Effect of tensor force on the shell structure of superheavy nuclei

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Abstract. We study the effect of tensor interaction on the shell structure of superheavy nuclei in the frame of the deformed Skyrme Hartree-Fock+BCS model. To this end, we adopt four different Skyrme interactions; SLy5 without tensor interaction, and SLy5+T, T24 and T44 with tensor interaction. The calculation with SLy5+T interaction reproduce the single particle spectra of protons and neutrons in ²⁴⁹Bk and ²⁵¹Cf, better than the other calculations with SLy5, T24 or T44 interaction. The large shell gaps of superheavy nuclei (SHE) are found at Z=114 and Z=120 for protons and N=184 for neutrons with the spherical shape irrespective of the tensor correlations. It is also found that Z=114 and N=164 shell gaps are more pronounced by SLy5+T interaction with the tensor correlations.

1. Introduction

Recent progress on superheavy nuclei attracts both theoreticians and experimenters in nuclear physics. Theoretically the major challenge is to predict the possible existence and location of the island of stability for the superheavy nuclei. In superheavy nuclei the level density of singleparticle energies is quite large, so that a small change of single-particle energy could be crucial to determine the shell stability. That is why till now we have not really known what is the next doubly magic nucleus beyond ²⁰⁸Pb. Different theoretical models give different results for this problem. Macroscopic-microscopic model [1, 2] predict the next proton shell gap at Z=114, resulting from a large splitting of the $2f_{7/2}$ and $2f_{5/2}$ spin-orbit partners. And the model predicts the neutron gap at N = 184. On the other hand, self-consistent mean field models predict extended region of shell stabilities. Skyrme Hartree-Fock (SHF) calculations favor gaps at Z=124, 126 and N=184 depending on the parameterizations [3, 4, 5], while relativistic mean field (RMF) calculations lead to shell stabilities around Z=120, N=172,184 or Z=126, N=184 [6, 7, 8] top of (Z=114, N=184) shell gap. It turns out that the differences in single-particle energies are responsible for the divergent predictions of magic numbers in the mass region of superheavy nuclei [9, 10] and a slight modification of the spin-orbit strength will give significant changes in the single-particle spectra and the stability of heavy and super-heavy nuclei [10].

One current topics is the role of tensor terms in the effective interactions on the spin-orbit splittings and the shell evolution of exotic nuclei far from the stability line. It is pointed out in Ref. [11] that the experimental isospin dependence of the spin-orbit splittings can not be described by the SHF calculations with standard Skyrme interactions, but is well reproduced

by including the tensor interaction [11]. The similar improvements were realized in the study of single particle energies of f and p orbits in nuclei around ⁴⁸Ca and ⁴⁶Ar [12]. These improvement of the single particle energies induced by tensor interactions suggests the inclusion of the tensor correlations for the study of shell gaps of nuclei in superheavy region.

Recently, the effect of tensor terms of Skyrme interactions was studied on the shell structure of superheavy elements in SHF model with spherical symmetry [13]. (see Refs.[4, 6, 14, 15], for the comparisons of the predictions of different Skyrme Hartree-Fock (SHF) and relativistic mean field (RMF) calculations). It is known however that the breaking of spherical symmetry in deformed nuclei leads to a strong modification of the single-particle spectra in Nilsson diagram and may change the shell evolution of heavy and superheavy nuclei. At the same time, the deformation potential itself will be modified in the mean field by inclusion of the tensor effect. On the other hand, the effect of tensor correlations becomes active only when the spin-orbit partners are partially occupied. In deformed nuclei, the ordering of single-particle states are changed drastically so that the tensor correlations might be very different compared with spherical nuclei. Thus we need mean field calculations for the stability of super heavy nuclei.

The effect of tensor interactions on deformed nuclei was studied in Ref. [16] for magic or semimagic nuclei with N or Z =20, 28, 40, 50, 82. We extend the work of Ref. [16] to study the effect of the tensor terms on deformation and shell structure of medium-heavy and superheavy nuclei and especially, in the predictions for SHE the quadrupole deformation is taken into account in the SHF model [17].

In this proceeding, we show the results of Ref. [17] about the effect of tensor interaction on the shell evolution of superheavy nuclei including tensor force in the SHF approach, especially focusing on the shell evolution at around the proton number Z=114.

2. Formalism

In the deformed Skyrme Hartree-Fock (SHF) calculation [18, 19], we take into account the triplet-even and triplet-odd zero-range tensor interactions,

$$V_{T} = \frac{T}{2} \{ [(\sigma_{1} \cdot \mathbf{k}')(\sigma_{2} \cdot \mathbf{k}') - \frac{1}{3} \mathbf{k'}^{2} (\sigma_{1} \cdot \sigma_{2})] + [(\sigma_{1} \cdot \mathbf{k})(\sigma_{2} \cdot \mathbf{k}) - \frac{1}{3} (\sigma_{1} \cdot \sigma_{2}) \mathbf{k}^{2}] \} \delta(\mathbf{r_{1}} - \mathbf{r_{2}}) + U \{ (\sigma_{1} \cdot \mathbf{k}') \delta(\mathbf{r_{1}} - \mathbf{r_{2}}) (\sigma_{2} \cdot \mathbf{k}) - \frac{1}{3} (\sigma_{1} \cdot \sigma_{2}) [\mathbf{k}' \cdot \delta(\mathbf{r_{1}} - \mathbf{r_{2}}) \mathbf{k}] \},$$
(1)

where \mathbf{k} and \mathbf{k}' are the momentum operators acting on right and left hand side, respectively. The time-even and time-odd tensor coupling constants T and U are free parameters in the SHF model. Since the contribution of the pseudo-tensor spin-orbit density is two order of magnitude smaller that the vector one[16], in the calculation, we only take into account the vector part in the Skyrme energy density functionals. And the vector part of the spin-orbit density in the SHF energy density is written as

$$\Delta H = \frac{1}{2}\alpha (J_n^2 + J_p^2) + \beta J_n J_p.$$
⁽²⁾

where the α and β have the contributions of central exchange (α_c and β_C) and tensor term (α_T and β_T),

$$\alpha = \alpha_C + \alpha_T; \ \beta = \beta_C + \beta_T, \tag{3}$$

$$\alpha_C = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1 x_1 + t_2 x_2); \ \beta_C = -\frac{1}{8}(t_1 x_1 + t_2 x_2),$$
(4)

$$\alpha_T = \frac{5}{12}U; \ \beta_T = \frac{5}{24}(T+U).$$
(5)

Then, with the contribution of tensor force, the spin-orbit potential is expressed as,

$$W_q = \frac{W_0}{2r} \left(2\frac{d\rho_q}{dr} + \frac{d\rho'_q}{dr} \right) + \left(\alpha \frac{J_q}{r} + \beta \frac{J'_q}{r} \right),\tag{6}$$

where the first term comes from the Skyrme spin-orbit interaction whereas the second one includes both the central exchange and tensor contributions. The interactions between like (unlike) particles are denoted q(q') in Eq. (6).

3. Shell structure of superheavy nuclei

We perform the deformed SHF+BCS calculations including the tensor contributions to the energy density function and also to the single particle potential. We choose four parameter sets in the following study: SLy5 force without tensor interactions and three parameter sets SLy5+T [11], T24 and T44 [20] with tensor interactions. These four sets of parameters can be considered to simulate the major effect of the tensor correlations in Skyrme interactions as described in Ref.[20]. The parameters for tensor interactions employed are taken from Refs. [11, 16]. The adopted parameters of tensor interactions T, U and also α, β are summarized in Table 1. The sign of α_T is negative for SLy5+T, while T24 and T44 have the positive values. On the other hand, the sign of β_T is always positive for these three parameter sets.

Table 1. Parameters of tensor interactions T, U and also α , β for Skyrme interactions SLy5, SLy5+T, T24 and T44. All the parameters are given in unit of MeV fm⁻⁵. See the text for details.

Skyrme parameters	Т	U	α_C	β_C	α_T	β_T	α	β
SLy5	0.0	0.0	80.20	-48.87	0.0	0.0	80.20	-48.87
SLy5+T	888.0	-408.0	80.20	-48.87	-170	100	-89.8	51.13
T24	33.74	59.22	95.33	-19.37	24.67	19.37	120	0
T44	520.98	21.52	111.03	6.98	8.97	113.02	120	120

In the calculation, a density-dependent surface delta pairing interaction

$$V(\mathbf{r_1}, \mathbf{r_2}) = \mathbf{V_0}' \left(\mathbf{1} - \frac{\rho(\mathbf{r})}{\rho_0} \right) \delta(\mathbf{r_1} - \mathbf{r_2}) , \qquad (7)$$

is used, where $\rho(r)$ is the HF density at $\mathbf{r} = (\mathbf{r_1} + \mathbf{r_2})/2$ and $\rho_0 = 0.16 \text{ fm}^{-3}$. We employ the BCS model with Lipkin-Nogami (LN) number projection for the calculations of pairing correlations. In order to obtain a proper estimate of the pairing correlations in superheavy nuclei, using the pairing strength $V'_0 = -1250 \text{ MeV fm}^{-3}$ from Ref. [16] we calculated the odd-even mass difference by the three-point formula $\Delta^{(3)}$ [21] for No isotopes. The results are given in Fig.1. We find that this pairing strength adopted in Ref. [16] is too large for superheavy nuclei, while the pairing strength $U'_0 = -625 \text{ MeV fm}^{-3}$ can well reproduce the odd-even difference compared with the experimental data. Therefore, in the following calculations, we use the pairing strength $V'_0 = -625 \text{ MeV fm}^{-3}$. For odd-A nucleus, the orbit occupied by the last odd nucleon is blocked, as described in Ref. [22]. The time-odd components of Skyrme energy density can be neglected in even-even nuclei. It was pointed, however, that the time-odd components have a substantial effect on the moment of inertia and thus on rotational spectra of deformed nuclei. Therefore, we take into account the time-odd components of central part of Skyrme energy density to study single-particle energies of odd super-heavy nuclei.



Figure 1. The odd-even mass difference for No isotopes. The three point formula is used to calculate the odd-even mass difference [21]. Experimental data are indicated by filled circles. The results for the pairing strength $V'_0 = -1250$ and -625 MeV fm⁻³ are indicated by open circles and open diamonds, respectively, while the corresponding results with odd-term fields are indicated by open squares and open triangles, respectively.

First we discuss the effect of tensor force on the proton and neutron single particle spectra of 249 Bk (Z=97) and 251 Cf (Z=98) known experimentally. The calculated results by the deformed SHF+BCS model with the four different Skyrme interactions are given in Fig. 2 (for 249 Bk) and Fig. 3 (for 251 Cf) compared with the experimental data [23, 24]. It is found that the energy minima are at the prolate deformation $\beta_2 \sim 0.3$ both in 249 Bk and 251 Cf for all the interactions. For 249 Bk, the SLy5+T interactions can reproduce the correct ordering of the experimental proton single particle energies, while the other interactions show the different level schemes for several single-particle states. Compared with the experimental Z=98 gap, the energy gaps around the Fermi surface appear at Z=96 for SLy5 and at Z=96, 98 and 100 for SLy5+T , respectively. Both the T24 and T44 interactions do not show the energy gap at Z=98 since [633]7/2⁺ level is much higher in energy compared with the empirical value.

Now we explain why the calculation with the SLy5+T interaction including tensor force produces Z=98 shell gap of ²⁴⁹Bk compared to that with the SLy5 interaction. Whether Z=98 and Z=96 shell gaps appear or not is determined by the relative positions of three proton states $[521]3/2^-$ (from $2f_{7/2}$ in the spherical limit), $[633]7/2^+$ (from $1i_{13/2}$ in the spherical limit) and $[521]1/2^{-}$ (from $2f_{5/2}$ in the spherical limit). However, the strength of spin-orbit splitting affects the relative position of single particle states and the tensor interactions have significant contributions to the spin-orbit splittings when one of the spin-orbit partners is unoccupied. In the case of SLy5+T interaction, we take $(\alpha_T, \beta_T) = (-170, 100)$ MeV fm⁵ and $(\alpha = \alpha_C + \alpha_T, \beta = \beta_C + \beta_T) = (-88.8, 51.1)$ MeV fm⁵. According to Eq. (6), when a $j_{>} = l + 1/2$ proton orbit is occupied and the spin-orbit partner $j_{<} = l - 1/2$ proton orbit is unoccupied, the negative α value increases the spin-orbit splitting of protons, while the positive β value decreases the spin-orbit splitting of neutrons. The effect is the same for the neutron configurations when one of the spin-orbit partners is unoccupied. Since the absolute value of α_T is larger than that of β_T , the net effect of tensor correlations in SLy5+T interaction makes a larger spin-orbit splitting for both neutrons and protons. In the spherical limit the proton orbits $2f_{7/2}$ and $1i_{13/2}$ in ²⁴⁹Bk are partially occupied, and due to the tensor correlations the proton spin-orbit splittings of $2f_{7/2} - 2f_{5/2}$ and $1i_{13/2} - 1i_{11/2}$ partners are enlarged. These energy changes of orbits $2f_{7/2}$, $2f_{5/2}$ and $1i_{13/2}$ in the spherical limit make the upward shift of $[521]1/2^-$ orbit and the downward shifts of $[521]3/2^-$ and $[633]7/2^+$ orbits compared with the results of SLy5 interaction. In this way, the tensor correlations create the Z=98 shell gap in ²⁴⁹Bk.

Concerning the neutron single particle energies of 251 Cf given in Fig. 3, the experimental data show the energy gaps at N=160 and 164. Calculated results of SLy5+T give also the same



Figure 2. Proton single particle energies of ²⁴⁹Bk calculated by deformed SHF+BCS model with L-N number projection. The parameter sets SLy5, SLy5+T, T24 and T44 are used. The deformation parameter is set to the calculated energy minimum at $\beta_2=0.3$. The long and short lines show the calculated results of single-particle energies without and with the timeodd terms in the HF hamiltonian, respectively. Experimental data is taken from Ref. [23].

shell structure as the experimental one, while the calculated gaps are larger than the empirical ones. The SLy5 gives also the gaps at N=160 and 164, but the positions of $[725]11/2^{-}$ (from $1j_{15/2}$ in the spherical limit) and $[750]1/2^{-}$ (from $1j_{13/2}$ in the spherical limit) are inverted in comparison with the empirical energy spectra. For SLy5+T interaction, there is no inversion of the two states. The different results calculated by SLy5 and SLy5+T interactions origin from the partial occupation of $1j_{15/2}$ orbit and the tensor forces of SLy5+T interaction enlarge the neutron spin-orbit splitting of $1j_{15/2} - 1j_{13/2}$ partner. The Skyrme parameters T24 and T44 have large gaps at N=160 and 164, respectively, but have no gap at N=164. This is mainly due to the fact that the positions of $[750]1/2^{-}$ and $[622]3/2^{+}$ states are inverted in the level diagram compared with the experimental data.



Figure 3. Neutron single particle energies of ²⁵¹Cf calculated by deformed SHF+BCS model with L-N number projection. The parameter sets SLy5, SLy5+T, T24 and T44 are used. The deformation parameter is set to the calculated energy minimum at $\beta_2=0.3$. The long and short lines show the calculated results of single-particle energies without and with the timeodd terms in the HF hamiltonian, respectively. Experimental data is taken from Ref. [24].

In the HF calculations of ²⁴⁹Bk and ²⁵¹Cf shown in Figs. 2 and 3, respectively, we take into account the time-odd components of Skyrme energy density functionals. The potential minimum is essentially the same for both calculations with and without the time-odd terms. The single particle energies are slightly changed by the time-odd terms in HF potential, but the changes are too small (about 200keV) to alter the ordering of single-particle levels.

Next, we study the shell structure of 254 No(Z=102), which is one of the heaviest even-even nuclei experimentally observed. Especially, we focus on the effect of tensor force. We perform the



Figure 4. (Color on line) The single-particle energies for neutrons (upper panels) and protons (lower panels) calculated for 254 No (Z=102, N=152) with SLy5 (left panels) and SLy5+T (right panels), respectively. The Fermi energy is indicated by the long-dashed line and the ground state deformation is indicated by the vertical dashed line.

DSHF+BCS calculations with SLy5 and SLy5+T interaction, respectively. Fig. 4 plot the single particle energies of neutrons (upper panels) and protons (lower panels) for ²⁵⁴No as a function of deformation parameter β_2 . For SLy5 interaction (left lower panel), around the energy minimum at $\beta_2 = 0.30$, there are several deformed proton shells, namely, Z=98, 102, 104 and 108. when including the tensor interaction, the deformed closed shells become Z=98 and 104 (right lower panel). In the spherical case, the proton gaps are Z=92, 100 and 120 for SLy5 interaction, but the gap at Z=114 is increased by SLy5+T interaction. Here, we find again the effect of tensor correlations on the shell structure of single particle states as were seen in case of ²⁴⁹Bk and 251 Cf. It origin from the partially occupation of the 1i13/2 proton orbit in the spherical limit and the tensor force enlarges the spin-orbit splittings of $(2f_{7/2}-2f_{5/2})$, $(1i_{13/2}-1i_{11/2})$, and $(1h_{11/2}-1h_{9/2})$ proton orbits, respectively. Therefore, the net effect of tensor correlation makes a large shell gap at Z=114, but a smaller one at Z=92 compared with those of SLy5 without tensor. A large shell gap at N=164 appears in the spherical limit in the case of SLy5+T (the upper right panel). It also origin from the enlarged $(2g_{9/2}-2g_{7/2})$ and $(1j_{15/2}-1j_{13/2})$ splittings caused by tensor force. Similarly, due to the effect of tensor force, in Fig. 4, we see N=152 and 162 deformed gaps for neutrons at $\beta_2 \sim 0.3$ with SLy5 interaction, while N=150, 152 and 160 shell gaps appear in the case of SLy5+T interaction.

In recent experimental observations of synthesis of heavy nuclei by cold fusion, the deformed shell gaps are pointed out at N=152 with $Z\sim100$ and N=162 with Z=108 [26]. The experimental results give a crucial test of the theoretical predictions of shell gaps. In the case of SLy5

interaction, we can see from Fig. 4 the magnified shell gaps at N=152 and 162 for neutrons and at Z=98, 102 and 108 for protons at the deformation $\beta_2 \sim 0.3$, which is identified to the empirical observations in the nuclear synthesis of heavy nuclei. However, for SLy5+T interaction the calculation gives the deformed shell gaps at N=150 and 160 for neutrons and Z=98 and 104 for protons. The shell gap at Z=108 is also somewhat quenched compared with the results of SLy5 interaction.



Figure 5. The calculated total binding energy as a function of deformation β_2 for ²⁹⁸114. The energy is normalized to the ground state energy ($\beta_2=0$) for each interaction.

Then, we calculate the total energies of the nucleus ²⁹⁸114₁₈₄. The results are shown in Fig. 5 as a function of quadrupole deformation β_2 with SLy5, SLy5+T, T24 and T44 interactions, respectively. One notices that the tensor force has an obvious effect on the energy surface. The SLy5+T interaction gives the spherical minimum lower by about 5MeV than the local minimum at $\beta_2 \sim 0.65$, while in the case of SLy5, T24 and T44 interactions, we see the competing ground and excited states at $\beta_2 \sim 0.55$ with smaller energy differences of 2.5 MeV, 1.4 MeV and 0.2 MeV, respectively. Furthermore, the barrier height in the case of SLy5+T is higher by about 2MeV than the other three interactions.

The single particle energies of protons in ²⁹⁸114₁₈₄ are plotted in Fig. 6 as a function of deformation parameter β_2 for the four Skyrme interactions. In all the cases, we find that Z=126 is the major shell gap in the spherical limit. For subshells, as discussed in the Ref.[25], whether the Z=114 or 120 shell gap appears depending on the relative positions of $2f_{5/2}$, $2f_{7/2}$ and $1i_{13/2}$ orbits. All the Skyrme calculations give the results in which the $1i_{13/2}$ orbit lies between $2f_{5/2}$ and $2f_{7/2}$ orbits. The shell gap energies at Z=114, Z=120 and Z=126 at the spherical minimum $\beta_2 = 0.0$ are listed in Table 2. We find the competing Z=114 and Z=120 shell gaps at in spherical limit for the SLy5, T24 and T44 interactions, while for SLy5+T force the energy gap at Z=114 is more pronounced and the Z=120 gap is almost disappeared.

Here, we see again the occupations of $1i_{13/2}$ and $2f_{7/2}$ orbits enhance the spin-orbit splittings and make the larger Z=114 shell gap for SLy5+T interaction than that for SLy5. In the case of T24 and T44 interaction, the tensor effect is not clearly seen in the Nilsson diagram of Fig. 6 as far as the gaps Z=114 and 120 are concerned. This is due to the large spin-orbit strengths W_0 for T24 and T44 interactions, i.e. $W_0=139.272$ and 161.367 MeV fm⁻⁵, respectively, compared to $W_0=126$ MeV fm⁻⁵ for SLy5 and SLy5+T. The positive values of α for T24 and T44 interactions or spin-orbit splittings. Thus, in Fig. 6, the tensor effects of T24 and T44 interactions compensate with the larger W_0 strength and the net effects give similar level schemes as that of SLy5 interaction. At the large prolate deformation, one notices that the SLy5 and SLy5+T interactions give clear energy gaps at Z=114 and 120, while T24 and T44 interactions show a shell gap at Z=122.

We give the single particle energies of neutrons in $^{298}114$ with the four interactions in Fig. 7.



Figure 6. (Color on line) The single-particle energies of protons in ²⁹⁸114 as a function of deformation parameter β_2 with SLy5, SLy5+T, T24 and T44. The Fermi energy is indicated by a long-dashed line. The position of local minimum is shown by a vertical dashed dashed line.

Table 2. The Z=114 and Z=120 gaps (in unit of MeV) at $\beta_2 = 0$ in ²⁹⁸114 for different Skyrme parameters SLy5, SLy5+T, T24 and T44. See the text for details.

Skyrme parameters	Z=114	Z=120	Z=126
SLy5	0.93	1.0	2.89
SLy5+T	1.89	0.36	3.18
T24	0.65	0.94	2.67
T44	0.54	0.75	2.41

A large energy gap at N=184 appears for the SLy5 and SLy5+T interactions in the spherical limit. We can see also that the N=164 gap becomes much larger for SLy5+T interaction because of the occupation of 1j15/2 orbit and the negative α value. At the large prolate deformation $\beta_2 \sim 0.6$, N=184 gap and somewhat smaller N=172 gap appear in the cases of T24, T44 and SLy5 interaction, while the deformed N=184 and N=172 gaps disappear for SLy5+T because $li_{11/2}$ orbit becomes close to $2g_{9/2}$ and $1j_{15/2}$ orbits due to the tensor interactions which kill the energy gaps at $\beta_2 \sim 0.6$.

Compared to the deformed shell gaps at N=152 and N=162, the gap at N=184 is a major gap in the spherical linit and may influence decay properties of a much wider charge and mass region of heavy nuclei. To synthesize nuclei with Z \geq 112 and N \geq 172, hot fusion reactions with massive nuclei are promising reaction mechanism rather than the well-studied cold fusion reactions. It



Figure 7. (Color on line) Same as Fig.6, but for the single particle energies of neutrons in $^{298}114$.

might be extremely interesting to study the decay properties of these synthesized heavy nuclei to establish a possible spherical shell gap at N=184.

4. Summary

In order to study the effect of the tensor interaction on the shell structure of superheavy nuclei, we performed the deformed SHF+BCS calculations with four different Skyrme interactions SLy5, SLy5+T, T24 and T44. There is no tensor interaction in SLy5, while the tensor interactions are included perturbatively in SLy5+T and, by the variational procedure in T24 and T44 parameter sets. We discussed first the single particle energies and the shell evolution of super-heavy nuclei 249 Bk, 251 Cf, 254 No and 298 114 are then discussed by the same model. It is shown that the single-particle spectra of 249 Bk and 251 Cf are largely influenced by the tensor correlations. The energies of proton and neutron single particle states are better described by SLy5+T interaction than by the other three interactions in comparison with the experimental data.

For superheavy nuclei ²⁹⁸114, we find the pronounced energy gaps at Z=114 and Z=120 at the spherical minimum in general. However, the tensor correlations of SLy5+T interaction make a larger shell gap at Z=114 and the Z=120 gap almost disappear. Near the deformed local minimum at $\beta_2 \sim 0.6$, the Z=114 and 120 shell gaps appear for the SLy5 and SLy5+T interactions. For neutron shells, we point out that the N=184 closure is robust for all the interactions. It is noticed that the tensor terms of SLy5+T interaction enhance the subshell closure at N=164 in the spherical limit. The large shell gap at N=184 appears both in the spherical limit and the deformed local minimum of $\beta_2 \sim 0.6$. The N=172 gap also appears at the deformed local minimum in the cases of SLy5, T24 and T44. However these two deformed

gaps disappear by the tensor correlations of SLy5+T interaction. The time-odd components of HF potential are examined in odd-nuclei ²⁴⁹Bk and ²⁵¹Cf. It is found that the effect is rather small on the single-particle energies and does not change the ordering of single-particle levels of both nuclei.

Although the parameter set SLy5+T has several good properties to describe the single particle energies and the shell structure, the fine agreement of mass systematics of the original SLy5 interaction is missing due to the perturbative adjustment of the tensor interactions. It is desperately desired to establish a new fitting protocol of Skyrme parameter set with the tensor terms to obtain both the fine mass systematics and the proper shell structure of heavy nuclei. Furthermore, the dynamical effect beyond the mean field approximation [27] may play a role to predict the shell structure of superheavy nuclei although it is expected smaller than the other mass region due to the less collectivity of low-lying excitations.

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