

**TESTING QCD AT HIGH ENERGY  $e^+e^-$  COLLIDERS:  
 $0.5 \leq Q \leq 2$  TeV\***

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ABSTRACT

A review is presented of the possibilities for making precise tests of Quantum Chromodynamics at high energy  $e^+e^-$  colliders operating at centre-of-mass energies in the range  $0.5 \leq Q \leq 2$  TeV.

1. Motivation

The physics case for construction of high energy  $e^+e^-$  colliders operating at centre-of-mass (c.m.) energies in the range  $0.5 \leq Q \leq 2$  TeV is a powerful one. With the recent discovery of the top quark at Fermilab, and determination of its mass to be around  $180 \text{ GeV}/c^2$ ,<sup>1</sup> it is apparent that a 500 GeV  $e^+e^-$  collider could serve as a 'top factory', allowing properties of the production and decay of top quarks to be studied. Furthermore, if the collider were run in the 350 - 400 GeV energy region the threshold behaviour of the  $t\bar{t}$  cross-section could be measured, potentially allowing accurate determinations of the top quark mass and decay width<sup>2,3</sup>. Similarly, searches for the Higgs boson(s)<sup>4</sup>, supersymmetric particles<sup>5</sup>, or strongly-interacting gauge bosons<sup>6</sup> could be made, with complementary 'discovery potential' to that at the Large Hadron Collider (LHC) planned for operation at CERN in the first decade of the next century.

With such glittering physics topics on the agenda it might be tempting to relegate to the pedestrian the possibility of testing our theory of strong interactions, Quantum Chromodynamics (QCD)<sup>7</sup>, at a high energy  $e^+e^-$  collider. Indeed, there are probably few who would argue that the capability to perform more precise and/or new tests of QCD is sufficient to justify the enormous investment of human resources required to build such a collider. However, given the compelling

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motivation to build a collider to study the origin of electroweak symmetry breaking, one can argue that tests of QCD are both enriching and essential to the programme of measurements to be made. To those who will read no further than this section I offer the following as a summary of motivation for QCD studies at such a collider:

- Since QCD is our theory of strong interactions it would be irresponsible not to test it at the highest energy scales available in different hard scattering processes. In this sense testing QCD at a 0.5–2.0 TeV  $e^+e^-$  collider is complementary to testing it at the 10–14 TeV LHC.

- Precise determination of the strong coupling  $\alpha_s$  is key to a better understanding of high energy physics. The current precision of  $\alpha_s(M_Z^2)$  measurements, limited to 5–10%, results in the dominant uncertainty on our prediction of the energy scale at which grand unification of the strong, weak and electromagnetic forces takes place<sup>8</sup>. An  $\alpha_s(M_Z^2)$  measurement of 1% precision may be possible at a high energy  $e^+e^-$  collider. Such a measurement would also allow improved determination of the mass and width of the top quark from the threshold behaviour of the  $t\bar{t}$  cross-section.

- Measurements of hadronic event properties at high energies, combined with existing lower energy data, would allow one to test the gauge structure of QCD by searching for anomalous ‘running’ of observables, such as the rate of production of events containing three jets, and to set limits on models which predict such effects, for example those involving light gluinos which are difficult to exclude by other means.

- Searches could be made for anomalous chromo-electric and chromo-magnetic moments of quarks, which effectively modify the rate and pattern of gluon radiation, and for which the phase space increases as the c.m. energy is raised.

- Gluon radiation in  $t\bar{t}$  events is expected to be strongly regulated by the large mass and width of the top quark;  $t\bar{t}g$  events will hence provide an exciting new domain for QCD studies. As a corollary, measurements of gluon radiation patterns in  $t\bar{t}g$  events may provide valuable additional constraints on the top quark decay width.

- Polarised electron beams will be exploited at high energy  $e^+e^-$  colliders and will allow tests of symmetries using multi-jet final states.
- Study of  $q\bar{q}$  events ( $q = u,d,s,c$  or  $b$ ), presently using Monte Carlo simulations, but eventually using the data themselves, will be vital to allow development of event selection cuts and background contamination estimates for analyses using samples of  $t\bar{t}$  or  $W^+W^-$  events, or involving searches for more exotic final states.
- Monte Carlo simulations of  $q\bar{q}$  events, involving predictions of jet properties and particle multiplicities and flows, are needed for the design and development of particle detectors optimised for measurements in the  $0.5 \leq W \leq 2.0$  TeV c.m. energy range.

In this review I shall discuss these topics in some detail and summarise the current state of studies for QCD analysis at high energies. Throughout, the acronym ‘HLC’, representing High energy Linear  $e^+e^-$  Collider, will be used to denote the set of colliders comprising NLC, JLC, CLIC, TESLA and VLEPP that have been proposed by the respective geo-political consortia to confront the supra-national physics that awaits us. It is assumed that the collider will be designed to deliver an integrated luminosity of  $50 \text{ fb}^{-1}$  per year of running at 500 GeV, and  $100 \text{ fb}^{-1}$  per year of running at 1 TeV.

## 2. Precise Measurement of $\alpha_s$

### 2.1. Current Status

QCD contains in principle only one free parameter, the strong interaction scale  $\Lambda$ . Tests of QCD hence comprise comparisons of measurements of  $\Lambda$  in different processes and at different hard scales  $Q$ . If one knows  $\Lambda$  one may calculate the strong coupling  $\alpha_s(Q^2)$  from the solution of the QCD renormalisation group equation<sup>9</sup>. Because of the large data samples taken in  $e^+e^-$  annihilation at the  $Z^0$  resonance, it has become conventional to use as a yardstick  $\alpha_s(M_Z^2)$ , where  $M_Z$  is the mass of the  $Z^0$  boson;  $M_Z \approx 91.2 \text{ GeV}$ .<sup>10</sup> Tests of QCD can therefore be quantified in terms of the consistency of the values of  $\alpha_s(M_Z^2)$  measured in different experiments.

Measurements of  $\alpha_s(M_Z^2)$  have been performed in  $e^+e^-$  annihilation, hadron-hadron collisions, and in deep-inelastic lepton-hadron scattering experiments, covering a range of  $Q^2$  from roughly 1 to  $10^4$  GeV<sup>2</sup>; for recent reviews see Refs. 11,12. These measurements are consistent within the errors. An average yields  $\alpha_s(M_Z^2) = 0.117 \pm 0.006$ ,<sup>12</sup> implying that QCD has been tested to a precision of about 5%. This precision is however rather modest compared with the achievement of sub-1% level tests of the electroweak theory<sup>13</sup>, and this is due primarily to the *theoretical uncertainties* that dominate most of the experimental measurements. These uncertainties are due to both the restriction of complete perturbative QCD calculations to low order, and non-perturbative ('hadronisation') effects that are presently incalculable in QCD.

## 2.2. Is a 1%-level Measurement of $\alpha_s(M_Z^2)$ Possible?

It is interesting to consider whether a measurement of  $\alpha_s(M_Z^2)$  at the 1%-level of precision is possible at the HLC. Consider a recent measurement from  $e^+e^-$  annihilation at the  $Z^0$  resonance by the SLD Collaboration, based on 15 hadronic event shape observables measured with a data sample comprising approximately 50,000 hadronic events<sup>14</sup>:

$$\alpha_s(M_Z^2) = 0.1200 \pm 0.0025 \text{ (exp.)} \pm 0.0078 \text{ (theor.)}$$

where the experimental error is composed of statistical and systematic components of about  $\pm 0.001$  and  $\pm 0.002$  respectively, and the theoretical uncertainty has components of  $\pm 0.003$  and  $\pm 0.007$  arising from hadronisation and missing higher order terms, respectively. Now consider 'scaling' this result to estimate the precision of a similar measurement at  $Q = 500$  GeV.

- **Statistical error:** At design luminosity the 500 GeV HLC would deliver roughly 100,000  $q\bar{q}$  ( $q=u,d,s,c,b$ ) events per year (Section 7), implying that a statistical error on  $\alpha_s(M_Z^2)$  well below  $\pm 0.001$  could be obtained.

- **Systematic error:** This results primarily from the uncertainty in modelling the jet resolution of the detector. The situation may be improved at the HLC by a combination of building better detectors and benefitting from improved calorimeter energy resolution for higher energy jets. It is not unreasonable to suppose that the

current systematic error of roughly  $\pm 0.002$  could be reduced by a factor of two, but more convincing demonstration of this point would require a simulation of the detector, as well as the event selection and analysis cuts (see Section 7).

- **Hadronisation uncertainty:** Since jets of final-state particles, rather than partons, are observed in detectors it is necessary to correct hadronic distributions for any smearing and bias effects that occur in the hadronisation process. These effects are usually estimated from Monte Carlo simulations incorporating hadronisation models. In  $Z^0$  decays such corrections are typically at the level of 10%.<sup>14</sup>

It can be argued that non-perturbative corrections to jet final states in  $e^+e^-$  annihilation can be parametrised in terms of inverse powers of the hard scale  $Q$ . For a generic observable  $X$ :

$$\delta X^{\text{non-pert}} \sim \sum_{n \geq 1} \frac{a_n}{Q^n}.$$

However, at leading order in perturbation theory:

$$X^{\text{pert}} \sim \frac{1}{\ln Q},$$

so that the ratio of non-perturbative to perturbative QCD contributions is dominated by a term of the form:

$$\frac{\delta X^{\text{non-pert}}}{X^{\text{pert}}} \sim \frac{\ln Q}{Q}.$$

Increasing  $Q$  from 91 GeV to 500 GeV causes this ratio to decrease by a factor of 5, implying that hadronisation corrections should be of order 2% at HLC. Assuming that these corrections can be estimated to better than  $\pm 50\%$ , the hadronisation uncertainty should contribute less than 1% to the error on  $\alpha_s(M_Z^2)$ .

A demonstration of this naive argument was provided in a Monte Carlo study<sup>15</sup> of jet resolution due to hadronisation. More recent theoretical work<sup>16</sup> has also added validity to the inverse power corrections approach, which for some observables appears to be in good agreement with current data. For example, the energy-dependence of the observable  $1 - T$  ( $T = \text{thrust}^{17}$ ) is shown in Fig. 1;<sup>16</sup>  $O(\alpha_s^2)$  QCD combined with a non-perturbative term of the form  $1/Q$  describes the data well.

- **Uncertainty due to missing higher orders:** Currently perturbative QCD calculations of hadronic event shapes are available complete up to  $O(\alpha_s^2)$ .<sup>18</sup> Since

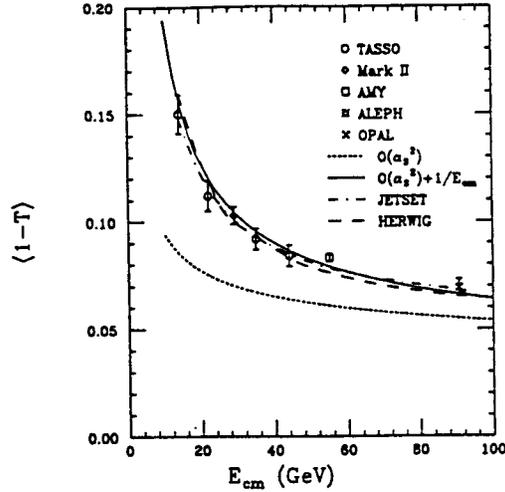


Figure 1: Mean value of 1-Thrust vs. c.m. energy.

the data contain knowledge of all orders one must estimate the possible bias inherent in measuring  $\alpha_s(M_Z^2)$  using the truncated QCD series. Though not universally accepted, it is customary to estimate this from the dependence of the fitted  $\alpha_s(M_Z^2)$  on the QCD renormalisation scale, yielding a large and dominant uncertainty of about  $\pm 0.007$ .<sup>14</sup> Since the missing perturbative terms are  $O(\alpha_s^3)$ , and since at  $W = 500$  GeV  $\alpha_s$  is expected to be about 25% smaller than its value at the  $Z^0$ , one naively expects the uncalculated terms to be almost a factor of two smaller at the higher energy, leading to an estimated uncertainty of  $\pm 0.004$  on  $\alpha_s(500 \text{ GeV})$ . However, translating to the yardstick  $\alpha_s(M_Z^2)$  yields an uncertainty of  $\pm 0.006$ , only slightly reduced compared with the current uncertainty.

From this simple analysis it seems reasonable to conclude that achievement of the luminosity necessary for ‘discovery potential’ at the HLC will result in a  $q\bar{q}$  event sample of sufficient size to measure  $\alpha_s(M_Z^2)$  with a statistical uncertainty of better than 1%. Construction of detectors superior in performance to those in operation today at SLC and LEP may be necessary in order to reduce systematic errors to the 1% level. Hadronisation effects should be significantly smaller, implying a sub-1% uncertainty. However, unless  $O(\alpha_s^3)$  contributions are calculated,

$\alpha_s(M_Z^2)$  measurements at 500 GeV will be limited by theoretical uncertainties to a precision of  $\pm 0.006$ , only marginally better than that achieved at present.

### 2.3. Top Quark Mass Determination and $\alpha_s$

It is clear that the value of  $\alpha_s$  controls the shape of the strong potential that binds quarkonia resonances. In the case of  $t\bar{t}$  production near threshold, the large top mass, and hence large decay width, ensure that the top quarks decay in a time comparable with the classical period of rotation of the bound system, making the toponium resonance a very short-lived phenomenon, and washing out most of the resonant structure in the cross-section. The shape of the  $t\bar{t}$  cross-section near threshold hence depends strongly not only on the top mass, but also on  $\alpha_s$ .

Fits to simulations of measurements of this cross-section have shown<sup>19</sup> that the top mass so determined is strongly correlated with the assumed value of  $\alpha_s(M_Z^2)$ . The European Top Quark Working Group has updated these simulations for the latest measured values of the top mass and has shown<sup>20</sup> that a simultaneous determination of  $m_t$  and  $\alpha_s(M_Z^2)$  by fitting to the threshold cross-section measured with one design-year of luminosity yields statistical precisions of  $\pm 250 \text{ MeV}/c^2$  and  $\pm 0.006$  on  $m_t$  and  $\alpha_s(M_Z^2)$ , respectively. Fixing  $\alpha_s(M_Z^2)$  to 0.120 reduces the error on  $m_t$  by a factor of 2. Since this technique would yield a measurement of  $\alpha_s(M_Z^2)$  no more precise than those made today, and since systematic uncertainties may be large and have not yet been considered, a more sensible strategy would be to measure  $\alpha_s(M_Z^2)$  as precisely as possible, as described in the previous section, and to use this value to allow better determination of the top quark parameters.

## 3. Energy Evolution Studies

The non-Abelian gauge structure of QCD implies that as the hard scattering scale  $Q$  increases, the strong coupling decreases roughly as  $1/\ln Q$ .<sup>9</sup> Existing hadronic final states data from  $e^+e^-$  annihilation at the PETRA, PEP, TRISTAN, SLC and LEP colliders span the range  $14 \leq Q \leq 91 \text{ GeV}$ , although hadronisation uncertainties are large on the data below 25 GeV.<sup>21</sup> A 2 TeV HLC would increase the lever-arm in  $1/\ln Q$  by almost a factor of two, hence allowing detailed study of the energy evolution of QCD observables that are proportional to  $\alpha_s$ , such as the rate of production of final states containing three hadronic jets,  $R_3$ . This would provide not

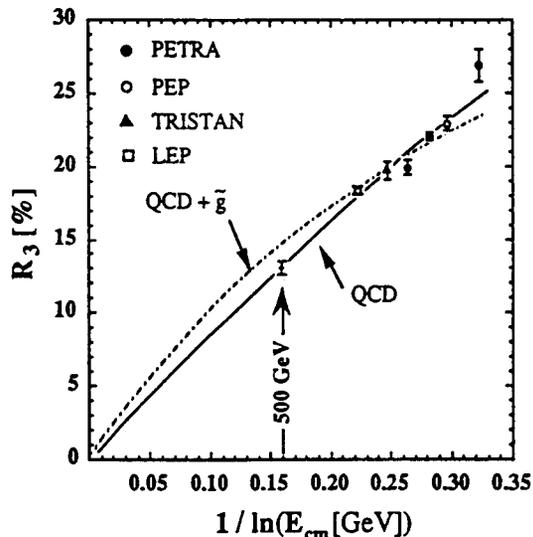


Figure 2: Three-jet rate vs.  $1/\ln E_{CM}$ .

only a test of the fundamental structure of the group describing strong interactions, but also a search-ground for new physics that might produce ‘anomalous’ running.

One such possibility is the existence of a light, electrically neutral coloured fermion that couples to gluons, often called a ‘light gluino’ and denoted by  $\tilde{g}$ . The existence of such a particle would manifest itself via a modification of gluon vacuum polarisation contributions involving fermion loops, effectively increasing the number of light fermions entering into the QCD  $\beta$ -function. At one-loop level the effective number of flavours would change from  $N_F$  to  $N_F + 3N_{\tilde{g}}$ , where  $N_{\tilde{g}}$  is the number of families of light gluinos, causing a decrease in the running of  $\alpha_s$  as a function of  $Q$ . The existence of a light gluino of mass between 2 and 5 GeV/ $c^2$  has not been excluded by searches with current data.<sup>22</sup> A simulated measurement of  $R_3$  at  $Q = 500$  GeV, corresponding to 20% of one design-luminosity-year, is shown in Fig. 2,<sup>22</sup> together with existing measurements, plotted as a function of  $1/\ln Q$ . The presence of one family of light gluinos of mass 2 GeV/ $c^2$  would cause an increase in the predicted value of  $R_3$  at 500 GeV by 10%. A 1%-level measurement of  $\alpha_s$ , as discussed in the previous section, would allow this difference to be measured with a significance of many standard deviations.

It should be noted, however, that data from a number of experiments at different  $e^+e^-$  colliders contribute to Fig. 2. Some of these data were recorded more than 10 years ago, were treated differently by the various experimental groups, and have relatively large systematic errors that are at least partly uncorrelated from point

to point. Furthermore, the sophistication and performance of particle detectors constructed in the last decade has improved significantly, and it is reasonable to assume that future detectors will be even better. In addition, our understanding of the modelling of hadronisation effects and theoretical uncertainties has improved enormously as a result of studies at the  $Z^0$ . Therefore, the precision of searches for anomalous running of QCD observables at HLC would be improved significantly if new data were taken at the lower c.m. energies with the *same* detector and analysis procedures.

In fact, if the luminosity of the 500 GeV HLC could be preserved at lower c.m. energies, very large data samples would be recorded. Table 1<sup>22</sup> shows the number of  $q\bar{q}$  events delivered per day at various c.m. energies by the HLC operating at the design luminosity of  $5 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$ . At each energy more luminosity would be delivered per day than was recorded in total by the original dedicated colliders! This argument is of course naive, in that a collider designed to operate at a luminosity of  $5 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$  at 500 GeV would not automatically be operable at the same luminosity at energies a factor of 5 or 10 lower; such capability would have to be designed from the outset. Furthermore, the requirements on the triggering and data processing capabilities of the detector are extreme by the standards of  $e^+e^-$  annihilation, and this would also have to be designed from the start. Nevertheless, the prospect of running the HLC at the  $Z^0$  resonance, or at even lower energies, for QCD studies, not to mention high-statistics electroweak physics measurements, is sufficiently attractive that a quantitative study of the resulting improvement in precision on anomalous running effects should be made.

Table 1. Number of  $q\bar{q}$  events per day delivered by an  $e^+e^-$  collider operating at a luminosity of  $5 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$ .

| c.m. energy $Q$ (GeV) | # $q\bar{q}$ events/day |
|-----------------------|-------------------------|
| 500                   | 1750                    |
| 91                    | 20,000,000              |
| 60                    | 75,000                  |
| 35                    | 150,000                 |

#### 4. Symmetry Tests Using Beam Polarisation

Production and transport of highly-polarised high-charge electron beams has been demonstrated at the SLAC Linear Collider (SLC), where stable operation at the  $Z^0$  resonance with a beam polarisation of magnitude 0.77 has been achieved. The  $Z^0$  bosons produced using longitudinally polarized electrons have polarization along the beam direction  $A_Z = (P_{e^-} - A_e)/(1 - P_{e^-} \cdot A_e)$ , where  $P_{e^-}$  is the electron beam polarization, defined to be negative (positive) for a left-(right-) handed beam, and  $A_e = 2v_e a_e/(v_e^2 + a_e^2)$  with  $v_e$  and  $a_e$  the electroweak vector and axial vector coupling parameters of the electron, respectively. In order to reduce systematic effects the electron spin direction is reversed randomly pulse-by-pulse, thus achieving higher sensitivities to polarization-dependent asymmetries.

For polarized  $Z^0$  decays to three hadronic jets one can define the triple-product:

$$\vec{S}_Z \cdot (\vec{k}_1 \times \vec{k}_2),$$

which correlates the  $Z^0$  boson polarization vector  $\vec{S}_Z$  with the normal to the three-jet plane defined by  $\vec{k}_1$  and  $\vec{k}_2$ , the momenta of the highest- and the second-highest-energy jets respectively. The triple-product is even under reversal of  $CP$ , and odd under  $T_N$ , where  $T_N$  reverses momenta and spin-vectors without exchanging initial and final states. Since  $T_N$  is not a true time-reversal operation a non-zero value does not signal  $CPT$  violation and is possible in a theory that respects  $CPT$  invariance. A similar triple product observable can be defined for  $e^+e^- \rightarrow q\bar{q}g$  events off the  $Z^0$  resonance. Indeed, a sizeable signal is expected<sup>23</sup> at c.m. energies below 40 GeV, although no experimental measurements have been performed since longitudinally polarized electron beams were not available.

The tree-level differential cross section for  $e^+e^- \rightarrow q\bar{q}g$  for a longitudinally polarized electron beam and massless quarks may be written<sup>23,24</sup>:

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_N} = \frac{9}{16} \left[ \left(1 - \frac{1}{3} \cos^2\theta_N\right) + \beta A_Z \cos\theta_N \right],$$

where  $\theta_N$  is the polar angle of the vector normal to the jet plane,  $\vec{k}_1 \times \vec{k}_2$ , w.r.t. the electron beam direction. With  $\beta|A_Z|$  representing the magnitude, the second term is proportional to the  $T_N$ -odd triple-product, and appears as a forward-backward

asymmetry of the jet-plane-normal relative to the electron polarization axis. It should be noted that the sign and magnitude of this term are different for the two beam helicities.

Recently Standard Model  $T_N$ -odd contributions of this form at the  $Z^0$  resonance have been investigated<sup>24</sup>. The triple-product vanishes identically at tree level, but non-zero contributions arise from higher order processes involving: a) QCD rescattering of massive quarks<sup>23</sup>, b) QCD triangle of massive quarks<sup>25</sup>, and c) electroweak rescattering via  $W$  and  $Z$  exchange loops. Due to various cancellations these contributions are found to be very small at the  $Z^0$  resonance and yield values of the correlation parameter  $|\beta| \leq 10^{-5}$ .<sup>24</sup> Because of this background-free situation, measurement of the cross section is sensitive to physics processes beyond the Standard Model that give  $\beta \neq 0$ . The first experimental study of this quantity has been made by the SLD Collaboration,<sup>26</sup> yielding limits:  $-0.022 < \beta < 0.039$ .

The dominant Standard Model contributions to  $\langle \cos\theta_N \rangle$  at the  $Z^0$  resonance arise from  $W^-$  and  $Z^0$ -rescattering processes<sup>27</sup>. The energy-dependence of these contributions is illustrated in Fig. 3. They are small in magnitude near the  $Z^0$  resonance and reach their maximum magnitude between 300 and 500 GeV. For any energy they remain less than 2 parts in  $10^5$ , which is immeasurably small. Therefore, at high energy, unless contributions from rescattering via pairs of vector bosons<sup>27</sup> turn out to be large,  $\theta_N$  is a potentially 'background-free' observable for searches for deviations from the Standard Model, for example due to rescattering of new gauge bosons that couple only to baryon number.<sup>28</sup>

## 5. Gluon Radiation in $t\bar{t}$ Events

The large mass and decay width of the top quark serve to make the study of gluon radiation in  $t\bar{t}$  events a new arena for testing QCD. The large mass acts as a cutoff for collinear gluon radiation, and the large decay width acts as a cutoff for soft gluon radiation, allowing reliable perturbative QCD calculations to be performed; these effects are of course correlated. The latter case is particularly interesting. If the top width were infinite, top quarks would decay immediately to bottom quarks, and any gluons would be radiated from the secondary b's. If the top width were zero, top quarks would live forever and all radiation would be from the primary t's. In the case of a large but finite width, expected to be around 2 GeV for a top mass

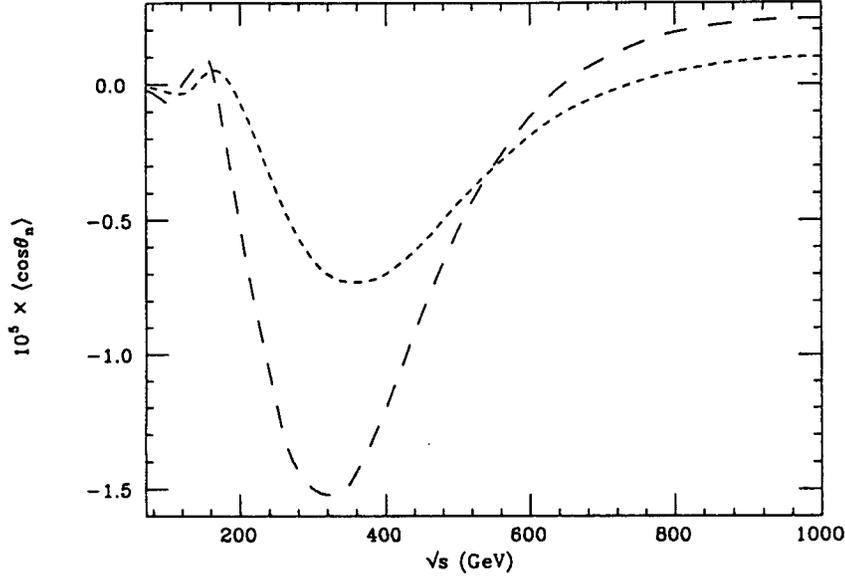


Figure 3: Expectation value of  $\cos\theta_N$  vs. c.m. energy; contributions from  $W$  (long dashes) and  $Z^0$  (short dashes) rescattering.

of  $180 \text{ GeV}/c^2$ , gluon radiation in  $t\bar{t}$  events will be a coherent sum of contributions from these two limiting cases, with a degree of coherence regulated by the top width itself.

A previous theoretical study<sup>29</sup> of  $t\bar{t}$  production above threshold, assuming  $m_t = 140 \text{ GeV}/c^2$ , demonstrated interference effects in the angular distribution of soft gluons w.r.t. the top quark flight direction. This study has been updated for  $m_t = 175 \text{ GeV}/c^2$  at  $Q = 1 \text{ TeV}$ , and the results are illustrated in Fig. 4.<sup>30</sup> This shows the angular distribution of 5 GeV gluons w.r.t. the  $t\bar{t}$  axis for the kinematic configuration in which the decay b-quark travels backwards w.r.t. the  $t$  flight direction. The dependence of the radiation pattern on the top decay width is strong. Similar effects are predicted in the spectrum of gluon radiation in  $t\bar{t}$  events around threshold.<sup>29</sup> Measurement of such effects would yield not only a dramatic demonstration of quantum interference in strong interactions, but might also provide an essential cross-check on the value of the top quark decay width, which may prove difficult to disentangle from measurements of the  $t\bar{t}$  threshold cross-section and top momentum distributions, which also depend on  $\alpha_s$  and  $m_t$  (section 2.3), as well as on the beam energy distribution.<sup>3</sup> This possibility is exciting, but will require a detailed hadron-level Monte Carlo simulation study, with the

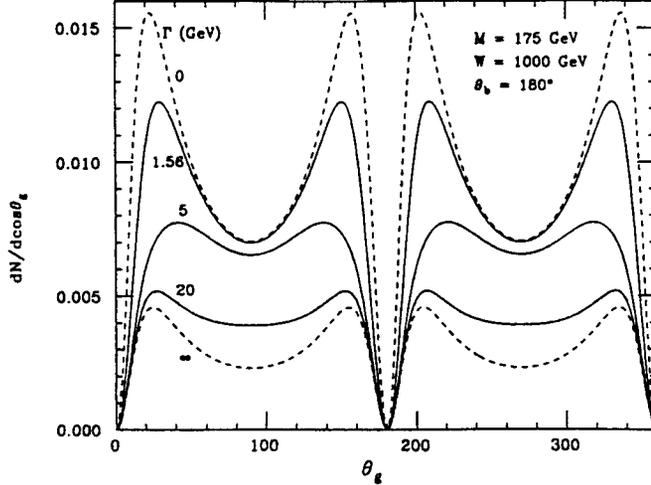


Figure 4: Angular distribution of 5 GeV gluons w.r.t.  $t\bar{t}$  axis, at  $Q = 1$  TeV.

inclusion of realistic detector effects, to demonstrate that the coherence effects can be measured with sufficient precision.

## 6. Anomalous Chromomagnetic Top Quark Couplings

The existence of anomalous couplings of quarks to gluons could manifest itself via a modification of the rate and pattern of emitted gluon radiation, beyond effects such as those discussed in the last section. A model-independent parametrisation of anomalous couplings in the strong-interaction Lagrangian may be written<sup>31</sup>:

$$\mathcal{L}^{q\bar{q}g} = g_s \bar{q} T_a \left( \gamma_\mu + \frac{i\sigma_{\mu\nu} k^\nu}{2m_t} (\kappa - i\tilde{\kappa}\gamma_5) \right) q G_a^\mu$$

where  $\kappa$  and  $\tilde{\kappa}$  represent anomalous ‘chromomagnetic’ and ‘chromoelectric’ dipole moments, respectively. The chromoelectric moment gives rise to CP-violating effects and will not be considered further here. The chromomagnetic case has been calculated at leading order in perturbation theory;<sup>31</sup> for HLC energies the perturbative approach remains reasonable for  $|\kappa| \leq 3$ .

In principle such moments may exist for quarks of any flavour, but new physics processes at high energy scales are more likely to couple to heavy quarks so that exploration of this scenario for  $t\bar{t}g$  events seems the most sensible first thing to do. This has been considered for  $m_t = 179$  GeV/ $c^2$  at  $Q = 500$  GeV and 1 TeV.<sup>31</sup> The phase space available for anomalous chromomagnetic effects increases with

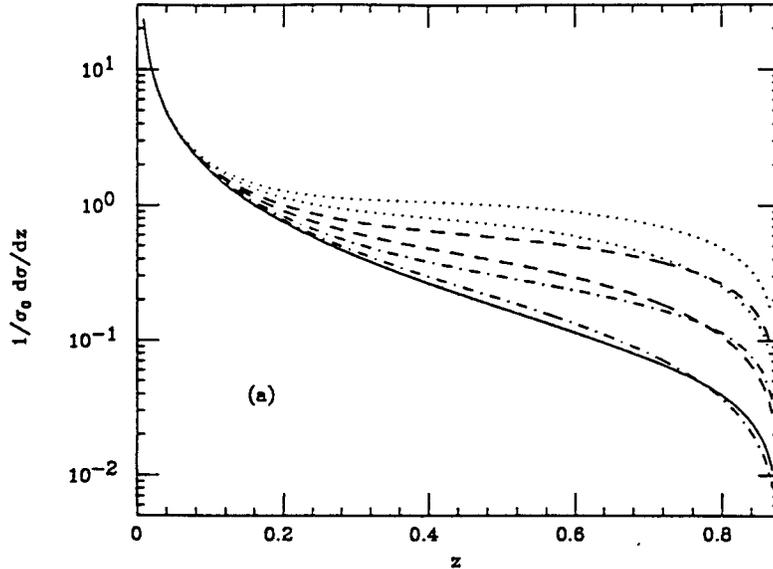


Figure 5: Distribution of scaled gluon energy  $z = E_{gluon}/E_{beam}$  at  $Q = 1$  TeV. QCD (solid)  $\kappa = \pm 1$  (dot-dash),  $\pm 2$  (dash),  $\pm 3$  (dots).

the c.m. energy; the gluon energy spectrum in the latter case is shown in Fig. 5 for various values of  $\kappa$ . A measurement of this spectrum for  $\kappa = 0$ , with a perfect detector and integrated luminosity corresponding to about 2 design years of running at 1 TeV, was simulated, and yielded 95% confidence-level limits of  $-0.12 \leq \kappa \leq 0.21$ . The limits obtainable at 500 GeV are about a factor of two worse, with the additional complication of a second minimum in  $\chi^2$  around  $\kappa = -2$  due to destructive interference between QCD and the anomalous contributions.<sup>31</sup>

Limits of comparable statistical precision may be obtainable from hadron-hadron colliders via measurement of the inclusive top quark production cross-section. At LHC, for example, this would require a measurement of  $\pm 20\%$  accuracy, but this does not take into account theoretical uncertainties in the cross-section due to higher-order QCD effects and limited knowledge of structure functions. In the  $e^+e^-$  case it would be valuable to simulate a realistic detector and to impose event selection cuts (Section 7) in order to understand better the limits that could be set on  $\kappa$ .

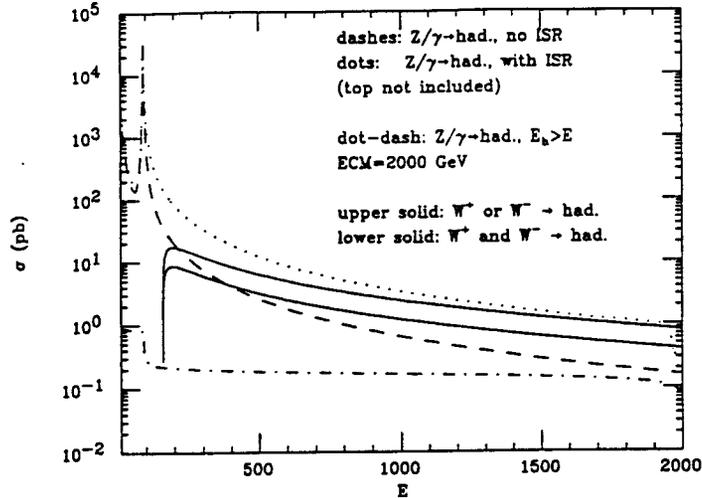


Figure 6: Production cross-sections vs. c.m. energy for  $q\bar{q}$  (dashes) and  $W^+W^-$  (solid) events.

## 7. Event Selection

### 7.1 The Problem

It is of extreme importance to demonstrate that samples of  $q\bar{q}$  ( $q=u,d,s,c,b$ ) and  $t\bar{t}$  events can be selected with good efficiency, high purity and low bias in order to perform the programme of QCD measurements just outlined. In QCD studies at and below the  $Z^0$  resonance the primary background sources to  $q\bar{q}$  events are  $e^+e^- \rightarrow$  lepton-pairs,  $e^+e^- \rightarrow \gamma\gamma$  with  $\gamma\gamma \rightarrow$  hadrons, cosmic rays, and beam-related background events. Application of simple requirements on the number of charged tracks, visible energy, and energy balance of events has allowed selection of  $q\bar{q}$  event samples of greater than 99% purity with over 90% efficiency (see *eg.* Ref. 14).

At higher energies the situation is less straightforward, as other physics processes are present that have comparable or larger cross-sections than  $q\bar{q}$  production, and which may also appear as hadronic final states. The most serious case is  $e^+e^- \rightarrow W^+W^-$ , in which one or both  $W$ 's may decay into hadrons. As shown in Fig. 6.<sup>32</sup> the cross-section for this process exceeds the  $q\bar{q}$  production cross-section at c.m. energies above about 400 GeV, and grows relatively as  $Q$  increases; at  $Q = 2$  TeV it is roughly a factor of two larger. This case is particularly troublesome as

the mass of a 1 TeV quark jet, roughly  $\alpha_s \times 1 \text{ TeV}$ , is comparable with the  $W$  mass, making separation of  $q\bar{q}$  and  $W^+W^-$  events on the basis of jet masses a difficult prospect. From the viewpoint of QCD studies, therefore, the aim is to develop a set of cuts to isolate separately  $q\bar{q}$  and  $t\bar{t}$  events and to exclude primarily  $W^+W^-$  and  $Z^0 Z^0$  events.

### *7.2 Initial State Radiation and Beamstrahlung*

Additional complications arise from the effects of initial state radiation and beamstrahlung, which serve to reduce the effective c.m. energy available for the  $e^+e^-$  annihilation, and to cause events to be Lorentz-boosted along the beam axis. This has been studied in Ref. 22, where it was shown that radiative effects are considerable and cause 'event pileup' at the  $Z^0$  resonance, where more events will be produced than at the nominal operating energy of the collider. In some sense, therefore, the energy scan discussed in Section 3 is delivered free by Nature itself. However, it is not clear that strongly forward-boosted events could be sufficiently well measured so as to be useful for QCD studies. This issue has serious implications for the design of the detector, perhaps requiring HERA-style detector elements in the forward regions, and hence deserves a detailed Monte Carlo study.

### *7.3 Selection Cuts*

Much progress has already been achieved with the development of a set of event selection cuts based on particle multiplicities, visible energy, the thrust-axis polar angle, and the invariant masses of the two event hemispheres.<sup>15</sup> The first two cuts are designed to reject lepton-pair,  $\gamma\gamma$  and beam-related final states; the third cut rejects some of the (t-channel)  $W^+W^-$  events; lower bounds on the hemisphere masses reject lepton pair events, whilst upper bounds reject  $t\bar{t}$ ,  $W^+W^-$  and  $Z^0 Z^0$  events. Assuming a detector with perfect efficiency within a polar-angle acceptance of  $|\cos\theta| < 0.98$ , these cuts are estimated to yield a sample of 85,000 hadronic events within 10% of the nominal c.m. energy per design year of running at 500 GeV. The sample comprises 83%  $q\bar{q}$ , 11%  $W^+W^-$ , 6%  $t\bar{t}$ , and  $< 1\%$   $\gamma\gamma$  events.<sup>15</sup> Using this sample to form distributions of standard QCD observables such as thrust, it was found that corrections at the level of 25% had to be applied in order to correct for the bias to the shape of the  $q\bar{q}$  distribution introduced by the

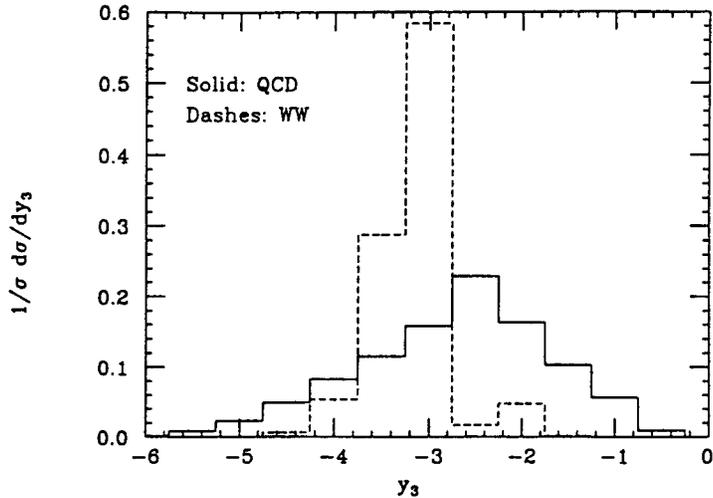


Figure 7: Distribution of  $\log_{10} y_3$  for  $q\bar{q}$  (solid) and  $W^+W^-$  (dashes) events at 2 TeV.

background contamination. Clearly this situation needs to be improved if a 1%-level measurement of  $\alpha_s(M_Z^2)$  is to be made. In this context it is interesting to note that the  $W^+W^-$  background can also be suppressed by selecting only events produced with right-handed electron beams.

An extension of this study to the more challenging environment at  $Q = 2$  TeV has recently been made.<sup>32</sup> The variable  $y_3$  was considered as a discriminator between  $q\bar{q}$  and  $W^+W^-$  events, where  $y_3$  is the jet-pair invariant mass, normalised by the event visible energy, at which an event changes its classification from 3-jet-like to 2-jet-like according to an iterative clustering algorithm, such as the Durham algorithm.<sup>33</sup> Distributions of  $\log_{10} y_3$  are shown in Fig. 7 for  $q\bar{q}$  and  $W^+W^-$  events; there are no  $W^+W^-$  events above  $y_3 \simeq 0.01$ . Since  $y_3$  is one of the most attractive observables used in determining  $\alpha_s$  (see eg. Ref. 14), the clean separation between  $q\bar{q}$  and  $W^+W^-$  events implies that an unbiased measurement of  $\alpha_s$  could be made from the  $y_3$  distribution for  $y_3 > 0.01$ .

## 8. Summary and Conclusions

Tests of QCD will enrich the physics programme at future high energy linear  $e^+e^-$  colliders. Measurement of  $\alpha_s(M_Z^2)$  at the 1% level of precision appears feasible experimentally, but will require considerable theoretical effort to calculate

$O(\alpha_s^3)$  contributions in QCD perturbation theory. A search for anomalous running of  $\alpha_s(Q^2)$ , by operating the collider at different c.m. energies, is an attractive prospect, but presents serious requirements on the design of both the collider and detectors. Longitudinal electron beam polarisation can be exploited to perform symmetry tests using multi-jet final states.

Quantum coherence is expected to give rise to interesting gluon radiation patterns in  $t\bar{t}$  events, which could be used to constrain the top quark decay width. Measurement of the gluon radiation spectrum would also constrain anomalous top quark chromomagnetic couplings. Realistic hadron-level Monte Carlo simulations, including detector effects, need to be performed to evaluate these possibilities quantitatively. Efficient separation of pure samples of  $q\bar{q}$  and  $t\bar{t}$  events will be complicated by high backgrounds from hadronic  $W^+W^-$  and  $Z^0 Z^0$  events, but good progress has been made in developing event selection cuts. Last, but by no means least, studies of the expected properties of jets at c.m. energies in the range  $0.5 \leq Q \leq 2$  TeV<sup>34</sup> need to be updated in order to specify the requirements on the performance of detectors for high energy linear  $e^+e^-$  colliders.

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