

Gravitational time delays along multiple light paths as a probe of physics beyond Einstein gravity

Hideki Asada¹

Faculty of Science and Technology, Hirosaki University, Hirosaki 036-8561, Japan

Abstract

The gravitational time delay of light is reexamined, allowing for various models of modified gravity. We clarify the dependence of the time delay (and induced frequency shift) on modified gravity models and investigate how to distinguish those models, when light propagates in static spherically symmetric spacetimes.

1 Introduction

Recent observations such as the magnitude-redshift relation of type Ia supernovae (SNIa) and the cosmic microwave background (CMB) anisotropy by WMAP strongly suggest a certain modification, in whatever form, in the standard cosmological model. We are forced to add a new component into the energy-momentum tensor in the Einstein equation or modify the theory of general relativity itself. Indeed, there have been a lot of proposals motivated by, for instance, scalar tensor theories, string theories, higher dimensional scenarios and quantum gravity. Therefore, it is of great importance to observationally test these models.

The theory of general relativity has passed “classical” tests, such as the deflection of light, the perihelion shift of Mercury and the Shapiro time delay, and also a systematic test using the remarkable binary pulsar “PSR 1913+16” [1]. In the twentieth century, these tests proved that the Einstein’s theory is correct with a similar accuracy of 0.1%.

Since the time delay effect along a light path in the gravitational field was first noticed in 1964 by Shapiro [2], this effect has successfully tested the Einstein’s theory [3]. A significant improvement was reported in 2003 from Doppler tracking of the Cassini spacecraft on its way to the Saturn, with $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [4]. Here, γ is one of parameters in the parameterized post-Newtonian (PPN) formulation of gravity [1]. The sensitivity in the Cassini experiment approaches the level at which, theoretically, deviations $10^{-6} - 10^{-7}$ are expected in some cosmological models [5, 6]. Therefore, it is important to investigate the Shapiro time delay with such a high accuracy.

Here, we discuss the dependence of the time delay (and induced frequency shift) on modified gravity models and investigate how to distinguish those models by using the Shapiro time delay [7]. An important point in this paper is that we allow for various modified gravity theories beyond the scope of the PPN formulation. Introducing a new energy or length scale (e.g. extra dimension scale) may make changes in functional forms of the gravitational field. Thus it is worthwhile to investigate how to probe such a modified functional form, by using the light propagation in the solar system. Throughout this paper, we take the units of $G = c = 1$.

2 Shapiro Time Delay

Let us assume that the electromagnetic fields propagate in four-dimensional spacetimes (even if the whole spacetime is higher dimensional). Thus photon paths follow null geodesics (as the geometrical optics approximation of Maxwell equation).

We shall consider a static spherically symmetric spacetime, in which light propagates, expressed as

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2d\Omega^2, \quad (1)$$

¹E-mail:asada@phys.hirosaki-u.ac.jp

where r and $d\Omega^2$ denote the circumference radius and the metric of the unit 2-sphere, respectively. The functions $A(r)$ and $B(r)$ depend on gravity theories.

The time lapse along a photon path is obtained as

$$t(r, r_0) = \int_{r_0}^r \frac{dr}{b} \sqrt{\frac{B(r)}{A(r)}} \frac{1}{\sqrt{\frac{A(r_0)}{r_0^2} - \frac{A(r)}{r^2}}}, \quad (2)$$

where b and r_0 denote the impact parameter and the closest point, respectively. Their relation is $b^2 = r_0^2/A(r_0)$.

For practical calculations, we keep only the leading term at a few AU in the corrections. Namely, $A(r)$ and $B(r)$ are approximated as

$$A(r) \approx 1 - \frac{2M}{r} + A_m r^m, \quad (3)$$

$$B(r) \approx 1 + \frac{2M}{r} + B_n r^n, \quad (4)$$

where M denotes the mass of the central body.

Examples of modified gravity theories are as follows. (1) $n = 1/2$, $A_n = -2B_n = -2\sqrt{M/r_c^2}$ for DGP model with r_c that is the extra scale within which gravity becomes five dimensional [8]. (2) $n = 3/2$, $A_n = (2/3)m_g^2\sqrt{2M/13}$ and $B_n = -m_g^2\sqrt{2M/13}$ with graviton mass m_g for one of massive gravity models [9, 10]. (3) $n = 2$, $A_n = -B_n = -\Lambda/3$ for the Schwarzschild-de Sitter spacetime, that is, general relativity with the cosmological constant Λ as a possible candidate for the dark energy, though this is not a manifest modification of gravity.

Up to the linear order, the extra contribution to time delay due to modified gravity is

$$\begin{aligned} \delta t &= r_0^{n+1} \left(\int_1^{R_E} + \int_1^{R_R} \right) dR \\ &\times \left(-A_n \frac{R^{n+3} - 2R^{n+1} + R}{(R^2 - 1)^{3/2}} + B_n \frac{R^{n+1}}{\sqrt{R^2 - 1}} \right), \end{aligned} \quad (5)$$

where we define $R \equiv r/r_0$.

It is convenient to use the relative change in the frequency, which is caused by the gravitational time delay. This frequency shift is defined as $y = -d(\Delta T)/dt$.

The general relativistic contribution is expressed as [1]

$$y_{GR} = 4 \frac{M}{b} \frac{db}{dt}. \quad (6)$$

For $n \neq 1$, the extra frequency shift becomes

$$\delta y = -\frac{A_n + B_n}{n-1} \{r_E^{n-1} + r_R^{n-1} - (n+1)r_0^{n-1}\} b \frac{db}{dt}, \quad (7)$$

while we obtain $\delta y = -(A_n + B_n)[\ln(r_E r_R / r_0^2) - 1] b db/dt$ for $n = 1$.

Here, we make an order-of-magnitude estimate of the frequency shift. First, we obtain $y_{GR} \sim 10^{-9}(M/M_\odot)(r_\odot/b)(\dot{b}/v_E)$, where the dot denotes the time derivative, and v_E is the orbital velocity of Earth (~ 30 km/s).

For a receiver at $r_R > r_E$, the extra frequency shift is

$$\begin{aligned} \delta y &\sim (A_n + B_n) r_R^n \frac{b}{r_R} \frac{db}{dt} \\ &\sim 10^{-17} \left(\frac{10\text{AU}}{r_\odot} \right)^n \left(\frac{(A_n + B_n) r_\odot^n}{10^{-10}} \right) \left(\frac{r_R}{10\text{AU}} \right)^{n-1} \left(\frac{b}{r_\odot} \right) \left(\frac{db/dt}{v_E} \right), \end{aligned} \quad (8)$$

where $10\text{AU}/r_\odot \sim 2 \times 10^3$. The larger the index of n , the longer the delay δy .

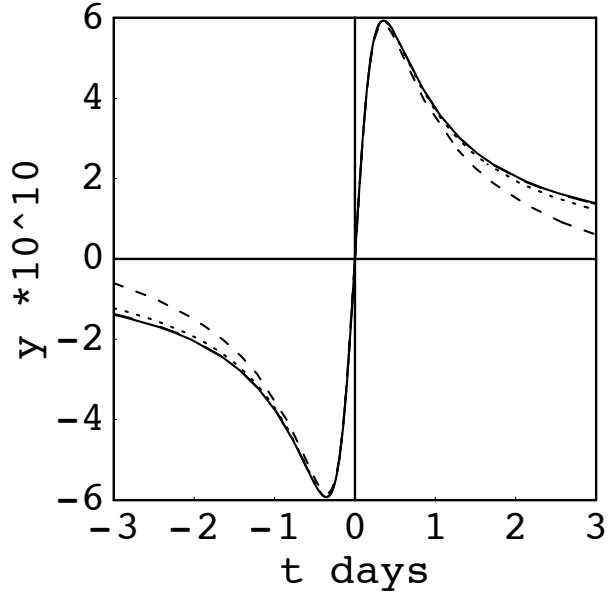


Figure 1: Dependence of the frequency shift on the distance r_R and the index n . The long dashed, short dashed and dotted curves denote the frequency shift for $(n, r_R) = (3/2, 10\text{AU})$, $(n, r_R) = (2, 10\text{AU})$, $(n, r_R) = (2, 1\text{AU})$, respectively. The long dashed curve for $n = 3/2$ and $r_R = 10$ AU is overlapped with the solid curve denoting the general relativistic case. Here, we assume $(A_n + B_n)r_\odot^n = 3 \times 10^{-11}$.

Figure 1 shows that an extra distortion due to δy would appear especially in the tail parts of $y - t$ curves. One can distinguish modified gravity models, which are characterized by various values of n , A_n , B_n , from observations using receivers at very different distances from Sun, as shown by Fig. 1.

Figure 2 shows the dependence of δy on n and $A_n + B_n$. Hence, one can put a constraint on n and $A_n + B_n$ from δy observed.

3 Multiple Paths

We consider three light paths, for which the impact parameters of the photon paths are almost the same (several times of the solar radius) for convenience sake. The locations of the receivers are denoted as r_{R1} , r_{R2} and r_{R3} , where the subscripts from 1 to 3 denote each light path. We assume that r_E is constant in time for simplicity. It is a straightforward task to take account of the eccentricity of the Earth orbit and a difference between the impact parameters.

We make use of a difference such as $y_2 - y_1$ and $y_3 - y_1$, in order to cancel out general relativistic parts. We find

$$y_2 - y_1 = \frac{A_n + B_n}{n - 1} (r_{R1}^{n-1} - r_{R2}^{n-1}) b \frac{db}{dt}. \quad (9)$$

It should be noted that $y_2 - y_1$ is proportional to $A_n + B_n$. Hence, the following ratio depends only on n as

$$\frac{y_3 - y_1}{y_2 - y_1} = \frac{r_{R1}^{n-1} - r_{R3}^{n-1}}{r_{R1}^{n-1} - r_{R2}^{n-1}}. \quad (10)$$

Thereby, one can determine the index n . Next, one obtains $A_n + B_n$ by substituting the determined n into Eq. (9).

4 Conclusion

In summary, we have clarified the dependence of the gravitational time delay on modified gravity models [7]. For neighboring light rays, the time delays become almost the same so that one can hardly distinguish

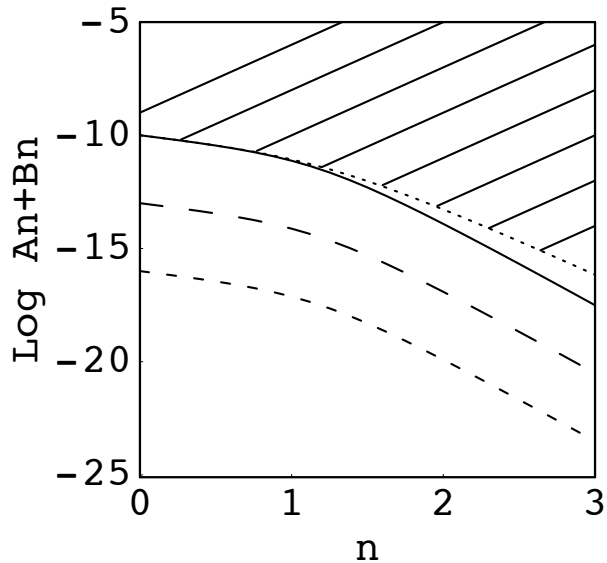


Figure 2: Contours of δy on the $n - |A_n + B_n|r_\odot^n$ plane. The solid, long-dashed and short-dashed curves correspond to $\delta y = 10^{-14}, 10^{-17}, 10^{-20}$, respectively, where we assume $r_E = 1$ AU, $r_R = 40$ AU, $b \sim r_\odot$ and $db/dt \sim v_E$. The limit due to the current technology is $\delta y \sim 10^{-17}$. The shaded region above the dotted curve ($\delta y = 10^{-14}$ for $r_R = 8.43$ AU) has been excluded by the Cassini experiment.

models of gravity. This implies that we should prepare receivers at very different distances from Sun.

Furthermore, b becomes the same order of r_E, r_R for future space-borne laser interferometric detectors such as LISA, DECIGO and especially ASTROD [11]. Namely, r_R and b change with time. Therefore, the sophisticated experiments by space-borne laser interferometric detectors, which are originally designed to detect time-dependent part of gravity, *i.e.* gravitational waves, could probe also a time-independent part of gravity at the relative level of $\Delta y \sim \Delta\nu/\nu \sim \Delta L/L < 10^{-20}$.

References

- [1] C. M. Will, *Theory and experiment in gravitational physics* (Cambridge Univ. Press, Cambridge, 1993).
- [2] I. I. Shapiro, Phys. Rev. Lett. **13**, 789 (1964).
- [3] C. M. Will, Living Rev. Relativity 9, 3 (2006), <http://relativity.livingreviews.org/Articles/lrr-2006-3>.
- [4] B. Bertotti, L. Iess and P. Tortora, Nature, **425**, 374 (2003).
- [5] T. Damour and A. M. Polyakov, Nucl. Phys. B **423**, 532 (1994).
- [6] T. Damour, F. Piazza and G. Veneziano, Phys. Rev. D **66**, 046007 (2002).
- [7] H. Asada, arXiv:0710.0477 [gr-qc]
- [8] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000).
- [9] A. I. Vainshtein, Phys. Lett. B **39**, 393 (1972).
- [10] T. Damour, I. I. Kogan and A. Papazoglou, Phys. Rev. D **67**, 064009 (2003).
- [11] W. T. Ni *et al.*, J. Phys. **32**, 154 (2006).