The Search for the Most Symmetric Superstring

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ABSTRACT

In this talk I discuss the question what is the maximal symmetry one can find in a string theory. I report on work together with Cederwall and Preitschopf in which we study the superconformal invariance of superstring theory. We show that in $D=3,4$ and 6 it is invariant under an $N=D-2$ superconformal algebra based on $\widehat{S^{D-3}}$. There is a direct relationship between this (world-sheet) symmetry and the super-Poincaré (target space) symmetry. This relationship is established using the light-cone gauge. We then show how the statement generalizes to $D=10$ and examine the properties of the $N=8$ superconformal algebra and the possible implications of its existence. I give some discussion on why this question is important in string theory.

String theory is the most promising candidate for a unified model of all interactions and for a theory of quantum gravity. Much work has been done over the years to check string theory and we are slowly learning the dynamics of it. Much work remains though and we have now pushed it to the borderlines of existing mathematics. If it is really a fundamental theory, it should have unique properties in some sense, and it is this issue that I will have in mind.

In order to understand the physics of a certain model and to classify it, we need to find its symmetries. The crucial step in the Veneziano model [1] was the discovery of the Virasoro algebra [2], the infinite-dimensional conformal symmetry. It led to the no-ghost theorem, and in the modern era it has been used *e.g.* to construct string field theory. It was also the extension of this symmetry to the superconformal one by Ramond [3] that led to the Ramond-Neveu-Schwarz model [3,4] we use today. However, in this formalism global supersymmetry is somewhat hidden and appears only after the GSO-projection [5]. To remedy this fact, Green and Schwarz proposed their formulation [6], where space-time supersymmetry is manifest. The superconformal symmetry, however, is unconventional [7], and leads to difficulties when one tries to quantize the theory covariantly [8]. It is not directly related to the conventional extended superconformal structures [9].

A string theory could either be regarded as a d-dimensional theory or as a 2-dimensional one. Both formulations should be fundamental and there should be a very close relation between them. In the formulations of the superstring mentioned above the target space symmetries are divorced from the world-sheet ones. This is really not satisfactory. If there is a "fundamental string theory", we expect the two sets of symmetries to have a common origin. There should be two equivalent principles for formulating the theory. On the one hand, starting with enough physical requirements on the space-time theory, the world-sheet symmetries should follow, while on the other hand the correct assumptions about the two-dimensional physics on the world-sheet should give the right target space behaviour.

In this talk I will report on work done by Martin Cederwall, Christian Preitschopf and myself [10] in which we are searching for the maximal (2-dimensional) superconformal symmetry and how it is related to a maximal (d-dimensional) target space symmetry. The discussion will be somewhat technical which cannot be avoided. To make progress in string theory one has to use advanced mathematics. Before going into these details let me first ask what we could gain by understanding the full symmetry? After the technical discussion I will so come back to these questions.

(*i*) It could give us ways to understand non-perturbative issues in string theory. This is certainly the most important question now and any way that will help us here will be instrumental.

(*ii*) It could help us understand perturbative finitenss better. There are very strong arguments that the perturbation theory for superstrings is finite order by order [11,12]. A more economic formulation using the full symmetry would most certainly be helpful in establishing an alternative proof.

(*iii*) It could also help us understand the early universe. It is obvious that non-perturbative quantum gravity is very important for this understanding and it has been argued that string theory indicates that there was a phase transition at Planck times [13, 14]. We have so far gained no information about "the early phase". It would be of great importance if one could gain insight here and a new formulation might help us understand the relevant order parameters.

(*iv*) It could give us ways to screen between d=4 models. With the knowledge we have today there is a huge number of d=4 superstring models that are candidates and we need more constraints to be able to argue that one model is better than the other. An identification and an understanding of the maximal symmetry could help us here. It will then be important to understand how this maximal symmetry is reflected in the d=4 theories and what the physical implications of it are.

In our work we have found that in the light-cone gauge formulation of string theory there is indeed a correlation between the superconformal symmetry and the target space one so it is a very natural formalism for this discussion. The light-cone formulation can in fact be obtained by using either of two principles alluded to above. The resulting theory is described by the transverse coordinates and is explicitely unitary. The difficulties arise in the (super-)Poincaré algebra which is non-linearly implemented [15]. In the quantum case anomalies occur, unless the critical dimension is chosen.

The Green-Schwarz string [6] in the light-cone gauge is described by an action (in a heterotic form)

$$S = \frac{1}{\pi}\int dzd\overline{z}\,(\partial\varphi^I\overline{\partial}\varphi^I + S^a\overline{\partial}S^a) \tag{1}$$

The index $I = 0,\ldots,D-3$ is vectorial, while $a = 0,\ldots,D-3$ is spinorial. The action (1) is classically super-Poincaré invariant for $D = 3,4,6$ and 10 [15]. Quantum mechanically matter has to be added for $D=3,4$ and 6 in order to avoid anomalies. Already at this point we would like to remark that the potential occurrence of an anomaly in the super-Poincaré algebra points towards a close connection between some of its generators and the world-sheet symmetry that carries the anomaly in a covariant formulation.

Before going into details on the super-Poincaré and superconformal algebras, we would like to introduce some division algebra formalism, by now well known to be related to the space-times with $D=3,4,6$ and 10 and to the supersymmetric structures appearing in these dimensionalities [16-20]. We denote by $\mathbf{K}_\nu$ the division algebra of dimension ν: $\mathbf{K}_1=\mathbf{R}$, the reals, $\mathbf{K}_2=\mathbf{C}$, the complex numbers, $\mathbf{K}_4=\mathbf{H}$, the quaternions, and $\mathbf{K}_8=\mathbf{O}$, the octonions. In the following, ν and $D-2$ are exchangeable. Conjugation of an element $x\in\mathbf{K}_\nu$ is denoted x^*, not to confuse with the complex structure of the world-sheet. Division algebra multiplication encodes the Clifford algebra of transverse space-time. So for example, is the equation $c^*=vs$ equivalent to $c_{\dot a}=v^I\gamma^I_{\dot a a}s_a$, where $v\in 8_v$, $s\in 8_s$ and $c\in 8_c$ of $SO(8)$, and analogously for lower dimensionalities. We also use the notation $[x]=\frac{1}{2}(x+x^*)$ and $\{x\}=\frac{1}{2}(x-x^*)$. Structure constants and associator coefficients are defined by $[e_i,e_j]=2\sigma_{ijk}e_k$, $[e_i,e_j,e_k]=2\rho_{ijkl}e_l$, where $\{e_i, i=1,\ldots,7\}$ are the imaginary units.

The action (1) can trivially be rewritten in this notation as

$$S=\frac{1}{\pi}\int dzd\bar z\,[\,\partial\varphi^*\bar\partial\varphi+S^*\bar\partial S\,] \tag{2}$$

In $D=3,4$ and 6, the light-cone superstring action is invariant under an $N=D-2$ extended superconformal algebra [20]. We only consider the part of the generators containing holomorphic fields, with

$$\begin{aligned}\varphi^I(z)\varphi^J(\zeta)&\sim\delta^{IJ}\ln(z-\zeta)\\ S^a(z)S^b(\zeta)&\sim\frac{\delta^{ab}}{z-\zeta}\end{aligned} \tag{3}$$

The generators of the algebras are [20]

$$\begin{aligned}\mathcal{J}&=\tfrac{1}{2}S^*S\\ \mathcal{G}&=\partial\varphi S\\ \mathcal{L}&=\tfrac{1}{2}[\partial\varphi^*\partial\varphi-S^*\partial S]\end{aligned} \tag{4}$$

with the operator product expansions

$$
\begin{aligned}
\mathcal{J}_\alpha(z)\mathcal{J}_{\alpha'}(\zeta) &\sim \frac{c/3}{(z-\zeta)^2}[\alpha\alpha'] + \frac{2}{z-\zeta}\mathcal{J}_{\alpha\alpha'} \\
\mathcal{J}_\alpha(z)\mathcal{G}_\Omega(\zeta) &\sim -\frac{1}{z-\zeta}\mathcal{G}_{\Omega\alpha} \\
\mathcal{G}_\Omega(z)\mathcal{G}_{\Omega'}(\zeta) &\sim \frac{2c/3}{(z-\zeta)^3}[\Omega^*\Omega'] + \frac{2}{(z-\zeta)^2}\mathcal{J}_{\Omega^*\Omega'}(\zeta) + \frac{1}{z-\zeta}(\partial\mathcal{J}_{\Omega^*\Omega'} + 2\,[\Omega^*\Omega']\mathcal{L}) \\
\mathcal{L}(z)\mathcal{J}(\zeta) &\sim \frac{1}{(z-\zeta)^2}\mathcal{J}(\zeta) + \frac{1}{z-\zeta}\partial\mathcal{J} \\
\mathcal{L}(z)\mathcal{G}(\zeta) &\sim \frac{3/2}{(z-\zeta)^2}\mathcal{G}(\zeta) + \frac{1}{z-\zeta}\partial\mathcal{G} \\
\mathcal{L}(z)\mathcal{L}(\zeta) &\sim \frac{c/2}{(z-\zeta)^4} + \frac{2}{(z-\zeta)^2}\mathcal{L}(\zeta) + \frac{1}{z-\zeta}\partial\mathcal{L}
\end{aligned}
\tag{5}
$$

where fields in ν- or $(\nu-1)$-dimensional representations are given indices by contractions $X_a = [a^*X]$, and where the anomaly c with these minimal field contents take the value $3\nu/2$. The Kac-Moody part of this algebra is $\widehat{S^{\nu-1}}$. A similar statement applies to $D=10$, as we will soon describe. By examining the gauge-fixing procedure of the Green-Schwarz superstring [6] to the light-cone gauge, and in particular the non-linear realization of the super-Poincaré algebra, we will give an interpretation of the world-sheet superconformal algebra in terms of space-time supersymmetry.

Consider the constraints derived from the Green-Schwarz action for the left-moving variables of a heterotic string (in $SL(2;\mathbf{K}_\nu)$-notation)[8]:

$$
\begin{aligned}
D_\alpha &\equiv \pi_{\dot\alpha} - \frac{1}{\sqrt{2}}\Pi_{\dot\alpha\alpha}\theta^\alpha + \tfrac{1}{4}(\theta\partial\theta^\dagger - \partial\theta\theta^\dagger)_{\dot\alpha\alpha}\theta^\alpha \approx 0 \\
L &\equiv \tfrac{1}{4}\Pi_{\dot\alpha\alpha}\Pi^{\alpha\dot\alpha} \approx 0
\end{aligned}
\tag{6}
$$

where $\Pi = \partial\varphi + \frac{1}{\sqrt{2}}(\theta\partial\theta^\dagger - \partial\theta\theta^\dagger)$ and π is the conjugate momentum of θ. The special property of the spinorial constraint is that it contains an equal number of first and second class constraints [7,8]. When a light-cone gauge is chosen, the separation comes about naturally, splitting the $SO(1,D-1)$ spinor into two spinors of the transverse group. The light-cone gauge amounts to choosing

$$
\begin{aligned}
\varphi^+(z) &= \alpha^+ \ln z \\
\theta^1 &= 0
\end{aligned}
\tag{7}
$$

The remaining part of the spinorial constraint reads $\pi^2 + \frac{\alpha^+}{z}\theta^2 \approx 0$ and is obviously second class. When we eliminate it and define $S = \sqrt{\frac{2\alpha^+}{z}}\theta^2$, the spinor correlator in eq.(3) is recovered. Then, one can solve for φ^- and π^1 through eq.(6) to obtain

$$\begin{aligned}\partial\varphi^- &= \frac{z}{2\alpha^+}[\partial\varphi^*\partial\varphi - S^*\partial S] \\ \pi^1 &= \frac{1}{2}\sqrt{\frac{z}{\alpha^+}}\partial\varphi S\end{aligned} \tag{8}$$

When we now go back to the superconformal generators of eq. (4), we notice that the variables eliminated by the gauge choices are exactly the superconformal generators $\mathcal{L}$ and $\mathcal{G}$. From the light-cone variables, we can construct the now non-linearly realized super-Poincaré generators. The crucial part, *i.e.* the part where anomalies may appear, contains P^-, J^{+-}, J^- and Q^-, the generators that take us out of the quantization surface. The complete set of generators is [15]

$$\begin{aligned}
P^+ &= \alpha^+ \\
P^- &= \frac{1}{2\alpha^+}\oint\frac{dz}{2\pi i}\, z\,[\partial\varphi^*\partial\varphi - S^*\partial S] = \frac{\mathcal{L}_0}{\alpha^+} \\
P &= p \\
J^{+-} &= x^+\frac{1}{2\alpha^+}\oint\frac{dz}{2\pi i}\, z\,[\partial\varphi^*\partial\varphi - S^*\partial S] + \alpha^+\frac{\partial}{\partial\alpha^+} = \frac{x^+}{\alpha^+}(\mathcal{L}_0 - \tfrac{1}{2}) + \alpha^+\frac{\partial}{\partial\alpha^+} \\
J^+ &= x^+p - \alpha^+x \\
J^- &= -p\frac{\partial}{\partial\alpha^+} - \frac{1}{2\alpha^+}\oint\frac{dz}{2\pi i}\, z\left(\tilde{\varphi}\,[\partial\varphi^*\partial\varphi - S^*\partial S - \frac{1}{z^2}] - \tfrac{1}{2}(\partial\varphi S)S^*\right) = \\
&= -p\frac{\partial}{\partial\alpha^+} - \frac{1}{\alpha^+}\oint\frac{dz}{2\pi i}\, z\left(\tilde{\varphi}(\mathcal{L} - \frac{1}{2z^2}) - \tfrac{1}{4}\mathcal{G}S^*\right) \\
J^{IJ} &= 2x^{[I}p^{J]} + \oint\frac{dz}{2\pi i}\left(\tilde{\varphi}^I\partial\tilde{\varphi}^J + \tfrac{1}{4}[S^*e_I^*(e_J S)]\right) \\
Q^+ &= 2^{1/4}\sqrt{\alpha^+}\oint\frac{dz}{2\pi i}\, z^{-1/2}S \\
Q^- &= \frac{1}{2^{1/4}\sqrt{\alpha^+}}\oint\frac{dz}{2\pi i}\, z^{1/2}\partial\varphi S = \frac{1}{2^{1/4}\sqrt{\alpha^+}}\mathcal{G}_0
\end{aligned} \tag{9}$$

(transverse indices are again suppressed, so that J^- contains the components J^{-I} *etc.*). For convenience, we have separated out the logarithmic mode of φ^μ according to $\tilde{\varphi}^\mu(z) = \varphi^\mu(z) - \ln z\oint\frac{d\zeta}{2\pi i}\partial\varphi^\mu(\zeta)$, and defined $x^\mu = \oint\frac{dz}{2\pi i z}\tilde{\varphi}^\mu(z)$. The important point is that knowledge of the super-Poincaré generators provides us with information about the superconformal generators, and vice versa. Explicit calculation of the anomaly in $[J^{-I}, J^{-J}]$ of course gives the result $\nu = 8$.

Compactification to $D=6$ yields the following changes:

$$
\begin{aligned}
P^- &= \frac{\mathcal{L}_0 + \mathcal{L}_0^{int}}{\alpha^+} \\
J^{+-} &= \frac{x^+}{\alpha^+}(\mathcal{L}_0 + \mathcal{L}_0^{int} - \tfrac{1}{2}) + \alpha^+ \frac{\partial}{\partial \alpha^+} \\
Q^- &= \frac{1}{2^{1/4}\sqrt{\alpha^+}}(\mathcal{G}_0 + \mathcal{G}_0^{int}) \\
J^- &= -p\frac{\partial}{\partial \alpha^+} - \frac{1}{\alpha^+}\oint \frac{dz}{2\pi i} z\Big(\tilde{\varphi}(\mathcal{L} + \mathcal{L}^{int} - \frac{1}{2z^2}) - \tfrac{1}{4}(\mathcal{G} + 2\mathcal{G}^{int})S^* + \\
&\qquad + \tfrac{1}{2}\mathcal{J}^{int}\partial\varphi\Big) \\
J^{IJ} &= 2x^{[I}p^{J]} + \oint \frac{dz}{2\pi i}\left(\tilde{\varphi}^I \partial \tilde{\varphi}^J + \tfrac{1}{4}[S^* e_I^*(e_J S)] + \tfrac{1}{2}[e_I^* e_J \mathcal{J}^{int}]\right)
\end{aligned}
\tag{10}
$$

where $\mathcal{J}^{int}$, $\mathcal{G}^{int}$ and $\mathcal{L}^{int}$ is an $N=4$ superconformal algebra for the internal degrees of freedom, and we are working with quaternions instead of octonions. An anomaly-free theory arises if $c^{int}=6$ and if the nullmode condition

$$
\oint \frac{dz}{2\pi i} z\Big(\mathcal{J}_I^{int}\mathcal{J}_K^{int} - \tfrac{1}{3}\delta_{IK}\mathcal{J}_L^{int}\mathcal{J}_L^{int}\Big) = 0 \tag{11}
$$

is satisfied. We note that while $\mathcal{J} = \frac{1}{2}S^*S$ contains the antiselfdual combination of spinors, we find in J^{IJ} the selfdual combination. Hence the internal algebra has the structure corresponding to an antiselfdual multiplet, while the transverse spacetime algebra corresponds to a selfdual multiplet. These two $N=4$ algebras are independent, and when one goes about constructing sigma-models, one will have to consider two independent hyperkähler structures, one in the internal sector and one in the noncompact sector.

If we compactify down to $D=4$, we obtain the same operators as in (10), except for J^-, which now reads

$$
\begin{aligned}
J^- = -p\frac{\partial}{\partial \alpha^+} - \frac{1}{\alpha^+}\oint \frac{dz}{2\pi i} z\Big(&\tilde{\varphi}(\mathcal{L} + \mathcal{L}^{int} - \frac{1}{2z^2}) - \tfrac{1}{4}(\mathcal{G} + 2\mathcal{G}^{int})S^* \\
&- \tfrac{1}{2}\mathcal{J}^{int}\partial\varphi + \tfrac{1}{2}\mathcal{A}^*\Big)
\end{aligned}
\tag{12}
$$

Now the internal algebra is more complicated. It contains the $N=2$, $c=9$ superconformal algebra $\mathcal{J}^{int}$, $\mathcal{G}^{int}$ and $\mathcal{L}^{int}$ as well as a complex chiral multiplet $(\mathcal{A}, \mathcal{R})$ of conformal weights

$(2, 3/2)$ with the following operator products:

$$
\begin{aligned}
\mathcal{G}_a^{int}(z)\mathcal{G}^{int}(\zeta) &\sim \frac{6e_a}{(z-\zeta)^3} + \frac{2e_a}{z-\zeta}\mathcal{L}^{int} - \frac{e_a}{(z-\zeta)^2}(\mathcal{J}^{int}(z) + \mathcal{J}^{int}(\zeta)) \\
\mathcal{J}^{int}(z)\mathcal{G}^{int}(\zeta) &\sim -\frac{i}{z-\zeta}\mathcal{G}^{int} \\
\mathcal{J}^{int}(z)\mathcal{J}^{int}(\zeta) &\sim \frac{1}{(z-\zeta)^2} \\
\\
\mathcal{G}_a^{int}(z)\mathcal{A}(\zeta) &\sim \frac{3e_a^*}{(z-\zeta)^2}\mathcal{R}(\zeta) + \frac{e_a^*}{z-\zeta}\partial\mathcal{R} \\
\mathcal{G}_a^{int}(z)\mathcal{R}(\zeta) &\sim \frac{e_a}{z-\zeta}\mathcal{A} \\
\mathcal{J}^{int}(z)\mathcal{A}(\zeta) &\sim -\frac{i}{z-\zeta}\mathcal{A} \\
\mathcal{J}^{int}(z)\mathcal{R}(\zeta) &\sim -\frac{2i}{z-\zeta}\mathcal{R} \qquad\qquad (13) \\
\\
\mathcal{R}_a(z)\mathcal{R}(\zeta) &\sim -\frac{4e_a}{(z-\zeta)^3} + \frac{2e_a}{(z-\zeta)^2}(\mathcal{J}^{int}(z) + \mathcal{J}^{int}(\zeta)) \\
&\quad - \frac{32e_a}{z-\zeta} : (\mathcal{J}^{int})^2 : \\
\mathcal{A}_j(z)\mathcal{R}(\zeta) &\sim \frac{2e_j}{(z-\zeta)^2}\mathcal{G}^{int}(\zeta) + \frac{2e_j}{z-\zeta} : \mathcal{G}^{int}\mathcal{J}^{int} : \\
\mathcal{A}_j(z)\mathcal{A}(\zeta) &\sim -\frac{12e_j}{(z-\zeta)^4} + \frac{4e_j^*}{(z-\zeta)^3}(\mathcal{J}^{int}(z) + \mathcal{J}^{int}(\zeta)) \\
&\quad - \frac{e_j}{(z-\zeta)^2}((: (\mathcal{J}^{int})^2(z) : + \mathcal{L}^{int}(z)) + (z \leftrightarrow \zeta)) \\
&\quad + \frac{e_j^*}{z-\zeta}(3\partial^2\mathcal{J}^{int} - 4 : \mathcal{L}^{int}\mathcal{J}^{int} : - : (\mathcal{G}^{int})^2 :)
\end{aligned}
$$

Expressions like $: (\mathcal{G}^{int})^2 :$ in the above expressions are normal ordered with respect to the modes of the currents. This prescription differs from the normal ordering with respect to the modes of, say, free component fields. The operator algebra in (13) replaces the nullmode condition (11). We do not know whether it is peculiar to some compactifications to $D=4$ or a more general property of the internal sector of $D=4$ superstrings. We have here an algebraic structure on the internal space without explicit appearance of coordinates. We have not checked, but it may well be possible to do so, whether this algebra, or some algebra of this type *has* to appear for J^- to be non-anomalous. If that is the case, one will have an instrument for treating the internal manifold in an abstract algebraic manner, that might be useful for extracting the physical consequences of specific choices for internal manifolds, and possibly for demonstrating "equivalence" between different manifolds with respect to their properties concerning string propagation.

What we still have not shown is that the interpretation of the super-Poincaré generators in terms of superconformal generators is valid also for the case $D = 10$. That will be the subject of the rest of this talk.

Let us now turn to the generalization to $N = 8$. We will give an intuitive step by step construction leading to the final form of the $N = 8$ superconformal generators and their algebra. Since on the light cone the spacetime supersymmetry algebra for $D = 10$ has the same structure as for $D = 6$ without a compact sector, one feels compelled to simply replace quaternions with octonions. The operator product of the imaginary currents $J = \frac{1}{2}S^*S$ is then given by

$$J^i(z)J^j(\zeta) \;=\; -\frac{4}{(z-\zeta)^2}\,\delta^{ij} \;+\; \frac{2}{z-\zeta}\left(\sigma_{ijk}\,J^k + \rho_{ij\alpha\beta}S^\alpha S^\beta\right) \tag{14}$$

Hence this current algebra does not close, and we may attribute this fact to the nonassociativity of the octonions, or equivalently to the fact that S^7 is not a group manifold. Using octonions, the 7-sphere is economically described by $S^7 = \{X \in \mathbb{O} \mid |X| = 1\}$, with tangent vectors Xe_i and normal X [21]. This defines a connection without curvature and torsion $T_{ijk}(X) = [(Xe_i)^*(Xe_j)e_k]$. Note that

$$T_{ijk}(X)|_{X=1} = \sigma_{ijk} \qquad \nabla_i T_{jkl}(X)|_{X=1} = 2\rho_{ijkl} \tag{15}$$

Hence we may move from the north pole $X = 1$ and form J by multiplication in a basis corresponding to another point on S^7, $J = \frac{1}{2}(XS)^*(XS)$, to obtain

$$J^i(z)J^j(\zeta) \;=\; -\frac{4}{(z-\zeta)^2}\,\delta^{ij} \;+\; \frac{2}{z-\zeta}\left(T^{ijk}(X)\,J^k + \nabla^i J^j\right) \tag{16}$$

The rest of the algebra has a similar structure: for $G = (X\varphi)^*(XS) = \sigma^I_{ab}(X)S_b\varphi^I$, with $\sigma^I_{ab}(X) = [e^I(e_aX^*)(Xe^*_b)]$, we obtain an algebra that closes modulo infinitesimal shifts on S^7, *i.e.* besides the "expected" terms there are terms containing ∇^i. By considering finite transformations generated by J, we see that we transform $Xe_a \to Y(Xe_a)$ for another unit octonion Y, i.e. we obtain a rotated basis of tangent vectors at $YX \in S^7$. Clearly we can also generate the basis $(X^*Y^*)(Y(Xe_i))$ at the northpole $Z = 1$. This basis is rotated with respect to the Xe_i. We conclude that the shifts operate not on the 7-sphere, but on an

SO(7)-bundle over S^7, i.e. on SO(8). The operator product of, say, $J_X = \frac{1}{2}(XS)^*(XS)$ and $J_{YX} = \frac{1}{2}(Y(XS))^*(Y(XS))$, does not fit into the simple scheme displayed above. But then, we would expect it to contain terms with an infinite number of S^7-derivatives. We will call the structure we found a current algebra that is "local on S^7".

Up to this point we have treated X as a number. For the algebra to close, we need a mechanism that takes care of the infinitesimal shifts on S^7. This is accomplished by letting the S^7 coordinate X be an operator, and adding an S^7 translation generator to J. More precisely, we introduce a pair of octonionic bosons (λ, ω) with conformal weights $(1/2, 1/2)$ and their superpartners (θ, π) of weights $(0,1)$, set $X = \lambda/|\lambda|$ and define

$$
\begin{aligned}
\mathcal{J} &= \{\omega^*\lambda\} + \tfrac{1}{2}(XS)^*(XS) \\
\mathcal{G} &= \pi^*\lambda - \partial\theta^*\omega + (X\partial\varphi)^*(XS) + \tfrac{1}{2}(XS)^*(\Lambda S) - \tfrac{1}{2}(\Lambda S)^*(XS) \\
\mathcal{L} &= \tfrac{1}{2}[\partial\lambda^*\omega - \lambda^*\partial\omega] - [\pi^*\partial\theta] + \tfrac{1}{2}[\partial\varphi^*\partial\varphi] - \tfrac{1}{2}[S^*\partial S]
\end{aligned}
\tag{17}
$$

where $\Lambda = |\lambda|^{-1}(\partial\theta - X[X^*\partial\theta])$ is the tangential part of $\partial\theta$. The algebra of these operators is soft [22], i.e. it closes with field dependent structure "constants" and anomaly terms. The classical algebra is

$$
\begin{aligned}
\mathcal{J}_\alpha(z)\mathcal{J}_{\alpha'}(\zeta) &\sim \frac{2}{z-\zeta}\mathcal{J}_{(\alpha X^*)(X\alpha')} \\
\mathcal{J}_\alpha(z)\mathcal{G}_\Omega(\zeta) &\sim -\frac{1}{z-\zeta}\Big(\mathcal{G}_{(\Omega X^*)(X\alpha)} + \mathcal{J}_{\lambda^{-1}((\partial\theta\Omega)\alpha - \partial\theta((\Omega X^*)(X\alpha)))}\Big) \\
\mathcal{G}_\Omega(z)\mathcal{G}_{\Omega'}(\zeta) &\sim \frac{2}{(z-\zeta)^2}\mathcal{J}_{(\Omega^* X^*)(X\Omega)} + \frac{2}{z-\zeta}\Big(\tfrac{1}{2}\partial(\mathcal{J}_{(\Omega^* X^*)(X\Omega')}) + [\Omega^*\Omega']\mathcal{L}\Big)
\end{aligned}
\tag{18}
$$

(Note that we still discuss the classical algebra). We note that only λ and θ enter into the field dependence, so that the structure functions (anti)commute. They have a natural interpretation in terms of the torsion tensor and its superpartner on S^7. The reduction to the $N<8$ algebras of eq.(5) is obvious: just remove all associator terms. If one replaces the term $\frac{1}{2}(XS)^*(XS)$ in $\mathcal{J}$ by $\{(X\omega')^*(X\lambda')\}$, where (λ', ω') is a conjugate pair of bosons of weights $(1/2, 1/2)$, and makes the corresponding replacements in $\mathcal{G}$ and $\mathcal{L}$, one finds the soft algebra Berkovits describes in the context of the twistor formulation of the superstring [24]. Thus we have found a natural generalization of the N=4 free field constructions.

We want to emphasize that we are working with explicit generators, and therefore automatically have the Jacobi identities fulfilled. If we on the right hand side of eq. (18) set $X = 1$, $\partial\theta = 0$, we get a non-associative algebra like the ones in [22,23,25]. The present formulation is stronger. A non-trivial feature is that, unlike what could be expected from a naive consideration of the properties of the octonions, the Kac-Moody part $\widehat{S^7}$ actually commutes with the $SO(8)$ of space rotations, and that our seven-sphere is therefore not the quotient of this group with an $SO(7)$ subgroup, but an additional symmetry. The $N=8$ generators of eq. (17) as they stand are not the ones that enter in the super-Poincaré generators (9). First the "parameter fields" (λ, ω) and (θ, π) must be removed — they are not physical fields. With our present understanding of the role of these fields we cannot make any certain statements about their physical interpretation in a covariant theory. We do not for example know what the constraints are that eliminate the parameter fields. A possible interpretation is that they are a remnant of a set of super-twistor variables from a combined space-time/twistorial formulation. For the moment we take a very pragmatic point of view and note that in order to reduce the field content to that of the light-cone superstring, we need some quantum mechanically consistent set of constraints (note that the superconformal generators cannot be set to zero with a quantum-mechanically nilpotent BRST charge). We may state $\omega \approx 0$ and $\pi \approx 0$, allowing for the gauge choices $X=1$, $\theta=0$, which of course takes us back to the situation in eqs. (14) and (16). The closure of the algebra is obstructed by the gauge choices. However, the role of the generators of the superconformal algebra in the super-Poincaré algebra is identical to that in the lower dimensionalities.

One may speculate in the ultimate role of the superconformal algebra in some kind of "covariant" formulation. We have a strong belief that the $N=D-2$ superconformal algebras have a fundamental significance, yet their generators enter very asymmetrically *e.g.* in the super-Poincaré generators, where $\mathcal{J}$ is not seen at all. It is tempting to think that the relation between space-time and worldsheet symmetries established in this paper gives a glimpse of the structure of an even bigger symmetry.

What is now the lesson so far? If we really insist on having the maximal superconformal symmetry we are led to a 10-dimensional string theory. We know since long that this theory is essentially unique while there are many seemingly consistent superstrings in lower dimensions. It is tempting to believe that the 10-dimensional symmetry is broken down to 4 dimensions in a way which keeps a maximal subsymmetry of the original one and that this could lead to a unique 4-dimensional model hence giving us the wanted screening among the 4-dimensional models. How this comes about and what physical principal that would lead to this result I do not know, but I find it a very intriguing question. With respect to the other questions raised in the beginning of the talk it is still too early to say how helpful our new insights are. It does remain to better understand the symmetry to find a way of implementing it covariantly and to get a physical insight into its origin. I hope to be able to find answers to all these questions at least before the Gold Jubilee of the Indian Association for General Relativity and Gravitation!

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