Redshift drift in LTB universes

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Abstract

We study the redshift drift, i.e., the time derivative of the cosmological redshift in the Lemaître-Tolman-Bondi (LTB) solution in which the observer is assumed to be located at the symmetry center. This solution has often been studied as an anti-Copernican universe model to explain the acceleration of cosmic volume expansion without introducing the concept of dark energy. One of decisive differences between LTB universe models and Copernican universe models with dark energy is believed to be the redshift drift. The redshift drift is negative in all known LTB universe models, whereas it is positive in the redshift domain $z \lesssim 2$ in Copernican models with dark energy. However, there have been no detailed studies on this subject. In the present paper, we prove that the redshift drift of an off-center source is always negative in the case of LTB void models. We also show that the redshift drift can be positive with an extremely large hump-type inhomogeneity. Our results suggest that we can determine whether we live near the center of a large void without dark energy by observing the redshift drift.

1 introduction

The standard cosmological model is based on the so-called Copernican principle that we are not located in a special position in the universe. This model can naturally explain almost all observational data, and consequently seems to imply that the Copernican principle is a reality. However, we should not blindly rely on this principle without observational justifications. Here, we should note that it is not clear at all how large the systematic errors would be in the determination of the cosmological parameters, if the Copernican principle is abandoned. Thus, it is an unavoidable task in observational cosmology to investigate possible "anti-Copernican" universe models and test if such models can be observationally excluded.

Almost all anti-Copernican universe models are based on the Lemaître-Tolman-Bondi (LTB) solution which describes the dynamics of a spherically symmetric dust. In order to check the LTB universe models observationally, it is crucial to find observable quantities which can reveal differences between the LTB universe models and Copernican universe models with the dark energy. One such quantity is believed to be the redshift drift, i.e., the time derivative of the cosmological redshift [1]. In the case of the ΛCDM model, which is the most likely Copernican model at present, the redshift drift is positive in the redshift domain $z \lesssim 2$, since the cosmological constant Λ causes repulsive gravity. By contrast, there is no exotic matter with the violation of the strong energy condition in the LTB solution. Thus, as long as there is no highly inhomogeneous structure, the redshift drift might be negative in LTB universe models. Although several authors have pointed out the importance of the redshift drift [1–4], there has been no detailed study of its general behavior in LTB universe models. It is the purpose of this paper to investigate it.

In Sec. 2, we briefly review the LTB solution. In Sec. 3, we derive the equation for the redshift drift. In Sec. 4, we define LTB void models and prove a theorem on the redshift drift in these models. In Sec. 5, we show that the redshift drift can be positive even in an LTB universe model, if an extremely large hump-type mass density distribution exists. Sec. 6 is devoted to the summary and discussion.

In this paper, we denote the speed of light and Newton's gravitational constant by c and G, respectively.

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2 the LTB solution

As mentioned in the introduction, we consider a spherically symmetric inhomogeneous universe filled with dust. This universe is described by an exact solution of the Einstein equations, which is known as the Lemaître-Tolman-Bondi (LTB) solution. The metric of the LTB solution is given by

$$ds^2 = -c^2dt^2 + \frac{\left(\partial_r R(t,r)\right)^2}{1-k(r)r^2}dr^2 + R^2(t,r)d\Omega^2, \tag{1}$$

where $k(r)$ is an arbitrary function of the radial coordinate r. The matter is dust whose stress-energy tensor is given by $T^{\mu\nu} = \rho u^\mu u^\nu$, where $\rho = \rho(t,r)$ is the mass density, and u^μ is the four-velocity of the fluid element. The coordinate system in Eq. (1) is chosen in such a way that $u^\mu = (c, 0, 0, 0)$.

The circumferential radius $R(t,r)$ is determined by one of the Einstein equations, $\left(\frac{\partial R}{\partial t}\right)^2 = \frac{2GM(r)}{R} - c^2kr^2$, where $M(r)$ is an arbitrary function related to the mass density ρ by $\rho(t,r) = \frac{1}{4\pi R^2 \partial_r R}\frac{dM}{dr}$. $M(r)$ is known as the Misner-Sharp mass that is the quasi-local mass naturally introduced into the spherically symmetric spacetime[5]. In this paper, we assume that the Misner-Sharp mass is a monotonically increasing function of r in the domain of interest. This assumption is equivalent to the one that $\partial_r R$ is positive if ρ is positive.

Following Ref. [6], we write the solution of Einstein equations in the form,

$$R(t,r) = (6GM)^{1/3}[t - t_{\rm B}(r)]^{2/3}S(x), \quad x = c^2kr^2\left(\frac{t-t_{\rm B}}{6GM}\right)^{2/3}, \tag{2}$$

where $t_{\rm B}(r)$ is an arbitrary function which determines the big bang time, and $S(x)$ is an analytic function in $x < (\pi/3)^{2/3}$ (see Ref.[6] for the definition of $\mathcal{S}(x)$).

As shown in the above, the LTB solution has three arbitrary functions, $k(r)$, $M(r)$ and $t_{\rm B}(r)$. One of them is a gauge degree of freedom for the rescaling of r. In this paper, since M is assumed to be a monotonically increasing function of r, we can fix this freedom by setting $M = \frac{4}{3}\pi\rho_0 r^3$, where ρ_0 is the mass density at the symmetry center at the present time t_0, i.e., $\rho_0 = \rho(t_0, 0)$.

3 Equation for the redshift drift

In order to study the cosmological redshift and the redshift drift, we consider ingoing radial null geodesics. The cosmological redshift z of a light ray from a comoving source at r to the observer at the symmetry center $r = 0$ is defined by $z(r) := k^t(\lambda(r))/k^t(\lambda(0)) - 1$, where k^t is the time component of the null geodesic tangent, and λ is the affine parameter which can be regarded as a function of r. From the geodesic equations, we have the equation for the redshift z as

$$\frac{dz}{dr} = \frac{(1+z)\partial_t\partial_r R}{c\sqrt{1-kr^2}}. \tag{3}$$

The null condition leads to

$$\frac{dt}{dr} = -\frac{\partial_r R}{c\sqrt{1-kr^2}}. \tag{4}$$

We denote the trajectories of light rays observed by the central observer at $t = t_0$ and $t = t_0 + \delta t_0$, respectively, by

$$\left\{\begin{array}{l} z = z_{\rm lc}(r; t_0) \\ t = t_{\rm lc}(r; t_0) \end{array}\right. \tag{5}$$

and

$$\left\{\begin{array}{l} z = z_{\rm lc}(r; t_0 + \delta t_0) =: z_{\rm lc}(r; t_0) + \delta z(r) \\ t = t_{\rm lc}(r; t_0 + \delta t_0) =: t_{\rm lc}(r; t_0) + \delta t(r) \end{array}\right. . \tag{6}$$

Here, by their definitions, we have $t_{\rm lc}(0; t_0) = t_0$, $z_{\rm lc}(0; t) = 0$, $\delta z(0) = 0$ and $\delta t(0) = \delta t_0$. Substituting Eq. (6) into Eqs. (3) and (4), and regarding $\delta z(r)$ and $\delta t(r)$ as infinitesimal quantities, we obtain

$$\frac{d}{dr}\delta z = \frac{\partial_t\partial_r R}{c\sqrt{1-kr^2}}\delta z + \frac{(1+z)\partial_t^2\partial_r R}{c\sqrt{1-kr^2}}\delta t, \quad \frac{d}{dr}\delta t = \frac{-\partial_t\partial_r R}{c\sqrt{1-kr^2}}\delta t, \tag{7}$$

where we have used the fact that (5) satisfies Eqs. (3) and (4), and the arguments of $\partial_t\partial_r R$ and $\partial_t^2\partial_r R$ are $t = t_{\rm lc}(r, t_0)$ and r.

Hereafter, we consider the case where the cosmological redshift z is monotonically increasing with r. We say that such a model is *z-normal.* Then, we replace the independent variable r by $z = z_{\rm lc}(r; t_0)$. By using $\frac{d}{dr} = \frac{dz}{dr}\frac{d}{dz} = \frac{(1+z)\partial_t\partial_r R}{c\sqrt{1-kr^2}}\frac{d}{dz}$, we have

$$\frac{d}{dz}\delta z = \frac{\delta z}{1+z} + \frac{\partial_t^2\partial_r R}{\partial_t\partial_r R}\delta t, \quad \frac{d}{dz}\delta t = -\frac{\delta t}{1+z}. \tag{8}$$

We can easily integrate the above equation to obtain $\delta t = \frac{\delta t_0}{1+z}$. By using the above result, we obtain

$$\frac{d}{dz}\left(\frac{\delta z}{1+z}\right) = \frac{1}{(1+z)^2}\frac{\partial_t^2\partial_r R}{\partial_t\partial_r R}\delta t_0. \tag{9}$$

4 The redshift drift in LTB void models

We call an LTB universe model the LTB *void* model, if the following three conditions are satisfied. 1 the mass density is non-negative; 2 the mass density is increasing with r increasing in the domain $r > 0$ on a spacelike hypersurface of constant t; 3 $\partial_r R$ is positive; 4 z-normality.

Proposition 1 In LTB void models, $\partial_t^2\partial_r R$ is negative.

Proof. By Einstein equations, we obtain

$$\partial_t^2\partial_r R(t,r) = -\frac{G\partial_r M}{R^2} + \frac{2GM\partial_r R}{R^3} = 4\pi G\frac{\partial_r R}{R^3}\left(-\rho R^3 + 2\int_0^r \rho(t,x)R^2(t,x)\partial_r R(t,x)dx\right). \tag{10}$$

Since $\partial_r R$ is positive by the definition of LTB void models, we may replace the integration variable x by $R = R(t,x)$ and obtain

$$\partial_t^2\partial_r R(t,r) = 4\pi G\frac{\partial_r R}{R^3}\left(-\rho R^3 + 2\int_0^{R(t,r)}\rho R^2 dR\right) = -4\pi G\frac{\partial_r R}{R^3}\int_0^{R(t,r)}\left(\frac{d\rho}{dR}R^3 + \rho R^2\right)dR. \tag{11}$$

Since $d\rho/dR = (\partial_r R)^{-1}\partial_r\rho$ is positive in the domain of $R > 0$, the integrand in the last equality of the above equation is positive. Q.E.D.

Theorem In LTB void models, the redshift drift of an off-center source observed at the symmetry center is negative.

Proof. Since the cosmological redshift z vanishes at $r = 0$, z is non-negative by the assumption of z-normality. Further, the z-normality leads to $\partial_t\partial_r R > 0$ through Eq. (3). Then, since $\delta t_0 > 0$, we see from Eq. (9) that Proposition 1 leads to the following inequality $\frac{d}{dz}\left(\frac{\delta z}{1+z}\right) < 0$. Since δz should vanish at $z = 0$, we have $\delta z < 0$ for $z > 0$ from the above inequality. Q.E.D.

5 Redshift drift in LTB universe models with a large hump

In the preceding section, we showed that the redshift drift observed at the symmetry center is negative for $r > 0$ in LTB void models. Conversely, if there is a domain in which the mass density is decreasing with increasing r, the redshift drift might be negative. In this section, we show that it is true with hump-type mass density distributions. We consider the following two LTB universe models, (i): $k(r) = 0$ and $t_{\rm B}(r) = f(r; a, r_1, r_2)$ with $a = -1.7H_0^{-1}$, $r_1 = 0.12cH_0^{-1}$ and $r_2 = 0.9cH_0^{-1}$, (ii): $t_{\rm B}(r) = 0$ and $k(r) = f(r; a, r_1, r_2)$ with $a = -100c^{-2}H_0^2$, $r_1 = 0.1cH_0^{-1}$ and $r_2 = 0.2cH_0^{-1}$, where $f(r; a, r_1, r_2) = 0$ for $r < r_1$, $f(r; a, r_1, r_2) = a\,(r - r_1)^3\left(r_1^2 - 5r_1r_2 + 10r_2^2 + 3r_1r - 15r_2r + 6r^2\right)/(r_2 - r_1)^5$ for $r_1 \le r < r_2$ and $f(r; a, r_1, r_2) = a$ for $r_2 \le r$. In Figs.1 and 2, we show the redshift drifts of these models. Although we do not show the energy densities of these models, a large hump in the mass density distribution exists in each model as well as in $t_{\rm B}(r)$ or $k(r)$. Although there is a redshift domain with positive redshift drift in each example, the distance-redshift relations of these models do not agree with the observational data, and further, the inhomogeneities need to be very large.

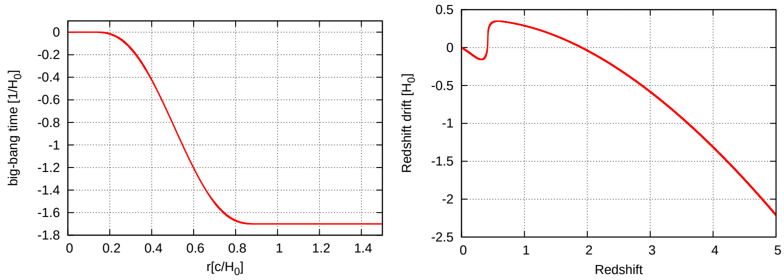

Figure 1: $t_{\rm B}(r)$(left panel) and $\delta z/\delta t_0$(right panel) of the model (i).

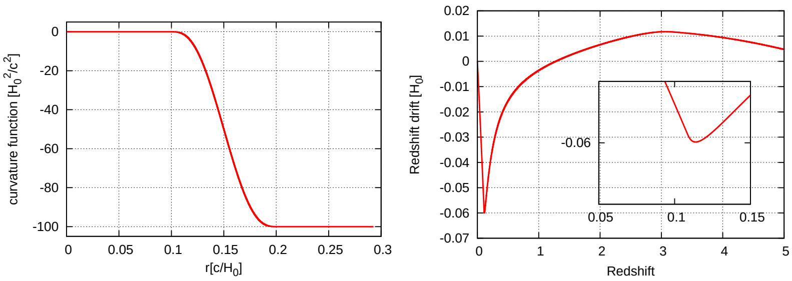

Figure 2: $k(r)$(left panel) and $\delta z/\delta t_0$(right panel) of the model (ii).

6 summary and discussion

In this paper, we studied the redshift drift in LTB universe models in which the observer is located at the symmetry center. We showed that, assuming that the mass density of the dust is positive, the redshift drift of an off-center source is negative if the mass density and the circumferential radius are increasing functions of the comoving radial coordinate. We also showed that if there is a very large hump structure around the symmetry center, the redshift drift can be positive. As a result, by observation of the redshift drift, we get a strong constraint on void-type universe models: if the redshift drift turns out to be positive in some redshift domain, LTB void models can be rejected.

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