

The normalized cross correlation $\zeta(\theta)$ v.s. the scalar mass m_s .

The Polarizations of Gravitational Waves

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Review

Seeing Black Holes: From the Computer to the Telescope

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Abstract: Astronomical observations are about to deliver the very first telescopic image of the massive black hole lurking at the Galactic Center. The mass of data collected in one night by the Event Horizon Telescope network, exceeding everything that has ever been done in any scientific field, should provide a recomposed image in 2018. All this, forty years after the first numerical simulations performed by the present author.

Keywords: black hole; numerical simulation; observation; general relativity

1. Introduction

According to the laws of general relativity (for recent overviews at the turn of its centennial, see, e.g., [1,2]), black holes are, by definition, invisible. Contrary to uncollapsed stars, their surface is neither a solid nor a gas; it is an intangible frontier known as the event horizon. Beyond this horizon, gravity is so strong that nothing escapes, not even light. Seen projected onto the background of the sky, the event horizon would probably resemble a perfectly black disk if the black hole is static (Schwarzschild black hole) or a slightly flattened disk if it is rotating (Kerr black hole).

A black hole however, be it small and of stellar mass or giant and supermassive, is rarely “bare”; in typical astrophysical conditions it is usually surrounded by gaseous matter. It forms an accretion disk in which the spinning gas is accelerated to large speeds by the huge gravity, releasing heat and high energy electromagnetic radiation. A giant black hole, as can be found in the centre of most galaxies, may also be surrounded by a cluster of stars, the orbital dynamics of which is strongly influenced by it. In essence, a black hole remains invisible, but in its own special way, it lights up the matter it attracts.

Logically, scientists have wondered what a black hole lit up by its surrounding matter would look like. Educational or artistic representations can be seen in popular science magazines in the form of a sphere seeming to float in a whirlpool of glowing gas. These images, although forceful, fail to convey the astrophysical reality. A black hole can be described correctly using computer simulations that take account of the complex distortions made by the gravitational field on space-time and on the paths of light rays that follow its fabric. These were performed for the first time in 1978 by the author of this article [3]. Today, progress in astronomical observation is about to deliver the first telescopic image of the shadow of a giant black hole, thanks to the ambitious Event Horizon Telescope (EHT) programme (for a popular account, e.g., [4]).

2. Black Holes Simulated

The notion of the black hole shadow was introduced for the first time in 1972 by James Bardeen at a Summer school in Les Houches, France [5]. He initiated research on gravitational lensing by spinning black holes by computing how the black hole's rotation would affect the shape of the shadow that the event horizon casts on light from a background radiating screen.

Next, Cunningham and Bardeen [6] calculated the optical appearance of a star in circular orbit in the equatorial plane of an extreme Kerr black hole, taking account of the Doppler effect due to relativistic motion of the star, and pointed out the corresponding amplification of the star's luminosity.

The calculation of the black hole shadow can be generalized to the more complex situation when the radiating source is an accretion disk, each emitting point of the disk being equivalent to a point-like source in circular orbit. To create the most realistic possible images of a black hole surrounded by an accretion disk, not only do we have to calculate the propagation of light rays emitted by the matter in the disk through the curved space-time geometry generated by the black hole, but we also have to know the physical properties of the accretion disk itself, in order to know the intrinsic flux emitted in its various regions. In 1978, I was a young scientist at the Paris-Meudon Observatory and performed the first accurate numerical simulation of the "photographic" appearance of a Schwarzschild black hole surrounded by a thin accretion disk. To do so, I used the IBM 7040 mainframe of the Paris-Meudon Observatory, an early transistor computer with punch card inputs. Without a computer visualisation tool, I had to create the final image by hand from the digital data. For this I drew directly on negative image paper with black India ink, placing dots more densely where the simulation showed more light. Next, I took the negative of my negative to get the positive, the black points becoming white and the white background becoming black.

This image (Figure 1) appeared first in the November issue of a French popular magazine [7] and concluded a 1979 article in a specialized journal, with all equations and technical details [3].

The top of the disk remains visible regardless of the viewing angle—in contrast to the typical views of Saturn's rings. Indeed, the gravitational field curves the light rays near the black hole so much that the rear part of the disk is "revealed". Even if the black hole hides what falls into it, it cannot mask what is behind it.

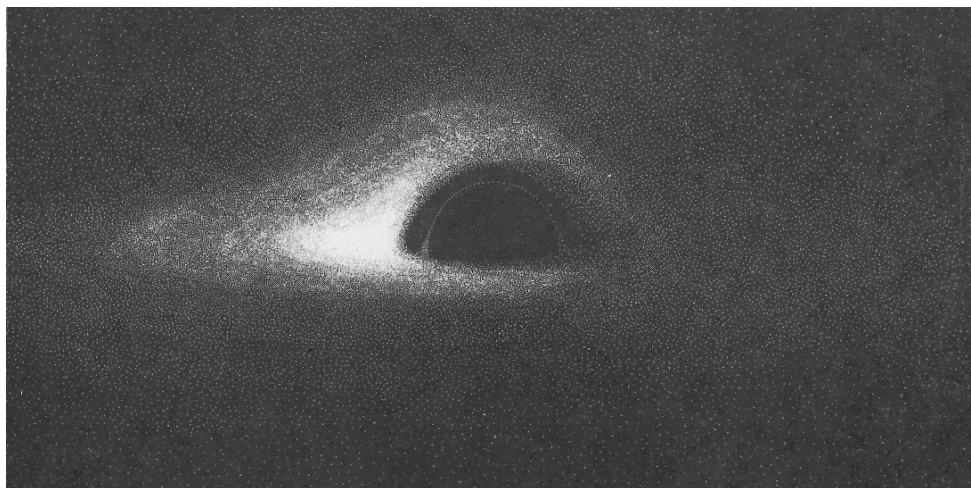


Figure 1. Simulated photograph of a spherical black hole with thin accretion disk. The system is seen from a great distance by an observer at 10° above the disk's plane, in a frame at rest with the black hole. © J.-P. Luminet, from [3].

The curving of the light rays also generates a secondary image that allows us to see the other side of the accretion disk, on the opposing side of the black hole from the observer. Very deformed optically, the rear part looks like a thin halo of light around the dark shadow of the black hole, which represents

the event horizon enlarged by a factor of $3\sqrt{3}/2 \approx 2.6$ due to the gravitational lens effect. Indeed the gravitational lensing generates an infinity of images of the disk, because the light rays can travel any number of times around the black hole before escaping from its gravitational field and being observed by a distant astronomer. The primary image shows the upper side, the secondary image shows the lower side, the third image shows the upper side again, and so on. However, multiple images of order higher than 2 are not relevant for observational purposes because they are stuck to the edge of the black hole shadow.

The main feature of this view of the black hole is the significant difference in luminosity between the various regions of the disk. On the one hand, the light shines maximally in the areas closest to the horizon, as the gas is hottest there since it moves more rapidly. On the other hand, the light received by a distant observer is considerably different from the light emitted, due to the combination of the Einstein and Doppler effects; the first caused by the gravity field, the latter by the rapid rotation of the accretion disk. For a distant observer, the light received is considerably amplified on the side of the image where the gas approaches the observer and is weaker on the side it is moving away from.

The virtual photo of the black hole was calculated “bolometrically”, i.e., displaying integrated light on the whole electromagnetic spectrum, from the radio to the gamma wave range. It is independent of the mass of the black hole and the flow of gas swallowed, on the condition that the accretion rate remains moderate and the disk is thin (in other situations, the structure of the accretion disk may be thick, take the form of a torus, etc.). This image may therefore describe a stellar black hole 10 km in radius, attracting the gas from an accompanying star, or a giant black hole lying at the centre of a galaxy and sucking in the interstellar gas in thin disk configurations.

This initial digital imaging work on black holes was then developed by numerous scientists, who benefited from the rapid progress in computer performances. Colours were added to the images (according to a specific coding dictated by variations in temperature) and background skies, to make the reconstitution as realistic as possible. Moreover, the observer was no longer assumed to be stationary and very distant from the black hole, but moving with it, which introduced a new distortion of the images by the Doppler effect due to the movement of the observer. Finally, the black hole can be rotating, such as in the Kerr solution, which is the most realistic astrophysical situation. However, although this rotation generates an additional asymmetry—the black hole event horizon is no longer strictly spherical—it remains small, even if rotating rapidly.

Of the numerous visualisations created, some of which can be found on the Internet [8,9], those made at the start of the 1990s by my colleague Jean-Alain Marck at the Paris-Meudon Observatory, in colour and animated, are the most remarkable (for a technical description, see [10]). We made a film [11] which shows the spectacle that would be seen from the window of a spacecraft falling freely towards the black hole on various trajectories (Figure 2).

In the autumn of 2014, the world’s media waxed lyrical on the representations of the supermassive black hole, “Gargantua”, imagined by the filmmaker, Christopher Nolan, for his film *Interstellar*. The American astrophysicist Kip Thorne, a renowned specialist in relativistic astrophysics (he received the Nobel Prize for physics in 2017 for his work on gravitational waves), was technical advisor to a team of 200 graphic animation experts, using the most sophisticated calculation and visualisation tools to model the appearance of a giant black hole measuring one hundred million solar masses, rotating rapidly and surrounded by an accretion disk.

The international press headlined the scientific realism of the calculated images, which resulted from a “simulation of unprecedented accuracy”. The most captivating view in the film is calculated for an observer located in the plane of the accretion disk in a frame at rest with the black hole (Figure 3). While it correctly describes the primary and secondary images distorted by the gravitational field, it mistakenly shows uniform luminosity of the disk.

In other words, it neglects all the physical effects due to its radiating structure and its rapid rotation. As Kip Thorne explained to me in a private email, the filmmaker decided that light asymmetry in the image would have been incomprehensible to the spectators. However, it is precisely this strong

asymmetry of apparent luminosity that is the main signature of a black hole, the only celestial object able to give the internal regions of an accretion disk a speed of rotation close to the speed of light and to induce a very strong Doppler effect.

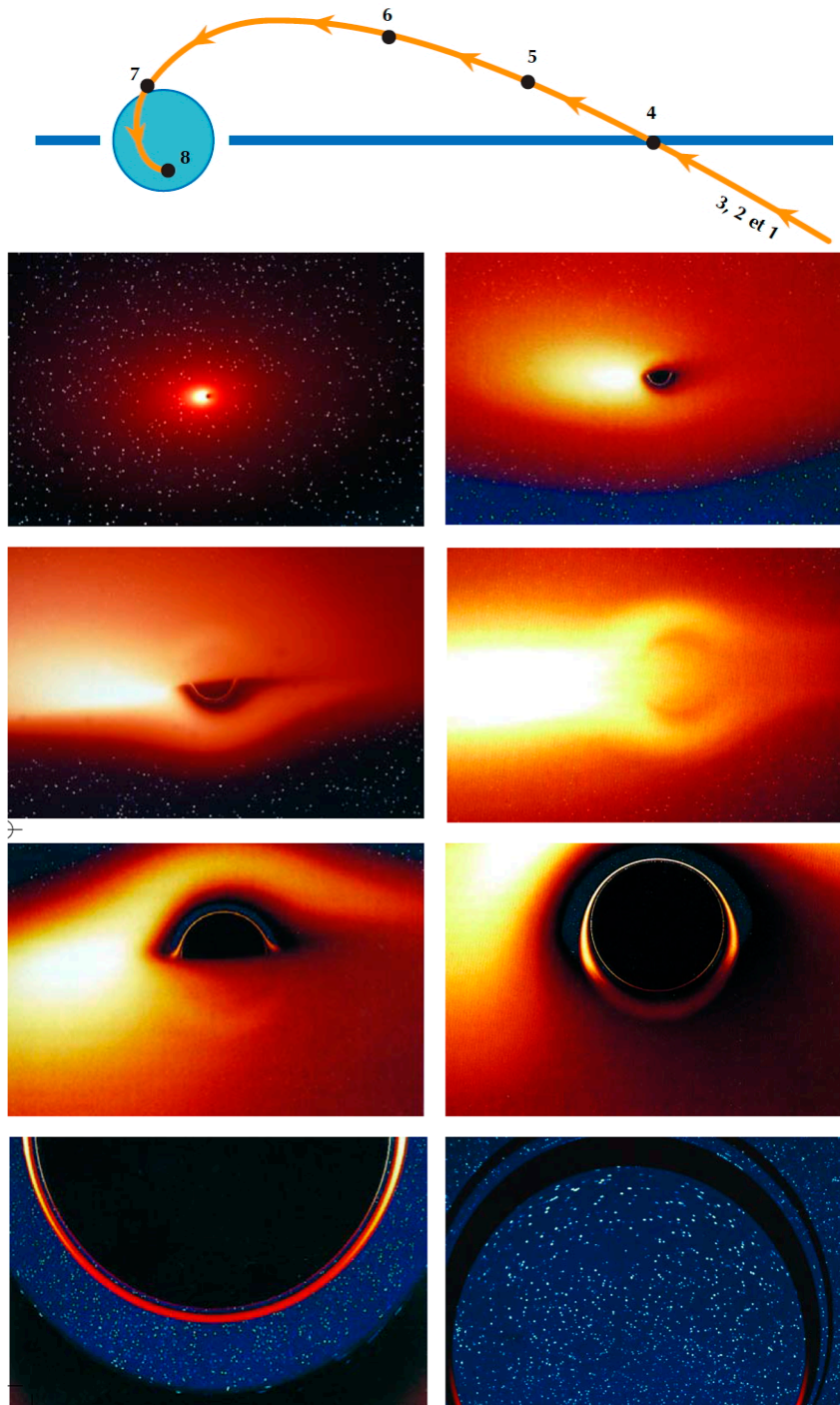


Figure 2. Colour simulations of a black hole accretion disk taking account of the Doppler and gravitational shifts. The images are calculated at positions 1 to 8 of the plunging trajectory schematized above. The last picture is taken from inside the black hole, the observer having rotated by 180° to watch the outside. © J.A Marck and J.-P. Luminet, from [12].

Slightly embarrassed by this bending of the scientific truth, Thorne and his colleagues subsequently published an image in a technical journal, taking account of the effects of the spectral

shifts (Figure 4), but still based on an artistic view of the accretion disk rather than on a physical model [13].

Indeed, from the computer simulations by Marck performed 20 years earlier, Image 4 from Figure 2 is much closer to the astrophysical reality, as it was also calculated as seen by an observer in the equatorial plane, with all the shift effects and the Page and Thorne [14] physical model of a thin accretion disk. For a more detailed criticism of *Interstellar* science, see [15].



Figure 3. Simulation of an accretion disk around a Kerr black hole as seen by an observer in the equatorial plane, shown in the movie *Interstellar*. © Double Negative artists/DNGR/™ & © Warner Bros.

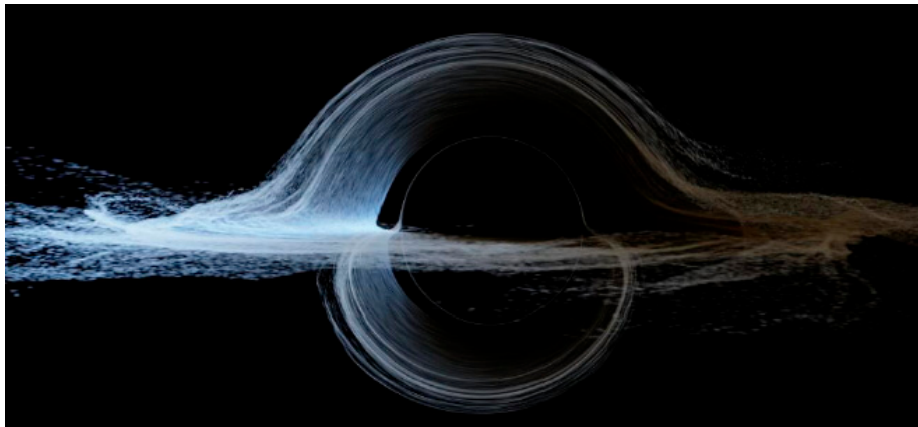


Figure 4. An « anemic » accretion disk around a Kerr black hole with spin $a/M = 0.6$, false colours, Doppler and gravitational shifts (from [13]).

As is well-known, the Kerr spacetime metric depends on two parameters, the black hole mass M and its normalized angular momentum a , but there is a critical angular momentum given by $a = M$ (in units where $G = c = 1$) above which the event horizon vanishes, leaving a naked singularity, also called a Kerr superspinner. Although such configurations are generally considered as unrealistic, they can be studied theoretically as their external field is governed by general relativity. The case of the appearance of Keplerian accretion disks orbiting a Kerr superspinner has been extensively studied in [16], which found that it differs significantly from those related to Kerr black holes.

Another way of visualising a “bare” black hole (without an accretion disk), is to calculate the gravitational mirage it causes on the background of stars. The most spectacular interpretations,

combining scientific accuracy and aesthetics, were obtained in 2006 by Alain Riazuelo [17]. He calculated the gravitational mirage caused by a black hole passing in front of a background of stars, the disk of our own galaxy or Magellanic clouds (Figure 5). In the same line, ref. [18] calculated the appearance of a cosmic microwave background to observers orbiting in close vicinity of Kerr black holes or superspinars.

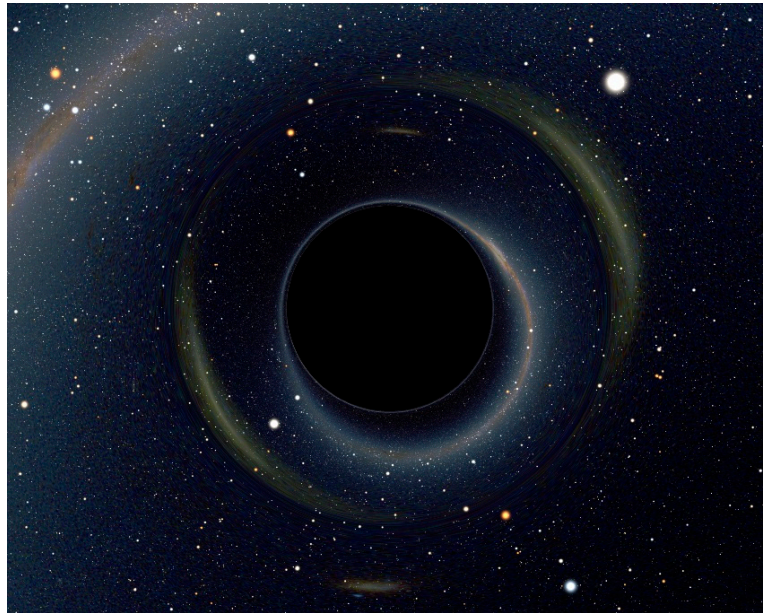


Figure 5. Gravitational lensing produced by a black hole in a direction almost centered on the Large Magellanic Cloud. Above it one easily notices the southernmost part of the Milky Way with, from left to right, Alpha and Beta Centauri, the Southern Cross. The brightest star close to the LMC is Canopus (seen twice). The second brightest star is Achernar, also seen twice. The two large arcs are the primary and secondary images of the LMC, the two smaller ones are those of the Small Magellanic Cloud. Courtesy Alain Riazuelo, CNRS/IAP.

3. From the Computer to the Telescope

All these illustrations are virtual images obtained using the equations of general relativity and more or less realistic physical models. But could we see a black hole directly? If astronomers had a sufficiently powerful telescope, they would be able to directly observe the shadow cast by the event horizon of a black hole and the hot mark of the accretion disk surrounding it. However, several technical challenges prevent the development of such an instrument. The main one is the tiny size of black holes seen from the Earth. The closest known stellar black hole, located in the binary X-ray source A0620-00 in our Galaxy, 3500 light-years away, has a diameter of just 40 km. So, we have to aim at close, supermassive black holes, knowing that their size is proportional to their mass. The two most promising candidates are Sagittarius A* (Sgr A*), located 26,000 light-years (8 kpc) away in the centre of our galaxy, with an estimated mass of 4.3 million solar masses [19], and the supergiant M87*, a monster of 6 or 7 billion solar masses, lying at the centre of the giant elliptical galaxy of M87, 55 million light-years (16.5 kpc) away [20]. Everything points to the fact that these black holes are surrounded by a rapidly rotating accretion disk, possibly formed of star debris previously broken up by tidal forces (Figure 6).

In terms of intrinsic size, the event horizons of Sgr A* and M87* are 25 million and 36 billion kilometres in diameter, respectively. However, as already mentioned in Section 2, the effect of the gravitational lens caused by the black hole amplifies the apparent size of the event horizon by a factor of 2.6. With all calculations done, it appears that the shadows cast by Sgr A* and M87* on the gaseous halos of their accretion disks have an apparent diameter of about 50 microarcseconds. This is the angle

under which we would see an apple on the Moon with the naked eye, requiring a resolution 2000 times greater than that of the Hubble space telescope.

The resolution of a telescope is proportional to its aperture (the diameter of its lens) and inversely proportional to the wavelength at which it is observing. Doubling the aperture would show details twice as accurately. Thus, we would need a telescope operating in the visible range 2 km in diameter to resolve the images of Sgr A* and M87*, this is not possible, even in the medium-term. Another difficulty is that the neighborhoods of black holes remain hidden from our view in most frequency bands of the electromagnetic spectrum. The galaxy centres are buried under dense clouds of dust that block out most of the radiation. To pierce through this fog, the wavelength needs to be increased.

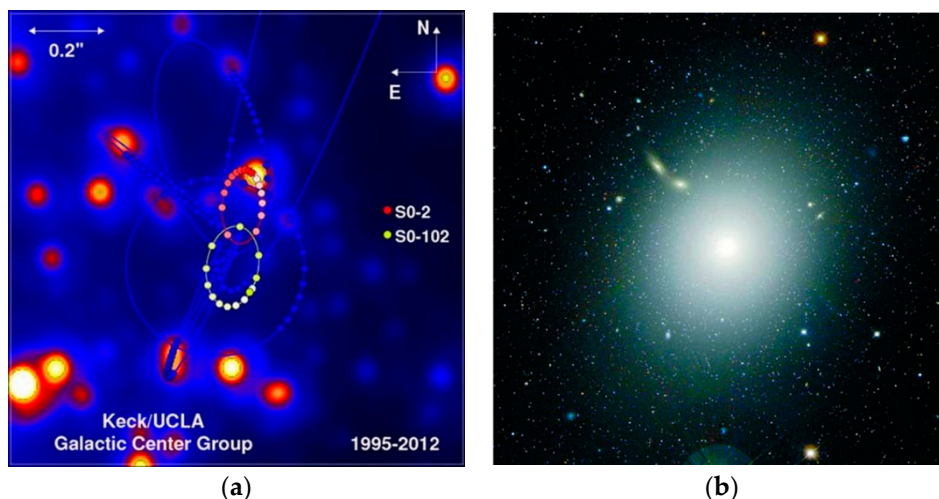


Figure 6. (a) The trajectories of several stars orbiting around the Galactic Center have been plotted from infrared observations continuously performed from 1995 to 2012 by the Keck telescopes in Hawaii. Their dynamical analysis implies the existence of a central massive black hole, Sgr A*, about 4 million solar masses; (b) The giant elliptical galaxy Messier 87, located in the Virgo local supercluster, shows a peak of luminosity at its very center, interpreted as the intense emission of gas falling into supermassive black hole about 6 billion solar masses.

Thus, at the millimetre wavelengths typical in radioastronomy, galaxy centres become almost transparent. The problem is that when the wavelength is doubled, resolution is divided by two, such that the size of the telescope needs to be increased further. So, to be able to observe the central black hole in our galaxy in the millimetre domain, we would need a radiotelescope about 5000 km in diameter. Impossible? Not at all, because at these wavelengths, astronomers can use very large base interferometry (VLBI), a technique that combines several observatories into a single virtual telescope with an aperture as large as the distance that separates them. A terrestrial-sized VLBI network is therefore just sensitive enough to resolve an image as small as 50 μ s of arc angle. This is how the EHT (Event Horizon Telescope) project was designed in the first decade of this century, combining millimetric radiotelescopes across the planet in a network, in the hope of capturing the first “real” images of giant black holes (Figure 7).

The idea of creating a worldwide network to observe the black hole at the centre of the galaxy started in 1999 at the initiative of the Dutch astronomer Heino Falcke [21,22], who works today at Nimègue University in the Netherlands. It was further developed by various radioastronomy groups [23], which merged in 2006 into the EHT consortium [24]. Gradually, the network grew to include several observatories to gain planetary reach. The eight radioastronomical stations currently in the network include Iram in Spain, the Large Millimeter Telescope (LMT) in Mexico, the Submillimeter Telescope (SMT) in Arizona, the James Clark Maxwell Telescope (JCMT) and the Submillimeter Array (SMA) in Hawaii, the South Pole Telescope (SPT) in Antarctica, the Atacama Large Millimeter Array

(Alma) and the Atacama Pathfinder Experiment (Apex) in Chile. Each of these instruments is located at altitude to reduce the atmospheric absorption of the signals. The whole set creates a virtual observatory with a 5000-km aperture. Two other observatories, the Greenland Telescope Project, in Greenland, and the Iram Noema interferometer, on the Bure plateau in the French Alps, will extend the network further and improve its performance.

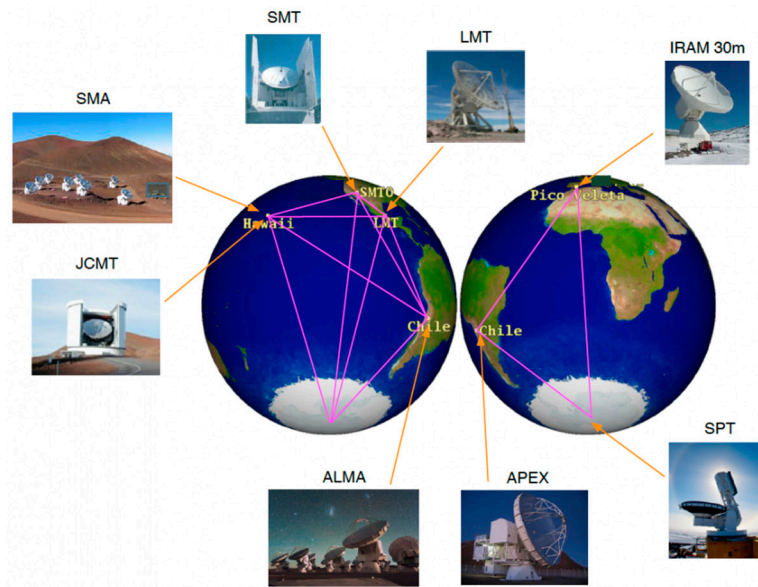


Figure 7. The very large base interferometry (VLBI) network of the Event Horizon Telescope. Courtesy EHT team.

4. A Long Wait

Once the concept was put forward, it then had to be achieved despite a multitude of observational and technical constraints. For example, weather conditions need to “cooperate” so that the VLBI network can avail of crystal skies simultaneously at eight places on four continents, as observations are done at the 1.3 mm wavelength, at which the signals are detected, is also absorbed and emitted by water. Thus, the main problem is the presence of water vapour in the atmosphere. Another constraint is imposed by the use of the Alma telescope in Chile, the most requested radio observatory in the world. Ultimately, the EHT teams only has a two-week window each year in which to attempt the group observations. They had to wait until April 2017 to have four full, clear nights, two for Sgr A* and two for M87*. However, there was no possibility of seeing an image directly on a screen. Building a high-resolution image by VLBI requires the combination of the signals captured by the various network aerials. To do so, atomic clocks are used to measure the arrival time of the signals to one tenth of a billionth of a second, to compare them in real time and triangulate with their point of origin to reconstitute an overall image. With eight observatories spread around the globe, including in places with poor Internet links, the EHT scientists had to record the data separately and store it on hard disks to combine them subsequently.

The mass of data collected exceeded anything that had ever been done in all scientific fields together: one night of observation collected 2 petabytes of data, as much as is collected in one complete year of experiments at the LHC, the CERN large-hadron collider that led to the discovery of the Higgs-Englert boson in 2012, following analysis of 4 million billion proton-proton collisions.

The hard disks of data stored in Antarctica then had to wait until December and the end of the long glacial winter, to be transported in secure flight conditions to join the several thousand hard disks centralised at the MIT Haystack Observatory in Massachusetts and the Max Planck Institute for Radio Astronomy in Bonn. There, clusters of supercomputers started to process the mountain of data, a task

that takes several months to obtain just a few pixels of an image of the massive black holes at the centre of the Milky Way and the farthest galaxy, M87.

The results will probably not be published until later in 2018. However, as suggested by the digital simulations recently done by the EHT scientists [25], the obtained recomposed image should resemble a brilliant crescent surrounding a black disk, placed on the side where the hot spot of the accretion disk is moving towards the observer (Figure 8).

As could already be predicted from our simulations forty years ago (Figure 1), by squinting your eyes to reduce the ocular resolution, the clear outline of a black hole surrounded by its accretion disk is indeed the black silhouette of its event horizon surrounded by the brilliant spot of the disk amplified by the Doppler effect. A very different, although improbable result, would mean that general relativity is incorrect in very strong gravitational fields, and new physics would be necessary.

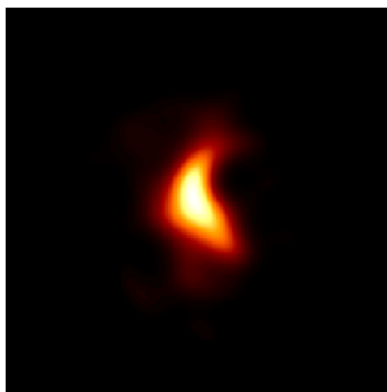


Figure 8. Synthetic image of the time variable black hole Sgr A* that could be recovered with an array of 8 radiotelescopes and averaging 8 epochs (from [25]).

5. The Golden Age of Relativistic Astrophysics

As well as the hope of capturing the first “photos” of black holes, EHT astrophysicists hope to garner a lot of information that will enable them to better understand the very special physics that operate in the environment close to black holes, namely the gigantic jets of particles and radiation that some project into space at speeds close to that of the speed of light [26]. This is the case for M87*, where the jets are larger than the galaxy itself (Figure 9). If Sgr A* produces jets, they are too small or not bright enough to be detected until now. The jets play an important role in the evolution of galaxies; for example, by heating interstellar space, they can prevent cooling of the gas that allows stars to form. The most probable physical explanation is that they are produced by the twisted magnetic fields associated with black holes. The existence of such magnetic fields, predicted by theoretical studies, involves a dynamo-type interaction between a rotating black hole and the inner parts of its accretion disk. Jets are expected to be fed energy either by an accretion disk, or by conversion of some of the rotational energy of the black holes themselves [27,28]. Black hole magnetic fields are confirmed by observations, and EHT telescopes are able to record the polarization signal (Faraday rotation) related to the strong magnetic field of the galactic black hole Sgr A*. As the amount of Faraday rotation is proportional to the integral of the magnetic field strength along the line of sight, such observations allow us to draw a map of the magnetic field near the event horizon of Sgr A*, which could perhaps reveal the physical mechanisms at the origin of the jets. VLBI observations in 2015 started to provide some clues as to the structure of Sgr A*'s magnetic field, hinting at the hypothesis of a rapidly rotating black hole [29].

The use of a set of very different instruments and methods promises even more spectacular developments in the near future, including an accurate description of black holes and their immediate environment. The optical interferometer Gravity [30], being built at the Very Large Telescope (VLT) at the European Southern Observatory in Chile, and the next generation of optical telescopes in the 30 m

diameter class, will be able to follow the stars around Sgr A* orbiting at only a few hundred times the radius of the black hole, and measure the precession of their pericentres to deduce the angular momentum (spin) of the black hole. In particular, it was expected that the follow-up observations of S2 star closely orbiting Sgr A* might allow us to measure the Schwarzschild-like gravitational field of Sgr A* [31]. Indeed, as the present article was just completed, the observation of the high velocity of S2 star at its passage to pericentre in May 2018 and the associated gravitational redshift (Einstein effect) was announced, confirming once again the validity of General Relativity in the regime of strong gravitational field [32]. In the next few years, the radio interferometer SKA (Square Kilometer Array) [33], built in South Africa and Australia, will be able to follow the orbits of pulsars around the galactic black hole, timing them ultra-precisely to test their properties. Eventually, the Lisa spatial interferometer (Laser Interferometer Space Antenna) [34], once in orbit, will be used to capture the gravitational waves emitted when small compact objects turn around supermassive black holes in nearby galaxies.

Relativistic astrophysics, still in its infancy in the 1970s due to a lack of experimental resources, is now entering a golden age. Already with EHT, by capturing the signals of what is happening very close to a supermassive black hole, astrophysicists will be able to test Einstein's theory of general relativity in the most extreme conditions. These measurements will complete historical detection of gravitational waves from 2015 [35], produced when pairs of stellar-mass black holes collide, providing the best evidence so far of the existence of black holes. EHT data will be able to give us the final proof.



Figure 9. The radio jet of the elliptical galaxy M87 is also visible in the optical range. Detailed observations from the Hubble Space Telescope show various substructures, perfectly aligned from the central nucleus up to distances much greater than the size of the galaxy itself (© STSci/NASA).

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