



Quantum corrections to thermodynamics of quasitopological black holes



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ARTICLE INFO

Article history:

Received 12 October 2017

Accepted 23 October 2017

Available online 28 October 2017

Editor: N. Lambert

ABSTRACT

Based on the modification to area-law due to thermal fluctuation at small horizon radius, we investigate the thermodynamics of charged quasitopological and charged rotating quasitopological black holes. In particular, we derive the leading-order corrections to the Gibbs free energy, charge and total mass densities. In order to analyze the behavior of the thermal fluctuations on the thermodynamics of small black holes, we draw a comparative analysis between the first-order corrected and original thermodynamical quantities. We also examine the stability and bound points of such black holes under effect of leading-order corrections.

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1. Overview and motivation

According to AdS/CFT duality, the Einstein general relativity in the bulk space–time corresponds to a gauge theory living on the boundary with a large N (number of colors) and large 't Hooft coupling [1]. Since the coupling constants in the gravity side relate to central charges in the gauge theory, therefore Einstein gravity has limited number of dual CFTs, in particular only those CFTs which have equal central charges, as Einstein gravity does not have enough free parameters. The presence of various higher-order derivatives in AdS gravity corresponds to new couplings among operators in the dual CFT. One well-known example of higher derivatives gravity theories is Gauss–Bonnet gravity. The Gauss–Bonnet gravity involves only one quadratic coupling term and therefore the corresponding range of dual theory is still limited. In order to improve this limitation of holographic studies to the classes of CFTs, one has to introduce the new higher order curvature terms, at least curvature-cubed terms, into gravity. One may achieve such curvature-cubed interactions by adding the cubic term in Lovelock gravity [2], but can not be very helpful as such term is topological in nature and becomes significant only in very high dimensions.

Recently, a new toy model for gravitation action has been proposed which contains not only the Gauss–Bonnet term but also a curvature-cubed interaction [3,4]. This is a quasitopological gravity model as the cubic terms do not have a topological origin like Lovelock gravity but contribute dynamically to the evolution

of fields in the bulk. This quasitopological gravity theory is endowed with two important properties. First, the equations of motion are generically of fourth order in derivatives of the metric which reduces to second order for spherically symmetric space-times, and second there exist the exact black hole solutions [4]. The holographic discussions for these black hole solutions with some recipes of AdS/CFT duality have been given in [5]. The basic thermodynamics of quasitopological Reissner–Nordström black holes are studied in Ref. [6]. Recently, the surface term of quasitopological gravity for space–time with flat boundary is introduced and the thermodynamic properties of these solutions have been investigated by using the relation between on-shell action and Gibbs free energy [7].

An important discovery that black holes behave as thermodynamic objects had affected our understanding of gravity theory and its relationship to quantum field theory considerably. Bekenstein and Hawking were first who proposed that black holes radiate as black bodies with characteristic entropy related to the area of the horizon [8]. In present scenario, it is more or less certain that black holes much larger than the Planck scale have entropy proportional to its horizon area [8–12]. So, this poses an interesting question that what could be the leading-order corrections when one reduces the size of the black holes. To answer this question, several attempts have been made. For instance, using a corrected version of the asymptotic Cardy formula for BTZ, string theoretic and all other black holes, whose microscopic degrees of freedom are described by an underlying CFT [13], the leading-order corrections have found logarithmic in nature. In fact, the consideration of matter fields in black hole backgrounds also yields logarithmic

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correction to the black holes entropy at the leading order [14]. The leading-order correction to black holes entropy is also logarithmic by considering string-black hole correspondence [15,16] and using Rademacher expansion of the partition function [17]. Furthermore, Das et al. in Ref. [19] showed that the leading-order corrections to the entropy of any thermodynamic system due to small statistical fluctuations around equilibrium are always logarithmic.

The study of leading-order correction to the black holes thermodynamics is a subject of current interests. In this direction, recently, the effects of quantum corrections on thermodynamics and stability of Gödel black hole [20], Schwarzschild–Beltrami–de Sitter black hole [21] and massive black hole in AdS space [22] have been studied. The corrected thermodynamics of a dilatonic black hole has also been discussed [23] which meets the same universal form of correction term. In another work, the corrected thermodynamics of a black hole is also studied from the partition function points of view [24]. The quantum gravity effects on the Hořava–Lifshitz black hole thermodynamics are analyzed and their stability is also discussed [25]. Similar investigation in case of the modified Hayward black hole is also made, where it has been found that correction term reduces the pressure and internal energy of the Hayward black hole [26]. We try to extend such study to the case of quasitopological black holes.

In this paper, we consider a charged quasitopological model which exhibits black hole solutions and discuss the effects of leading-order correction on thermodynamics which becomes significant for small size of the black holes. First, we compute the leading-order correction to the entropy of charged quasitopological black hole and plot a graph to make a comparative analysis between corrected and uncorrected entropy densities for smaller black holes. Here, we find that for (negative-)positive correction parameter (α) there exists a (positive-)negative peak for the corrected entropy density at sufficiently small black holes. The corrected entropy density becomes negative valued for the positive correction parameter, which is not physical and therefore can be forbidden. We see that two critical points exist for the entropy density. The correction term affects significantly the entropy densities in between these critical points. Furthermore, we derive the first-order corrected Gibbs free energy density and discuss the effects of correction terms. We observe that the correction terms with negative correction parameter make Gibbs free energy density (more-)less negative valued for the (smaller-)larger black holes. However, the correction terms with positive correction parameter make Gibbs free energy density more positive valued for the black holes with smaller horizon radius. For the larger values of charge and AdS radius, the deviation of corrected Gibbs free energy density with their original value becomes less. We also calculated the corrected expression for the total charge of the quasitopological black holes which coincides with their original expression in limit $\alpha \rightarrow 0$. Moreover, we evaluate the first-order corrected expression for the mass density of this black hole. We find that a critical point exists for total mass density below which corrected terms with the positive correction parameter shows opposite behavior. We also check the stability and bound point of black holes by calculating specific heat at constant chemical potential and plot with respect to horizon radius. We find that the phase transition does not occur due to the correction term with positive correction parameter and black holes are in stable state. The correction term with negative parameter causes instability for such black holes. Furthermore, in the same fashion, we investigate the effects of thermal fluctuation on the thermodynamics of charged quasitopological black holes endowed with global rotation.

The paper is organized as follows. In section 2, we derive the corrected expression for entropy density due to the thermal fluctuations when the size of the black holes is reduced to the Planck

scale. In section 3, we discuss the effects of quantum corrections due to thermal fluctuations on the thermodynamics of charged quasitopological black holes. Within this section, we study the influence of leading-order correction on stability of such black holes. In section 4, we consider a charged topological black holes endowed with global rotation and discuss the effects of thermal fluctuations on the thermodynamics of it. We also study the stability and bound points of charged rotating quasitopological black holes under the influence of thermal fluctuations. We summarize our results with concluding remarks in the last section 5.

2. Thermodynamics under (quantum) thermal instability: Preliminaries

In this section, we review the corrections to thermodynamic entropy density of the quasitopological black holes when small stable fluctuations around equilibrium are taken into account. In this connection, one may assume that the system of quasitopological black holes is characterized by the canonical ensemble. In order to begin the analysis, let us first define the density of states with fixed energy as [27,28]

$$\rho(E) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{S(\beta)} d\beta. \quad (1)$$

Here $S(\beta)$ refers to the exact entropy density which is not just its value at equilibrium and depends on temperature $T = 1/\beta$ explicitly. The exact entropy density corresponds to the sum of entropy densities of subsystems of the thermodynamical system, which are small enough to be considered in equilibrium. In order to solve the complex integral (1), we utilize the method of steepest descent around the saddle point $\beta_0 (= 1/T_H)$ such that $\left(\frac{\partial S(\beta)}{\partial \beta}\right)_{\beta=\beta_0} = 0$. We assume that the quasitopological black hole is in equilibrium at Hawking temperature T_H . Now, the Taylor expansion of exact entropy density around the saddle point $\beta = \beta_0$ yields

$$S(\beta) = S_0 + \frac{1}{2}(\beta - \beta_0)^2 \left(\frac{\partial^2 S(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0} + (\text{higher order terms}), \quad (2)$$

where $S_0 = S(\beta_0)$ refers the leading-order entropy density. Now, by plugging this $S(\beta)$ (2) into (1), and solving integral by choosing $c = \beta_0$ for positive $\left(\frac{\partial^2 S(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0}$ leads to [19]

$$\rho(E) = \frac{e^{S_0}}{\sqrt{2\pi \left(\frac{\partial^2 S(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0}}}. \quad (3)$$

The logarithm of the above density of states yields the corrected microcanonical entropy density at equilibrium (obtained by incorporating small fluctuations around thermal equilibrium)

$$S = S_0 - \frac{1}{2} \log \left(\frac{\partial^2 S(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0} + (\text{sub-leading terms}). \quad (4)$$

By considering the most general form of the exact entropy density, $S(\beta)$, the form of $\left(\frac{\partial^2 S(\beta)}{\partial \beta^2}\right)_{\beta=\beta_0}$ can be determined. The generic expression for leading-order correction to Bekenstein–Hawking formula is calculated by [19,18]

$$S = S_0 + \alpha \ln(S_0 T_H^2), \quad (5)$$

where α is a (constant) correction parameter. One should note that we considered a general correction parameter α because this is not

fixed valued and takes different values in different circumstances. Eventually, we observe that the leading-order corrections to the entropy density of any thermodynamic system (quasitopological black holes) due to small statistical fluctuations around equilibrium are logarithmic in nature. Now, we shall study the effects of such correction term on the thermodynamics of both the charged and charged rotating quasitopological black holes.

3. Charged quasitopological black holes: Thermal instability

The general action for the quasitopological gravity with cosmological constant Λ in $(d + 1)$ space-time dimensions in the presence of the electromagnetic field (A_b) can be given as [4,7]

$$I = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x \sqrt{-g} \left[R - \Lambda + \frac{\lambda l^2}{(d-2)(d-3)} \mathcal{X}_2 + \frac{8(2d-1)\mu l^4}{(d-2)(d-5)(3d^2-9d+4)} \mathcal{X}_3 - \frac{1}{4} F_{ab} F^{ab} \right], \quad (6)$$

where $\Lambda = -d(d-1)/2l^2$, Maxwell field-strength tensor $F_{ab} = \partial_a A_b - \partial_b A_a$, λ is the Gauss–Bonnet coupling constant and μ is the quasitopological coupling constant. Here, \mathcal{X}_2 and \mathcal{X}_3 are the Gauss–Bonnet and quasitopological terms, respectively, with the following explicit expressions:

$$\mathcal{X}_2 = R_{abcd} R^{abcd} - 4 R_{ab} R^{ab} + R^2, \quad (7)$$

$$\mathcal{X}_3 = R_a^c R_b^d R_c^e R_d^f R_e^a R_f^b + \frac{1}{(2d-1)(d-3)} \left[\frac{3(3d-5)}{8} R_{abcd} R^{abcd} R - 3(d-1) R_{abcd} R^{abc} R^{de} + 3(d+1) R_{abcd} R^{ac} R^{bd} + 6(d-1) R_a^b R_b^c R_c^a - \frac{3(3d-1)}{2} R_a^b R_b^a R + \frac{3(d+1)}{8} R^3 \right]. \quad (8)$$

Now, in order to study the thermodynamics of quasitopological black hole described by the action (6), we consider a $(d + 1)$ -dimensional static metric with a flat boundary as follows,

$$ds^2 = N^2(r) f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 \sum_{i=1}^{d-1} d\phi_i^2, \quad (10)$$

where $N(r)$ is a lapse function. Here we should note that the field equations of quasitopological gravity are second-order differential equations only for this metric. It has been shown in Ref. [7] that $N(r)$ must be a constant and therefore without loss of generality can be set to unit (i.e. $N(r) = 1$). By considering the gauge potential ansatz $A_a = h(r) \delta_a^t$ to have radial electric field, the equation of motion will have the following solution: $h(r) = -\sqrt{\frac{2(n-1)}{n-2}} \frac{q}{r^{d-2}}$. Now, the solution for metric function is given by [7]

$$f(r) = \frac{r^2}{l^2} - \frac{m}{r^{d-2}} + \frac{q^2}{r^{2(d-2)}}, \quad (11)$$

where q and m are integration constants, respectively, related to the electric charge and total mass of the quasitopological black holes. The integration constant m can easily be evaluated from the metric function on the horizon ($f(r=r_+) = 0$) as

$$m = \frac{r_+^d}{l^2} + \frac{q^2}{r_+^{d-2}}. \quad (12)$$

Now, exploiting relations (11) and (12), the Hawking temperature of the event horizon can be calculated by [7]

$$T_H = \frac{f'(r)}{4\pi} \Big|_{r=r_+} = \frac{dr_+ - (d-2)q^2 l^2 r_+^{3-2d}}{4\pi l^2}, \quad (13)$$

where r_+ is the outer horizon radius. The Gibbs free energy per unit volume can be identified with the Euclidean action per volume times the temperature [29]. Corresponding to the resulting Gibbs free energy per unit volume, the leading entropy density of charged quasitopological black holes is calculated as [7]

$$S_0 = \frac{1}{4} r_+^{d-1}. \quad (14)$$

The expression for electric potential, measured at infinity with respect to the horizon, for static case is given by [7,30]

$$\Phi = \sqrt{\frac{2(d-1)}{d-2}} \frac{q}{r_+^{d-2}}. \quad (15)$$

With the help of relations (13) and (15), the Hawking temperature of quasitopological black holes can be expressed in terms of electric potential as follows:

$$T_H = \frac{dr_+}{4\pi l^2} - \frac{(d-2)^2 \Phi^2}{8\pi (d-1) r_+}, \quad (16)$$

which leads to the horizon radius in terms of temperature and electric potential as

$$r_+ = \frac{2\pi l^2}{d} T + 2 \left[\frac{\pi^2 l^4}{d^2} T^2 + \frac{(d-2)^2 l^2}{8d(d-1)} \Phi^2 \right]^{1/2}. \quad (17)$$

Utilizing the relations (13) and (14), the first-order corrected entropy per volume (5) for the charged quasitopological black hole due to the thermal fluctuation is computed as

$$S = \frac{1}{4} r_+^{d-1} + \alpha \log \left[\frac{d^2 r_+^{d+1} + (d-2)^2 q^4 l^4 r_+^{5-3d} - 2d(d-2) q^2 l^2 r_+^{3-d}}{64\pi^2 l^4} \right]. \quad (18)$$

This can further be expressed in terms of electric potential as follows:

$$S = \frac{1}{4} r_+^{d-1} + \alpha \log \left[\frac{1}{64\pi^2} \left(\frac{d^2 r_+^{d+1}}{l^4} + \frac{(d-2)^4}{4(d-1)^2} \Phi^4 r_+^{d-3} - \frac{d(d-2)^2}{(d-1)} \frac{\Phi^2 r_+^{d-1}}{l^2} \right) \right]. \quad (19)$$

The effects of leading-order correction on behavior of the entropy per volume with respect to horizon radius can be seen in Figs. 1 and 2. For negative correction parameter α , the first maxima (positive peak) occurs in between the critical points for the entropy per volume at sufficiently small black holes. Also, one can see in figures that in four space-time dimensions a negative region for the entropy density occurs for quasitopological black holes before second critical point corresponding to the positive values of correction parameter α . After the second critical point, the entropy density is an increasing function only. For larger values of charge and AdS radius, the critical value of entropy density increases and occurs at larger horizon radius. In five space-time dimensions case, there exists only one critical point and the first maxima/minima (peak) occurs after the critical point. In this case, for larger values of charge and AdS radius, the corrected entropy density diverges just after the critical point.

The Gibbs free energy per unit volume for charged quasitopological black holes can be calculated utilizing the standard relation, $G(T_H, \Phi) = -\int S dT_H$, as follows

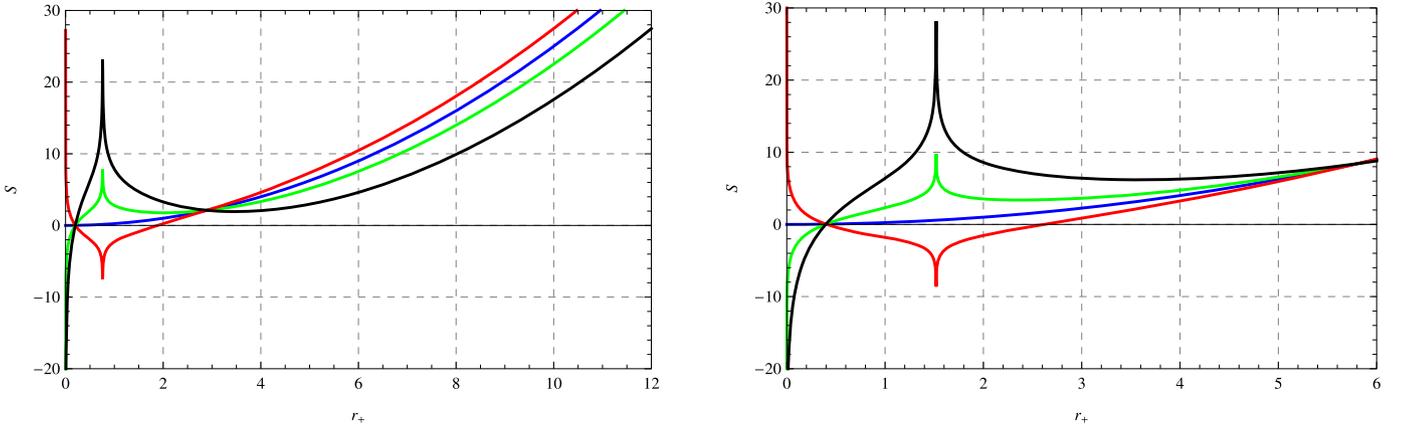


Fig. 1. Left: Entropy per volume vs. the black hole horizon radius for $d = 3$, $l = 1$ and $q = 1$. Right: Entropy per volume vs. the black hole horizon radius for $d = 3$, $l = 2$ and $q = 2$. Here, $\alpha = 0$ denoted by blue line, $\alpha = -0.5$ denoted by green line, $\alpha = 0.5$ denoted by red line, and $\alpha = -1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

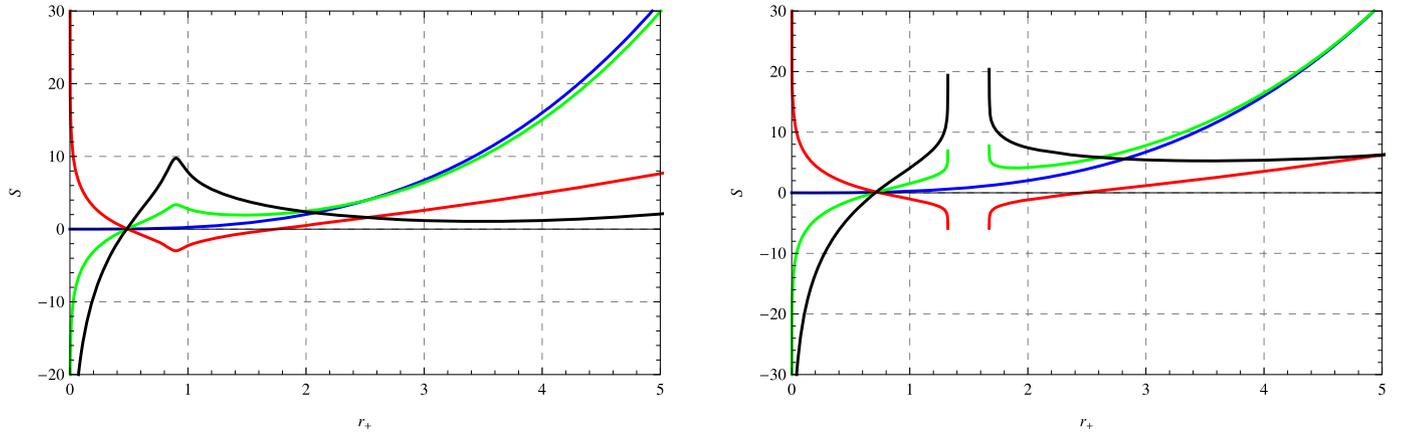


Fig. 2. Left: Entropy per volume vs. the black hole horizon radius for $d = 4$, $l = 1$ and $q = 1$. Right: Entropy per volume vs. the black hole horizon radius for $d = 4$, $l = 2$ and $q = 2$. Here, $\alpha = 0$ denoted by blue line, $\alpha = -0.5$ denoted by green line, $\alpha = 0.5$ denoted by red line, and $\alpha = -1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$\begin{aligned}
 G(T_H, \Phi) = & -\frac{1}{16\pi l^2} \left[r_+^d + \frac{d-2}{2(d-1)} l^2 \Phi^2 r_+^{d-2} \right] \\
 & + \alpha \frac{d(1+d)r_+}{4\pi l^2} + \alpha \frac{(d-3)(d-2)^2 \Phi^2}{8\pi(d-1)} \frac{1}{r_+} \\
 & - \alpha \frac{2d(d-1)r_+^2 - (d-2)^2 l^2 \Phi^2}{8\pi(d-1)l^2 r_+} \\
 & \times \log \left[\frac{1}{64\pi^2} \left(\frac{d^2 r_+^{d+1}}{l^4} + \frac{(d-2)^4}{4(d-1)^2} \Phi^4 r_+^{d-3} \right. \right. \\
 & \left. \left. - \frac{d(d-2)^2 \Phi^2 r_+^{d-1}}{l^2} \right) \right]. \quad (20)
 \end{aligned}$$

Here, it is evident that in the limit $\alpha \rightarrow 0$, this coincides with the original expression calculated in Ref. [7]. The effects of leading-order correction terms on the Gibbs free energy per volume with respect to the black hole horizon radius in four space-time dimensions can be seen from Fig. 3. We observe that the Gibbs free energy per volume is a decreasing function with respect to horizon radius. The Gibbs free energy density without any correction is negligibly small for smaller black holes and becomes negative valued when horizon radius increases. However, the correction terms with negative correction parameter makes it finite negative valued for the smaller black holes. However, the correction terms with positive correction parameter makes the Gibbs free energy density

positive valued for the smaller black holes, falls more sharply to take negative value along with increasing horizon radius. For horizon radius $r_+ \rightarrow 0$, asymptotic behavior of corrected Gibbs free energy per volume with negative α is completely opposite to that of the uncorrected and corrected ones with positive α . In fact, for sufficiently larger size of black hole the corrected Gibbs free energy per volume coincides the uncorrected one as expected. For the larger values of charge and AdS radius, the corrected Gibbs free energy per volume behaves more closely to the uncorrected one.

The corrected charge density of charged quasitopological black holes under the influence of statistical fluctuations can be calculated as follows:

$$\begin{aligned}
 Q = & -\left(\frac{\partial G}{\partial \Phi} \right)_T, \\
 = & \frac{1}{16\pi} \sqrt{2(d-1)(d-2)} q \\
 & + \frac{\alpha(d-2)^2}{4\pi} \sqrt{\frac{2(d-2)}{d-1}} \left[\frac{3d - (3d-1)q^2 l^2 r_+^{2-2d}}{d + (d-2)q^2 l^2 r_+^{2-2d}} \right] \frac{q}{r_+^{d-1}}. \quad (21)
 \end{aligned}$$

In limit $\alpha \rightarrow 0$, the above expression reduces to the original one obtained in [7]. Here, we notice that for space dimensions $d < 3$, one can not have charged quasitopological black holes.

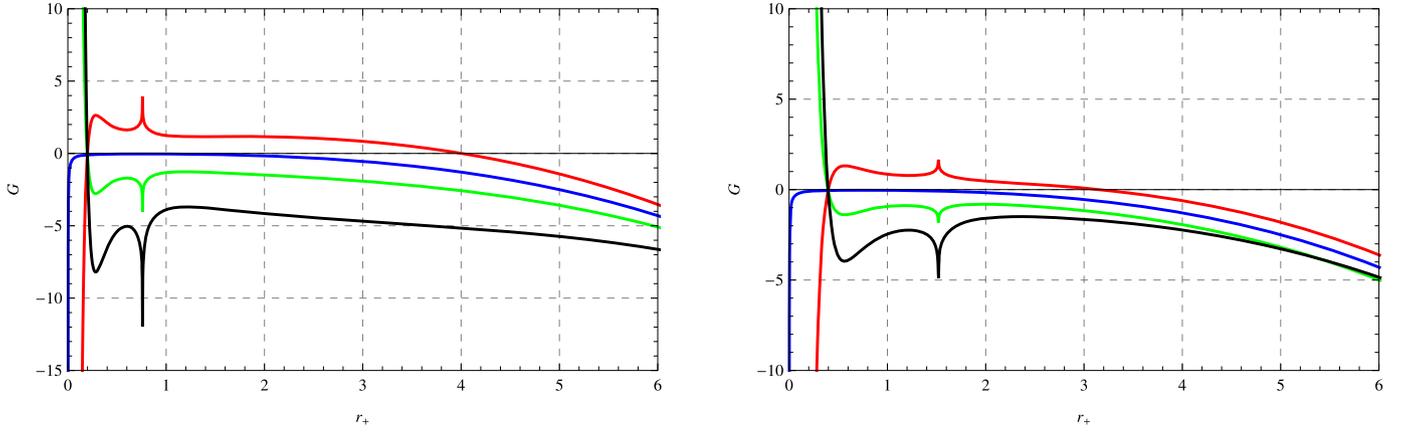


Fig. 3. Left: Gibbs free energy per volume vs. the black hole horizon radius for $d=3$, $l=1$ and $q=1$. Right: Gibbs free energy per volume vs. the black hole horizon radius for $d=3$, $l=2$ and $q=2$. Here, $\alpha=0$ denoted by blue line, $\alpha=-0.5$ denoted by green line, $\alpha=0.5$ denoted by red line, and $\alpha=-1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

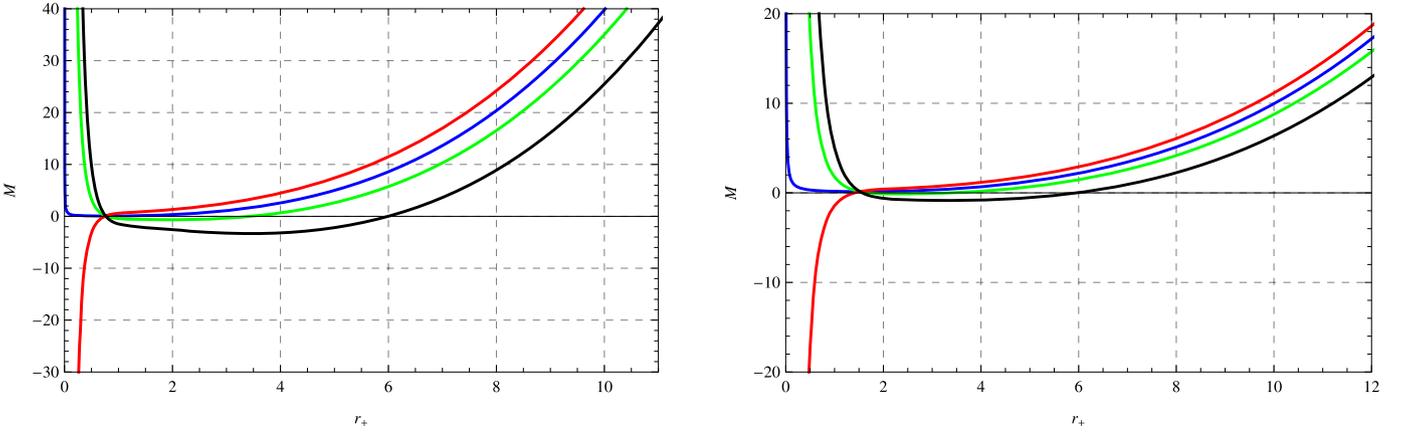


Fig. 4. Left: Mass per volume vs. the black hole horizon radius for $d=3$, $l=1$ and $q=1$. Right: Mass per volume vs. the black hole horizon radius for $d=3$, $l=2$ and $q=2$. Here, $\alpha=0$ denoted by blue line, $\alpha=-0.5$ denoted by green line, $\alpha=0.5$ denoted by red line, and $\alpha=-1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The corrected expression for the mass per volume of the charged quasitopological black holes can be easily calculated from the definition, $M = G + TS + \Phi Q$, as follows

$$\begin{aligned}
 M &= \frac{(d-1)}{16\pi} m \\
 &+ \alpha \frac{(d-2)}{4\pi} \left[\frac{d(7d-15) - (d-2)(5d-1)q^2 l^2 r_+^{2-2d}}{d + (d-2)q^2 l^2 r_+^{2-2d}} \right] \frac{q^2}{r_+^{2d-3}} \\
 &+ \alpha \frac{d(d+1)}{4\pi l^2} r_+. \quad (22)
 \end{aligned}$$

A comparative analysis of corrected and uncorrected mass per volume can be seen in Fig. 4. One can see, for sufficiently large size of black holes, the corrected and uncorrected mass per volume show same behavior as expected. However, when horizon radius tends to zero value, the corrected mass per volume with positive correction parameter shows opposite behavior and takes negative asymptotic value. The larger values of charge and AdS radius minimize the differences of the corrected and uncorrected mass per volume. We note that a critical value exists for the mass per volume for small black holes after that the mass density becomes an increasing function.

3.1. Stability of charged quasitopological black holes

Now, we discuss thermal stability of the charged quasitopological black holes. It is well-known that the stability conditions in canonical ensemble depend on sign of the specific heat. A change of sign may appear whether when specific heat meets root(s) or divergence(s). The root of specific capacity (or temperature) confirms a bound point. This bound point divides physical solutions (which corresponds to positive temperature) from non-physical solutions (which corresponds to negative temperature). However, the divergences of specific heat represent to the phase transition points. The negative specific heat represents to the unstable solutions which may encounter a phase transition to acquire a stable state.

The specific heat per volume with a fixed chemical potential (Φ) is given by

$$\begin{aligned}
 C_\Phi &= T \left(\frac{\partial S}{\partial T} \right)_\Phi, \\
 &= \frac{2\pi(d-1)l^2 r_+^d}{2d(d-1)r_+^2 + (d-2)l^2 \Phi^2} + 2\alpha, \\
 &= \frac{\pi(d-1)l^2 r_+^{d-1}}{dr_+^{2d-2} + (d-2)l^2 q^2 r_+^{3-2d}} + 2\alpha. \quad (23)
 \end{aligned}$$

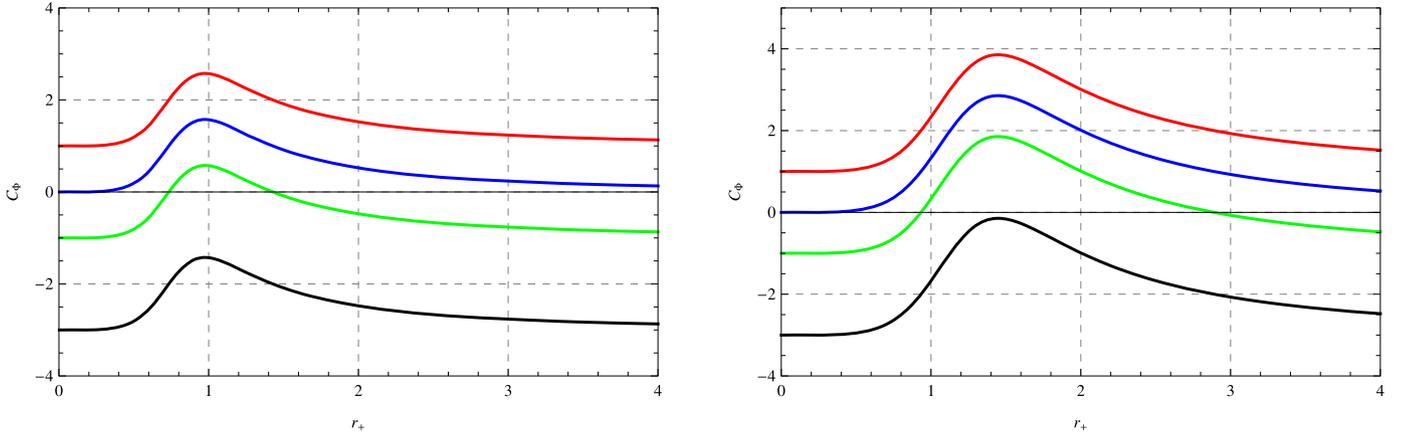


Fig. 5. Left: Specific heat per volume vs. the black hole horizon radius for $d = 3$, $l = 1$ and $q = 1$. Right: Specific heat vs. the black hole horizon radius for $d = 3$, $l = 2$ and $q = 2$. Here, $\alpha = 0$ denoted by blue line, $\alpha = -0.5$ denoted by green line, $\alpha = 0.5$ denoted by red line, and $\alpha = -1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In order to get bound points, we solve the denominator of above expression with respect to horizon radius and get

$$r_c = \left[-\frac{(d-2)l^2q^2}{d} \right]^{1/4d-3}. \quad (24)$$

However, to get phase transition points, one can solve the numerator of above expression with respect to horizon radius which seems a cumbersome task for an arbitrary space-time dimensions in presence of correction parameter.

From the Fig. 5, we observe that for quasitopological black holes in four space-time dimensions there exists no phase transition point corresponding to both the uncorrected and corrected specific heat per volume with positive α and black holes are stable. Interestingly, we find that the correction term with negative correction parameter causes instability to the black holes and a stable state exists only in case of corrected specific heat density with smaller value of negative correction parameter.

4. Charged rotating quasitopological black holes: Thermal instability

In order to describe the charged rotating quasitopological black holes, we equip our charged static solution with a global rotation. The metric for a $(d+1)$ -dimensional asymptotically AdS rotating solution with k rotation parameters can be written as [7]

$$ds^2 = N^2(r)f(r) \left(\Delta dt - \sum_{i=1}^k a_i d\phi_i \right)^2 + \frac{r^2}{l^4} \sum_{i=1}^k (a_i dt - \Delta l^2 d\phi_i)^2 + \frac{dr^2}{f(r)} - \frac{r^2}{l^2} \sum_{i < j}^k (a_i d\phi_j - a_j d\phi_i)^2 + r^2 \sum_{i=k+1}^{d-1} d\phi_i^2, \quad (25)$$

where $\Delta^2 = 1 + \sum_{i=1}^k \frac{a_i^2}{l^2}$ and the angular coordinates can have the following range: $0 \leq \phi_i < 2\pi$. Also, the gauge potential corresponding to this metric has the following form: $A_a(r) = -\sqrt{\frac{2(n-1)}{n-2}} \frac{q}{r^{d-2}} (\Delta dt - \sum_{i=1}^k a_i d\phi_i)$.

The Hawking temperature from the area law is calculated by

$$T_H = \frac{f'(r)}{4\pi \Delta} \Big|_{r=r_+} = \frac{dr_+ - (d-2)q^2 l^2 r_+^{3-2d}}{4\pi \Delta l^2}. \quad (26)$$

The horizon radius in terms of the intensive quantities can be written as

$$r_+ = (1 - l^2 \Omega^2)^{-1/2} \times \left\{ \frac{2\pi l^2}{d} T + 2 \left[\frac{\pi^2 l^4}{d^2} T^2 + \frac{(d-2)^2 l^2}{8d(d-1)} \Phi^2 \right]^{1/2} \right\}, \quad (27)$$

where Φ is the electric potential, measured at infinity with respect to the horizon, with the following explicit form [7]:

$$\Phi = \sqrt{\frac{2(d-1)}{d-2}} \frac{q}{\Delta r_+^{d-2}}. \quad (28)$$

The entropy density of charged rotating quasitopological black hole without any thermal fluctuation can be calculated with the help of Gibbs free energy function and the temperature as [7]

$$S_0 = \frac{\Delta}{4} r_+^{d-1}. \quad (29)$$

Due to thermal fluctuation around equilibrium induces a correction to the original entropy density. We calculate this first-order corrected entropy density as

$$S = \frac{\Delta}{4} r_+^{d-1} + \alpha \log \left[\frac{d^2 r_+^{d+1} + (d-2)^2 q^4 l^4 r_+^{5-3d} - 2d(d-2)q^2 l^2 r_+^{3-d}}{64\pi^2 \Delta l^4} \right], \quad (30)$$

where relations (5), (26) and (29) have been utilized. A comparative study of leading-order corrected and uncorrected entropy densities with respect to horizon radius for four and five space-time dimensions can be seen in Figs. 6 and 7 respectively. For negative correction parameter α there exists first maximum (positive peak) for the entropy per volume in between the critical points. However, there exists a negative region for entropy density with a minimum (negative peak) corresponding to positive α which is physically irrelevant and can be ignored. One can see that the correction terms do not play an important role for the entropy per volume at sufficiently larger horizon radius. Also, there exist critical entropy densities at horizon radii $r_+ \approx 0.2$ and $r_+ \approx 3$ for four dimensional black holes. For larger values of charge and AdS radius, the second critical value of entropy density increases and occurs at larger horizon radius. For five space-time dimensions case, there exists only one critical point for entropy density and the corrected entropy density with large negative parameter falls

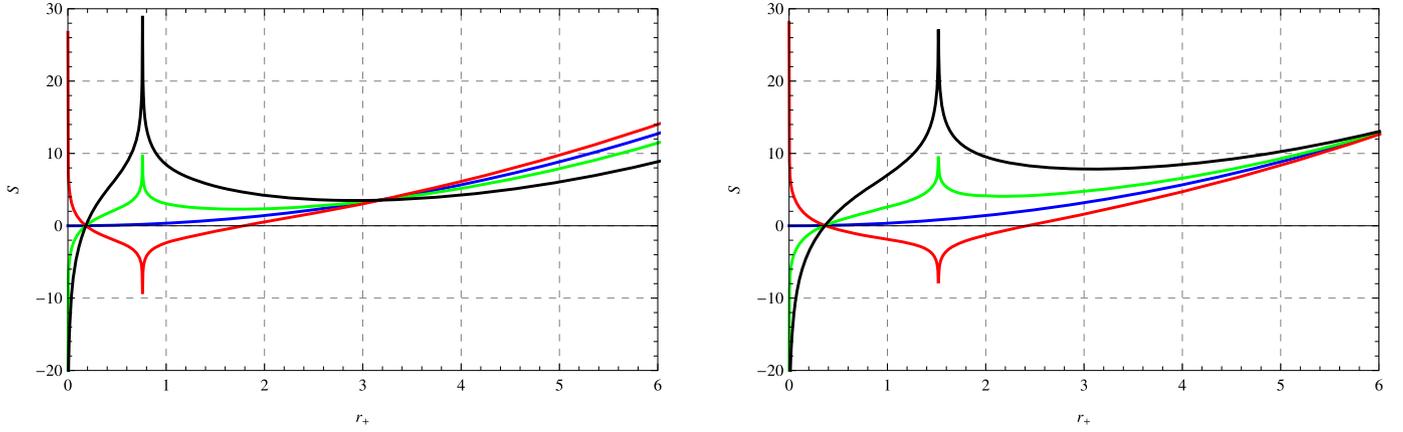


Fig. 6. Left: Entropy per volume vs. the black hole horizon radius for $\Delta^2 = 2$, $d = 3$, $l = 1$ and $q = 1$. Right: Entropy per volume vs. the black hole horizon radius for $\Delta^2 = 2$, $d = 3$, $l = 2$ and $q = 2$. Here, $\alpha = 0$ denoted by blue line, $\alpha = -0.5$ denoted by green line, $\alpha = 0.5$ denoted by red line, and $\alpha = -1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

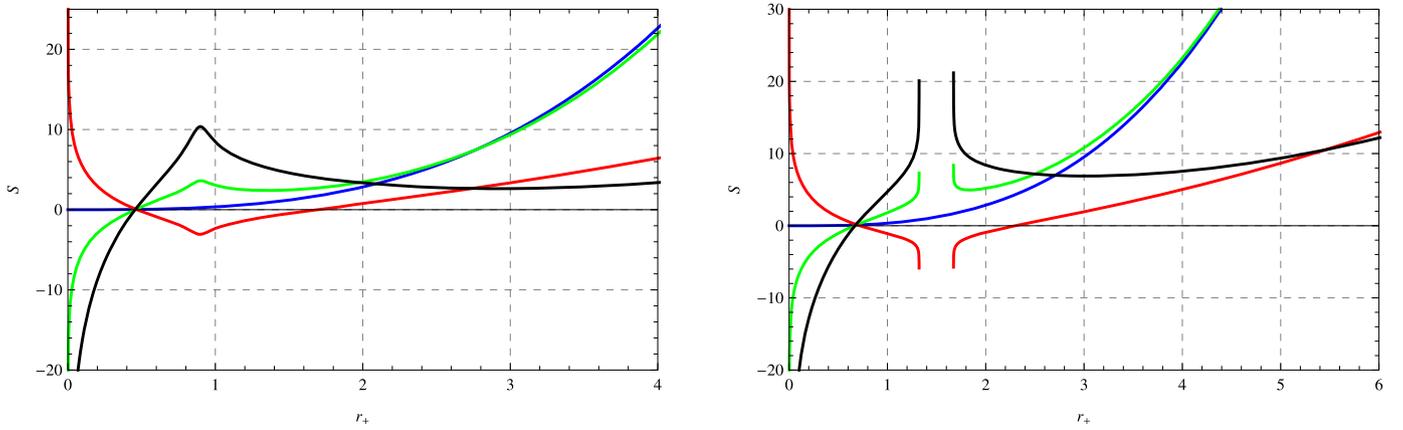


Fig. 7. Left: Entropy per volume vs. the black hole horizon radius for $\Delta^2 = 2$, $d = 4$, $l = 1$ and $q = 1$. Right: Entropy per volume vs. the black hole horizon radius for $\Delta^2 = 2$, $d = 4$, $l = 2$ and $q = 2$. Here, $\alpha = 0$ denoted by blue line, $\alpha = -0.5$ denoted by green line, $\alpha = 0.5$ denoted by red line, and $\alpha = -1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

more sharply. As the charge and AdS radius take larger values, the corrected entropy density diverges after the first critical point.

The corrected expression for the Gibbs free energy per unit volume is calculated by

$$\begin{aligned}
 G(T_H, \Phi, \Omega) = & -\frac{1}{16\pi l^2} \left[r_+^d + \frac{d-2}{2(d-1)(1-l^2\Omega^2)} l^2 \Phi^2 r_+^{d-2} \right] \\
 & + \alpha \frac{d(1+d)r_+}{4\pi \Delta l^2} + \alpha \frac{(d-3)(d-2)^2 \Phi^2}{8\pi(1-d)\Delta} \frac{1}{r_+} \\
 & - \alpha \frac{2d(d-1)r_+^2 - (d-2)^2 l^2 \Phi^2}{8\pi(1-d)\Delta l^2 r_+} \times \\
 & \log \left[\frac{1}{64\pi^2} \left(\frac{d^2 r_+^{d+1}}{l^4} + \frac{(d-2)^4}{4(d-1)^2} \Phi^4 r_+^{d-3} \right. \right. \\
 & \left. \left. - \frac{d(d-2)^2 \Phi^2 r_+^{d-1}}{l^2} \right) \right] \\
 & + \alpha \frac{\log[\Delta]}{\Delta} T_H, \tag{31}
 \end{aligned}$$

where Ω is the angular velocity of the Killing horizon and has the following form: $\Omega_i = a_i/\Delta l^2$. We draw a plot in Fig. 8 for the Gibbs free energy density with respect to horizon radius to make a comparative discussion between the corrected and uncorrected the Gibbs free energy densities. In this figure, we see that

the Gibbs free energy density is a negative valued function for the larger horizon radius. The leading-order correction terms with (negative-)positive α make it (more-)less negative valued for larger horizon radius. For small horizon radius, the corrected Gibbs energy density with positive α is positive valued. In the limit $r_+ \rightarrow 0$, the corrected Gibbs free energy density with negative α shows opposite asymptotic behavior in comparison to uncorrected and corrected ones with positive α . Two critical points occur for the Gibbs free energy density. For the larger values of charge and AdS radius, after critical points the corrected Gibbs free energy per volume with negative α becomes less negative.

Now, we calculate the charge density of charged rotating quasi-topological black hole under the influence of statistical fluctuations. This is given by

$$\begin{aligned}
 Q = & -\left(\frac{\partial G}{\partial \Phi} \right)_T, \\
 = & \frac{1}{16\pi} \sqrt{2(d-1)(d-2)} \Delta q \\
 & + \frac{\alpha(d-2)^2}{4\pi \Delta} \sqrt{\frac{2(d-2)}{d-1}} \left[\frac{3d - (3d-1)q^2 l^2 r_+^{2-2d}}{d + (d-2)q^2 l^2 r_+^{2-2d}} \right] \frac{q}{r_+^{d-1}}. \tag{32}
 \end{aligned}$$

From the above expression, the original expression of total charge density given in [7] can be recovered in the $\alpha \rightarrow 0$ limit. Also, we

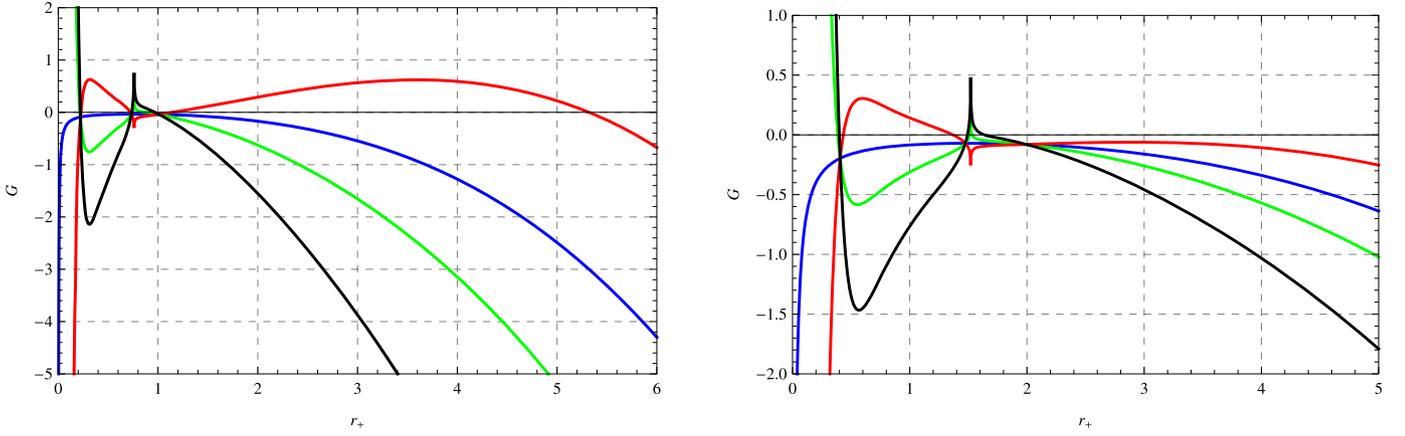


Fig. 8. Left: Gibbs free energy per volume vs. the black hole horizon radius for $d=3$, $\Delta^2=2$, $l=1$ and $q=1$. Right: Gibbs free energy per volume vs. the black hole horizon radius for $d=3$, $\Delta^2=2$, $l=2$ and $q=2$. Here, $\alpha=0$ denoted by blue line, $\alpha=-0.5$ denoted by green line, $\alpha=0.5$ denoted by red line, and $\alpha=-1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

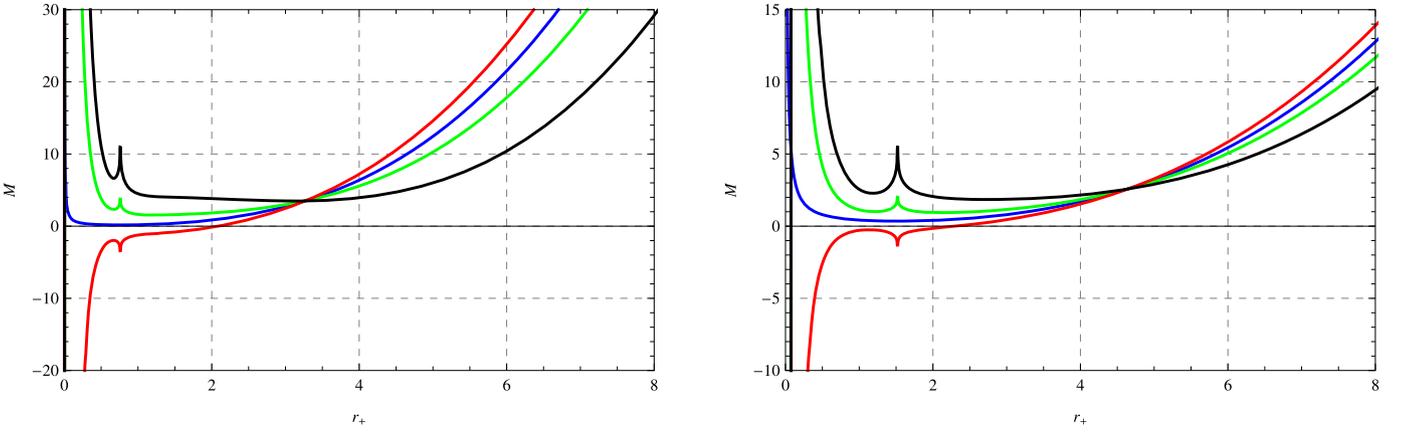


Fig. 9. Left: Mass per volume vs. the black hole horizon radius for $d=3$, $\Delta^2=2$, $l=1$ and $q=1$. Right: Mass per volume vs. the black hole horizon radius for $d=3$, $\Delta^2=2$, $l=2$ and $q=2$. Here, $\alpha=0$ denoted by blue line, $\alpha=-0.5$ denoted by green line, $\alpha=0.5$ denoted by red line, and $\alpha=-1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

see that the charged rotating quasitopological black hole does not exist for space dimensions $d < 3$.

Since the black hole solution is endowed with a global rotation, therefore, this possesses an associated angular momentum also. We compute the first-order corrected angular momentum per volume as follows,

$$\begin{aligned}
 J_i &= - \left(\frac{\partial G}{\partial \Omega_i} \right)_{T, \Phi}, \\
 &= \frac{d}{16\pi} \Delta m a_i - \frac{\alpha}{2\pi} \left[\frac{d^2 r_+}{l^2} - (d-2) \frac{q^2}{r_+^{2d-3}} \right] a_i \\
 &+ \frac{\alpha}{4\pi} \left[\frac{dr_+}{l^2} + \frac{(d-2)q^2}{r_+^{2d-3}} \right] \\
 &\times a_i \log \left[\frac{d^2 r_+^{d+1} + (d-2)^2 q^4 l^4 r_+^{5-3d} - 2d(d-2)q^2 l^2 r_+^{3-d}}{64\pi^2 \Delta l^4} \right].
 \end{aligned} \tag{33}$$

This expression also coincides with the original one calculated in [7], when we switch off thermal fluctuations (i.e. $\alpha=0$).

Now, utilizing the standard relation, $M = G + TS + \Phi Q + \sum_{i=1}^k \Omega_i J_i$, we are able to calculate the first-order corrected to-

tal mass per volume of the charged rotating quasitopological black hole as follows:

$$\begin{aligned}
 M &= \frac{1}{16\pi} (d\Delta^2 - 1)m + \frac{\alpha}{4\pi \Delta l^2} d(3d - 2d\Delta^2 + 1)r_+ \\
 &+ \alpha \frac{\log[\Delta]}{\Delta} T_H - \frac{\alpha(d-2)}{4\pi \Delta} \left[\frac{1 - 2\Delta^2(\Delta^2 - 1)}{\Delta^2} \right. \\
 &- 2(d-2) \left. \left(\frac{3d - (3d-1)q^2 l^2 r_+^{2-2d}}{d + (d-2)q^2 l^2 r_+^{2-2d}} \right) \right] \frac{q^2}{r_+^{2d-3}} \\
 &+ \alpha \frac{(\Delta^2 + 1)dr_+ + (\Delta^2 - 1)\Delta^{-2}(d-2)q^2 l^2 r_+^{3-2d}}{4\pi \Delta l^2} \times \\
 &\log \left[\frac{d^2 r_+^{d+1} + (d-2)^2 q^4 l^4 r_+^{5-3d} - 2d(d-2)q^2 l^2 r_+^{3-d}}{64\pi^2 \Delta l^4} \right].
 \end{aligned} \tag{34}$$

This expression of corrected total mass density is also consistent with the one calculated originally in [7] in the limit $\alpha \rightarrow 0$.

In order to discuss the effects of thermal fluctuations on the total mass density as one reduces the size of the charged rotating quasitopological black hole, we plot Fig. 9. We see that the corrected total mass density with negative α is an decreasing function before the critical point and increasing function after the critical

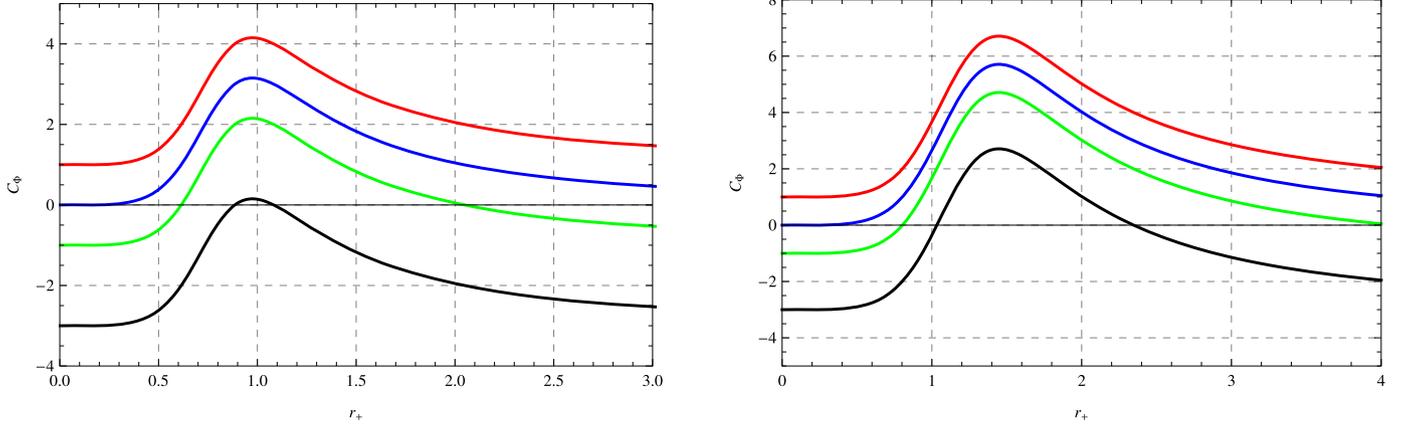


Fig. 10. Left: Specific heat per volume vs. the black hole horizon radius for $\Delta^2 = 2$, $d = 3$, $l = 1$ and $q = 1$. Right: Specific heat vs. the black hole horizon radius for $\Delta^2 = 2$, $d = 3$, $l = 2$ and $q = 2$. Here, $\alpha = 0$ denoted by blue line, $\alpha = -0.5$ denoted by green line, $\alpha = 0.5$ denoted by red line, and $\alpha = -1.5$ denoted by black line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

point, but not a negative valued function. However, the corrected mass density with positive α is an increasing function only but a negative valued function before the critical point. The larger values of charge and AdS radius decrease the critical value of mass density a bit and occurs at bit larger horizon radius.

4.1. Stability of charged rotating quasitopological black holes

Here, in order to discuss the thermal stability for charged rotating quasitopological black holes, we would analyze the sign of the specific heat as the negative specific heat represents to the unstable solutions which may encounter a phase transition to acquire a stable state.

The specific heat per volume with a fixed chemical potential (Φ) for the charged rotating quasitopological black holes is calculated by

$$\begin{aligned} C_\Phi &= T \left(\frac{\partial S}{\partial T} \right)_\Phi, \\ &= \frac{2\pi(d-1)^2 l^2 \Delta^2 r_+^d}{2d(d-1)r_+^2 + (d-2)^2 l^2 \Phi^2} + 2\alpha, \\ &= \frac{\pi(d-1)l^2 \Delta^2 r_+^{d-1}}{dr_+^{2d-2} + (d-2)l^2 q^2 r_+^{3-2d}} + 2\alpha. \end{aligned} \quad (35)$$

The bound points can be obtained by solving the denominator of above expression with respect to horizon radius. By doing so, we obtain

$$r_c = \left[-\frac{(d-2)l^2 q^2}{d} \right]^{1/4d-3}, \quad (36)$$

which is an exactly same point as obtained in the case of charged quasitopological black holes without rotation.

In order to see the effects of thermal fluctuations on the stability of charged rotating quasitopological black hole, we plot the Fig. 10. We see that due to the correction terms with negative correction parameters instabilities occur for the charged rotating quasitopological black holes. However, the correction terms with positive correction parameter make the specific heat more positive valued and therefore more stable. The larger values of charge and AdS radius improves the stability of such black holes corresponding to the correction terms with negative correction parameters.

5. Concluding remarks

It is well-known that quasitopological gravity is a new gravitational theory, including Gauss–Bonnet term and curvature-cubed interactions, which possesses exact black hole solutions. Here, we have considered both the charged and charged rotating quasitopological gravity with black hole solutions to study the effects of thermal fluctuation on thermodynamics of small black holes.

First, we have evaluated the leading-order correction to the entropy density of charged quasitopological black hole and made a comparative analysis between corrected and uncorrected entropy densities through plots for small sizes of the black holes. We have found that corresponding to (negative-)positive correction parameters there exist (positive-)negative peaks for the corrected entropy density in between the critical points. Also, corrected entropy density becomes negative valued corresponding to the positive values of correction parameter, which is not physical and therefore can be forbidden. The correction term plays a crucial role for entropy densities in between these critical points. Furthermore, we have computed the first-order corrected Gibbs free energy density. We have plotted graph to make the comparative analysis and found that the leading-order corrected Gibbs free energy density with negative correction parameter makes it (more-)less negative valued for the (smaller-)larger black holes. In spite of that the corrected Gibbs free energy density with positive correction parameter became more positive valued. The higher values of charge and AdS radius decrease the deviation of corrected Gibbs free energy density to that of the uncorrected one. We have also calculated the corrected expression for total charge of quasitopological black holes. Finally, we have evaluated the more exact expression for the total mass density of such black holes. A critical horizon radius has been found for total mass density and before which the positive correction parameter causes opposite behavior. We have also discussed the stability of such black holes by calculating the corrected specific heat density with fixed chemical potential and have found a bound point. We noticed that a phase transition does not exist for quasitopological black holes under the influence of thermal fluctuation with positive α and therefore black holes are in stable state. However, due to the thermal fluctuation with negative α an instability occurs such black holes.

Furthermore, we have considered a charged quasitopological black holes endowed with the global rotation and have computed the Hawking temperature and leading-order correction to the entropy density. We have found that a maximum (positive peak)

occurs for the corrected entropy density with negative α in between the critical horizon radii. However, the corrected entropy density becomes negative for positive α which is physically irrelevant and can be ignored. This indicates that only negative valued correction parameter α is physically relevant. We also noted that for larger values of charge and AdS radius the second critical point occurs at larger horizon radius. We have obtained the corrected expressions for Gibbs free energy, charge, angular momentum and total mass densities. The correction terms with (negative-)positive α make the Gibbs free energy density (more-)less negative valued. The corrected Gibbs free energy density with negative α shows opposite asymptotic behavior. The larger values of charge and AdS radius make Gibbs free energy per volume more negative valued. We have noticed that the correction terms with negative α increase total mass density before critical point and decrease after critical point. However, the corrected mass density with positive α takes negative asymptotic value as horizon radius tends to zero. Also, the larger values of charge and AdS radius increase the value of critical horizon radius. We have calculated the corrected specific heat with fixed chemical potential in case of charged rotating quasitopological black holes also and discussed their stability. It would be interesting to investigate the effects of thermal fluctuation on the $P - V$ criticality of quasitopological black holes where negative cosmological constant could play the role of thermodynamic pressure.

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