



# On the Maxwell supergravity and flat limit in $2 + 1$ dimensions

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## ARTICLE INFO

### Article history:

Received 4 July 2018

Received in revised form 23 August 2018

Accepted 27 August 2018

Available online 30 August 2018

Editor: M. Cvetič

## ABSTRACT

The construction of the three-dimensional Chern–Simons supergravity theory invariant under the minimal Maxwell superalgebra is presented. We obtain a supergravity action without cosmological constant term characterized by three coupling constants. We also show that the Maxwell supergravity presented here appears as a vanishing cosmological constant limit of a minimal AdS–Lorentz supergravity. The flat limit is applied at the level of the superalgebra, Chern–Simons action, supersymmetry transformation laws and field equations.

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## 1. Introduction

The three-dimensional (super)gravity is considered as an interesting toy model for approaching richer (super)gravity models, which in general are non-trivial. Furthermore, the three-dimensional theory shares many interesting properties with higher-dimensional theories such as black hole solutions [1]. Remarkably, supergravity theory in three spacetime dimensions [2,3] can be expressed as a gauge theory using the Chern–Simons (CS) formalism for the AdS [4] or Poincaré supergroup [5]. In the last decades, diverse three-dimensional supergravity models have been studied in [6–24]. In particular, there has been a growing interest to extend AdS and Poincaré supergravity theories to other symmetries.

The Maxwell symmetry has become an interesting alternative for generalizing Einstein gravity. The Maxwell algebra has been introduced to describe a particle moving in a four-dimensional background in the presence of a constant electromagnetic field [25–27]. Further applications of the Maxwell group have been developed in [28–30]. The Maxwell symmetry and its generalizations have been useful to recover General Relativity (GR) as a particular limit using the CS and Born–Infeld (BI) gravity formalism [31–35]. In four dimensions, Maxwell gravity models have been successfully constructed in [36–38]. More recently, there has been a growing interest in studying the three-dimensional CS gravity theory invariant under the Maxwell group [39–43]. This theory, as the Poincaré one, is asymptotically flat and does not contain a cosmological constant term. Interestingly, a novel asymptotic structure appears for the

three-dimensional Maxwell CS gravity [43]. Such asymptotic symmetry corresponds to a deformed  $\mathfrak{bms}_3$  algebra and has been first introduced in [44] by using the semigroup expansion procedure [45]. Furthermore, it has been shown that the vacuum energy and angular momentum are influenced by the presence of the gravitational Maxwell field [43].

The Maxwell algebra also appears as an İnönü–Wigner (IW) contraction [46,47] of an enlarged algebra known as AdS–Lorentz algebra. The AdS–Lorentz algebra, also known as the semisimple extended Poincaré algebra, was introduced in [48,49] and has been studied in the context of gravity in diverse dimensions [40, 50,51]. Such symmetry can be written as the semidirect sum  $\mathfrak{so}(d-1, 2) \oplus \mathfrak{so}(d-1, 1)$  which motivates the name of AdS–Lorentz.

At the supersymmetric level, a minimal extension of the Maxwell algebra appears to describe the geometry of a four-dimensional superspace in presence of a constant abelian supersymmetric gauge field background [52]. Subsequently in [53,54], the Maxwell superalgebra has been used to construct a pure supergravity action in four dimensions using geometric methods. Further applications of the Maxwell superalgebra in the context of supergravity can be found in [55,56]. Such superalgebra has the particularity of having more than one spinor charge and has been obtained in [57,58] through algebraic expansion mechanisms [45, 59]. More recently, a CS gravity action for a generalized Maxwell superalgebra has been presented in [60,61] in three spacetime dimensions. Although the CS supergravity theory is appropriately constructed, it requires to consider a large amount of gauge fields whose origin is due to the methodology. To our knowledge, the construction of a CS supergravity action for the minimal Maxwell superalgebra in absence of extra fields has not been presented yet.

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A well-defined minimal Maxwell supergravity theory would allow to study more features of the three-dimensional Maxwell supergravity, like its asymptotic structure and non-relativistic limit.

In this paper, we present the minimal three-dimensional Maxwell CS supergravity theory without considering a large amount of extra fields. In particular, we present two alternative procedures to obtain it. We first derive the minimal Maxwell superalgebra and the invariant tensor applying the expansion method to a supersymmetric extension of the Lorentz algebra. Then, we present the corresponding CS action, supersymmetry transformations and field equations. In the second part, we recover the minimal Maxwell CS supergravity as a flat limit of a minimal AdS–Lorentz CS supergravity.

The present work is organized as follows: In Section 2, we give a brief review of Maxwell CS gravity theory in three spacetime dimensions. In Section 3, we present a minimal Maxwell CS supergravity theory in three dimensions. The Section 4 is devoted to the obtention of the Maxwell supergravity theory through a flat limit of a minimal AdS–Lorentz CS supergravity theory. In Section 5, we end our work with some comments about future possible developments.

## 2. Maxwell Chern–Simons gravity theory in 2 + 1 dimensions

In this section, we briefly review the three-dimensional CS gravity theory invariant under the Maxwell algebra [39–43]. The explicit commutators of the Maxwell algebra can be obtained as a deformation and enlargement of the Poincaré one. In particular, the Maxwell algebra is spanned by the set  $\{J_a, P_a, Z_a\}$  whose generators satisfy the following non-vanishing commutation relations:

$$\begin{aligned} [J_a, J_b] &= \epsilon_{abc} J^c, & [J_a, P_b] &= \epsilon_{abc} P^c, \\ [J_a, Z_b] &= \epsilon_{abc} Z^c, & [P_a, P_b] &= \epsilon_{abc} Z^c, \end{aligned} \quad (2.1)$$

where  $a, b, \dots = 0, 1, 2$  are raised and lowered with the Minkowski metric  $\eta_{ab}$  and  $\epsilon_{abc}$  is the Levi-Civita tensor. Note that, unlike the Poincaré algebra, the commutator of the translational generators  $P_a$  is no longer zero but proportional to the new abelian generator  $Z_a$ .

In order to construct a CS action

$$I_{CS} = \frac{k}{4\pi} \int_{\mathcal{M}} \left( AdA + \frac{2}{3} A^3 \right), \quad (2.2)$$

invariant under the Maxwell group, we require the Maxwell connection one-form  $A = A_\mu dx^\mu$  and the corresponding non-vanishing components of the invariant tensor.

The gauge connection one-form for the Maxwell algebra reads

$$A = \omega^a J_a + e^a P_a + \sigma^a Z_a, \quad (2.3)$$

where  $\omega^a$  corresponds to the spin connection,  $e^a$  is the vielbein and  $\sigma^a$  is the gravitational Maxwell gauge field [43]. On the other hand, the non-vanishing components of the invariant tensor for the Maxwell algebra are given by

$$\begin{aligned} \langle J_a J_b \rangle &= \alpha_0 \eta_{ab}, \\ \langle J_a P_b \rangle &= \alpha_1 \eta_{ab}, \\ \langle J_a Z_b \rangle &= \alpha_2 \eta_{ab}, \\ \langle P_a P_b \rangle &= \alpha_2 \eta_{ab}, \end{aligned} \quad (2.4)$$

where  $\alpha_0, \alpha_1$  and  $\alpha_2$  are real constants. In particular, the Maxwell algebra has the following quadratic Casimir [29,42],

$$C = \alpha_0 J^a J_a + \alpha_1 P^a J_a + \alpha_2 (P^a P_a + J^a Z_a). \quad (2.5)$$

Then considering the connection one-form (2.3) and the invariant tensor (2.4), the CS gravity action invariant under the Maxwell symmetry reads

$$\begin{aligned} I_{CS} &= \frac{k}{4\pi} \int_M \left[ \alpha_0 \left( \omega^a d\omega_a + \frac{1}{3} \epsilon_{abc} \omega^a \omega^b \omega^c \right) + 2\alpha_1 R_a e^a \right. \\ &\quad \left. + \alpha_2 (T^a e_a + 2R^a \sigma_a) \right], \end{aligned} \quad (2.6)$$

where the Lorentz curvature and torsion two-forms are given respectively by

$$\begin{aligned} R^a &= d\omega^a + \frac{1}{2} \epsilon^{abc} \omega_b \omega_c, \\ T^a &= de^a + \epsilon^{abc} \omega_b e_c. \end{aligned} \quad (2.7)$$

Here  $k = \frac{1}{4G}$  is the CS level of the theory and it is related to the gravitational constant  $G$ . Note that the term proportional to  $\alpha_0$  is the exotic Lagrangian also known as the Lorentz Lagrangian. The second term corresponds to the Einstein–Hilbert (EH) term while the term proportional to  $\alpha_2$  contains the explicit gravitational Maxwell field  $\sigma_a$ . Note that each term is invariant under the Maxwell symmetry. In particular, the local gauge transformations  $\delta A = d\Lambda + [A, \Lambda]$ , with gauge parameter  $\Lambda = \rho^a J_a + \varepsilon^a P_a + \gamma^a Z_a$ , are given by

$$\begin{aligned} \delta \omega^a &= D\rho^a, \\ \delta e^a &= D\varepsilon^a - \epsilon^{abc} \rho_b e_c, \\ \delta \sigma^a &= D\gamma^a + \epsilon^{abc} (e_b \varepsilon_b - \rho_b \sigma_c), \end{aligned} \quad (2.8)$$

where  $Du^a = du^a + \epsilon^{abc} \omega_b u_c$  is the Lorentz covariant derivative.

The equations of motion derived from the action (2.6) read

$$\begin{aligned} \delta \omega^a: & \quad \alpha_0 R_a + \alpha_1 T_a + \alpha_2 \left( D\sigma_a + \frac{1}{2} \epsilon_{abc} e^b e^c \right) = 0, \\ \delta e^a: & \quad \alpha_1 R_a + \alpha_2 T_a = 0, \\ \delta \sigma^a: & \quad \alpha_2 R_a = 0. \end{aligned} \quad (2.9)$$

In particular, when  $\alpha_2 \neq 0$ , the field equations are given by the vanishing of every curvature two-form

$$\begin{aligned} R^a &= 0, \\ T^a &= 0, \\ F^a &= D\sigma_a + \frac{1}{2} \epsilon_{abc} e^b e^c = 0. \end{aligned} \quad (2.10)$$

On the other hand, let us note that the EH dynamics is recovered in the limit  $\alpha_2 = 0$ . Such interesting feature is proper of the Maxwell like symmetry  $\mathfrak{B}_k$  [50,62] which corresponds to the Maxwell algebra for  $k = 4$ . As was shown in [32–35], GR can be recovered under a particular limit from a CS and BI gravity theory invariant under the Maxwell like groups.

## 3. Minimal Maxwell Chern–Simons supergravity in three-dimensional spacetime

In this section, following a similar procedure used in [60], we present a novel way of finding the so called minimal Maxwell superalgebra  $s\mathcal{M}$ . Moreover the construction of the CS action invariant under this superalgebra is presented. The minimal Maxwell superalgebra in  $D = 4$  dimensions was first introduced in [52].

Then, in  $D = 3$  dimensions, a generalized minimal Maxwell superalgebra was derived as a semigroup expansion ( $S$ -expansion) of the  $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$ , and it was used in order to study the corresponding CS theory [60]. Although the procedure was useful to derive a three-dimensional CS formalism, the generalized minimal Maxwell superalgebra obtained there contains a large set of bosonic generators  $\{J_{ab}, P_a, Z_{ab}, \tilde{Z}_{ab}, \tilde{Z}_a\}$ . In particular the bosonic content  $\{\tilde{Z}_{ab}, \tilde{Z}_a\}$  implies a large number of extra terms in the CS action whose presence is due to the considered starting superalgebra.

Here we show that using a smaller structure as the original algebra, the minimal Maxwell superalgebra can be obtained through the  $S$ -expansion [45]. As we shall see, the extra bosonic generators do not appear if a super-Lorentz algebra is considered as the starting superalgebra. In addition, such method allows not only to recover the complete set of (anti-)commutation relations of the minimal Maxwell superalgebra but also the invariant tensor which is required for the construction of a CS action.

The most natural minimal supersymmetric extension of the Lorentz algebra in three spacetime dimensions is spanned by Lorentz generators  $M_a$  and Majorana fermionic generators  $Q_\alpha$  ( $\alpha = 1, 2$ ) [63]. The (anti-)commutation relations read

$$\begin{aligned} [M_a, M_b] &= \epsilon_{abc} M^c, \\ [M_a, Q_\alpha] &= \frac{1}{2} (\Gamma_a)^\beta_\alpha Q_\beta, \\ \{Q_\alpha, Q_\beta\} &= -\frac{1}{2} (C\Gamma^a)_{\alpha\beta} M_a, \end{aligned} \quad (3.1)$$

where  $a, b, \dots = 0, 1, 2$  are the Lorentz indices,  $\Gamma_a$  are the Dirac matrices in three dimensions, and  $C$  represents the charge conjugation matrix,

$$C_{\alpha\beta} = C^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (3.2)$$

One can easily check the consistency of the (anti-)commutation relations of the super-Lorentz algebra  $s\mathcal{L}$  (3.1) by checking the Jacobi identities, and by considering that  $C^T = -C$  and  $C\Gamma^a = (C\Gamma^a)^T$ .

Let  $S_E^{(4)} = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$  be the abelian semigroup whose elements satisfy the multiplication law

$$\lambda_\alpha \lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{when } \alpha + \beta \leq 5, \\ \lambda_5, & \text{when } \alpha + \beta > 5, \end{cases} \quad (3.3)$$

where  $\lambda_5 = 0_s$  is the zero element of the semigroup.

Following the definitions of [45], after extracting a resonant subalgebra of  $S_E^{(4)} \times s\mathcal{L}$  and applying its  $0_s$ -reduction, one finds a new superalgebra whose generators  $\{J_a, P_a, Z_a, Q_\alpha, \Sigma_\alpha\}$  are related to the super-Lorentz ones as

$$\begin{aligned} J_a &= \lambda_0 M_a, & \ell P_a &= \lambda_2 M_a, & \ell^2 Z_a &= \lambda_4 M_a, \\ \ell^{1/2} Q_\alpha &= \lambda_1 Q_\alpha, & \ell^{3/2} \Sigma_\alpha &= \lambda_3 Q_\alpha. \end{aligned} \quad (3.4)$$

Using the multiplication law of the semigroup and the super-Lorentz (anti-)commutators, one can see that the expanded generators satisfy the following non-vanishing (anti-)commutation relations

$$\begin{aligned} [J_a, J_b] &= \epsilon_{abc} J^c, & [J_a, P_b] &= \epsilon_{abc} P^c, \\ [J_a, Z_b] &= \epsilon_{abc} Z^c, & [P_a, P_b] &= \epsilon_{abc} Z^c, \\ [J_a, Q_\alpha] &= \frac{1}{2} (\Gamma_a)^\beta_\alpha Q_\beta, \\ [J_a, \Sigma_\alpha] &= \frac{1}{2} (\Gamma_a)^\beta_\alpha \Sigma_\beta, \end{aligned} \quad (3.5)$$

$$\begin{aligned} [P_a, Q_\alpha] &= \frac{1}{2} (\Gamma_a)^\beta_\alpha \Sigma_\beta, \\ \{Q_\alpha, Q_\beta\} &= -\frac{1}{2} (C\Gamma^a)_{\alpha\beta} P_a, \\ \{Q_\alpha, \Sigma_\beta\} &= -\frac{1}{2} (C\Gamma^a)_{\alpha\beta} Z_a. \end{aligned}$$

The (anti-)commutation relations (3.5) correspond to the minimal Maxwell superalgebra  $s\mathcal{M}$ , introduced in [64] in three spacetime dimensions. This supersymmetric extension of the Maxwell symmetry is characterized by the introduction of a new Majorana spinor charge  $\Sigma$  which is required to satisfy the Jacobi identity  $(P, Q, Q)$ . The presence of a second abelian spinorial generators is not new in the literature and has already been studied in the context of  $D = 11$  supergravity [65] and superstring theory [66].

The non-vanishing components of the invariant tensor of the Maxwell superalgebra can be obtained in terms of the super-Lorentz ones using the definitions of the  $S$ -expansion method,

$$\begin{aligned} \langle J_a J_b \rangle &= \mu_0 \langle M_a M_b \rangle_{s\mathcal{L}} = \mu_0 \eta_{ab}, \\ \langle J_a P_b \rangle &= \frac{\mu_2}{\ell} \langle M_a M_b \rangle_{s\mathcal{L}} = \frac{\mu_2}{\ell} \eta_{ab}, \\ \langle J_a Z_b \rangle &= \frac{\mu_4}{\ell^2} \langle M_a M_b \rangle_{s\mathcal{L}} = \frac{\mu_4}{\ell^2} \eta_{ab}, \\ \langle P_a P_b \rangle &= \frac{\mu_4}{\ell^2} \langle M_a M_b \rangle_{s\mathcal{L}} = \frac{\mu_4}{\ell^2} \eta_{ab}, \\ \langle Q_\alpha Q_\beta \rangle &= \frac{\mu_2}{\ell} \langle Q_\alpha Q_\beta \rangle_{s\mathcal{L}} = \frac{\mu_2}{\ell} C_{\alpha\beta}, \\ \langle Q_\alpha \Sigma_\beta \rangle &= \frac{\mu_4}{\ell^2} \langle Q_\alpha Q_\beta \rangle_{s\mathcal{L}} = \frac{\mu_4}{\ell^2} C_{\alpha\beta}, \end{aligned} \quad (3.6)$$

where  $\mu_0, \mu_2$  and  $\mu_4$  are arbitrary constants. For convenience, we will redefine the  $\mu$ 's as follows

$$\mu_0 \rightarrow \alpha_0, \quad \mu_2 \rightarrow \ell \alpha_1, \quad \mu_4 \rightarrow \ell^2 \alpha_2,$$

and the invariant tensor takes the form

$$\begin{aligned} \langle J_a J_b \rangle &= \alpha_0 \eta_{ab}, & \langle P_a P_b \rangle &= \alpha_2 \eta_{ab}, \\ \langle J_a P_b \rangle &= \alpha_1 \eta_{ab}, & \langle Q_\alpha Q_\beta \rangle &= \alpha_1 C_{\alpha\beta}, \\ \langle J_a Z_b \rangle &= \alpha_2 \eta_{ab}, & \langle Q_\alpha \Sigma_\beta \rangle &= \alpha_2 C_{\alpha\beta}. \end{aligned} \quad (3.7)$$

The connection one-form reads

$$A = \omega^a J_a + e^a P_a + \sigma^a Z_a + \bar{\psi} Q + \bar{\xi} \Sigma, \quad (3.8)$$

where  $\omega^a$  is the spin connection one-form,  $e^a$  corresponds to the vielbein one-form,  $\sigma^a$  is the Maxwell gravity field one-form while  $\psi$  and  $\xi$  are fermionic fields.

The corresponding curvature two-form is given by

$$F = R^a J_a + \mathcal{T}^a P_a + \mathcal{F}^a Z_a + \nabla \bar{\psi} Q + \nabla \bar{\xi} \Sigma, \quad (3.9)$$

with

$$\begin{aligned} R^a &= d\omega^a + \frac{1}{2} \epsilon^{abc} \omega_b \omega_c, \\ \mathcal{T}^a &= de^a + \epsilon^{abc} \omega_b e_c + \frac{1}{4} \bar{\psi} \Gamma^a \psi, \\ \mathcal{F}^a &= d\sigma^a + \epsilon^{abc} \omega_b \sigma_c + \frac{1}{2} \epsilon^{abc} e_b e_c + \frac{1}{2} \bar{\psi} \Gamma^a \xi. \end{aligned} \quad (3.10)$$

Here, the covariant derivative  $\nabla = d + [A, \cdot]$  acting on spinors read

$$\begin{aligned} \nabla \psi &= d\psi + \frac{1}{2} \omega^a \Gamma_a \psi, \\ \nabla \xi &= d\xi + \frac{1}{2} \omega^a \Gamma_a \xi + \frac{1}{2} e^a \Gamma_a \psi. \end{aligned} \quad (3.11)$$

The CS supergravity action can be written considering the non-vanishing component of the invariant tensor (3.7) and the gauge connection one-form (3.8),

$$I_{s\mathcal{M}} = \frac{k}{4\pi} \int \alpha_0 \left( \omega^a d\omega_a + \frac{1}{3} \epsilon_{abc} \omega^a \omega^b \omega^c \right) + \alpha_1 (2e^a R_a - \bar{\psi} \nabla \psi) + \alpha_2 (2R^a \sigma_a + e^a T_a - \bar{\psi} \nabla \xi - \bar{\xi} \nabla \psi), \quad (3.12)$$

where  $T^a = De^a$  is the usual torsion two-form.

Note that the term proportional to  $\alpha_0$  contains just the exotic Lagrangian. The piece along  $\alpha_1$  contains the EH and the Rarita–Schwinger terms while the term proportional to  $\alpha_2$  is of particular interest since it contains the Maxwell gravitational field  $\sigma^a$  plus a torsional term. Let us note that the fermionic terms contribute only to the  $\alpha_1$  and  $\alpha_2$  sectors of the bosonic action (2.6). In addition, the CS action (3.12) reproduces the pure three-dimensional supergravity action when the exotic CS terms are neglected ( $\alpha_0 = \alpha_2 = 0$ ). Such feature also appears on four-dimensional supergravity action based on the Maxwell supergroup [54] using the MacDowell–Mansouri formalism [67].

By construction, the CS action (3.12) is invariant under the gauge transformation  $\delta A = d\Lambda + [A, \Lambda]$ . In particular, the action is invariant under the following local supersymmetry transformation laws

$$\begin{aligned} \delta \omega^a &= 0, \\ \delta e^a &= \frac{1}{2} \bar{\epsilon} \Gamma^a \psi, \\ \delta \sigma^a &= \frac{1}{2} \bar{\epsilon} \Gamma^a \xi + \frac{1}{2} \bar{\varrho} \Gamma^a \psi, \\ \delta \psi &= d\epsilon + \frac{1}{2} \omega^a \Gamma_a \epsilon, \\ \delta \xi &= d\varrho + \frac{1}{2} \omega^a \Gamma_a \varrho + \frac{1}{2} e^a \Gamma_a \epsilon, \end{aligned} \quad (3.13)$$

where the gauge parameter is  $\Lambda = \rho^a J_a + \varepsilon^a P_a + \gamma^a Z_a + \bar{\epsilon} Q + \bar{\varrho} \Sigma$ .

The equations of motion derived from (2.6) are

$$\begin{aligned} \delta \omega^a: \quad & \alpha_0 R_a + \alpha_1 \mathcal{T}_a + \alpha_2 \mathcal{F}_a = 0, \\ \delta e^a: \quad & \alpha_1 R_a + \alpha_2 \mathcal{T}_a = 0, \\ \delta \sigma^a: \quad & \alpha_2 R_a = 0, \\ \delta \bar{\psi}: \quad & \alpha_1 \nabla \psi + \alpha_2 \nabla \xi = 0, \\ \delta \bar{\xi}: \quad & \alpha_2 \nabla \psi = 0. \end{aligned} \quad (3.14)$$

As in the bosonic case, the field equations reduce to the vanishing of the curvature two-forms provided  $\alpha_2 \neq 0$ ,

$$R^a = 0, \quad \mathcal{T}^a = 0, \quad \mathcal{F}^a = 0, \quad \nabla \psi = 0, \quad \nabla \xi = 0. \quad (3.15)$$

As was recently shown in the Maxwell gravity case [43],  $\sigma^a$  is not simply an extra field but modifies the asymptotic charges of the solutions. It would be interesting to extend the results obtained in [43] to the Maxwell supergravity theory presented here. As in the bosonic theory, one could expect a deformation of the super  $\mathfrak{bms}_3$  algebra [68,69] as the corresponding asymptotic symmetry. In particular, as in the finite Maxwell superalgebra, two infinite-dimensional fermionic generators should appear in the asymptotic structure.

As an ending remark, the CS theory presented here offers us an alternative three-dimensional minimal supergravity with vanishing

cosmological constant. Then, analogously to the Poincaré CS supergravity which appears as a flat limit of the AdS one, it is natural to expect that the Maxwell CS supergravity obtained here can also be found as a flat limit of a particular supergravity theory. The next section is devoted to the obtention of the Maxwell supergravity theory as a flat limit of an AdS–Lorentz supergravity.

#### 4. Maxwell supergravity theory as a flat limit of the AdS–Lorentz supergravity

It has been shown that the Maxwell symmetry can be alternatively obtained as an IW contraction of an enlarged symmetry denoted as the AdS–Lorentz algebra [50]. Such algebra and its generalizations have been useful to relate diverse (pure) Lovelock gravity theories [70–72]. At the supersymmetric level, numerous studies have been done in four and three dimensions leading to interesting AdS–Lorentz supergravities [73–76]. Recently, it was shown in three dimensions that a generalized Maxwell supergravity can be obtained as an IW contraction of a generalized AdS–Lorentz supergravity model [61]. Nevertheless, as was discussed in the previous section, the field content of such theories is large and the physical interpretation of the extra fields remains unknown. In particular, the extra fields related to the set of generators  $\{\tilde{Z}_{ab}, \tilde{Z}_a\}$  appear as a consequence of the procedure used to construct the supergravity theories.

In this section we show that the minimal Maxwell supergravity theory presented previously can be recovered as a flat limit of a minimal AdS–Lorentz supergravity. To this purpose, let us first consider a novel supersymmetric extension of the AdS–Lorentz algebra in three dimensions. We extend the AdS–Lorentz algebra generated by  $\{J_a, P_a, Z_a\}$  with the fermionic generators  $\{Q_\alpha, \Sigma_\alpha\}$ , and we get the following superalgebra

$$\begin{aligned} [J_a, J_b] &= \epsilon_{abc} J^c, & [J_a, P_b] &= \epsilon_{abc} P^c, \\ [P_a, P_b] &= \epsilon_{abc} Z^c, & [J_a, Z_b] &= \epsilon_{abc} Z^c, \\ [Z_a, Z_b] &= \frac{1}{\ell^2} \epsilon_{abc} Z^c, & [P_a, Z_b] &= \frac{1}{\ell^2} \epsilon_{abc} P^c, \\ [J_a, Q_\alpha] &= \frac{1}{2} (\Gamma_a)^\beta_\alpha Q_\beta, & [J_a, \Sigma_\alpha] &= \frac{1}{2} (\Gamma_a)^\beta_\alpha \Sigma_\beta, \\ [P_a, Q_\alpha] &= \frac{1}{2} (\Gamma_a)^\beta_\alpha \Sigma_\beta, & [P_a, \Sigma_\alpha] &= \frac{1}{2\ell^2} (\Gamma_a)^\beta_\alpha Q_\beta, \\ [Z_a, Q_\alpha] &= \frac{1}{2\ell^2} (\Gamma_a)^\beta_\alpha Q_\beta, & [Z_a, \Sigma_\alpha] &= \frac{1}{2\ell^2} (\Gamma_a)^\beta_\alpha \Sigma_\beta, \\ \{Q_\alpha, Q_\beta\} &= -\frac{1}{2} (C\Gamma^a)_{\alpha\beta} P_a, & \{Q_\alpha, \Sigma_\beta\} &= -\frac{1}{2} (C\Gamma^a)_{\alpha\beta} Z_a, \\ \{\Sigma_\alpha, \Sigma_\beta\} &= -\frac{1}{2\ell^2} (C\Gamma^a)_{\alpha\beta} P_a. \end{aligned} \quad (4.1)$$

We will refer to this superalgebra as the minimal AdS–Lorentz superalgebra in three dimensions. Note that the supersymmetric extension of the AdS–Lorentz algebra is not unique. However, to our knowledge, this corresponds to the minimal supersymmetric extension of the AdS–Lorentz algebra containing two spinor charges.

One can see that the limit  $\ell \rightarrow \infty$  leads appropriately to the minimal Maxwell superalgebra (3.5) considered in the previous section. As we shall see, the flat limit can also be directly performed at the level of the action, supersymmetry transformations and field equations.

In order to write down a CS action for the minimal AdS–Lorentz superalgebra, let us consider the connection one-form

$$A = \omega^a J_a + e^a P_a + \sigma^a Z_a + \bar{\psi} Q + \bar{\xi} \Sigma, \quad (4.3)$$

and the corresponding curvature two-form

$$F = R^a J_a + \mathcal{T}^a P_a + \mathcal{F}^a Z_a + \nabla \bar{\psi} Q + \nabla \bar{\xi} \Sigma, \quad (4.4)$$

where

$$\begin{aligned} R^a &= d\omega^a + \frac{1}{2} \epsilon^{abc} \omega_b \omega_c, \\ \mathcal{T}^a &= de^a + \epsilon^{abc} \omega_b e_c + \frac{1}{\ell^2} \epsilon^{abc} \sigma_b e_c + \frac{1}{4} \bar{\psi} \Gamma^a \psi + \frac{1}{4\ell^2} \bar{\xi} \Gamma^a \xi, \\ \mathcal{F}^a &= d\sigma^a + \epsilon^{abc} \omega_b \sigma_c + \frac{1}{2\ell^2} \epsilon^{abc} \sigma_b \sigma_c + \frac{1}{2} \epsilon^{abc} e_b e_c + \frac{1}{2} \bar{\psi} \Gamma^a \xi. \end{aligned} \quad (4.5)$$

Furthermore,

$$\begin{aligned} \nabla \psi &= d\psi + \frac{1}{2} \omega^a \Gamma_a \psi + \frac{1}{2\ell^2} \sigma^a \Gamma_a \psi + \frac{1}{2\ell^2} e^a \Gamma_a \xi, \\ \nabla \xi &= d\xi + \frac{1}{2} \omega^a \Gamma_a \xi + \frac{1}{2\ell^2} \sigma^a \Gamma_a \xi + \frac{1}{2} e^a \Gamma_a \psi. \end{aligned} \quad (4.6)$$

Note that the curvatures reduce to the Maxwell ones (3.10) and (3.11) by performing the limit  $\ell \rightarrow \infty$ .

The non-vanishing components of the invariant tensor are given by (3.7) along with

$$\begin{aligned} \langle P_a Z_b \rangle &= \frac{\alpha_1}{\ell^2} \eta_{ab}, \\ \langle Z_a Z_b \rangle &= \frac{\alpha_2}{\ell^2} \eta_{ab}, \\ \langle \Sigma_\alpha \Sigma_\beta \rangle &= \frac{\alpha_1}{\ell^2} C_{\alpha\beta}. \end{aligned} \quad (4.7)$$

Thus, the most general quadratic Casimir of the minimal AdS–Lorentz superalgebra is

$$\begin{aligned} C &= \alpha_0 J^a J_a + \alpha_1 \left( P^a J_a + \frac{1}{\ell^2} P^a Z_a + \bar{Q} Q + \frac{1}{\ell^2} \bar{\Sigma} \Sigma \right) \\ &\quad + \alpha_2 \left( P^a P_a + J^a Z_a + \frac{1}{\ell^2} Z^a Z_a + \bar{Q} \Sigma \right). \end{aligned} \quad (4.8)$$

Let us note that the flat limit  $\ell \rightarrow \infty$  at the level of the invariant tensor of the AdS–Lorentz superalgebra leads us to the non-degenerate invariant tensor of the minimal Maxwell superalgebra (3.7). Then, using the connection one-form (4.3) and the invariant tensors (3.7) and (4.7) in the CS action (2.2), we get

$$\begin{aligned} I_{sAdS-\mathcal{L}} &= \frac{k}{4\pi} \int \alpha_0 \left( \omega^a d\omega_a + \frac{1}{3} \epsilon_{abc} \omega^a \omega^b \omega^c \right) \\ &\quad + \alpha_1 \left( 2e^a R_a + \frac{2}{\ell^2} e^a F_a + \frac{1}{3\ell^2} \epsilon_{abc} e^a e^b e^c \right. \\ &\quad \left. - \bar{\psi} \nabla \psi - \frac{1}{\ell^2} \bar{\xi} \nabla \xi \right) + \alpha_2 \left( 2R^a \sigma_a + \frac{2}{\ell^2} F^a \sigma_a + e^a T_a \right. \\ &\quad \left. + \frac{1}{\ell^2} \epsilon_{abc} e^a \sigma^b e^c - \bar{\psi} \nabla \xi - \bar{\xi} \nabla \psi \right), \end{aligned} \quad (4.9)$$

where  $T^a = de^a + \epsilon^{abc} \omega_b e_c$  is the torsion two-form and

$$F^a = d\sigma^a + \epsilon^{abc} \omega_b \sigma_c + \frac{1}{2\ell^2} \epsilon^{abc} \sigma_b \sigma_c. \quad (4.10)$$

Although the field content of the AdS–Lorentz supergravity theory is the same as in the super Maxwell one, the CS action is much richer. As in the four-dimensional case [74], this symmetry allows the presence of a cosmological constant term which was

absent in the super Maxwell theory. This is due to the presence of the component  $\langle P_a Z_b \rangle$  of the invariant tensor which vanishes in the Maxwell case. Interestingly, from the minimal AdS–Lorentz supergravity action (4.9) we see that the vanishing cosmological constant limit  $\ell \rightarrow \infty$  leads us to the minimal Maxwell supergravity action (3.12) introduced in the previous section. This is very similar to the flat behavior present in AdS supergravity and in the bosonic AdS–Lorentz gravity theory. For completeness we apply the flat limit at the level of the supersymmetry transformations and equations of motion.

The CS action (4.9) is invariant by construction under the gauge transformation  $\delta A = d\Lambda + [A, \Lambda]$ . In particular, the supersymmetry transformation laws read

$$\begin{aligned} \delta \omega^a &= 0, \\ \delta e^a &= \frac{1}{2} \bar{\epsilon} \Gamma^a \psi + \frac{1}{2\ell^2} \bar{\varrho} \Gamma^a \xi, \\ \delta \sigma^a &= \frac{1}{2} \bar{\epsilon} \Gamma^a \xi + \frac{1}{2} \bar{\varrho} \Gamma^a \psi, \\ \delta \psi &= d\epsilon + \frac{1}{2} \omega^a \Gamma_a \epsilon + \frac{1}{2\ell^2} \sigma^a \Gamma_a \epsilon + \frac{1}{2\ell^2} e^a \Gamma_a \varrho, \\ \delta \xi &= d\varrho + \frac{1}{2} \omega^a \Gamma_a \varrho + \frac{1}{2} e^a \Gamma_a \epsilon + \frac{1}{2\ell^2} \sigma^a \Gamma_a \varrho, \end{aligned} \quad (4.11)$$

with gauge parameter  $\Lambda = \rho^a J_a + \varepsilon^a P_a + \gamma^a Z_a + \bar{\epsilon} Q + \bar{\varrho} \Sigma$ . Note that the supersymmetry transformations (4.11) reduce to the Maxwell ones of eq. (3.13) in the limit  $\ell \rightarrow \infty$ .

The field equations are given by

$$\begin{aligned} \delta \omega^a: \quad & \alpha_0 R_a + \alpha_1 \mathcal{T}_a + \alpha_2 \mathcal{F}_a = 0, \\ \delta e^a: \quad & \alpha_1 \left( R_a + \frac{1}{\ell^2} \mathcal{F}_a \right) + \alpha_2 \mathcal{T}_a = 0, \\ \delta \sigma^a: \quad & \frac{\alpha_1}{\ell^2} \mathcal{T}_a + \alpha_2 \left( R_a + \frac{1}{\ell^2} \mathcal{F}_a \right) = 0, \\ \delta \bar{\psi}: \quad & \alpha_1 \nabla \psi + \alpha_2 \nabla \xi = 0, \\ \delta \bar{\xi}: \quad & \alpha_2 \nabla \psi + \frac{\alpha_1}{\ell^2} \nabla \xi = 0. \end{aligned} \quad (4.12)$$

When  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$  are independent, the equations of motion reduce to the vanishing of the curvature two-forms,

$$\begin{aligned} R^a &= 0, \quad \mathcal{T}^a = 0, \quad \mathcal{F}^a = 0, \\ \nabla \psi &= 0, \quad \nabla \xi = 0. \end{aligned} \quad (4.13)$$

It is simple to verify that the vanishing cosmological constant limit  $\ell \rightarrow \infty$  in (4.12) leads to the super Maxwell field equations (3.14).

It is important to clarify that an alternative supersymmetric extension of the AdS–Lorentz algebra can be obtained with one spinor generator  $\tilde{Q}_\alpha$  such that

$$\begin{aligned} [\tilde{J}_a, \tilde{J}_b] &= \epsilon_{abc} \tilde{J}^c, & [\tilde{J}_a, \tilde{P}_b] &= \epsilon_{abc} \tilde{P}^c, \\ [\tilde{P}_a, \tilde{P}_b] &= \epsilon_{abc} \tilde{Z}^c, & [\tilde{J}_a, \tilde{Z}_b] &= \epsilon_{abc} \tilde{Z}^c, \\ [\tilde{Z}_a, \tilde{Z}_b] &= \frac{1}{\ell^2} \epsilon_{abc} \tilde{Z}^c, & [\tilde{P}_a, \tilde{Z}_b] &= \frac{1}{\ell^2} \epsilon_{abc} \tilde{P}^c, \\ [\tilde{J}_a, \tilde{Q}_\alpha] &= \frac{1}{2} (\Gamma_a)^\beta_\alpha \tilde{Q}_\beta, \\ [\tilde{P}_a, \tilde{Q}_\alpha] &= \frac{1}{2} (\Gamma_a)^\beta_\alpha \tilde{Q}_\beta, \\ [\tilde{Z}_a, \tilde{Q}_\alpha] &= \frac{1}{2\ell^2} (\Gamma_a)^\beta_\alpha \tilde{Q}_\beta, \\ \{\tilde{Q}_\alpha, \tilde{Q}_\beta\} &= -\frac{1}{2} (C\Gamma^a)_{\alpha\beta} \tilde{Z}_a - \frac{1}{2} (C\Gamma^a)_{\alpha\beta} \tilde{P}_a. \end{aligned} \quad (4.14)$$

This superalgebra can be seen as a supersymmetric extension of a semisimple generalization of the Poincaré algebra [48]. Although this superalgebra contains the bosonic AdS–Lorentz subalgebra, its supersymmetrization is quite different from the minimal one presented previously (see eqs. (4.1)–(4.2)). One can see that there is no redefinition of the generators allowing to relate them. Moreover, a reinterpretation of the  $P_a$  and  $Z_a$  generators would modify drastically the bosonic algebra. More recently, a three-dimensional CS supergravity action invariant under this super AdS–Lorentz algebra has been presented in [73] allowing to introduce a generalized cosmological term to the EH term.

In particular, a non-standard Maxwell superalgebra [29,77] can be obtained applying the IW contraction to (4.14). Although a supergravity CS action can be constructed from this alternative super AdS–Lorentz version, the IW contraction does not reproduce a supergravity theory. Indeed, in the non-standard Maxwell superalgebra we have

$$\{\tilde{Q}_\alpha, \tilde{Q}_\beta\} = -\frac{1}{2} (C\Gamma^a)_{\alpha\beta} \tilde{Z}_a. \quad (4.15)$$

Here, the four-momentum generators  $\tilde{P}_a$  are no more expressed as bilinear expressions of the fermionic generators  $\tilde{Q}$  which leads to an exotic supersymmetric action.

## 5. Discussion

The minimal supersymmetric extension of the three-dimensional Maxwell Chern–Simons gravity has been presented by expanding a Lorentz superalgebra. The Maxwell superalgebra obtained is the minimal one presented in [52] containing two spinor generators  $Q_\alpha$  and  $\Sigma_\alpha$ . The methodology considered here allows us to construct the CS supergravity action invariant under the minimal superMaxwell which is characterized by three coupling constants. In particular, the gravitational Maxwell field  $\sigma_a$  appears in the  $\alpha_2$  sector. In addition, the supergravity theory presented here does not contain additional bosonic fields as those appearing in the generalized Maxwell superalgebra [60,61].

Interestingly, the present Maxwell supergravity theory has also been obtained by applying a vanishing cosmological constant limit to a minimal AdS–Lorentz supergravity theory. The flat limit was presented not only in the commutation relations but also in the action, field equations and supersymmetry transformation laws. This flat behavior appears also at the bosonic level, including its infinite-dimensional symmetries [43,44] and higher-spin extension of the Maxwell and AdS–Lorentz gravity [41].

Having a well defined Maxwell supergravity theory in three spacetime dimensions, it would be interesting to go further and analyze the influence of the gravitational Maxwell field in the asymptotic symmetry of this supergravity theory. Analogously to the bosonic case, one could expect a deformation and enlargement of the super  $\mathfrak{bms}_3$  algebra [work in progress]. One could go even further and study the asymptotic symmetry of the AdS–Lorentz supergravity in three dimensions and analyze the existence of a flat limit.

Another important aspect that deserves further investigation is the study of the non-relativistic (NR) limit of the minimal Maxwell CS supergravity presented here. It has been pointed out that the NR gravities could play an important role to the understanding of non-relativistic coupled systems in the boundary. Recently, the NR limit of a three-dimensional Maxwell CS gravity has been considered in [42] where a generalization of the Extended Bargmann gravity has been obtained. Therefore, it would be interesting to extend the approach of [42] to our supergravity case.

## Acknowledgements

This work was supported by the Chilean FONDECYT Projects No. 3170437 (P.C.) and No. 3170438 (E.R.). D.M.P. acknowledges DIVERIA for financial support through Proyecto Postdoctorado 2018 VRIEAPUCV. The authors would like to thank to L. Ravera for valuable discussions and comments.

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