Bilinear(s)	Models [Refs.]
2	scalar	$\mathbf{1}^s_{-1}$	$\overline{L^c} i\tau^2 L$	Zee model [121, 123, 127], $\not{R}_p$SUSY [221]
	scalar	$\mathbf{1}^s_{-2}$	$\overline{E^c} E$	
	scalar	$\mathbf{3}^s_{-1}$	$\overline{L^c} i\tau^2 \tau^a L$	Left-right sym. [93, 222–224]
	vector	$\mathbf{2}^v_{-3/2}$	$\overline{E^c} \gamma^\rho L$	331 model [225–227]
0	vector	$\mathbf{1}^v_0$	$\bar{L}\gamma^\rho L$, $\bar{E}\gamma^\rho E$	Models with Z' [177]
	vector	$\mathbf{3}^v_0$	$\bar{L}\gamma^\rho \tau^a L$	Models with W' [177]
	scalar	$\mathbf{2}^s_{1/2}$	$\bar{E} L$	$\not{R}_p$SUSY [221]

Table 5.1: Possible SM bilinear field combinations involving only leptons.

5.2 Model analysis of $d = 6$ operators

In this section, we discuss the model-building implications of requesting large $d = 6$ NSI induced by theories of physics beyond the Standard Model. We specifically highlight the basic principles, which can be found in the $d = 8$ case as well. However, as we shall see later, the $d = 8$ case is technically somewhat more challenging.

In order to shed light on model building, let us analyze the operators according to the possible tree-level mediator. This is most efficiently done by listing all possible SM bilinear field combinations, and combining them in all possible ways [173, 174].

We therefore show in Table 5.1 the possible bilinears constructed from leptons only, which can lead to the $d = 6$ NSI operators in Eqs. (5.2)-(5.5). It is obvious from the table that the bilinears carry the mediator information and that they can therefore be directly associated with specific models (as illustrated). The mediators are denoted – all through the paper – by $\mathbf{X}^{\mathcal{L}}_Y$, where

- $\mathbf{X}$ denotes the $SU(2)$ nature, *i.e.*, singlet $\mathbf{1}$, doublet $\mathbf{2}$, or triplet $\mathbf{3}$.
- $\mathcal{L}$ refers to the Lorentz nature, *i.e.*, scalar (s), vector (v), left-handed (L) or right-handed (R) fermion [5].
- Y refers to the hypercharge $Y = Q - I^W_3$.

Table 5.2 shows in turn all possible $d = 6$ operators which can be constructed from the SM bilinear field combinations in Table 5.1. The coefficients of the $d = 6$ operators obtained by this procedure are denoted by $(c^{\mathbf{X}\,\mathcal{L}})^{\alpha\gamma}_{\beta\delta}$ and $(f^{\mathbf{X}\,\mathcal{L}})^{\alpha\gamma}_{\beta\delta}$, where c (f) indicates

[4] There can be also subdominant effects in detection chains. For example, in OPERA, the taus resulting from hadronic interactions decay into muons or electrons. It implies for instance $\epsilon^m_{\tau\tau} = -\epsilon^s_{\tau\tau}$, which means that tau decay into electrons is in this case connected with matter NSI. Note that NSI and SM contributions add coherently to the $\tau^- \to e^- \nu_\tau \bar{\nu}_e$ width.

[5] Fermionic mediators will appear explicitly later on, when discussion $d = 8$ effective interactions

$d=6$ operators	Mediator	$\mathcal{C}_{LE}$	$\mathcal{C}^{\mathbf{1}}_{LL}$	$\mathcal{C}^{\mathbf{3}}_{LL}$	$\mathcal{C}_{EE}$
$\boldsymbol{\bar{L}E\bar{E}L}$					
$(c^{\mathbf{2}v}/\Lambda^2)((\overline{E^c})_\gamma\gamma^\rho L_\alpha)(\bar{L}^\beta\gamma_\rho(E^c)^\delta)$	$\mathbf{2}^v_{-3/2}$	$2c^{\mathbf{2}v}$			
$(f^{\mathbf{1}v}_{LE}/\Lambda^2)(\bar{L}^\beta\gamma^\rho L_\alpha)(\bar{E}^\delta\gamma_\rho E_\gamma)$	$\mathbf{1}^v_0$	$-2f^{\mathbf{1}v}_{LE}$			
$(f^{\mathbf{2}s}/\Lambda^2)(\bar{L}^\beta E_\gamma)(\bar{E}^\delta L_\alpha)$	$\mathbf{2}^s_{1/2}$	$f^{\mathbf{2}s}$			
$\boldsymbol{\bar{L}L\bar{L}L}$					
$(c^{\mathbf{1}s}_{LL}/\Lambda^2)((\overline{L^c})_\alpha i\tau^2 L_\gamma)(\bar{L}^\beta i\tau^2(L^c)^\delta)$	$\mathbf{1}^s_{-1}$		$\frac{1}{4}c^{\mathbf{1}s}$	$-\frac{1}{4}c^{\mathbf{1}s}$	
$(c^{\mathbf{3}s}/\Lambda^2)((\overline{L^c})_\alpha i\tau^2\vec{\tau}L_\gamma)(\bar{L}^\beta\vec{\tau}i\tau^2(L^c)^\delta)$	$\mathbf{3}^s_{-1}$		$-\frac{3}{4}c^{\mathbf{3}s}$	$-\frac{1}{4}c^{\mathbf{3}s}$	
$(f^{\mathbf{1}v}_{LL}/\Lambda^2)(\bar{L}^\beta\gamma^\rho L_\alpha)(\bar{L}^\delta\gamma_\rho L_\gamma)$	$\mathbf{1}^v_0$		$f^{\mathbf{1}v}_{LL}$		
$(f^{\mathbf{3}v}/\Lambda^2)(\bar{L}^\beta\gamma^\rho\vec{\tau}L_\alpha)(\bar{L}^\delta\gamma_\rho\vec{\tau}L_\gamma)$	$\mathbf{3}^v_0$			$f^{\mathbf{3}v}$	
$\boldsymbol{\bar{E}E\bar{E}E}$					
$(c^{\mathbf{1}s}_{EE}/\Lambda^2)((\overline{E^c})_\alpha E_\gamma)(\bar{E}^\beta(E^c)^\delta)$	$\mathbf{1}^s_{-2}$				$\frac{1}{2}c^{\mathbf{1}s}_{EE}$
$(f^{\mathbf{1}v}_{EE}/\Lambda^2)(\bar{E}^\beta\gamma^\rho E_\alpha)(\bar{E}^\delta\gamma_\rho E_\gamma)$	$\mathbf{1}^v_0$				$f^{\mathbf{1}v}_{EE}$

Table 5.2: Possible $d=6$ operators obtained by combining the lepton bilinears in Table 5.1.

that the corresponding operator results from the exchange of particles carrying two (zero) lepton number, and $\mathbf{X}$ and $\mathcal{L}$ refer again to the $SU(2)$ and Lorentz nature, respectively. Any subscript refers to the combination of bilinears involved. The last four columns contain the contribution of the analyzed operators to the $d=6$ operator coefficients in the BW basis in Eqs. (5.2)-(5.5). The flavour structure for any coefficient in the table is understood to be $(\,)^{\alpha\gamma}_{\beta\delta}$, see main text for further explanations.

At this point it is important to note that the operators obtained from the mediators do not constitute a basis. Instead they are not independent, but linear combinations of those in the BW basis, Eqs. (5.2)-(5.5). Therefore, it might be more accurate to call them "mediator-operators" or "operator combinations". We will not make this special distinction, but the reader should keep that in mind. Re-writing the individual effective operators from Table 5.2 in the BW basis, we find the coefficients given in the last four columns of Table 5.2. For example, the first line of the second group, mediated by $\mathbf{1}^s_{-1}$, reads (including flavor indices)

$$\delta\mathscr{L}^{d=6}_{\text{eff}} = \frac{(c^{\mathbf{1}s}_{LL})^{\alpha\gamma}_{\beta\delta}}{\Lambda^2}((\overline{L^c})_\alpha i\tau^2 L_\gamma)(\bar{L}^\beta i\tau^2(L^c)^\delta) = \frac{1}{4}\frac{(c^{\mathbf{1}s}_{LL})^{\alpha\gamma}_{\beta\delta}}{\Lambda^2}(\mathcal{O}^{\mathbf{1}}_{LL})^{\beta\delta}_{\alpha\gamma} - \frac{1}{4}\frac{(c^{\mathbf{1}s}_{LL})^{\alpha\gamma}_{\beta\delta}}{\Lambda^2}(\mathcal{O}^{\mathbf{3}}_{LL})^{\beta\delta}_{\alpha\gamma}\,. \tag{5.33}$$

Conversely, the decomposition of the operator $\mathcal{O}^{\mathbf{1}}_{LL}$ of the BW basis can be read off from the column labeled $\mathcal{C}^{\mathbf{1}}_{LL}$, in terms of the relative weights of the mediator-operators:

$$(\mathcal{C}^{\mathbf{1}}_{LL})^{\alpha\gamma}_{\beta\delta} = \frac{1}{4}(c^{\mathbf{1}s}_{LL})^{\alpha\gamma}_{\beta\delta} - \frac{3}{4}(c^{\mathbf{3}s})^{\alpha\gamma}_{\beta\delta} + (f^{\mathbf{1}v}_{LL})^{\alpha\gamma}_{\beta\delta}\,. \tag{5.34}$$

Note that the flavor indices in the first column of Table 5.2 are arranged such that the flavor indices of all coefficients and of the BW operators are the same as in Eqs. (5.33) and (5.34). Therefore, we show the flavor indices explicitly only in the first column.

In order to have large NSI without four charged lepton interactions, the $d = 6$ cancellation conditions Eq. (5.28) must now be implemented. One can directly read off now from Table 5.2 that these conditions can be re-written as

$$2c^{\mathbf{2}v} - 2f^{\mathbf{1}v}_{LE} + f^{\mathbf{2}s} = 0 \qquad (\text{from } \mathcal{C}_{LE} = 0)\,, \tag{5.35}$$

$$-c^{\mathbf{3}s} + f^{\mathbf{1}v}_{LL} + f^{\mathbf{3}v} = 0 \qquad (\text{from } \mathcal{C}^{\mathbf{1}}_{LL} + \mathcal{C}^{\mathbf{3}}_{LL} = 0)\,, \tag{5.36}$$

$$c^{\mathbf{1}s}_{EE} + 2f^{\mathbf{1}v}_{EE} = 0 \qquad (\text{from } \mathcal{C}_{EE} = 0)\,, \tag{5.37}$$

in the mediator picture. The operators contributing to the first equation will not produce any NSI (since $\mathcal{C}_{LE} = 0$ in Eq. (5.19)), while the operators present in the second equation lead to NSI if $\mathcal{C}^{\mathbf{1}}_{LL} = -\mathcal{C}^{\mathbf{3}}_{LL} \neq 0$ (*cf.*, Eq. (5.20)).

One approach to use Table 5.2 is to discuss departures from the SM couplings. For example, for a hypothetical experimental departure pointing towards a four-lepton coupling such as that in operator $\mathcal{O}^{\mathbf{3}}_{LL}$ in Eq. (5.4), Table 5.2 indicates directly that a new heavy scalar triplet could induce it at tree-level, while a scalar doublet wouldn't.

From the model building perspective, it is illustrative to consider again the case of the operator mediated by $\mathbf{1}^s_{-1}$ leading to Eq. (5.33). The table shows that it is the only $d = 6$ possibility using only one mediator which directly satisfies the cancellation condition of pure charged lepton interactions Eq. (5.28) (or their tree-level equivalent Eqs. (5.35)-(5.37)). This antisymmetric combination of the basis elements was first found in Ref. [174].

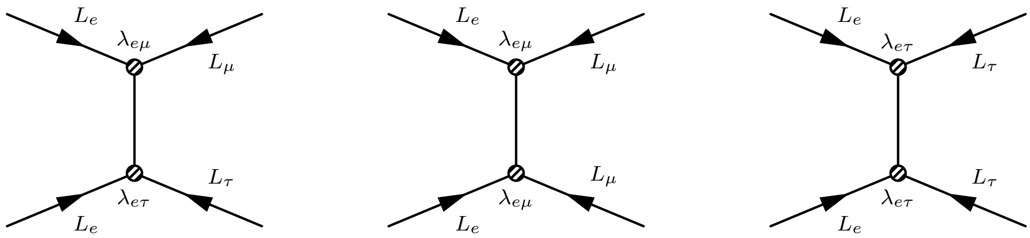

Figure 5.1: Diagrams mediated by a bilepton $\mathbf{1}^s_{-1}$.

This example serves to illuminate the power of the mediator analysis (see Ref. [174] and also Ref. [81]). The $\mathbf{1}^s_{-1}$ exchange leading to the originally proposed operator is depicted in Fig. 5.1, left. Once a certain mediator is assumed for a certain operator, contributions to other operators are simultaneously induced, though, as illustrated at the center and right of Fig. 5.1, *i.e.*,

$$\frac{|(c^{\mathbf{1}s}_{LL})^{e\mu}_{e\tau}|^2}{\Lambda^4} = \frac{|\lambda_{e\mu}|^2|\lambda_{e\tau}|^2}{M^4_{\mathbf{1}s}} = \frac{\left|(c^{\mathbf{1}s}_{LL})^{e\mu}_{e\mu}\right|\left|(c^{\mathbf{1}s}_{LL})^{e\tau}_{e\tau}\right|}{\Lambda^4}\,, \tag{5.38}$$

where $\lambda_{\alpha\beta}$ is the coupling for the lepton-bilepton interaction and $M_{\mathbf{1}s}$ is the mass of the bilepton. A coherent contribution to G_F and a violation of the lepton universality is then induced by the diagrams at the middle and right of Fig. 5.1. From the strict experimental bounds on these quantities $\epsilon^m_{\mu\tau}$ has been constrained to $|\epsilon^m_{\mu\tau}| \lesssim 1.9 \cdot 10^{-3}$ (90% CL), using this particular mediator [81]. The bound from a neutrino factory on $|\epsilon^m_{\mu\tau}|$ would be $1.8 \cdot 10^{-2}$ for complex $\epsilon^m_{\mu\tau}$ [210]. If it was assumed to be real, which does not describe the most general class of models, the bound would be $3.7 \cdot 10^{-4}$ (90% CL) [210]. However, since this is a model-dependent assumption, we do not use this bound.

The antisymmetric operator discussed in the previous paragraphs is not, however, the only possibility to build a model satisfying Eqs. (5.35)-(5.37). For example, one may choose bosonic triplets $\mathbf{3}^s_{-1}$ and $\mathbf{3}^v_0$, for which the coefficients can be chosen independently, in order to satisfy Eq. (5.36) without suppressing completely the $d = 6$ NSI operator coefficient. In particular, if the simplest possibility is experimentally constrained, one may consider models with more than one mediator.

At this point, we would like to clarify that cancellations or fine-tuning of operator coefficients cannot be an argument in itself for judging the naturalness and complexity of a model. It depends on the field content and the symmetries of the model. Consider for instance once again the antisymmetric operator in the left-hand side of Eq. (5.33), induced at tree-level by the exchange of just one mediator, $\mathbf{1}^s_{-1}$, illustrated in Fig. 5.1 left. That equation shows that, in the BW basis, the antisymmetric operator appears to be constructed from the combination of two BW operators with specific (fine-tuned?) coefficients. In the effective operator picture, "fine-tuning" is thus a basis and model-dependent qualification. We therefore define the simplest model to be the one with the fewest mediators. In the $d = 6$ case, it is the antisymmetric operator in Eq. (5.33) with only one mediator. In the case that the NSI come only through $d = 8$ (or higher dimension) effective operators, we will demonstrate that the simplest viable models require at least two mediators. Once the field content is chosen, any relative precise adjustment of the couplings of the mediators can be considered a fine-tuning, unless the symmetries of the model ensure it. It will be left to the model builder to eventually explore possible symmetries, whenever such cancellations will turn out to be required below for phenomenologically viable NSI.

In general, it is easy to show that all NSI from $d = 6$ operators are strongly constrained when the possible mediators are taken into account. There is, however, one exception. The present experimental constraints allow the condition $\mathcal{C}_{LE} = 0$ in Eq. (5.28), which cancels interactions among four charged leptons, to be substantially violated for certain combinations of flavour indices. In particular, the coefficient of the flavor conserving (BW) operator $(\bar{L}^\tau E_e)(\bar{E}^e L_\tau)$, is not very strongly constrained [79, 219]. The mediators $\mathbf{2}^v_{-3/2}$ or $\mathbf{2}^s_{1/2}$ (*cf.*, Table 5.2) can generate such an operator, leading to the following effective interactions, *cf.*, Eq. (5.19):

$$\delta\mathscr{L}_{\text{eff}}^{d=6} = -\frac{(\mathcal{C}_{LE})_{\tau e}^{\tau e}}{2\Lambda^2}\left(\left(\bar{\nu}^{\tau}\gamma^{\rho}\mathrm{P}_L\nu_{\tau}\right)\left(\bar{e}\gamma_{\rho}\mathrm{P}_R e\right) - \left(\bar{\tau}\gamma^{\rho}\mathrm{P}_L\tau\right)\left(\bar{e}\gamma_{\rho}\mathrm{P}_R e\right)\right) + \text{h.c.}\,. \tag{5.39}$$

The coefficient is constrained by (see Eq.(14) in Ref. [79])

$$|\epsilon_{\tau\tau}^{m}| = \frac{v^2}{4\Lambda^2}\,|(\mathcal{C}_{LE})_{\tau e}^{\tau e}| = |\kappa_{\tau R}| \lesssim 0.1\,. \tag{5.40}$$

If the possibility of large $SU(2)_L$ breaking effects was considered in addition, a possible gain of almost an order of magnitude could be allowed for the NSI $\epsilon_{\tau\tau}^{m}$ strength [174]. In conclusion, large (order unity) values for $\epsilon_{\tau\tau}^{m}$ resulting from $d = 6$ effective interactions are not excluded.

Table 5.2 also shows that the relationship between mediator and coefficient is unique at the $d = 6$ level, except for $\mathbf{1}_0^v$. If a model uses this mediator, then there will be three different $d = 6$ operator contributions, which are independent in the BW basis. In particular, one cannot neglect $\mathcal{O}_{EE}$, which can induce physics effects while not resulting in NSI.

5.3 Model analysis of $d = 8$ operators

We consider all possible $d = 8$ operators which can induce purely leptonic NSI, analyzing them from the point of view of their possible tree-level mediators. We will focus on the systematic analysis of all possible products of SM bilinears, which may result from exchanging mediators which only couple to pairs of SM fields. Such an analysis was performed for $d = 6$ operators in Refs. [173, 174], and we extend it here to the $d = 8$ case. Other scenarios leading to some of the $d = 8$ operators will be briefly analyzed afterwards.

A convenient basis of linearly independent $d = 8$ operators has been given in Eqs. (5.9) to (5.16), *i.e.*, the BR basis. In order to suppress four charged lepton interactions, both the cancellation conditions for $d = 8$ operators in Eq. (5.29) and the cancellation conditions for $d = 6$ operators in Eq. (5.28) are now required to be satisfied. Under these conditions, if any $d = 6$ operator remains, it is expected to dominate the new physics and, as discussed in the previous section, only effects related to $\epsilon_{\tau\tau}^{m}$ are then allowed to be experimentally sizeable. In this section, we instead focus on NSI which stem exclusively from $d = 8$ (and higher) operators and their implications for model building. In particular, we are interested in the $\mathcal{O}_{\text{NSI}}$ without four charged lepton interactions, *i.e.*, satisfying Eq. (5.29), which has been object of intense speculations in the literature.

When the mediators couple only to SM bilinears we have the following options with respect to the undesired $d = 6$ operators:

1. The required mediators do not induce any $d = 6$ operator involving four leptons (in other words, the mediators differ from those in Table 5.2).

2. The $d = 6$ couplings induced by different mediators turn out to explicitly cancel among themselves.

As we will illustrate later, there is no simple possibility for which the first option works. For the second option to happen, the coefficients for the BW operators in Eqs. (5.2)-(5.5) have to vanish independently, because they constitute a basis:

$$\mathcal{C}_{LE} = 0\,, \quad \mathcal{C}^{\mathbf{1}}_{LL} = 0\,, \quad \mathcal{C}^{\mathbf{3}}_{LL} = 0\,, \quad \mathcal{C}_{EE} = 0\,. \tag{5.41}$$

Their implementation in the mediator picture can be read off from the columns in Table 5.2. They are given by Eqs. (5.35) and (5.37), together with

$$\frac{1}{4}c^{\mathbf{1}s}_{LL} - \frac{3}{4}c^{\mathbf{3}s} + f^{\mathbf{1}v}_{LL} = 0 \qquad (\text{from } \mathcal{C}^{\mathbf{1}}_{LL} = 0)\,, \tag{5.42}$$

$$-\frac{1}{4}c^{\mathbf{1}s}_{LL} - \frac{1}{4}c^{\mathbf{3}s} + f^{\mathbf{3}v} = 0 \qquad (\text{from } \mathcal{C}^{\mathbf{3}}_{LL} = 0)\,, \tag{5.43}$$

which replace Eq. (5.36) of that set. For example, if a model introduces two bosonic doublets $\mathbf{2}^v_{-3/2}$ and $\mathbf{2}^s_{1/2}$, one can satisfy Eq. (5.35) (to which Eq. (5.41) simplifies in this case) by achieving $2c^{\mathbf{2}v} + f^{\mathbf{2}s} = 0$.

Note that the introduction of exotic fermions in the game potentially leads to the additional $d = 6$ operators in Eqs. (5.6)-(5.8), which are made out of two lepton fields and two Higgs doublets. In accordance with the main line of this section, we do not consider constraints from those operators, which means that, unless explicitly stated otherwise, when mentioning $d = 6$ operators in this section we refer exclusively to those in Eqs. (5.2)-(5.5).

5.3.1 A toy model

In order to estimate the theoretical price to pay for obtaining large NSI from exotic particles coupling to SM bilinears, without large charged lepton flavour violation, we show here a toy model in a bottom-up fashion, which precisely generates the $d = 8$ operator $\mathcal{O}_{\rm NSI}$ in Eq. (5.30) and no $d = 6$ operator. Then we will provide a systematic analysis, from which we will recover the toy model as the simplest possibility in a top-down approach.

Consider the following toy Lagrangian for the underlying theory, which adds both a new scalar doublet ($\mathbf{2}^s_{1/2}$) Φ and a vector doublet ($\mathbf{2}^v_{-3/2}$) V_μ to the SM Lagrangian, with general couplings to the SM fields y, g and λ's,

$$\begin{aligned}\mathscr{L} = \mathscr{L}_{\rm SM} &- (y)_\beta{}^\gamma\,(\bar{L}^\beta)^i E_\gamma \Phi_i - (g)_{\beta\delta}\,(\bar{L}^\beta)^i\gamma^\rho (E^c)^\delta (V_\rho)_i \\ &+ \lambda_{\mathbf{1}s}(H^\dagger H)(\Phi^\dagger\Phi) + \lambda_{\mathbf{3}s}(H^\dagger\vec{\tau}H)(\Phi^\dagger\vec{\tau}\Phi) \\ &+ \lambda_{\mathbf{1}v}(H^\dagger H)(V_\rho^\dagger V^\rho) + \lambda_{\mathbf{3}v}(H^\dagger\vec{\tau}H)(V_\rho^\dagger\vec{\tau}V^\rho) + \text{h.c.} + ...\end{aligned} \tag{5.44}$$

where the dots refer to other bosonic interactions not relevant for this work. After integrating out the intermediate particles, the following $d = 6$ effective interactions involving leptons are induced (see Table 5.2):

$$\delta\mathscr{L}_{\rm eff}^{d=6} = \frac{(c^{\mathbf{2}v})^{\alpha\gamma}_{\beta\delta}}{\Lambda^2}\,(\overline{E^c}_\gamma\gamma^\rho L_\alpha)(\bar{L}^\beta\gamma_\rho E^{c\,\delta}) + \frac{(f^{\mathbf{2}s})^{\alpha\gamma}_{\beta\delta}}{\Lambda^2}\,(\bar{L}^\beta E_\gamma)(\bar{E}^\delta L_\alpha)\,, \tag{5.45}$$

where now

$$\frac{(c^{\mathbf{2}v})^{\alpha\gamma}_{\beta\delta}}{\Lambda^2} = -\frac{(g^\dagger)^{\gamma\alpha}(g)_{\beta\delta}}{M_V^2}\,, \quad \frac{(f^{\mathbf{2}s})^{\alpha\gamma}_{\beta\delta}}{\Lambda^2} = \frac{(y^\dagger)_\delta{}^\alpha(y)_\beta{}^\gamma}{M_\Phi^2}\,. \tag{5.46}$$

For simplicity of notation and illustrative purposes we can assume $M_\Phi \simeq M_V \equiv M(=\Lambda)$. The $d = 6$ cancellation conditions on four charged lepton transitions in Eq. (5.28), or its equivalent in the mediator picture Eq. (5.35), translate into

$$-\,2(g^\dagger)^{\gamma\alpha}(g)_{\beta\delta} + (y^\dagger)_\delta{}^\alpha(y)_\beta{}^\gamma = 0\,. \tag{5.47}$$

The relevant effective $d = 8$ Lagrangian induced reads

$$\begin{aligned}\delta\mathscr{L}_{\rm eff}^{d=8} =& \frac{1}{M^4}[\lambda_{\mathbf{1}s}\left(\bar{L}\,y\,E\right)\left(\bar{E}\,y^\dagger\,L\right)(H^\dagger H) + \lambda_{\mathbf{3}s}\left(\bar{L}\,y\,E\right)\vec{\tau}\left(\bar{E}\,y^\dagger L\right)\left(H^\dagger\vec{\tau}H\right)\\ &+\lambda_{\mathbf{1}v}\left(\bar{L}\,g\,\gamma^\rho E^c\right)\left(\overline{E^c}\gamma_\rho\,g^\dagger L\right)(H^\dagger H) + \lambda_{\mathbf{3}v}\left(\bar{L}\,g\,\gamma^\rho E^c\right)\vec{\tau}\left(\overline{E^c}\gamma_\rho\,g^\dagger L\right)\left(H^\dagger\vec{\tau}H\right)],\end{aligned} \tag{5.48}$$

where flavour indices have been omitted and each expression in brackets is to be understood as a flavour singlet. Eq. (5.48) can be rewritten in terms of the operators of the BR basis in Eqs. (5.9) and (5.10), as

$$\delta\mathscr{L}_{\rm eff}^{d=8} = -\,\frac{1}{\Lambda^4}\,(\mathcal{C}^{\mathbf{1}}_{LEH}\mathcal{O}^{\mathbf{1}}_{LEH} + \mathcal{C}^{\mathbf{3}}_{LEH}\mathcal{O}^{\mathbf{3}}_{LEH})\,, \tag{5.49}$$

where

$$\mathcal{C}^{\mathbf{1}}_{LEH} = \lambda_{\mathbf{1}v}\,(g^\dagger)^{\gamma\alpha}(g)_{\beta\delta} + \frac{1}{2}\lambda_{\mathbf{1}s}\,(y^\dagger)_\delta{}^\alpha(y)_\beta{}^\gamma\,, \tag{5.50}$$

$$\mathcal{C}^{\mathbf{3}}_{LEH} = \lambda_{\mathbf{3}v}\,(g^\dagger)^{\gamma\alpha}(g)_{\beta\delta} + \frac{1}{2}\lambda_{\mathbf{3}s}\,(y^\dagger)_\delta{}^\alpha(y)_\beta{}^\gamma\,. \tag{5.51}$$

In order not to produce interactions between four charged leptons, it is necessary to satisfy Eq. (5.29), *i.e.*, the condition $\mathcal{C}^{\mathbf{1}}_{LEH} = \mathcal{C}^{\mathbf{3}}_{LEH} \neq 0$, so that the effective $d = 8$ interaction in Eq. (5.49) reduces precisely to $\mathcal{O}_{\rm NSI}$ in Eq. (5.18). Together with the $d = 6$ cancellation condition, Eq. (5.47), it is finally required that

$$\lambda_{\mathbf{1}s} + \lambda_{\mathbf{1}v} = \lambda_{\mathbf{3}s} + \lambda_{\mathbf{3}v} \neq 0\,. \tag{5.52}$$

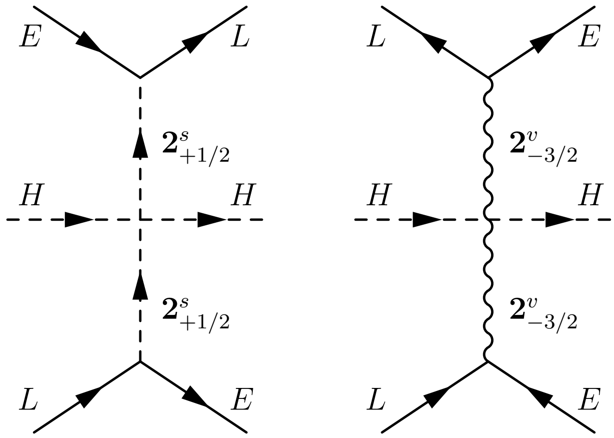

Figure 5.2: Dimension eight operator decomposed into dimension four interactions

As a consequence, the NSI in matter can be substantial for all flavours. While source and detection NSI cannot be created from our toy model, the epsilon matter parameter reads

$$\left|\epsilon^{m,R}_{\beta\alpha}\right| = \frac{v^4}{2M^4}\left|(\lambda_{\mathbf{1}s} + \lambda_{\mathbf{1}v})(g^\dagger)^{e\alpha}(g)_{\beta e}\right| . \tag{5.53}$$

In resume, by adding both an $SU(2)$ doublet scalar and a doublet vector to the SM content, and imposing two relations to their couplings, Eqs. (5.47) and (5.52), a toy model for viable large NSI has resulted. The model interactions are visualized in Fig. 5.2, where the first two effective interactions in Eq. (5.48) correspond to the diagram on the left – mediated by $\mathbf{2}^s_{1/2}$ – and the last two interactions to the diagram on the right – mediated by $\mathbf{2}^v_{-3/2}$. In fact, other combinations of just one of the first two operators in Eq. (5.48) together with one of the last two operators in that equation would have been enough for the purpose [6]. As we will demonstrate below, our toy model is the most general possible model involving only two mediators, when the exotic particles couple only to SM bilinears.

We keep dubbing the construction above as "toy" because, to begin with, the presence of a vector field, which is not a gauge boson, implies that it is non-renormalizable. The toy Lagrangian, Eq. (5.44), can thus only be considered as an effective theory of some larger construction, such as for instance models of extra dimensions in which the vector doublet could be a component of a higher dimensional gauge theory.

Moreover, its phenomenological analysis is beyond the scope of the present work: the constraints from electroweak precision tests need to be analyzed for each specific

[6]For instance a combination involving $\lambda_{\mathbf{1}s}$ and $\lambda_{\mathbf{3}v}$, or alternatively $\lambda_{\mathbf{3}s}$ and $\lambda_{\mathbf{1}v}$, would be suitable.

model, in particular the *oblique* corrections [229–231] it may induce. The new couplings may also have a relevant impact on other flavour changing transitions at the loop level, although considering large values for the quartic couplings λ and small values for the elements of the g and y flavour matrices, it will probably remain phenomenologically safe.

The toy model demonstrates that it is possible to achieve the desired $d = 8$ interactions, without inducing simultaneously $d = 6$ ones, by fixing the coefficients of the new fields in the Lagrangian. It requires ad-hoc cancellations, though, and it is left as an open question for the model builder whether some symmetry can justify them.

5.3.2 Systematic analysis

In this subsection, a systematic analysis of all possible effective NSI $d = 8$ operators is performed. The full decomposition of any combination of $d = 8$ operators, constructed from combining bilinear combinations of SM fields, leads to a large number of possibilities. We will first consider the cases which are conceptually similar to the toy model above, *i.e.*, new fundamental interactions involving exactly *two* SM fields, which are the SM bilinears according to our earlier definition. Then we will discuss new interactions involving only one SM field.

Mediators coupling to SM bilinears

We summarize these possibilities for the $\bar{L}L\bar{E}E$-type operators in Table 5.3 and for the $\bar{L}L\bar{L}L$-type operators in Table 5.4, which are one of the main results of this study. The notation used has been described in Sect. 3. The tables show, from left to right in each row:

- an ordinal assigned to each operator,
- the operator itself,
- the value of the operator coefficients of the BR basis needed to reconstruct it,
- whether the $d = 8$ cancellation conditions in Eq. (5.29) are directly fulfilled ("$\mathcal{O}_{\rm NSI}$?"),
- the required mediators, with those inducing additional $d = 6$ interactions of four charged leptons (Table 5.2) highlighted in boldface.

Obviously, the number of possible mediators of $d = 8$ interactions is much larger than for the $d = 6$ case in Table 5.2. In particular, fermions are now possible mediators, unlike for $d = 6$. We illustrate the operator decomposition for operator #2 from Table 5.3, showing the corresponding Feynman diagram in Fig. 5.3.

Notice that only the minimal mediator content necessary to obtain each possible $d = 8$ operator is shown in Tables 5.3 and 5.4. In other words, although there is always a particular set of exotic particles whose exchange induces at tree-level the $d = 8$ operators considered, this set might not be unique. Nevertheless, for each operator, the particle content shown in the tables is contained in all other possible sets of mediators leading to it.

From both Tables 5.3 and 5.4, and from Table 5.2, one can easily read off the following key results for the operators considered :

- There is no way to write down a $d = 8$ operator without involving a mediator (pinpointed in boldface) which also generates $d = 6$ four-lepton interactions.
- In order to build $\mathcal{O}_{\rm NSI}$ *and* to cancel the dangerous (or all) NSI $d = 6$ contributions, at least two new fields are needed.

This implies that fine-tuning – or hopefully symmetries – will be required if all $d = 6$ NSI are to be cancelled, Eq. (5.41) [7]. For model building, one may use the tables as follows: in order to generate a pure $\mathcal{O}_{\rm NSI}$-type operator, it is necessary to choose effective operators such that Eq. (5.29) is fulfilled, *i.e.*, interactions with four charged leptons are suppressed, and that Eq. (5.41) is satisfied, *i.e.*, the NSI contributions from $d = 6$ operators cancel. The two simplest methods to build a model leading to a pure $\mathcal{O}_{\rm NSI}$ interaction are:

1. To select from the tables those effective operators marked as $\mathcal{O}_{\rm NSI}$.
2. To linearly combine the effective operators in either of the tables to obtain an $\mathcal{O}_{\rm NSI}$ structure. One possibility is to choose any combination of at least two non-$\mathcal{O}_{\rm NSI}$ operators which are linearly independent in the BR basis (not considering $\mathcal{O}^{\mathbf{333}}_{LLH}$).[8]

The necessary mediators can then be directly read off from the tables; as the next step, the $d = 6$ cancellation conditions should be translated into relations among the couplings.

[7] Recall that this condition ensures that, in addition to avoiding lepton flavour violation among four charged fermions, other putatively dangerous $d = 6$ couplings are suppressed, such as for instance possible contributions to the very precise measurement of G_F determined from muon decay. Note as well that, in principle, one could avoid to impose such a strong cancellation condition by assuming very large couplings among the new heavy fields, and very small values for the couplings between those heavy fields and the SM fields which induce $d = 6$ operators. However, since the product between these two types of couplings will be present in the $d = 8$ operator (as in our toy model), the $d = 8$ couplings would be effectively suppressed as well and extreme fine-tuning would be needed.

[8] In short, the linear combination of two vectors involves only one free parameter (aside from the normalization). The condition in Eq. (5.29) amounts then to a linear equation with only one parameter, which can always be solved for. Since the vectors are linearly independent, they cannot cancel each other, which means that there will be non-vanishing NSI. If, on the other hand, one chooses linearly-dependent vectors, there will be no $d = 8$ operator at all – neither $\mathcal{O}_{\rm NSI}$, nor the harmful one.

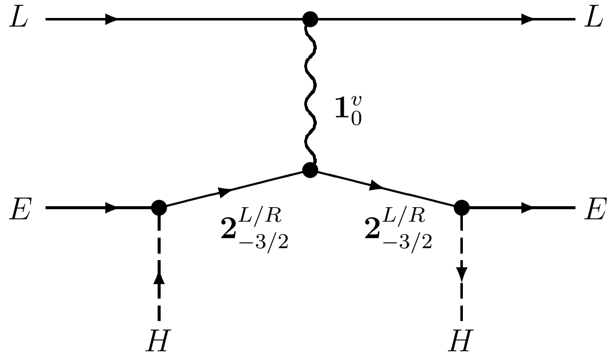

Figure 5.3: Example for a fully decomposed operator. The diagram corresponds to #2 of Table 5.3.

Note that, in addition, there might be flavor dependent conditions and other constraints, which means that our tables can only serve as hints on how to build the simplest models. For example, one may have to worry about electroweak precision data, flavour changing neutral currents, non-unitarity of the PMNS matrix, loop constraints, and chiral anomalies if exotic fermions are introduced [9]. Also, vectorial scalar $SU(2)$ doublets call for a deeper theory when present, as discussed earlier.

Other such constraints can result from interactions of the $\bar{E}E\bar{E}E$-type, which we show in Table 5.5. Although these interactions do not produce NSI, care is mandatory when one introduces mediators which could induce such interactions. For example, operator #36 not only produces NSI, but will also lead to potential non-unitarity (through the operator in Eq.(5.6)) and other unwanted $d = 6$ effects, and the operator #61 from Table 5.5 potentially leads to charged lepton flavor violation.

For the operators in Table 5.3, our toy model is seen to be the only possibility using only two new fields, namely $\mathbf{2}^s_{+1/2}$ and $\mathbf{2}^v_{-3/2}$. It combines operators #7, #8, #13, and #14, which correspond to the four effective interactions in Eq. (5.48) in our toy model. The table also allows to conclude that it is as well the most general version of the model with only two fields, while a simpler version might, for instance, only include #7 and #14. Recall that source and detection NSI cannot be created from our toy model, while matter NSI for all flavours are allowed. All $\mathcal{O}_{\rm NSI}$ operators obtained in Table 3 correspond to the combination of operators of the BR basis in Eq. (5.18) which are thus equivalent to Eq. (5.30)

[9]This concerns for instance several examples in Table 5.3. In general, in order to cancel the chiral anomaly new vector-like fermions may be introduced. In the tables we just show the smallest number of mediators which can induce the $d = 8$ operators.

#	Dim. eight operator	$\mathcal{C}^{\mathbf{1}}_{LEH}$	$\mathcal{C}^{\mathbf{3}}_{LEH}$	$\mathcal{O}_{\text{NSI}}$?	Mediators
Combination $\boldsymbol{\bar{L}L}$					
1	$(\bar{L}\gamma^\rho L)(\bar{E}\gamma_\rho E)(H^\dagger H)$	1			$\mathbf{1}^v_0$
2	$(\bar{L}\gamma^\rho L)(\bar{E}H^\dagger)(\gamma_\rho)(HE)$	1			$\mathbf{1}^v_0 + 2^{L/R}_{-3/2}$
3	$(\bar{L}\gamma^\rho L)(\bar{E}H^T)(\gamma_\rho)(H^* E)$	1			$\mathbf{1}^v_0 + 2^{L/R}_{-1/2}$
4	$(\bar{L}\gamma^\rho \vec{\tau} L)(\bar{E}\gamma_\rho E)(H^\dagger \vec{\tau} H)$		1		$\mathbf{3}^v_0 + \mathbf{1}^v_0$
5	$(\bar{L}\gamma^\rho \vec{\tau} L)(\bar{E}H^\dagger)(\gamma_\rho \vec{\tau})(HE)$		1		$\mathbf{3}^v_0 + 2^{L/R}_{-3/2}$
6	$(\bar{L}\gamma^\rho \vec{\tau} L)(\bar{E}H^T)(\gamma_\rho \vec{\tau})(H^* E)$		1		$\mathbf{3}^v_0 + 2^{L/R}_{-1/2}$
Combination $\boldsymbol{\bar{E}L}$					
7	$(\bar{L}E)(\bar{E}L)(H^\dagger H)$	$-1/2$			$\mathbf{2}^s_{+1/2}$
8	$(\bar{L}E)(\vec{\tau})(\bar{E}L)(H^\dagger \vec{\tau} H)$		$-1/2$		$\mathbf{2}^s_{+1/2}$
9	$(\bar{L}H)(H^\dagger E)(\bar{E}L)$	$-1/4$	$-1/4$	✓	$\mathbf{2}^s_{+1/2} + 1^R_0 + 2^{L/R}_{-1/2}$
10	$(\bar{L}\vec{\tau} H)(H^\dagger E)(\vec{\tau})(\bar{E}L)$	$-3/4$	$1/4$		$\mathbf{2}^s_{+1/2} + 3^{L/R}_0 + 2^{L/R}_{-1/2}$
11	$(\bar{L}\mathrm{i}\tau^2 H^*)(H^T E)(\mathrm{i}\tau^2)(\bar{E}L)$	$1/4$	$-1/4$		$\mathbf{2}^s_{+1/2} + 1^{L/R}_{-1} + 2^{L/R}_{-3/2}$
12	$(\bar{L}\vec{\tau}\mathrm{i}\tau^2 H^*)(H^T E)(\mathrm{i}\tau^2 \vec{\tau})(\bar{E}L)$	$3/4$	$1/4$		$\mathbf{2}^s_{+1/2} + 3^{L/R}_{-1} + 2^{L/R}_{-3/2}$
Combination $\boldsymbol{\overline{E^c}L}$					
13	$(\bar{L}\gamma^\rho E^c)(\overline{E^c}\gamma_\rho L)(H^\dagger H)$	-1			$\mathbf{2}^v_{-3/2}$
14	$(\bar{L}\gamma^\rho E^c)(\vec{\tau})(\overline{E^c}\gamma_\rho L)(H^\dagger \vec{\tau} H)$		-1		$\mathbf{2}^v_{-3/2}$
15	$(\bar{L}H)(\gamma^\rho)(H^\dagger E^c)(\overline{E^c}\gamma_\rho L)$	$-1/2$	$-1/2$	✓	$\mathbf{2}^v_{-3/2} + 1^R_0 + 2^{L/R}_{+3/2}$
16	$(\bar{L}\vec{\tau} H)(\gamma^\rho)(H^\dagger E^c)(\vec{\tau})(\overline{E^c}\gamma_\rho L)$	$-3/2$	$1/2$		$\mathbf{2}^v_{-3/2} + 3^{L/R}_0 + 2^{L/R}_{+3/2}$
17	$(\bar{L}\mathrm{i}\tau^2 H^*)(\gamma^\rho)(H^T E^c)(\mathrm{i}\tau^2)(\overline{E^c}\gamma_\rho L)$	$-1/2$	$1/2$		$\mathbf{2}^v_{-3/2} + 1^{L/R}_{-1} + 2^{L/R}_{+1/2}$
18	$(\bar{L}\vec{\tau}\mathrm{i}\tau^2 H^*)(\gamma^\rho)(H^T E^c)(\mathrm{i}\tau^2 \vec{\tau})(\overline{E^c}\gamma_\rho L)$	$-3/2$	$-1/2$		$\mathbf{2}^v_{-3/2} + 3^{L/R}_{-1} + 2^{L/R}_{+1/2}$
Combination $\boldsymbol{H^\dagger L}$					
19	$(\bar{L}E)(\bar{E}H)(H^\dagger L)$	$-1/4$	$-1/4$	✓	$\mathbf{2}^s_{+1/2} + 1^R_0 + 2^{L/R}_{-1/2}$
20	$(\bar{L}E)(\vec{\tau})(\bar{E}H)(H^\dagger \vec{\tau} L)$	$-3/4$	$1/4$		$\mathbf{2}^s_{+1/2} + 3^{L/R}_0 + 2^{L/R}_{-1/2}$
21	$(\bar{L}H)(\gamma^\rho)(H^\dagger L)(\bar{E}\gamma_\rho E)$	$1/2$	$1/2$	✓	$\mathbf{1}^v_0 + 1^R_0$
22	$(\bar{L}\vec{\tau} H)(\gamma^\rho)(H^\dagger \vec{\tau} L)(\bar{E}\gamma_\rho E)$	$3/2$	$-1/2$		$\mathbf{1}^v_0 + 3^{L/R}_0$
23	$(\bar{L}\gamma^\rho E^c)(\overline{E^c}H)(\gamma^\rho)(H^\dagger L)$	$-1/2$	$-1/2$	✓	$\mathbf{2}^v_{-3/2} + 1^R_0 + 2^{L/R}_{+3/2}$
24	$(\bar{L}\gamma^\rho E^c)(\overline{E^c}H)(\gamma^\rho)(H^\dagger L)$	$-3/2$	$1/2$		$\mathbf{2}^v_{-3/2} + 3^{L/R}_0 + 2^{L/R}_{+3/2}$
Combination $\boldsymbol{HL}$					
25	$(\bar{L}E)(\mathrm{i}\tau^2)(\bar{E}H^*)(H^T \mathrm{i}\tau^2 L)$	$1/4$	$-1/4$		$\mathbf{2}^s_{+1/2} + 1^{L/R}_{-1} + 2^{L/R}_{-3/2}$
26	$(\bar{L}E)(\vec{\tau}\mathrm{i}\tau^2)(\bar{E}H^*)(H^T \mathrm{i}\tau^2 \vec{\tau} L)$	$3/4$	$1/4$		$\mathbf{2}^s_{+1/2} + 3^{L/R}_{-1} + 2^{L/R}_{-3/2}$
27	$(\bar{L}\mathrm{i}\tau^2 H^*)(\gamma^\rho)(H^T \mathrm{i}\tau^2 L)(\bar{E}\gamma_\rho E)$	$-1/2$	$1/2$		$\mathbf{1}^v_0 + 1^{L/R}_{-1}$
28	$(\bar{L}\vec{\tau}\mathrm{i}\tau^2 H^*)(\gamma^\rho)(H^T \mathrm{i}\tau^2 \vec{\tau} L)(\bar{E}\gamma_\rho E)$	$-3/2$	$-1/2$		$\mathbf{1}^v_0 + 3^{L/R}_{-1}$
29	$(\bar{L}\gamma^\rho E^c)(\mathrm{i}\tau^2)(\overline{E^c}H^*)(\gamma_\rho)(H^T \mathrm{i}\tau^2 L)$	$1/2$	$-1/2$		$\mathbf{2}^v_{-3/2} + 1^{L/R}_{-1} + 2^{L/R}_{+1/2}$
30	$(\bar{L}\gamma^\rho E^c)(\vec{\tau}\mathrm{i}\tau^2)(\overline{E^c}H^*)(\gamma_\rho)(H^T \mathrm{i}\tau^2 \vec{\tau} L)$	$3/2$	$1/2$		$\mathbf{2}^v_{-3/2} + 3^{L/R}_{-1} + 2^{L/R}_{+1/2}$

Table 5.3: Complete list of $\bar{L}L\bar{E}E$-type $d = 8$ interactions which involve two SM fields at any possible vertex of interaction (field bilinears within brackets).

#	Dim. eight operator	$\mathcal{C}_{LLH}^{\mathbf{111}}$	$\mathcal{C}_{LLH}^{\mathbf{331}}$	$\mathcal{C}_{LLH}^{\mathbf{133}}$	$\mathcal{C}_{LLH}^{\mathbf{313}}$	$\mathcal{C}_{LLH}^{\mathbf{333}}$	$\mathcal{O}_{\mathrm{NSI}}$?	Mediators
Combination $(\bar{L}^{\beta} L_{\alpha})(\bar{L}^{\delta} L_{\gamma})(H^{\dagger} H)$								
31	$(\bar{L}\gamma^{\rho}L)(\bar{L}\gamma_{\rho}L)(H^{\dagger}H)$	1						$\mathbf{1}_0^v$
32	$(\bar{L}\gamma^{\rho}\vec{\tau}L)(\bar{L}\gamma_{\rho}\vec{\tau}L)(H^{\dagger}H)$		1					$\mathbf{3}_0^v$
33	$(\bar{L}\gamma^{\rho}L)(\bar{L}\gamma_{\rho}\vec{\tau}L)(H^{\dagger}\vec{\tau}H)$			1				$\mathbf{1}_0^v + \mathbf{3}_0^v$
34	$(\bar{L}\gamma^{\rho}\vec{\tau}L)(\bar{L}\gamma_{\rho}L)(H^{\dagger}\vec{\tau}H)$				1			$\mathbf{1}_0^v + \mathbf{3}_0^v$
35	$(-\mathrm{i}\epsilon^{abc})(\bar{L}\gamma^{\rho}\tau^a L)\times$					1	✓	$\mathbf{3}_0^v$
	$(\bar{L}\gamma_{\rho}\tau^b L)(H^{\dagger}\tau^c H)$							
Combination $(\bar{L}^{\beta} L_{\alpha})(\bar{L}^{\delta} H)(H^{\dagger} L_{\gamma})$								
36	$(\bar{L}\gamma^{\rho}L)(\bar{L}H)(\gamma_{\rho})(H^{\dagger}L)$	1/2		1/2			✓	$\mathbf{1}_0^v + 1_0^R$
37	$(\bar{L}\gamma^{\rho}L)(\bar{L}\vec{\tau}H)(\gamma_{\rho})(H^{\dagger}\vec{\tau}L)$	3/2		−1/2				$\mathbf{1}_0^v + 3_0^{L/R}$
38	$(\bar{L}\gamma^{\rho}\vec{\tau}L)(\bar{L}\vec{\tau}H)(\gamma_{\rho})(H^{\dagger}L)$		1/2		1/2	1/2	✓	$\mathbf{1}_0^v + 1_0^R + 3_0^{L/R}$
39	$(\bar{L}\gamma^{\rho}\vec{\tau}L)(\bar{L}H)(\gamma_{\rho})(H^{\dagger}\vec{\tau}L)$		1/2		1/2	−1/2	✓	$\mathbf{1}_0^v + 1_0^R + 3_0^{L/R}$
40	$(-\mathrm{i}\epsilon^{abc})(\bar{L}\gamma^{\rho}\tau^a L)\times$		1		−1			$\mathbf{3}_0^v + 1_0^R + 3_0^{L/R}$
	$(\bar{L}\tau^b H)(\gamma_{\rho})(H^{\dagger}\tau^c L)$							
Combination $(\bar{L}^{\beta} L_{\alpha})(\bar{L}^{\delta} H^{\dagger})(L_{\gamma} H)$								
41	$(\bar{L}\gamma^{\rho}L)(\bar{L}\mathrm{i}\tau^2 H^*)(\gamma_{\rho})(H^T\mathrm{i}\tau^2 L)$	−1/2		1/2				$\mathbf{1}_0^v + 1_{-1}^{L/R}$
42	$(\bar{L}\gamma^{\rho}L)(\bar{L}\vec{\tau}\mathrm{i}\tau^2 H^*)(\gamma_{\rho})(H^T\mathrm{i}\tau^2\vec{\tau}L)$	−3/2		−1/2				$\mathbf{1}_0^v + 3_{-1}^{L/R}$
43	$(\bar{L}\gamma^{\rho}\vec{\tau}L)(\bar{L}\vec{\tau}\mathrm{i}\tau^2 H^*)(\gamma_{\rho})(H^T\mathrm{i}\tau^2 L)$		−1/2		1/2	1/2		$\mathbf{3}_0^v + 1_{-1}^{L/R} + 3_{-1}^{L/R}$
44	$(\bar{L}\gamma^{\rho}\vec{\tau}L)(\bar{L}\mathrm{i}\tau^2 H^*)(\gamma_{\rho})(H^T\mathrm{i}\tau^2\vec{\tau}L)$		−1/2		1/2	−1/2		$\mathbf{3}_0^v + 1_{-1}^{L/R} + 3_{-1}^{L/R}$
45	$(-\mathrm{i}\epsilon^{abc})(\bar{L}\gamma^{\rho}\tau^a L)\times$		−1		−1		✓	$\mathbf{3}_0^v + 3_{-1}^{L/R}$
	$(\bar{L}\tau^b\mathrm{i}\tau^2 H^*)(\gamma_{\rho})(H^T\mathrm{i}\tau^2\tau^c L)$							
Combination $(\bar{L}^{\beta}(L^c)^{\delta})((\overline{L^c})_{\alpha} L_{\gamma})(H^{\dagger} H)$								
46	$(\bar{L}\mathrm{i}\tau^2 L^c)(\overline{L^c}\mathrm{i}\tau^2 L)(H^{\dagger}H)$	1/4	−1/4				✓	$\mathbf{1}_{-1}^s$
47	$(\bar{L}\vec{\tau}\mathrm{i}\tau^2 L^c)(\overline{L^c}\mathrm{i}\tau^2\vec{\tau}L)(H^{\dagger}H)$	−3/4	−1/4					$\mathbf{3}_{-1}^s$
48	$(\bar{L}\mathrm{i}\tau^2 L^c)(\overline{L^c}\mathrm{i}\tau^2\vec{\tau}L)(H^{\dagger}\vec{\tau}H)$			1/4	−1/4	−1/4	✓	$\mathbf{1}_{-1}^s + \mathbf{3}_{-1}^s$
49	$(\bar{L}\vec{\tau}\mathrm{i}\tau^2 L^c)(\overline{L^c}\mathrm{i}\tau^2 L)(H^{\dagger}\vec{\tau}H)$			−1/4	1/4	−1/4	✓	$\mathbf{1}_{-1}^s + \mathbf{3}_{-1}^s$
50	$(-\mathrm{i}\epsilon^{abc})(\bar{L}\tau^a\mathrm{i}\tau^2 L^c)\times$			−1/2	−1/2			$\mathbf{3}_{-1}^s$
	$(\overline{L^c}\mathrm{i}\tau^2\tau^b L)(H^{\dagger}\tau^c H)$							
Combination $(\bar{L}^{\beta} H^{\dagger})((L^c)^{\delta} H)((\overline{L^c})_{\alpha} L_{\gamma})$								
51	$(\bar{L}\mathrm{i}\tau^2 H^*)(H^T L^c)(\overline{L^c}\mathrm{i}\tau^2 L)$	1/8	−1/8	1/8	−1/8	1/8	✓	$\mathbf{1}_{-1}^s + 1_0^L + 1_{-1}^{L/R}$
52	$(\bar{L}\vec{\tau}\mathrm{i}\tau^2 H^*)(H^T L^c\vec{\tau})(\overline{L^c}\mathrm{i}\tau^2 L)$	−3/8	3/8	1/8	−1/8	1/8	✓	$\mathbf{1}_{-1}^s + 3_0^{L/R} + 1_{-1}^{L/R}$
53	$(\bar{L}\vec{\tau}\mathrm{i}\tau^2 H^*)(H^T L^c)(\overline{L^c}\mathrm{i}\tau^2\vec{\tau}L)$	−3/8	−1/8	−3/8	−1/8	1/8	✓	$\mathbf{3}_{-1}^s + 1_0^L + 3_{-1}^{L/R}$
54	$(\bar{L}\mathrm{i}\tau^2 H^*)(H^T\vec{\tau}L^c)(\overline{L^c}\mathrm{i}\tau^2\vec{\tau}L)$	3/8	1/8	−1/8	−3/8	−1/8		$\mathbf{3}_{-1}^s + 3_0^{L/R} + 1_{-1}^{L/R}$
55	$(-\mathrm{i}\epsilon^{abc})(\bar{L}\tau^a\mathrm{i}\tau^2 H^*)\times$	3/4	1/4	−1/4	1/4	1/4		$\mathbf{3}_{-1}^s + 3_0^{L/R} + 1_{-1}^{L/R}$
	$(H^T\tau^b L^c)(\overline{L^c}\mathrm{i}\tau^2\tau^c L)$							
Combination $(\bar{L}^{\beta}(L^c)^{\delta})(H^{\dagger}(\overline{L^c})_{\alpha})(L_{\gamma} H)$								
56	$(\bar{L}\mathrm{i}\tau^2 L^c)(\overline{L^c}H^*)(H^T\mathrm{i}\tau^2 L)$	1/8	−1/8	−1/8	1/8	1/8	✓	$\mathbf{1}_{-1}^s + 1_0^L + 1_{-1}^{L/R}$
57	$(\bar{L}\vec{\tau}\mathrm{i}\tau^2 L^c)(\overline{L^c}\vec{\tau}H^*)(H^T\mathrm{i}\tau^2 L)$	3/8	1/8	−3/8	−1/8	−1/8		$\mathbf{3}_{-1}^s + 3_0^{L/R} + 1_{-1}^{L/R}$
58	$(\bar{L}\mathrm{i}\tau^2 L^c)(\overline{L^c}\vec{\tau}H^*)(H^T\mathrm{i}\tau^2\vec{\tau}L)$	−3/8	3/8	−1/8	1/8	1/8	✓	$\mathbf{1}_{-1}^s + 3_0^{L/R} + 3_{-1}^{L/R}$
59	$(\bar{L}\vec{\tau}\mathrm{i}\tau^2 L^c)(\overline{L^c}H^*)(H^T\mathrm{i}\tau^2\vec{\tau}L)$	−3/8	−1/8	−1/8	−3/8	1/8	✓	$\mathbf{3}_{-1}^s + 1_0^L + 3_{-1}^{L/R}$
60	$(-\mathrm{i}\epsilon^{abc})(\bar{L}\tau^a\mathrm{i}\tau^2 L^c)\times$	3/4	1/4	1/4	−1/4	1/4		$\mathbf{3}_{-1}^s + 3_0^{L/R} + 3_{-1}^{L/R}$
	$(\overline{L^c}\tau^b H^*)(H^T\mathrm{i}\tau^2\tau^c L)$							

Table 5.4: Same as Table 5.3, but for the $\bar{L}L\bar{L}L$-type operators. Note that in this case the relationship between flavor structure and symbol is not unique. We show the flavor structure for each group separately.

#	Dim. eight operator	$\mathcal{C}_{EEH}$	Mediators
61	$(\bar{E}^{\beta}\gamma^{\rho}E_{\alpha})(\bar{E}^{\delta}\gamma_{\rho}E_{\gamma})(H^{\dagger}H)$	1	$\mathbf{1}^{v}_{0}$
62	$(\bar{E}\gamma^{\rho}E)(\bar{E}H^{T})(\gamma_{\rho})(H^{*}E)$	1	$\mathbf{1}^{v}_{0} + \mathbf{2}^{L/R}_{-1/2}$
63	$(\bar{E}\gamma^{\rho}E)(\bar{E}H^{\dagger})(\gamma_{\rho})(HE)$	1	$\mathbf{1}^{v}_{0} + \mathbf{2}^{L/R}_{-3/2}$
64	$(\bar{E}^{\beta}E^{c\delta})(\overline{E^{c}}_{\alpha}E_{\gamma})(H^{\dagger}H)$	1/2	$\mathbf{1}^{s}_{-2}$
65	$(\bar{E}H^{\dagger})(E^{c}H)(\overline{E^{c}}E)$	1/2	$\mathbf{1}^{s}_{-2} + \mathbf{2}^{L/R}_{-3/2} + \mathbf{2}^{L/R}_{+1/2}$
66	$(\bar{E}E^{c})(H^{\dagger}\overline{E^{c}})(EH)$	1/2	$\mathbf{1}^{s}_{-2} + \mathbf{2}^{L/R}_{-3/2} + \mathbf{2}^{L/R}_{+1/2}$

Table 5.5: Effective $d = 8$ operators of the $\bar{E}E\bar{E}E$-type.

In Table 5.4, the simplest possibility to build a pure $\mathcal{O}_{\rm NSI}$ and no $d = 6$ interaction requires at least three fields, namely $\mathbf{1}^{s}_{-1}$, $\mathbf{3}^{v}_{0}$, and $\mathbf{3}^{s}_{-1}$, which may come from a large number of possible operator combinations. For example, one may combine operators #35 and #48. As discussed in Sec. 5.1, such a model could have correlations between source and matter NSI. Note that neither these models, nor our toy model, involve fermions [10], which means that they cannot generate corrections to the unitarity of the PMNS matrix (through contributions to the operators in Eqs. (5.6) and (5.7)) nor to electroweak data (through contributions to the operators in Eqs. (5.7) and (5.8)), or at least not at leading order.

New interactions involving only one SM field

Beyond the operators in the tables above, a much larger number of effective operators is obtained if, in addition to the interactions with SM bilinears, couplings between one SM field and two exotic fields are allowed in the fundamental theory [81]. The resulting $d = 8$ operators are diagrammatically illustrated in Fig. 4 and fall in three categories, which contain the following SM bilinears at the external vertices:

1. (LE)- or (LL)-type interactions with new fields. At least one of the mediators will necessarily induce some of the $d = 6$ interactions among four leptons discussed earlier (corresponding to the external vertices in the figure), and the couplings will thus be subject to the corresponding constraints. The fundamental interactions describing the internal vertices, however, may not be related to the previously discussed $d = 6$ interactions.

2. (LH)-type interactions. In this case, the mediators do not necessarily induce any dangerous $d = 6$ operator involving four leptons, even if there are some common mediators. The connections previously studied linking $d = 6$ and $d = 8$ operators do not need to hold. Nevertheless, these type of interactions involve exotic fermions

[10] More precisely, they do not involve Yukawa couplings linking the exotic and standard fermions.

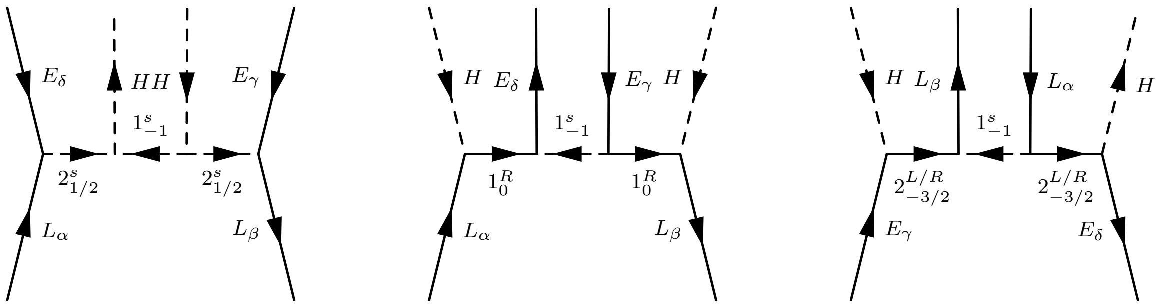

Figure 5.4: Examples for each category of diagrams which lead to $d = 8$ operators and require couplings of the new fields *both* to SM bilinears and to only one SM field.

($SU(2)$ singlets or triplets) and are constrained by non-unitary contributions to the PMNS matrix and some of them also by electroweak precision data, see, *e.g.*, Ref. [81]: Fig. 5.4, center, illustrates that this class of diagrams is connected to one of the $d = 6$ operators in Eqs.(5.6) and (5.7), or a combination of them.

3. (EH)-type interactions. These type of interactions are suggestive. The mediators may not induce dangerous four-fermion $d = 6$ operators. Furthermore, they do not introduce corrections to the PMNS matrix at leading order. They involve exotic leptons, however, which are typically strongly constrained by electroweak precision tests [168]. Fig. 5.4, right, illustrates that this class of diagrams is connected to the $d = 6$ operators in Eq. (5.8).

Possible "mixed" diagrams, that is, diagrams involving two different SM bilinear couplings, will combine the corresponding properties. For instance, a model containing both (LE) and (LH) couplings to exotic mediators will simultaneously induce some of the $d = 6$ operators in Table 2 *and* some of the operators in Eqs. (10)-(12) which induce non-unitarity.

It is easy to show that the vertex involving just one SM field (L, E or H) requires that the two exotic particles attached to it have different $SU(2) \times U(1)$ charges. Indeed, we have explicitly checked that *all* of these possibilities require at least two new fields to be phenomenologically viable, *i.e.*, are not simpler than the cases discussed prior to this subsection.

The scenarios in diagram #2 and specially #3 in Fig. 5.4 are appealing alternatives, as none of them is correlated to harmful $d = 6$ interactions (*i.e.*, four charged-fermion lepton couplings), and #3 does not induce non-unitarity either. Furthermore, these two examples are $\mathcal{O}_{\rm NSI}$ operators. Indeed, the exchange of a singlet fermion $\mathbf{1}^R_0$ and a charged scalar $\mathbf{1}^s_{-1}$ shown in #2 gives schematically

$$(\bar{L}H)(E)(\bar{E})(H^\dagger L) = -\frac{1}{4}(\mathcal{O}^{\mathbf{1}}_{LEH}) - \frac{1}{4}(\mathcal{O}^{\mathbf{3}}_{LEH})\,. \tag{5.54}$$

Here the projection onto the BR basis shows that it complies with the $d = 8$ cancellation conditions, Eq. (5.29). The mediator $\mathbf{1}^s_{-1}$ could induce in addition $d = 6$ effective interactions if it would also couple to SM lepton doublets, as shown in Table 5.2, but such couplings are not mandatory. In contrast, the PMNS unitarity constraints should be relevant, as a singlet exotic fermion is involved.

Turning now to type #3 and the scenario with an exotic doublet fermion $\mathbf{2}^{L/R}_{-3/2}$ and a charged scalar $\mathbf{1}^s_{-1}$, the resulting effective operator for this example is of the form

$$(EH)(\bar{L})(L)(H^\dagger \bar{E}) = -\frac{1}{4}(\mathcal{O}^{\mathbf{1}}_{LEH}) - \frac{1}{4}(\mathcal{O}^{\mathbf{3}}_{LEH}) \,, \tag{5.55}$$

and is thus again of the $\mathcal{O}_{\rm NSI}$ type. Furthermore, in this case the interactions neither lead to non-unitarity, nor any $d = 6$ operator in Table 5.2 needs to be induced if the charged scalar does not couple to SM lepton doublets (in other words, the $d = 6$ complete cancellation conditions in Eq. (5.41) can be implemented as well). Other scenarios of the kind just discussed do not necessarily have to lead by themselves to $\mathcal{O}_{\rm NSI}$ structures: for them, cancellations similar to those in our toy model could be considered. However, it remains to be explored how difficult is to circumvent the constraints which electroweak precision tests impose on exotic leptons, and whether the necessary cancellations are feasible without running into extreme fine-tunings, for instance enlarging the scalar sector of the theory.

During the completion of this work, Ref. [81] appeared. It explores (but is not limited to) the possible exchange of exotic fields which in our notation have quantum numbers of a scalar $\mathbf{1}^s_{-1}$ (to obtain $d = 6$ NSI) and of a fermion $\mathbf{1}^R_0$ (to obtain $d = 8$ NSI). The latter induces also $d = 6$ interactions, which lead to non-unitary contributions to the PMNS matrix, as it is well known and is further explored in that reference. Ref. [81] performs a systematic topological scan of the $d = 8$ operators, based on Feynman diagrams, trying to obtain the interaction $\mathcal{O}_{\rm NSI}$ directly from just one Feynman diagram while avoiding *any* harmful $d = 6$ and $d = 8$ contribution. Our tables correspond to the topologies 2 and 3 in this reference, whereas the previous paragraph in this subsection would correspond to their topology 1. Since all possibilities in our tables contain at least one mediator leading to harmful $d = 6$ effects if one does not allow for cancellations, Ref. [81] effectively exclude topologies 2 and 3 in their scan (apart from our #46, which does not induce harmful $d = 6$ four charged lepton interactions, but the mediator $\mathbf{1}^s_{-1}$ is constrained otherwise, as we and Ref. [81] discussed before). Therefore, our work is complementary to that reference. Note that they find that the NSI in matter and the NSI at source or detector are correlated in all of their examples by the non-unitary effects of the heavy fermions, whereas it is easy to see that uncorrelated scenarios are achievable when one allows to combine different operators from our Table 5.3 (such as #7, #8, #13 and #14). As the most important difference, we have related the operators obtained from mediator exchanges to a complete basis of independent operators, which allows us to deduce the general cancellation conditions.

Conclusions

The Standard Model of Particle Physics continues to be as successful experimentally as it is intriguing for theoreticians. Among other unsettling issues we have the Hierarchy Problem and The Flavour Puzzle. While we ruminate these ancient problems over and over we wait, hoping for the experiment that will break down the SM (the LHC, maybe?).

A decade ago something of the like happened when neutrino masses were discovered. This was indeed the evidence of particle physics beyond the SM but it was much more than that. It was the first indication that Lepton Number should not be a symmetry of the universe. And it was also a handle with which to grasp those questions that had been posed for so long. A protuberance in the mighty rock of the Flavour Puzzle. We clung to it desperately.

Ten years later, neutrino physics is still a boiling hot subject. This thesis, which reflects my work on the issue, presented three original contributions. Although there is a common horizon to all three - we want to understand neutrinos and certainly we want to understand flavour -, there is no strong tie among them. This thesis was in that sense exploratory in nature.

Maybe the most interesting result of our work was the identification of an extremely simple Seesaw model of neutrino masses requiring only 2 heavy neutrinos. The Yukawa couplings of this model can be determined in terms of the light neutrino mass matrix and, therefore, a relation exists between the coefficient of the $d = 5$ Weinberg operator, Eq.(1.35), and the coefficients of $d = 6$ operators involved in flavour processes. The flavour violating rates induced by the $d = 6$ couplings can be reconstructed - including CP phases - from the parameters in the light neutrino mass matrix, except for: 1) a global normalization and 2) discrete degeneracies in the Majorana phase. We also provided some phenomenological implications of the model that might be at reach for near future experiments. This was the case of the rate for rare flavour-changing lepton decays or neutrinoless double beta decay

More generally, we explored the relation between MFV and seesaw models. A key condition imposed was the existence of some approximate $U(1)_{LN}$ lepton number symmetry implying two distinct scales, $\Lambda_{LN} \gg \Lambda_{FL}$. The LN scale Λ_{LN} was necessary to suppress all operators violating lepton number, such as the $d = 5$ Weinberg's operator, and thus leading to small neutrino masses. The flavour scale Λ_{fl} was useful to suppress to a lesser extent flavour violating but lepton number conserving processes, such as $l_i \to l_j \gamma$, mediated by $d = 6$ effective operators.

Within this framework we were able to find examples among the Seesaw models of both the *minimal* and *extended* types of MFV that were developed in previous literature:

- The Seesaw Type II is an example of minimal MFV with a natural separation of scales $\Lambda_{LN} \gg \Lambda_{fl}$.
- The separation of scales is not achieved in the minimal fermionic Seesaws, Type I and III. However, enlarging the exotic fermion sector they can be made to fulfill the conditions of extended MFV

Without abandoning neutrino mass models, a less conventional approach was taken in the middle section of this thesis. The objective was to show that the standard lore regarding neutrino masses coming from the $d = 5$ Weinberg operator wasn't a mandatory option. We were able to prove that neutrino mass can be generated solely from higher than $d = 5$ operators even at the tree level. The idea is that, combining LN with another - continuous or discrete - abelian symmetry, the $d = 5$ couplings may be forbidden and neutrino masses only appear at $d = 7$ or higher. We showed that, in order for this mechanism to produce small neutrino masses naturally, an extension of the SM, at energies around the TeV is necessary.

In our work, we focused on the Two Higgs Doublet Model extension of the Standard Model which naturally yields neutrino masses from effective operators of odd dimension. The posible high-energy models that lead to neutrino masses through $d > 5$ operators were also determined systematically. This is interesting because, after EW symmetry breaking, these models lead to generalizations of the typical seesaw scenarios and, in particular, they can provide an ultraviolet completion of inverse seesaws at the TeV scale.

Finally we were able to find one example of a model that induces neutrino masses at $d = 7$ but only radiatively. Neutrino masses here were shown to be proportional to the breaking of a continuous new $U(1)$ symmetry to a discrete $\mathbb{Z}_5$. The additional suppression due to the loops makes the new physics scale of order of the TeV even with order one couplings. Thus it is possible a universal scale responsible for neutrino masses *and* flavour processes and still have the latter sizable. In this case we do not advocate any model in particular but rather show some examples of the principle: combining LN with discrete symmetries can lower the Seesaw scale.

In the final part of this thesis we took neutrino masses for granted, and hence flavour violation in the lepton sector. Among exotic flavour violating processes we studied NSNIs which, admittedly, haven't been linked to Seesaw models but remain nonetheless a possibility of new physics. These interactions however cannot escape from respecting the gauge symmetries of nature, in particular, $SU(2)_W$ gauge invariance which relates them to scattering processes between four charged leptons. These processes are measured to coincide with the SM prediction with great accuracy, a fact that poses strong indirect constraints on NSNIs

The aim was to gauge the theoretical price of achieving phenomenologically viable, large neutrino NSIs. We established the minimal constraints that models have to respect for this purpose by imposing that interactions with four charged leptons have to be absent or highly suppressed since these would lead to charged lepton flavor violation or corrections to G_F.

NSNIs appear from operators of even dimension, $d = 6, 8$, etc. In the case of $d = 6$, all effective operators lead to charged lepton processes except for one, namely, the popular antisymmetric four-lepton operator. The model that generates it at tree-level is nevertheless very well constrained. At $d = 8$ there are many operators that yield NSNIs without inducing interactions among four charged leptons. However, imposing the requirement that the contributions to $d = 6$ operators should cancel one finds the important result that BSM models of NSNIs must include at least two exotic fields. A systematic analysis of the mediators and cancellation conditions leading to successful NSNIs was also carried. These cancellation conditions generally translate into fine-tuning of the model parameters.

All in all, the future for NSNIs doesn't seem to be bright. From this work we conclude that in principle, it is not enough to extend the SM but also strong fine-tuning is almost a necessity for NSNIs to be realistic. On the other hand, we did not exclude them and cancellations might appear due to new symmetries. That's the glass half-full.

Flavour physics in the lepton sector is a fascinating subject. Neutrino masses open up a universe of phenomena that could be there but we have yet to measure. In summary we have shown, through rather model-independent techniques, how these phenomena are correlated, both among themselves and with the renormalizable parameters couplings that already appear in the SM Lagrangian. Furthermore, we have clarified what type of symmetries are to be expected to have observability in the flavour sector in the present and foreseeable experiments. Also we showed how a particularly simple and predictive model of neutrino masses can lead to signals that might be around the corner of today's experiments.

Appendix A

Naturalness

We address here the question of naturalness and the stability of the scales present in the models considered, which is an issue as they include at least one scale larger than the electroweak one.

In models of type A as in eqs. (3.15) and (3.50), the quantum corrections induced on the size of the electroweak scale by the presence of Λ of $\mathcal{O}(TeV)$ are not significant, because they have to be proportional to the small parameters ϵ or μ, μ'.

While the smallness of the ϵ entries in eq. (3.15) can be technically natural, as discussed in section 4, a naturalness problem arises instead in the type A models in eq. (3.50), when the zeros are justified as due to a conserved global lepton number, which is then spontaneously broken, i.e. by the vev of a singlet scalar field S with interactions given in eq. (3.56). The scalar potential $V(S, \phi)$,

$$V(S,\phi) = \lambda_\phi(\phi^\dagger\phi)^2 + \lambda_S(S^\dagger S)^2 + \mu_\phi^2\phi^\dagger\phi + \mu_S^2 S^\dagger S + \lambda(\phi^\dagger\phi)(S^\dagger S)\,, \tag{A.1}$$

leads to

$$< S >= \sqrt{\frac{(\lambda\mu_\phi^2 - 2\lambda_\phi\mu_S^2)}{4\lambda_\phi\lambda_S - \lambda^2}}\,. \tag{A.2}$$

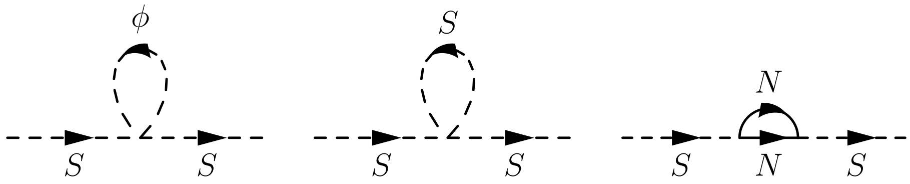

Figure A.1: Loop corrections to mass of the scalar S in type A models with spontaneously broken lepton number.

This vev has to be small compared to Λ, as $\mu = g < S >$, $\mu' = g' < S >$, see eqs. (3.50) and (3.56). The problem arises because, for instance, μ_S is destabilized at one-loop by contributions sensitive to high scales and only weighted by the couplings g, g', λ_S or λ. As an example, the contribution from the three diagrams in Fig. 6 are, respectively,

$$\delta\mu_S^2 \sim \frac{\lambda}{(4\pi)^2}\left[\Lambda_c^2 - m_\phi^2 \ln\left(1+\frac{\Lambda_c^2}{m_\phi^2}\right)\right], \tag{A.3}$$

$$\delta\mu_S^2 \sim \frac{3\lambda_S}{(4\pi)^2}\left[\Lambda_c^2 - m_S^2 \ln\left(1+\frac{\Lambda_c^2}{m_S^2}\right)\right], \tag{A.4}$$

$$\delta\mu_S^2 \sim \frac{(g+g')^2}{4(4\pi)^2}\left[\Lambda_c^2 + \Lambda^2 \ln\left(1+\frac{\Lambda_c}{\Lambda}\right)\right], \tag{A.5}$$

where Λ_c is a cutoff scale to be removed by renormalization, after which finite contributions will still remain proportional to physical scales such as the Higgs mass m_ϕ, the scalar mass m_S or the flavour scale Λ. A fine-tuning is thus necessary to preserve the desired hierarchy, unless the dimensionless couplings g, g', λ_S and λ turn out to be small.

Type B models involve at least two large scales, represented by Λ and Λ', typically with $\Lambda' \gg \Lambda$. The class of models in eq. (3.57) taken by themselves is free from naturalness problems. To illustrate it, it suffices to take the simpler case $\mu_1 = \mu_2 = \Lambda$,

$$\begin{aligned}\mathscr{L} = \mathscr{L}_{SM} + i\bar{N}\partial\!\!\!/ N + i\bar{N'}\partial\!\!\!/ N' + i\bar{N''}\partial\!\!\!/ N'' - \Big[Y_N \bar{N}\tilde{\phi}^\dagger \ell_L + \frac{\Lambda'}{2}\bar{N''}N''^{\,c} + \\ + \frac{\Lambda}{2}\Big(\bar{N}N''^{\,c} + \bar{N'}N^c + \bar{N}N''^{\,c} + \bar{N''}N^c + \bar{N''}N''^{\,c} + \bar{N''}N'^{\,c}\Big) + \text{h.c.}\Big],\end{aligned} \tag{A.6}$$

which becomes, in the basis of mass eigenstates denoted N_1, N_2, N_3,

$$\begin{aligned}\mathscr{L} = \mathscr{L}_{SM} + i\bar{N}_1\partial\!\!\!/ N_1 + i\bar{N}_2\partial\!\!\!/ N_2 + i\bar{N}_3\partial\!\!\!/ N_3 - \Big[Y_N(\alpha\bar{N}_1 + \beta\bar{N}_2 + \gamma\bar{N}_3)\tilde{\phi}^\dagger L + \\ + \frac{\Lambda}{2}(\bar{N}_1 N_1^c + \Lambda\bar{N}_2 N_2^c) + \frac{\Lambda'}{2}\bar{N}_3 N_3^c + \text{h.c.}\Big],\end{aligned} \tag{A.7}$$

where $(N_1, N_2, N_3)^T = U\,(N, N', N'')^T$, U being unitary. α, β and γ are functions of Λ and Λ' which, up to order $\frac{\Lambda}{\Lambda'}$, read

$$\alpha = \frac{i}{\sqrt{2}}, \quad \beta = -\frac{1}{\sqrt{2}}, \quad \gamma = \frac{\Lambda}{\Lambda'}. \tag{A.8}$$

In this basis, it is directly seen that the coupling of the Higgs to the heaviest field N_3 is suppressed by the factor $\frac{\Lambda}{\Lambda'}$, a fact that could already be guessed from eq. (3.57). Also, for instance, the amplitude of the loop diagram depicted in Fig. 7, can be written as

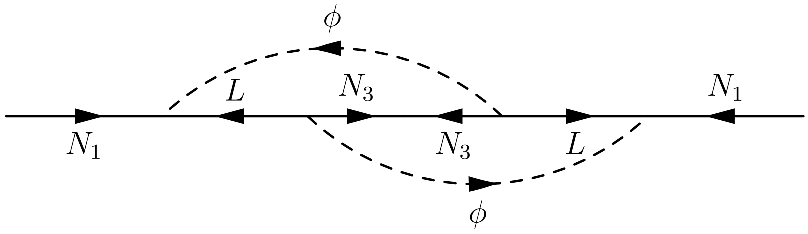

Figure A.2: Two loop correction to the N_1 mass on type B model.

$$\mathcal{M}(p)_A^C = i\frac{Y^4}{2}\frac{\Lambda^2}{\Lambda'}\int \frac{d^4l\, d^4k}{(2\pi)^8}\frac{l_\mu k_\nu \sigma^\mu_{A\dot{B}}(\bar{\sigma}^\nu)^{\dot{B}C}}{l^2k^2[(l+k-p)^2-\Lambda'^2](p-l)^2(p-k)^2}, \tag{A.9}$$

where p is the incoming momentum and where we have neglected the mass of the Higgs and the lepton running inside the loop. The integral in eq.(A.9) yields a logarithmic contribution of order one, hence the suppression factor $\gamma^2 = (\Lambda/\Lambda')^2$ guarantees no higher order correction to mass of the N_1. Furthermore, it is clear that this type of suppression always appears when the N_3 field runs inside a loop, and no naturalness problem results in this model.

The trouble is that the zeros in eq. (3.57) appear to be an ad hoc constraint. Again, they can be justified if lepton number is a symmetry of the Lagrangian, spontaneously broken by the vev of some scalar field(s), i.e. a singlet scalar S, to induce the entries μ_1, μ_2 while the null entries remain protected by the symmetry. This solution rises questions of naturalness, though, as quantum corrections may push the value of Λ towards that of the higher scale Λ'. We will illustrate it in what follows.

Let us promote the Lagrangian corresponding to eq. (3.57) to the lepton number conserving one

$$\begin{aligned}\mathscr{L} = {} & \mathscr{L}_{SM} + i\bar{N}\partial\!\!\!/ N + i\bar{N}'\partial\!\!\!/ N' + i\bar{N}''\partial\!\!\!/ N'' \\ & - V(S,\phi) - \Bigg[\frac{\Lambda}{2}(\bar{N}'N^c + \bar{N}N''^{\,c} + \frac{\Lambda'}{2}\bar{N}''N''^c + \\ & \qquad + Y_N\bar{N}\tilde{\phi}^\dagger L + \frac{f_1}{2}S(\bar{N}'N''^c + \bar{N}''N'^c) + \frac{f_2}{2}S^\dagger(\bar{N}N''^c + \bar{N}''N^c) + \text{h.c.}\Bigg],\end{aligned} \tag{A.10}$$

where S is a new scalar field with charge -1 under lepton number symmetry. Note that the symmetry is only violated after S acquires a vev, resulting in μ_1, μ_2 in eq. (3.57) given by $\mu_1 \equiv f_1\langle S\rangle$ and $\mu_2 \equiv f_2\langle S\rangle$. Due to the couplings of the S field new quantum corrections arise. The diagram in Fig. 8 induces a correction to the scale Λ given by

$$\delta\Lambda \sim \frac{f_1f_2}{(4\pi)^2}\Lambda'. \tag{A.11}$$

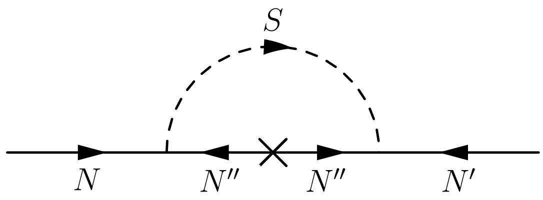

Figure A.3: Loop corrections to mass of N_1 in type B models with spontaneously broken lepton number.

where logarithms of order one have been neglected. This correction could suffice to destabilize the Λ scale. Note though that it does not need to be the case if the dimensionless coupling f_2, which does not enter in eq. (3.59), turns out to be sufficiently small.

In summary, naturalness issues arise in those models in which the justification of the vanishing or smallness of some couplings calls for a spontaneous breaking of lepton number symmetry. In the scenarios of this type analyzed, the problem can be evaded if certain dimensionless new couplings take small values. If this is the case, although we have not identified a symmetry reason justifying such small values, the protection of the size of the scales is technically natural.

Appendix B

On non-standard four neutrino interactions

Although interactions among four neutrinos hardly contribute to laboratory processes, there has been some interest in the literature in the context of flavor oscillations in astrophysical environments, such as dense neutrino gases; see *e.g.* Ref. [232] and references therein. The direct laboratory bounds on these interactions are naturally extremely weak, see Refs. [233, 234]. In this appendix, we discuss these four neutrino interactions in our gauge invariant framework.

B.1 Effective operator formalism

Since the four neutrino interactions require interactions with four lepton doublets, they only appear for the $\bar{L}L\bar{L}L$ operators. In this case, Eq. (5.20) reads, including the four neutrino interactions,

$$
\begin{aligned}
\delta\mathscr{L}_{\text{eff}} = & \frac{1}{\Lambda^2}\left(\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L}\right)_{\beta\delta}^{\alpha\gamma}\left(\bar{\nu}^\beta\gamma^\rho \mathrm{P}_L\nu_\alpha\right)\left(\bar{\ell}^\delta\gamma^\rho \mathrm{P}_L\ell_\gamma\right) \\
& + \frac{1}{\Lambda^2}\left(\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}} + \frac{v^2}{2\Lambda^2}\left(\mathcal{C}_{LLH}^{\mathbf{111}} + \mathcal{C}_{LLH}^{\mathbf{331}} - \mathcal{C}_{LLH}^{\mathbf{133}} - \mathcal{C}_{LLH}^{\mathbf{313}}\right)\right)_{\beta\delta}^{\alpha\gamma}\left(\bar{\ell}^\beta\gamma^\rho \mathrm{P}_L\ell_\alpha\right)\left(\bar{\ell}^\delta\gamma^\rho \mathrm{P}_L\ell_\gamma\right) \\
& + \frac{1}{\Lambda^2}\left(\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}} + \frac{v^2}{2\Lambda^2}\left(\mathcal{C}_{LLH}^{\mathbf{111}} + \mathcal{C}_{LLH}^{\mathbf{331}} + \mathcal{C}_{LLH}^{\mathbf{133}} + \mathcal{C}_{LLH}^{\mathbf{313}}\right)\right)_{\beta\delta}^{\alpha\gamma}\left(\bar{\nu}^\beta\gamma^\rho \mathrm{P}_L\nu_\alpha\right)\left(\bar{\nu}^\delta\gamma^\rho \mathrm{P}_L\nu_\gamma\right) \\
& + \text{h.c.}\,.
\end{aligned} \tag{B.1}
$$

The first point one notices is that the four charged lepton and four neutrino interactions share for $d = 6$ the same coefficient $\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}}$. This means that for $d = 6$, any bound from charged lepton flavor violation *etc.* can be directly translated into the four neutrino interactions. This is illustrated here with one example. For $\beta = \mu$ and

$\alpha = \gamma = \delta = e$, the bound from $\mu \to eee$ can, apart from some SU(2) symmetry breaking effects, be directly transferred to the four neutrino interactions. In our notation, one has

$$\mathrm{Br}(\mu \to 3e) = \frac{1}{G_F^2}\left(\frac{\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}}}{\Lambda^2}\right)^2 = \frac{F^2}{G_F^2}\,, \tag{B.2}$$

where the non-standard parameter is defined as $F \equiv (\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}})/\Lambda^2$ – as often done in the literature. The current bound $\mathrm{Br}(\mu \to 3e) < 10^{-12}$ (90% CL) [235] then directly translates into $F \lesssim 10^{-6}\,G_F$, which is far below any laboratory bound or even the bound from primordial nucleosynthesis. Of course, it is dependent on the participating flavors and somewhat looser for combinations involving the τ, but this procedure illustrates the generic argument. Note that the bound for a vector mediated interaction, such as often discussed in the literature, turns out to be the same in this case.

As discussed in Sec. 5.1, Eq. (5.28) should be satisfied for any realistic model in order to avoid these bounds. As we can read off from Eq. (B.1), however, the $d = 6$ coefficients for the four charged lepton and four neutrino interactions are exactly the same, which means that there will not be any four neutrino interactions in that case. As a consequence, one has to go to $d = 8$ with the interactions being suppressed by Λ^4.

For $d = 8$, the corresponding Eq. (5.29) to suppress the harmful interactions among four charged fermions can be implemented in qualitatively different ways. For example, if $\mathcal{C}_{LLH}^{\mathbf{111}} = -\mathcal{C}_{LLH}^{\mathbf{331}}$ and $\mathcal{C}_{LLH}^{\mathbf{133}} = -\mathcal{C}_{LLH}^{\mathbf{313}}$, there will be no four neutrino interactions but NSI, whereas for $\mathcal{C}_{LLH}^{\mathbf{111}} + \mathcal{C}_{LLH}^{\mathbf{331}} = \mathcal{C}_{LLH}^{\mathbf{133}} + \mathcal{C}_{LLH}^{\mathbf{313}} \neq 0$, there will be both four neutrino interactions and NSI. As it is demonstrated below, both possibilities can be realized within the model framework in this study.

B.2 Model analysis

In order to find models for large four neutrino interactions at $d = 8$, the same argumentation as in Sec. 5.3 is needed. First of all, Eq. (5.29) has to be satisfied to suppress the four charged lepton processes. Second, the $d = 6$ contributions to the NSI have to be cancelled, since there are strong bounds, *i.e.*, Eq. (5.41) has to be satisfied. As an additional condition, one can **not** have (*cf.*, Eq. (B.1))

$$\mathcal{C}_{LLH}^{\mathbf{111}} + \mathcal{C}_{LLH}^{\mathbf{331}} + \mathcal{C}_{LLH}^{\mathbf{133}} + \mathcal{C}_{LLH}^{\mathbf{313}} = 0 \tag{B.3}$$

because such an operator will not contribute to the four neutrino interactions. The relevant decomposed operators can be found in Table 5.4, where one can easily read off if Eq. (B.3) is satisfied. Furthermore, note that operators which only induce $\mathcal{C}_{LLH}^{\mathbf{333}}$ will not be useful for the four neutrino interactions. We find from the table that operators #35, #40, #41, #43, #44, #46, #48, #49, #51, #52, #54, #56, #57, and #58 do not contribute to the four neutrino interactions. This implies that the possibility pointed out in the main text, *i.e.*, to combine #35 and #48, does not lead to four neutrino interactions. One has to use more complicated combinations by the combination of different operators, such as #32 and #50 to satisfy Eq. (5.29), and #48 (which satisfies Eq. (5.29)) to introduce an additional mediator to cancel the $d = 6$ NSI. Then the four neutrino interactions can be constructed with three different mediators, where only #32 and #50 contribute to the four neutrino interactions. Constructions with less mediators are, under the assumptions in this study, not possible, which is different from the NSI, which can be generated from two mediators.

As soon as a specific model is known, the relationship among source and production NSI, matter NSI, and four neutrino interactions can be easily calculated using Sec. 5.1 and Eq. (B.1).

In summary, for the $d = 6$ four neutrino interactions, gauge invariance implies that they face the stringent bounds from charged lepton flavor violation, such as from μ to three electrons. Therefore, large four neutrino interactions have to come from $d = 8$ effective operators. From the model point of view, having four neutrino interactions is even more complicated than having large NSI, since at least three different mediators are needed in the framework discussed in this study.

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