COMPOSITE MODEL OF ELEMENTARY PARTICLES

Y. Ohnuki
Nagoya University, Nagoya, Japan

1. Looking back upon the history of physics, especially the development from atom to nucleus and from nucleus to elementary particles, one sees that the elements of matter once regarded as ultimate have come to be considered to be constructed of more fundamental ones, i.e., the atom is constructed of nucleus and electrons, and the nucleus is constructed of protons and neutrons. This tells us that the meaning of the word " elementary " or " basic " is not very absolute; it seems to change as our knowledge of nature increases. In this sense, it is impossible to say that all of the present elementary particles will be the most basic elements of matter, because they are recognized as elementary on the basis of our present knowledge of nature, so they will not necessarily stand on the same footing at the next stage of physics; some of them may still be elementary, but others may not. Accordingly it may be expected that they are constructed of a smaller number of elements, by which their characteristic properties can be explained consistently.

In fact, Fermi and Yang [1] pointed out a possibility of explaining the π meson as a compound state of a nucleon and an antinucleon. Especially, the recent progress in experimental physics has led us to a group of new particles, and it seems a realistic problem to examine whether or not they are elementary, if one is to explain the remarkable properties of their mutual interactions.

As is well known, all of the present elementary particles except the photon can be classified into two families, i.e., the baryon-meson family and the lepton family, according to their interaction properties. The mutual transitions of particles in the baryon-meson family are very peculiar and the phenomenological rule for them was found by Gell-Mann, Nakano and Nishijima [2].

In order to understand this rule in a fundamental way, in 1955 Sakata [3] proposed a composite model for particles belonging to the baryon-meson family. According to it, three particles, proton, neutron and Λ particle are regarded as basic particles at the present stage, and others (belonging to this family) are assumed to be compound objects constructed out of them. He showed that the Gell-Mann-Nakano-Nishijima rule could be reproduced in a consistent way, and the strangeness, which was introduced *ad hoc* into this rule, could have a realistic meaning, namely the number of Λ particles.

Of course, Sakata's model is not concerned with the lepton family and the photon. So, it is not a unified or final description of all elementary particles. But it seems very difficult or rather impossible to build up the final theory on the basis of the present knowledge of elementary particles. Furthermore, even if a more unified theory is made at the next stage of physics, it will only be a theory based on our knowledge at that particular stage. Accordingly we must search step by step for a profound meaning behind phenomena at each stage of physics. For this purpose, it now seems necessary to make clear the characteristic properties of each family, and further to grasp connections among them.

2. As was mentioned already, in Sakata's model proton, neutron and Λ particle are regarded as the basic particles in the boson-baryon family, and other members of the family are compound particles constructed of them.

Now, it is considered that Sakata's model presents the following two problems: the first is concerned with the kinematical aspect of this model, and the second with the dynamics appropriate to the composite state. But the construction of the dynamics has not been successful. This seems to be due to our meager knowledge of the behavior of the bound baryons. Hence we shall, for the present, discuss mainly the first problem.

In Sakata's model there exist the following properties of the basic baryons:

(I) Each mass is of the same order of magnitude.

(II) The spin is ½.

(III) They have strong interactions in common which satisfy conservation laws

$$\Delta N_p = \Delta N_n = \Delta N_\Lambda = 0$$

where ΔN_p (ΔN_n, ΔN_Λ) means the change of the proton (neutron, Λ particle) number through the interactions.

It will be expected that these similarities among the basic baryons appear also in the compound states. Of course, as the nucleon mass differs somewhat from that of the Λ, the similarities do not hold exactly. Therefore, in the actual case we must take into account such a mass difference. Since its existence means that there would be an unknown mechanism inducing it, it seems necessary to examine to what extent the other similarities hold. We shall examine the composite model with the assumption that there exists a complete symmetry [4] among the basic baryons.

3. We shall introduce the following assumption:

" In the limit of $m_N = m_\Lambda$ the theory satisfies the invariance under exchanges of Λ and proton, and of Λ and neutron in addition to the usual charge independence and conservation of the baryon number."

Then it is easily seen that this is equivalent to the following statement:

" The theory is invariant under the transformation

$$\chi_i'(x) = \sum_{j=1}^{3} U_{ij}\chi_j(x)\,, \quad (i = 1,2,3)$$
$$\chi_1(x) = p(x)\,, \quad \chi_2(x) = n(x)\,, \quad \chi_3(x) = \Lambda(x) \tag{3.1}$$

in the limiting case of $m_N = m_\Lambda$, where $p(x)$, $n(x)$, $\Lambda(x)$ stand for the proton, neutron and Λ particle fields respectively and U is an arbitrary unitary matrix of degree three." Thus if the irreducible representation of this group $U(3)$ is obtained, all states belonging to it should have the same energy, spin and parity in the limit of $m_N = m_\Lambda$. So by examining the properties of $U(3)$, we shall make clear some aspects of the kinematical features of Sakata's model.

Now it can be shown that the irreducible representations of $U(3)$ are specified by a set of three quantum numbers, one of which is the baryon number N_B and others are defined by

$$M = \frac{1}{2}\sum_{i,j=1,2,3} X_{ij}X_{ij}$$

and

$$M' = \frac{1}{8}\sum_{i,j,k=1,2,3} X_{ij}(\{X_{ik},\, X_{jk}\} + \{X_{ik},\, X_{kj}\} + \{X_{kj},\, X_{ki}\} - \{X_{jk},\, X_{ki}\}) \tag{3.2}$$

$$\{X, X'\} = XX' + X'X$$

where $X_{ij}(i, j, k = 1, 2, 3)$ are nine hermitian generators of this group which satisfy the commutation relations

$$[X_{ij},\, X_{kl}] = i(\delta_{ik}X_{[jl]} - \delta_{lj}X_{[k,l]} + \delta_{il}X_{(jk)} - \delta_{kj}X_{(li)})$$
$$X_{(ij)} = \frac{X_{ij}+X_{ji}}{2}, \qquad X_{[ij]} = \frac{X_{ij}-X_{ji}}{2}. \tag{3.3}$$

Basic vectors of the irreducible representation and the associated quantum numbers are given in Tables I and II.

Table I shows the composite states constructed of one baryon and one antibaryon. In this case, there

Table I. Configuration of the system of one baryon and one antibaryon

Class I. $M = 3$, $M' = 0$

$B_2^1(-1, ½)$	$\bar{K}^0$ K^-	$-\Lambda\bar{n}$ $\Lambda\bar{p}$
$B_2^1(0, 0)$	$\pi^{0'}$	$\{-p\bar{p}-n\bar{n}+2\Lambda\bar{\Lambda}\}/\sqrt{6}$
$B_2^1(0, 1)$	π^+ π^0 π^-	$p\bar{n}$ $\{p\bar{p}-n\bar{n}\}/\sqrt{2}$ $n\bar{p}$
$B_2^1(1, ½)$	K^+ K^0	$p\bar{\Lambda}$ $n\bar{\Lambda}$

Class II. $M = 0$, $M' = 0$

$B_2^2(0, 0)$	$\{p\bar{p}+n\bar{n}+\Lambda\bar{\Lambda}\}/\sqrt{3}$

Table II. Configuration of the system of two baryons and one antibaryon

Class 1. $M = {}^{11}/_2$, $M' = {}^{17}/_2$

$F^1_3(-2, \frac{1}{2})$	$-\Lambda\Lambda\bar{n}$ $\Lambda\Lambda\bar{p}$
$F^1_3(-1, 0)$	$\{-(p\Lambda)\bar{p}-(n\Lambda)\bar{n}+2\Lambda\Lambda\bar{\Lambda}\}/2\sqrt{2}$
$F^1_3(-1, 1)$	$(p\Lambda)\bar{n}/\sqrt{2}$ $\{(p\Lambda)\bar{p}-(n\Lambda)\bar{n}\}/2$ $(n\Lambda)\bar{p}/\sqrt{2}$
$F^1_3(0, \frac{1}{2})$	$\{-2pp\bar{p}-(pn)\bar{n}+3(p\Lambda)\bar{\Lambda}\}/2\sqrt{6}$ $\{-(pn)\bar{p}-2nn\bar{n}+3(n\Lambda)\bar{\Lambda}\}/2\sqrt{6}$
$F^1_3(0, {}^3/_2)$	$-pp\bar{n}$ $\{pp\bar{p}-(pn)\bar{n}\}/\sqrt{3}$ $\{(pn)\bar{p}-nn\bar{n}\}/\sqrt{3}$ $nn\bar{p}$
$F^1_3(1, 1)$	$pp\bar{\Lambda}$ $(pn)\bar{\Lambda}/\sqrt{2}$ $nn\bar{\Lambda}$

Class 2. $M = {}^3/_2$, $M' = {}^5/_2$

$F^2_3(-1, 0)$	$\{(p\Lambda)\bar{p}+(n\Lambda)\bar{n}+2\Lambda\Lambda\bar{\Lambda}\}/2\sqrt{2}$
$F^2_3(0, \frac{1}{2})$	$\{2pp\bar{p}+(pn)\bar{n}+(p\Lambda)\bar{\Lambda}\}/2\sqrt{2}$ $\{(pn)\bar{p}+2nn\bar{n}+(n\Lambda)\bar{\Lambda}\}/2\sqrt{2}$

Class 3. $M = {}^7/_2$, $M' = \frac{1}{2}$

$F^3_3(-1, 1)$	$[p\bar{\Lambda}]\bar{n}/\sqrt{2}$ $\{[p\Lambda]\bar{p}-[n\Lambda]\bar{n}\}/2$ $[n\Lambda]\bar{p}/\sqrt{2}$
$F^3_3(0, \frac{1}{2})$	$\{[pn]\bar{n}+[p\Lambda]\bar{\Lambda}\}/2$ $\{[pn]\bar{p}+[n\Lambda]\bar{\Lambda}\}/2$
$F^3_3(1, 0)$	$[pn]\bar{\Lambda}/\sqrt{2}$

Class 4. $M = {}^3/_2$, $M' = {}^5/_2$

$F^4_3(-1, 0)$	$\{[p\Lambda]\bar{p}+[n\Lambda]\bar{n}\}/2$
$F^4_3(0, \frac{1}{2})$	$\{[pn]\bar{n}+[p\Lambda]\bar{\Lambda}\}/2$ $\{-[pn]\bar{p}+[n\Lambda]\bar{\Lambda}\}/2$

where $(AB) = A(x)B(y)+B(x)A(y)$

$[AB] = A(x)B(y)-B(x)A(y)$.

are two irreducible representations which we call class I and class II. Since both kaons and pions belong to class I, they must have the same mass, spin and parity in the limit of $m_N = m_\Lambda$. Thus, if it can be assumed that their kinematical properties (except mass value) do not change by adiabatically decreasing the mass of the Λ particle to the actual value, the kaon should be a pseudoscalar particle.

Here it is to be noted that there exists a neutral meson with zero isotopic spin in class I. Of course this is also a pseudoscalar meson if the above adiabatic process is allowed. We shall call it $\pi^{0\prime}$. Though experimental evidence for its existence has not yet been given, Sawada and Yonezawa [5] analyzed the energy spectrum of electrons in K_{e3} decay assuming the $\pi^{0\prime}$ mass to be about 350 MeV. The result is shown in Fig. 1, in which the $(V-A)$ type of the weak interaction is adopted.

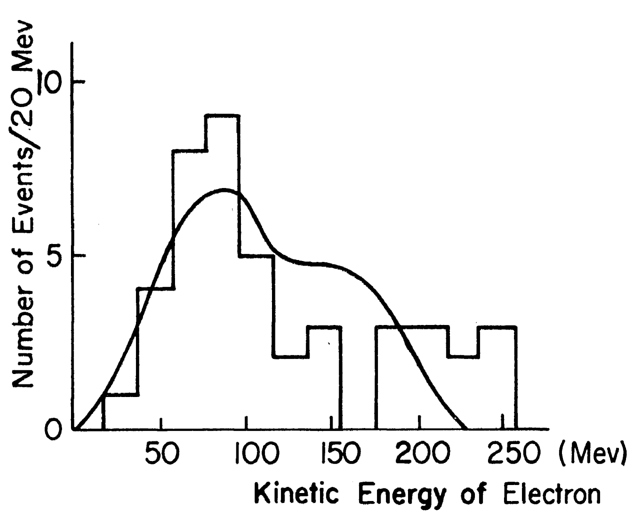

Fig. 1 K_{e3} decay spectrum.

If the $\pi^{0'}$ is not assumed, the maximum of the energy spectrum of the electron appears in the right half of Fig. 1. Obviously this is inconsistent with experimental data, which has two peaks in the right and left halves respectively. But now if the $\pi^{0'}$ exists, the kaon should have a decay mode K'_{e3} other than the usual K_{e3}. So the combined spectrum of both decay modes gives the curve in Fig. 1, where the first peak is due to the K'_{e3}. This may be a fine improvement on the $V-A$ theory. In the case of the usual K_{e3} only, the χ^2 probability is 0.0004, whereas in the combined case of K_{e3} and K'_{e3} it becomes about 0.4.

Next we shall mention other properties of this meson. Its main decay modes are thought to be

$$\pi^{0'} \to \pi^+ + \pi^- + \gamma$$

and (3.4)

$$\pi^{0'} \to 2\gamma$$

If the first decay would occur, the $\pi^{0'}$ could be easily observed. But, according to a perturbation calculation, the decay rate is given by

$$\frac{w(\pi^{0'} \to \pi^+ + \pi^- + \gamma)}{w(\pi^{0'} \to 2\gamma)} \simeq 4.2 \times 10^{-4} \qquad (3.5)$$

So, the most of $\pi^{0'}$ mesons should decay into 2γ, and therefore in low energy phenomena its observation may be difficult.

For a one-baryon-one-antibaryon system, there is another irreducible representation (class II), to which only one state belongs. But due to the kinematical aspect of our discussions we cannot make clear whether or not it is a stable state.

Now we shall enter the examination of two-baryon-one antibaryon systems (Table II). In this case there are four irreducible representations called classes 1, 2, 3, and 4 respectively. Classes 2 and 4 have the same quantum numbers N_B, M and M' as those of proton, neutron and Λ particle. Therefore the basic vectors of these classes may be regarded as the higher configurations of the single basic particles.

The state with isotopic spin 3/2 and strangeness 0 in class 1 will be the so-called (3/2, 3/2) resonating state in π-N scattering.

The state with isotopic spin 1/2 and strangeness -2 appears also in this class; it can be regarded as Ξ particle, and its spin may be 3/2, since it will have the same spin as the (3/2, 3/2) state, if the "adiabatic" process is allowed.

We shall take the state with isotopic spin 1/2 and strangeness -1 in class 3 as the Σ-particle, and regard all other states as resonances in K-N and π-N scatterings.

In fact, Sawada and Yonezawa [6] have examined them by using a semi-empirical mass formula, given already by Matsumoto [7], which is written

$$\begin{aligned}\mathbf{M} = &(n_N + n_{\bar{N}})m_N + (n_{N\bar{N}} - n_{NN} - n_{\bar{N}\bar{N}})V(N\bar{N}) + \\ &+ (n_\Lambda + n_{\bar{\Lambda}})m_\Lambda + (n_{N\bar{\Lambda}} + n_{\Lambda\bar{N}} - n_{N\Lambda} - n_{\bar{N}\bar{\Lambda}})V(N\bar{\Lambda}) + \\ &+ (n_{\Lambda\bar{\Lambda}} - n_{\Lambda\Lambda} - n_{\bar{\Lambda}\bar{\Lambda}})V(\Lambda\bar{\Lambda}),\end{aligned}$$

$$V(N\bar{\Lambda}) = V(N\bar{N}) + \Delta V, \quad V(\Lambda\bar{\Lambda}) = V(N\bar{N}) + 2\Delta V,$$

$$\Delta V = 182\,\text{MeV}, \quad V(N\bar{N}) = -1740\,\text{MeV} \qquad (3.6)$$

where $n_N(n_\Lambda)$ and $n_{\bar{N}}(n_{\bar{\Lambda}})$ are the numbers of nucleon (Λ particle) and antinucleon (anti Λ-particle) respectively and $n_{N\bar{N}}$, etc., are the numbers of nucleon-antinucleon pairs in the composite state. In deriving this formula, Matsumoto postulated a short range repulsive force between two baryons (two antibaryons) and an attractive force with almost the same magnitude between a baryon and an antibaryon; the unknown parameters V and ΔV were determined to give the actual masses of the pion and the kaon. Then we see that the expectation values of $\mathbf{M}$ calculated by using the states in classes 1 and 3 almost correspond to the resonant levels observed in π-N and K-N scatterings. They are given in Figs. 2, 3 and 4. Of course, our discussions are phenomenological, but the results so far obtained seem to suggest to us that our approach has a close connection with reality.

4. We have so far been concerned only with the strongly interacting particles, but not with leptons. Moreover, even in the case of the strong interactions, the charge independence or the full symmetry among the three basic particles has been introduced in a formal way. Why should the strong interaction have such invariance properties? To answer this question and, further, to describe both baryon and lepton in a unified way, we now go beyond Sakata's model.

At the Kiev conference last year, Gamba, Marshak and Okubo [8] pointed out the interesting fact that the weak interaction is invariant under the simultaneous transformations

$$p \leftrightarrow \nu, \ n \leftrightarrow e^-, \ \Lambda \leftrightarrow \mu^- \tag{4.1}$$

We shall call this the G.M.O. symmetry. The existence of the G.M.O. symmetry tells us that with respect to the weak interactions the proton, neutron and Λ-particle, which are basic particles in Sakata's model, have similar properties to those of the neutrino, electron and muon respectively. In other words, it suggests a connection between the baryon and the lepton.

If we imagine that p, n, and Λ are the compound states of positively charged matter B^+ [9] with ν, e^- and μ^- respectively, all weak interactions will be reduced to those of leptons, and the strongly

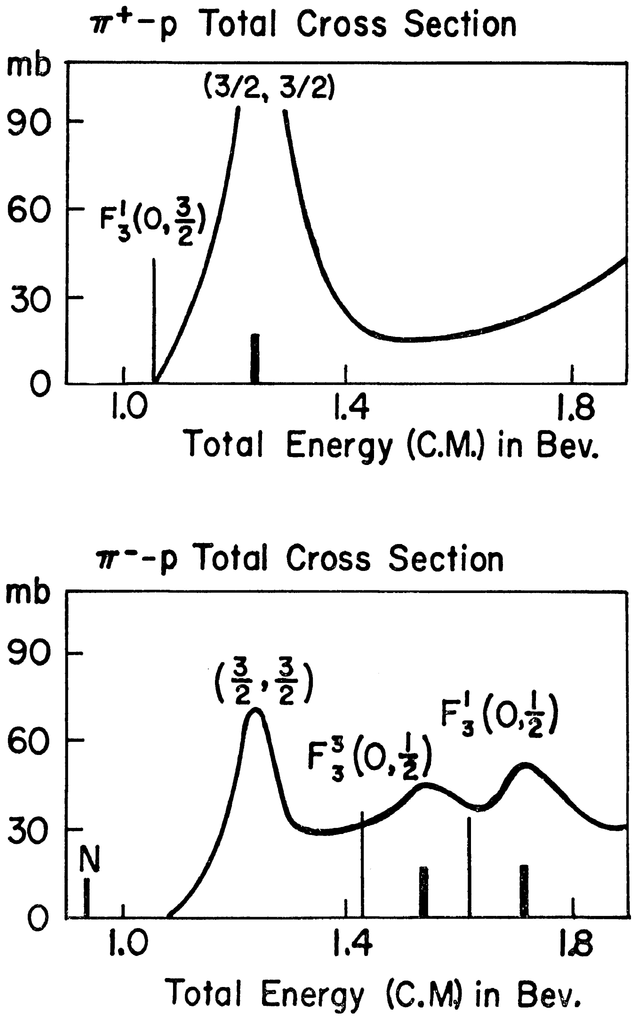

Fig. 2 $\pi^\pm p$ total cross sections. Thick and thin lines indicate the experimental levels of resonance and the levels given by the mass formula respectively.

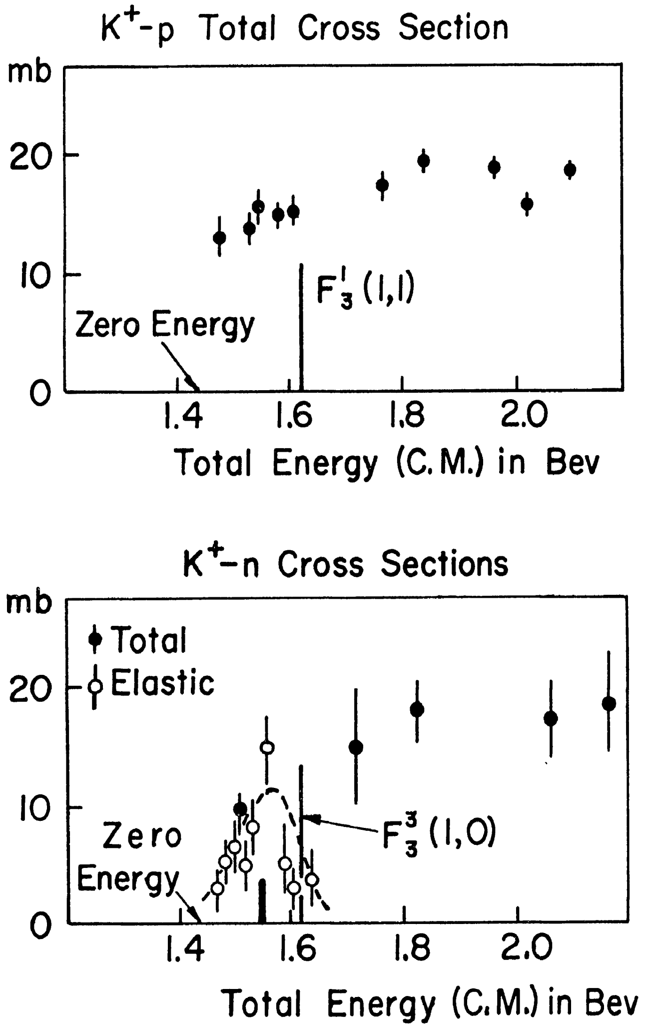

Fig. 3 K^+p total cross sections. Thick and thin lines indicate the experimental levels of resonance and the levels given by the mass formula respectively.

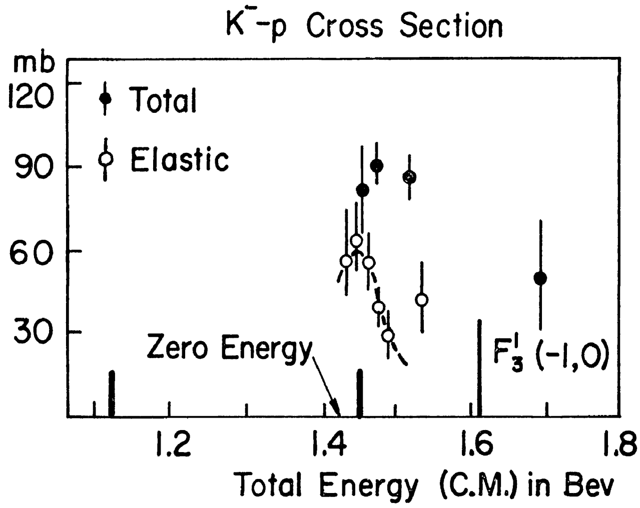

Fig. 4 K^-p cross sections. Thick and thin lines indicate the experimental levels of resonance and the levels given by the mass formula respectively.

interacting properties of the baryon will be regarded as a property of B^+. At the present time, however, the nature of B^+ as well as the mechanism by which B^+ is bound to the lepton are entirely unknown. Indeed, it is an open question whether B^+ behaves like ordinary matter or not. But independent of its real nature, B^+ should have the following properties:

(I) B^+ is bound to only one lepton, but not to any antilepton. (II) Separation of a baryon into B^+ and lepton is very hard or may be impossible. (III) All of the particle-like properties of the baryon (spin, statistic and so on) come from those of the lepton. The role of B^+ within the baryon is to make it massive and strongly interacting. (IV) B-matter must be conserved for all processes. This accounts for the conservation of the baryon number.

Now we shall discuss the weak interaction. Here it is assumed that the lepton is the only source of weak interactions; we describe the primary interaction by an expression of the form:

$$\mathrm{H}_{\mathrm{weak}} = j_\mu j_\mu^\dagger \tag{4.2}$$

where

$$j_\mu = f\{(e^-\gamma_\mu(1+\gamma_5)\nu)+(\mu^-\gamma_\mu(1+\gamma_5)\nu)\} \tag{4.3}$$

If B^+ is bound to the lepton, j_μ takes the form

$$J_\mu = \langle j_\mu \rangle_B = f\{(\bar{n}\gamma_\mu(1+\gamma_5)p)+(\bar{\Lambda}\gamma_\mu(1+\gamma_5)p)\}$$

Accordingly, the effective Hamiltonian including both baryon and lepton will turn out to be

$$\mathrm{H}_{\mathrm{weak}}^{\mathrm{eff}} = \mathbf{J}_\mu \mathbf{J}_\mu^\dagger \tag{4.4}$$

with effective current

$$\mathbf{J}_\mu = j_\mu + J_\mu . \tag{4.5}$$

On the other hand, through the phenomenological analysis of the weak interactions we already know the Hamiltonian [10]

$$\mathrm{H}_{\mathrm{weak}} = \tilde{\mathbf{J}}_\mu \tilde{\mathbf{J}}_\mu^\dagger$$

$$\tilde{\mathbf{J}}_\mu = f_1(\bar{\nu}\gamma_\mu(1+\gamma_5)e)+f_2(\bar{\nu}\gamma_\mu(1+\gamma_5)\mu^-)+$$
$$+f_3(\bar{p}\gamma_\mu(1+\gamma_5)n)+f_4(\bar{p}\gamma_\mu(1+\gamma_5)\Lambda)$$

and (4.6)

$$f_1 \simeq f_2 \simeq f_3 , \left(\frac{f_4}{f_1}\right)^2 \simeq \frac{1}{10} \simeq \frac{1}{100} .$$

The non-leptonic part of Eq. (4.6) does not satisfy the iso-selection rule $\Delta I = \frac{1}{2}$, and it seems difficult to account for the branching ratio in $K_{\pi 2}$ decay.[11] But recently Oneda, Pati and Sakita [12] have shown a possibility of explaining it. We shall regard Eq. (4.6) as the phenomenological Hamiltonian for the weak interactions. According to Eq. (4.6) the coupling constant f_4 is smaller than the others. Therefore Eq. (4.4) should be an approximate Hamiltonian. We may, however, expect that such differences among the coupling constants will be explained, for instance, by taking into account the unknown mechanism of binding B^+ to the lepton, which is not considered here.

Now we shall discuss the strong interactions. In this case, it is assumed that the strength of the strong interactions among the basic baryons should be determined by the nature of B-matter, and be independent or almost independent of the kind of lepton to which B is bound. Moreover, the lepton in the baryon cannot change its kind through the strong interaction; such a transition happens only when the lepton suffers a weak interaction. Then it is conceivable that, by virtue of the property (II) of B^+, the currents for each basic particle

$$I_p = (\bar{p}Op)$$
$$I_n = (\bar{n}On)$$
$$I_\Lambda = (\bar{\Lambda}O\Lambda)$$

play an essential role in providing the strong interaction. Graphically this is represented in Fig. 5

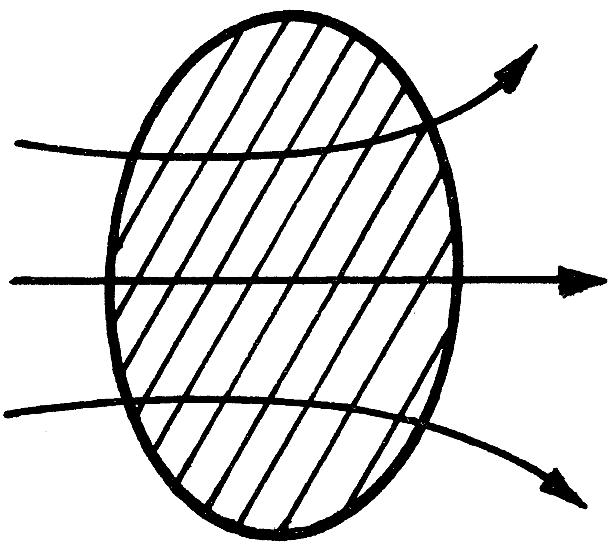

Fig. 5 The strong interaction.

where each arrow line stands for a baryon current, and it cannot make transition to the other through the strong interactions. From these assumptions one can easily conclude that the effective Hamiltonian of the strong interactions can be expressed by a function of $(I_p+I_n+I_A)$. Then the resulting Hamiltonian satisfies not only charge independence, but also the full symmetry among the basic baryons in Sakata's model.

These properties have hitherto been explained on the basis of the hypothetical charge space, whereas in this model such symmetries are a direct consequence derived from the simple properties of B-matter.

Now we shall mention the type of strong interaction, which is a function of $\{(\bar{p}Op)+(\bar{n}On)+(\bar{\Lambda}O\Lambda)\}$. We can easily show that, if O is the scalar, axial vector or pseudoscalar type of Dirac matrix, the pion cannot be constructed. In this case of S, A or P type, we can write $\{(\bar{p}Op)+(\bar{n}On)+(\bar{\Lambda}O\Lambda)\}$ in the form

$$\{(\bar{p}Op)+(\bar{n}On)+(\bar{\Lambda}O\Lambda)\}=\frac{1}{2}\sum_{i=1}^{4}(\psi_i\beta O\psi_i)+(\bar{\Lambda}O\Lambda) \tag{4.7}$$

where ψ_i is the i^{th} component of the real field ψ,

$$\psi=\left|\begin{array}{c} p_2(x) \\ p_1(x) \\ -n_2(x) \\ -n_1(x) \end{array}\right| \tag{4.8}$$

and p_i, $n_i (i=1, 2)$ are the Majorana fields defined by

$$p(x)=\frac{p_1(x)+ip_2(x)}{\sqrt{2}}, \quad n(x)=\frac{n_1(x)+in_2(x)}{\sqrt{2}} \tag{4.9}$$

Here we have used the Majorana representation for the Dirac matrix O. On the other hand, the free Lagrangian is

$$\mathfrak{L}^0=-\frac{1}{2}\sum_{i=1}^{4}(\psi_i\beta[\gamma_\mu\partial_\mu+m]\psi_i)-(\bar{\Lambda}[\gamma_\mu\partial_\mu+m]\Lambda). \tag{4.10}$$

Therefore, in the case of S, A or P type interaction the theory is invariant under rotations in a 4-dimensional Euclidean space:

$$\psi_i'=\sum_j R_{ij}\psi_j, \quad \text{where } \sum_i R_{ij}R_{ik}=\delta_{jk} \tag{4.11}$$

(R_{ij} real)

That is ψ is a vector in the representation space $D_{\frac{1}{2},\frac{1}{2}}$. Then it can be shown that the 3-dimensional space corresponding to one of the suffixes of $D_{\frac{1}{2},\frac{1}{2}}$ is the usual isospace. Now we shall call the 3-dimensional space corresponding to the other suffix of $D_{\frac{1}{2},\frac{1}{2}}$ the K-space,[13] whose generators are denoted by $K_i (i=1, 2, 3)$. Then K_3 is one half of the nucleon number, and the charge conjugation operator is expressed as

$$C=\exp\{i\pi(\tau_2+K_2)\} \tag{4.12}$$

where τ_2 is the second component of the isotopic spin vector. From this it follows that through the S, A or P type interaction the pion cannot be constructed. In fact, if it could be made, it should have eigenvalues $T=1$ and $K=0$, because if $K=1$, the invariance of the K-space rotation leads us to a state with pion mass, whose nucleon number is two. But a state with $T=1$ and $K=0$ cannot be identified with the pion either. To see this, let us consider the state with $T=1$, $T_3=0$, $K=0$, which would be a π^0-meson. Then its charge conjugation parity should be odd, since as is seen from Eq. (4.12), charge conjugation is a rotation through π around the second axis of the isospace multiplied by a rotation through π around the second axis of the K-space. Therefore it cannot decay into two photons, so it is not a π^0 meson. Thus, it can be said that vector or tensor type interaction plays an important role to provide the pion in Sakata's model.

In fact, in the recent work by Matsumoto and Nakagawa[14] to understand the mass formula on the basis of the B-matter, the vector type interaction is postulated although their work is essentially based on a classical picture.

By neglect of Δm and ΔV the main part of the mass formula for a compound state Eq. (3.7) is reduced to

$$\mathbf{M}=m_B(n_B+n_{\bar{B}})-V(n_{B\bar{B}}-n_{BB}-n_{\bar{B}\bar{B}}) \tag{4.13}$$

This formula can be derived under the following assumptions:

(1) B-matter has a charge-like[15] character and distributions in ν, e and μ which are like spheres of radius l_0. (II) There exists a very short range ($\ll l_0$) interaction among B-matters, which is given by

$C\bar{C}\int \rho(\mathbf{x})v(\mathbf{x}-\mathbf{x}')\bar{\rho}(\mathbf{x}')d\mathbf{x}d\mathbf{x}'$ between B and anti$B$ matters,

$CC\int \rho(\mathbf{x})v(\mathbf{x}-\mathbf{x}')\rho(\mathbf{x}')d\mathbf{x}d\mathbf{x}'$ between B matters.

$\bar{C}\bar{C}\int \bar{\rho}(\mathbf{x})v(\mathbf{x}-\mathbf{x}')\bar{\rho}(\mathbf{x}')d\mathbf{x}d\mathbf{x}'$ between anti$B$ matters,

$$\bar{C} = -C, \tag{4.14}$$

where ρ, $\bar{\rho}$ denote densities of B and anti$B$ matter respectively and $v(\mathbf{x}-\mathbf{x}')$ is a very short range interaction. (III) The energy of the ground state of the compound particle is given by the classical minimum energy.

It is seen that the relativistic and quantum mechanical interaction corresponding to Eq. (4.14) is of the vector type discussed before.

From these assumptions we get easily the baryon mass

$$m_B = \frac{1}{2}C^2\int \rho(\mathbf{x})v(\mathbf{x}-\mathbf{x}')\rho(\mathbf{x}')d\mathbf{x}d\mathbf{x}' \simeq \frac{3\alpha}{8\pi}\frac{C^2}{l_0} \tag{4.15}$$

if $\quad v(\mathbf{x}-\mathbf{x}') = \alpha l_0^2\delta(\mathbf{x}-\mathbf{x}')$.

Furthermore the interaction energy between the i^{th} baryon (or antibaryon) and the j^{th} baryon (or antibaryon) is

$$V^{ij}(\mathbf{x}_i-\mathbf{x}_j) = C^iC^j\int d\mathbf{x}_i d\mathbf{x}_j \rho_i(\mathbf{x}_i)v(\mathbf{x}_i-\mathbf{x}_j)\rho_j(\mathbf{x}_j) \simeq$$

$$\simeq \begin{cases} V^{ij}(0)\left[\dfrac{1}{16}\dfrac{|\mathbf{X}_i-\mathbf{X}_j|}{l_0^3}-\dfrac{3}{4}\dfrac{|\mathbf{X}_i-\mathbf{X}_j|}{l_0}+1\right] & \text{for } |\mathbf{X}_i-\mathbf{X}_j|<2l_0, \\ 0 & \text{for } |\mathbf{X}_i-\mathbf{X}_j|>2l_0 \end{cases} \tag{4.16}$$

if $\rho(\mathbf{x})$ is assumed to be uniform. Here $\mathbf{X}_i$ is the center of mass coordinate of the i^{th} baryon (or antibaryon). Then the classical minimum of the total energy becomes

$$E_{\min} = \sum_{i=1}^{n} m_i + \sum_{i<j}^{n} V^{ij}(0). \tag{4.17}$$

Since $\quad V^{ij}(0) = 2m_B\varepsilon^{ij}$,

$$\varepsilon^{ij} = \begin{cases} 1, & \text{if both } i \text{ and } j \text{ indicate baryons or antibaryons,} \\ -1 & \text{if } i \text{ and } j \text{ indicate baryon and antibaryon respectively.} \end{cases} \tag{4.18}$$

Eq. (4.17) is just equivalent to the formula (4.13). This suggests that the vector type interaction is essential on the one hand and the quantum mechanical effects would be suppressed at small distance on the other, because, as is seen above, a classical treatment of the energy is essentially important to derive the mass formula.

LIST OF REFERENCES AND NOTES

1. Fermi, E. and Yang, C. N. Phys. Rev. **76**, p. 1739 (1948). Calculations for the compound state have been done by many authors : Tanaka, S. Progr. Theoret. Phys. **16**, pp. 625 and 631 (1956). Maki, Z. Progr. Theoret. Phys. **16**, p. 667 (1956). Polubarinov, I. V. Nuclear Phys. **8**, p. 44 (1958). Ihara, C. and Hatano, S. Progr. Theoret. Phys. **20**, p. 356 (1958). Yamamoto, H. Progr. Theoret. Phys. **23**, p. 110 (1960).
2. Gell-Mann, M. and Pais, A. Proceedings of the Glascow Conference 1954 (London). Nakano, T. and Nishijima, K. Progr. Theoret. Phys. **10**, p. 581 (1953).
3. Sakata, S. Progr. Theoret. Phys. **16**, p. 686 (1956).
4. Ogawa, S. Progr. Theoret. Phys. **21**, p. 209 (1959). Ikeda, M., Ogawa, S. and Ohnuki, Y. Progr. Theoret. Phys. **22**, p. 715 (1959). Yamaguchi, Y. Progr. Theoret. Phys. Suppl. **11**, pp. 1 and 37 (1959). From another viewpoint the following investigations on the symmetry of three fields were made : Thirring, W. Proc. 9th Conf. High Energy Nucl. Phys., Kiev (1959). Wess, J. Nuovo Cimento **15**, p. 52 (1960).
5. Sawada, S. and Yonezawa, M. Progr. Theoret. Phys. **22**, p. 610 (1959).
6. Sawada, S. and Yonezawa, M. Progr. Theoret. Phys. **23**, p. 662 (1960).
7. Matsumoto, K. Progr. Theoret. Phys. **16**, p. 583 (1956).
8. Gamba, A., Marshak, R. E. and Okubo, S. Proc. 9th Conf. High Energy Nucl. Phys., Kiev (1959); Proc. Nat. Acad. Sci. **45**, p. 881 (1959).
9. Maki, Z., Nakagawa, M., Ohnuki, Y. and Sakata, S. Progr. Theoret. Phys. **23**, p. 1174 (1960).
10. Okubo, S., Marshak, R. E. and Sudarshan, E. C. G. Phys. Rev. **113**, p. 944 (1959); Okun', L. B. JETP **34**, p. 476 (1958).
11. Maki, Z. Progr. Theoret. Phys. **23** (1960); Sakita, B. Phys. Rev. **114**, p. 1959 (1959).
12. Oneda, S., Pati, J. C. and Sakita, B. (Preprint).
13. Hara, O., Fujii, Y. and Ohnuki, Y. Progr. Theoret. Phys. **19**, p. 129 (1958).
14. Matsumoto, K. and Nakagawa, M. Progr. Theoret. Phys. **23**, p. 1181 (1960).
15. A charge-like character of B-matter was first proposed by Taketani, M., comment at the " Symposium of the Weak Interaction " held at the Research Institute for Fundamental Physics (Kyoto) in Dec. 1959.
Taketani, M. and Katayama, Y. Progr. Theoret. Theor. Phys. (to be published).

DISCUSSION

NOYES: I wonder if there is an experimentalist here who is familiar with the details of the K_{e3} spectrum? As I recall, most methods have a very strong experimental bias against measurements of high energy electrons in this decay. I wonder if someone can make an experimental comment on that point, as the evidence given for a 350 MeV $\pi^{0\prime}$ depends entirely on whether or not such bias is present.

HEISENBERG: There seems to be no answer.

FEINBERG: I would like to make a comment on another point. The $\pi^{0\prime}$ could be looked for by the same type of experiment that was described the other day, to look for violations of isotopic spin conservation, in particular by the reaction $d+d \to \alpha+\pi^{0\prime}$. If the $\pi^{0\prime}$ interacts strongly, then one would expect that the cross section for this, well above threshold, would be, maybe, 10^{-29} cm^2, which is much larger than the cross section for production of α+photon, so I think finding this particle, if it exists, might be possible by such an experiment.

(The following remark by Dr. Chamberlain was added after the discussion.)

CHAMBERLAIN: If the $\pi^{0\prime}$ mass is greater than two pion masses, it will be difficult to distinguish its production from double π^0 production in the $d+d$ reaction.

RECENT RESEARCH ON THE NONLINEAR SPINOR THEORY OF ELEMENTARY PARTICLES

W. Heisenberg

Max-Planck-Institut für Physik und Astrophysik, München, Germany

During the last year research on the nonlinear spinor theory has been carried out in Munich mainly along three lines which may be characterized by the three topics: indefinite metric in Hilbert space, group theory, and approximation methods for the calculation of eigenvalues; some work has also been done on the analytical behavior of matrix-elements.

1. INDEFINITE METRIC IN HILBERT SPACE

(a) The question of whether the probability interpretation of quantum theory is compatible with the use of an indefinite metric in Hilbert space has been taken up from a general mathematical viewpoint in three papers by Schlieder. Starting from given symmetry groups Schlieder studies bilinear forms which are invariant under the given transformations, and constructs the corresponding fundamental metric tensor. He then states supplementary conditions which are sufficient for the probability interpretation of the asymptotic waves. While for finite and compact groups one comes back in this way essentially to the conventional theory, infinite and non-compact groups may lead to more general representations in a space with indefinite metric which still are compatible with the probability interpretation of the asymptotic waves. Besides the special cases that have been studied in connection with Quantum electrodynamics by Bleuler and Gupta, and in connection with the Lee model by the author of the present report, Schlieder mentions the case of systems in which superselection rules exist. These rules divide the space of states into different incoherent sectors. It is