FIELD THEORY OF MATTER

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The strongly interacting particles exhibit various approximate regularities that appear to have rather special dynamical origins. Non-leptonic decays are governed by the $\Delta T = 1/2$ rule but with appreciable $\Delta T = 3/2$ admixture, which is not naturally explained as a symmetry property of the weak interactions. Electromagnetic mass differences show a dominant $\Delta T = 1$ effect in the approximately equal spacing of Σ^-, Σ^0 and Σ^+, but $\Delta T = 2$ must also be significant to account for the very nature of the pion mass spectrum ($\pi^+ = \pi^- \neq \neq \pi^0$). The baryon ($1/2^+$) and baryon resonance ($3/2^+$) multiplets are approximately equally spaced when they are regarded as U or V multiplets, in the sense of the three isotopic spins T, U, V that characterize SU_3 symmetry. Thus, equal spacing for the $U = 1$ baryon multiplet N^0, $1/2\,(3^{1/2}\Lambda + \Sigma^0)$, Ξ^0 or the $V = 1$ multiplet Ξ^-, $1/2(3^{1/2}\Lambda - \Sigma^0)$, N^+ expresses the G(ell-Mann)—O(kubo) mass relation. It is a directly observable property of, say, the $U = 3/2$ resonance multiplet N^*, Y^*, Ξ^*, Ω^-. This means that the mechanism predominantly responsible for the mass splittings obeys $\Delta U = = 1$ and $\Delta V = 1$.

We shall give a qualitative dynamical interpretation of these regularities in the framework of a recently proposed field theory of matter [1]. In doing this we face another such question, but one that, for the moment, lacks a definite experimental basis. How does the breakdown of the underlying W_3 symmetry produce predominantly SU_3 symmetry?

1. The first in the hierarchy of interactions couples the gauge field B to the nucleonic charge-bearing fields ψ_a ($N = \pm 1$) and V_a ($N = \pm 2$), $a = 1, 2, 3$. That interaction is invariant with respect to the two independent internal symmetry groups $U_3(\psi)$ and $U_3(V)$. Such is the underlying symmetry W_3. It is broken by the interaction that exchanges nucleonic charge between ψ and V in accordance with the unitary structure $\bar{\psi}_3(\psi V)$ and its adjoint. Although this is a strong interaction, it will be expedient to discuss its implications in the language of perturbation theory.

The low-lying states that one identifies with degenerate families of physical particles are created by combinations of fields with opposite signs of nucleonic charge, as in $\bar{\psi}_a\psi_b$, $\bar{\psi}_a V_b$, and $\overline{\psi_a\psi_b}\psi_c V_d$. All these combinations are characterized by invariance under common phase transformations of ψ and V,

$$\psi_a \to e^{i\alpha}\psi_a, \quad V_a \to e^{i\alpha} V_a, \quad a = 1, 2, 3,$$

as distinguished from the phase transformation of nucleonic charge. Thus the total nucleonic charges associated with the ψ and V fields in these states are related by

$$1/2 N(V) = -N(\psi) = N.$$

If we assume that all other types of states (such as the triplets generated by ψ_a and V_a) are much more massive, the most important effect on the low-lying states, of the W_3 destroying interaction, comes from the part that is invariant under the common phase transformation. This is represented by the set of perturbation terms symbolized as

$$(\bar{\psi}_3\psi_3)^n\,[(\bar{\psi}V)(\bar{V}\psi)]^n, \quad n = 1, \ldots .$$

The product of $2n$ field operators at distinct space-time points that is implied by $(\bar{\psi}_3\psi_3)^n$ can be decomposed into irreducible parts by means of the vacuum expectation value. Thus, $(\bar{\psi}_3\psi_3)^n$ is additively represented by a number $\langle(\bar{\psi}_3\psi_3)^n\rangle$; irreducible operator pairs

$$[\bar{\psi}_3\psi_3] = \bar{\psi}_3\psi_3 - \langle\bar{\psi}_3\psi_3\rangle,$$

multiplied by numbers which are the vacuum expectation values $\langle(\bar{\psi}_3\psi_3)^{n-1}\rangle$; and so forth. When all terms in the perturbation series are rearranged in this way, the resulting functional form is

$$f_0((\bar{\psi}V)(\bar{V}\psi)) + [\bar{\psi}_3\psi_3]\,f_1((\bar{\psi}V)(\psi)) + \ldots \quad (1)$$

A vacuum expectation value becomes singular as two points coincide. If the symmetry destroying interaction is sufficiently localized in space and time, owing to the dominance of very massive states, the consecutive terms in the series (1), which involve successively fewer replacements of operator products by vacuum expectation values, may be of diminishing numerical importance. Under such circumstances, the primary effect of the mechanism that breaks W_3 symmetry is to introduce a dynamical regime of SU_3 symmetry, as the symmetry group that governs common* transformations of ψ_a and V_a in the coupling term $f_0\,[(\bar{\psi}V)(\bar{V}\psi)]$.

The same mechanism has the secondary effect of breaking down SU_3 symmetry. This aspect of the perturbation is dominated by the term with $[\psi_3\bar{\psi}_3]$ as a factor. The latter responds as a component of a three-dimensional Euclidean vector under the SU_3 subgroups that govern transformations in the 23 plane (U isotopic spin) and the 31 plane (V isotopic spin). Accordingly, matrix elements of this perturbation will obey just those selection rules, $\Delta U = 1$, $\Delta V = 1$, that are observed in the dominant part of the mass splitting that violates SU_3 symmetry. A relative measure of the higher perturbation terms can be had by comparing the 8 Mev excess in the mass of Λ, over the value given by the GO formula, with the average of the mass splittings between Σ and N, Ξ and Σ. This ratio is 1 : 23. If a similar ratio connects the first two terms of (1), the intervals between the SU_3 multiplets of a common W_3 representation should be several Gev. The W_3 concept will acquire an observational basis when one finds some evidence of this larger pattern among the particles.

The dynamical processes that result in broken SU_3 symmetry possess the residual symmetry that is described by the invariance group U_2, generated by the isotopic spin $T\,(SU_2)$ and the hypercharge.

2. The electric current vector is formed additively from the operator products $\psi_1\bar{\psi}_1$ and $\bar{V}_1V_1$ (omitting spinor and vector indices). A part of the current is proportional to the electromagnetic vector potential A. Electromagnetic mass displacements and the breakdown of SU_2 symmetry are produced by electromagnetic vacuum fluctuations, through this induced current, together with the second order effect of the linearly coupled potential. The nature of the two mechanisms is symbolized by

$$(\bar{\psi}_1\psi_1+\bar{V}_1V_1)+(\bar{\psi}_1\psi_1+\bar{V}_1V_1)^2. \qquad (2)$$

The operators $\psi_1\bar{\psi}_1$ and $\bar{V}_1V_1$ behave as components of three-dimensional Euclidean vectors with respect to T isotopic spin transformations (12 plane). Therefore they have matrix elements obeying $\Delta T = 1$. The products of these operators also possess matrix elements with $\Delta T = 2$. But such operator products can be decomposed into irreducible parts through the introduction of the vacuum expectation value. A term such as $[\bar{\psi}_1\psi_1]\,\langle\bar{\psi}_1\psi_1\rangle$ is characterized by $\Delta T = 1$. These vacuum expectation value contributions, and the $\Delta T = 1$ effect, will dominate to the extent that the electromagnetic mechanism is localized. Thus, the long range nature of electromagnetic action would seem to explain the comparative importance of $\Delta T = 2$ effects, as measured by the 1 : 3 (mass)^2 of $\pi^+ - \pi^0$ and $K - K^+$.

3. The weak interactions of the strongly interacting particles are described by a coupling of the charged vector field Z with currents of the form indicated by $\bar{\psi}_1\psi_2 + \bar{V}_1V_2$ ($\Delta T = 1$, $\Delta Y = 0$) and $\bar{\psi}_1\psi_3 + \bar{V}_1V_3$ ($\Delta T = 1/2$, $|\Delta Y| = 1$). The self-action of these currents through the intermediary of the Z-field contains a part symbolized by

$$(\bar{\psi}_2\psi_1+\bar{V}_2V_1)\,(\bar{\psi}_1\psi_3+\bar{V}_1V_3)$$

and its adjoint. This perturbation destroys the conservation of hypercharge ($|\Delta G| = 1$) and induces isotopic spin transitions with $\Delta T = 3/2$ and $1/2$. The decomposition of the operator by means of vacuum expectation values gives the term

$$\bar{\psi}_2\,\langle\psi_1\bar{\psi}_1\rangle\,\psi_3+\bar{V}_2\,\langle V_1\bar{V}_1\rangle\,V_3 \qquad (3)$$

and its adjoint, which is characterized by $|\Delta Y| = 1$ and $\Delta T = 1/2$. We conclude from the observed dominance of the latter effect that the current self-coupling is effectively localized, or that the Z-field excitations are quite massive. The relative importance of the $\Delta T = 3/2$ effect is measured by the amplitude ratio for $K^+ \to \pi^+ + \pi^0$ and $K_1^0 \to \pi^+ + \pi^-$, which is 1 : 23.

In order to make some further comments about the weak interaction (3), we recall the detailed structure of the fermion current that is coupled to $\bar{Z}_\mu$,

$$\bar{\psi}_1\gamma^\mu\,(1+i\gamma_5)\,\psi_2+\bar{\psi}_1\gamma^\mu\,(1-i\gamma_5)\,\psi_3.$$

* The additional possibility of independent phase transformations for ψ and V is described by the nucleonic charge phase transformation and by the common phase transformation that leaves invariant the lowlying states.

The opposite signs of $i\gamma_5$ in the two terms should be noted. The $\Delta T = 1/2$ contribution derived from the second order effect of this coupling contains the Green's function of the Z-field and of the ψ_1 field. When the product of the Green's functions is approximated by a four-dimensional delta function, the resulting localized interaction is proportional to

$$-(1/8)\,\bar{\psi}_2\gamma^\mu(1+i\gamma_5)\gamma_\mu(1-i\gamma_5)\psi_2$$

with its adjoint, which equals

$$\bar{\psi}_2(1-i\gamma_5)\psi_3+\bar{\psi}_3(1+i\gamma_5)\psi_2. \quad (4)$$

These leading terms would have vanished if the same sign fo $i\eta$ had been used in the $\overline{\psi_1}\psi_2$ and $\psi_1\psi_3$ currents.

The complete $\Delta T = 1/_{29}$, CP invariant perturbation is obtained by adding the parity preserving contribution of the V field, which is represented by

$$\bar{V}_2^\mu V_{\mu 3}+\bar{V}_3^\mu V_{\mu 2}. \quad (5)$$

No general statement can be made about the relative magnitude of the parity-violating and parity-preserving effects, particularly since the strong interactions of the broken SU_3 symmetry scheme are of decisive influence on the observed phenomena *.

4. In view of the serious technical obstacles to performing field theory calculations from first principles, it is useful to convert the characteristic ideas of this field theory of matter into a corresponding phenomenology. Such a program is suggested by the structure of the electromagnetic interaction. The electromagnetic potential A_μ is coupled to the electric current vector

$$j^\mu = e\,[-\bar{\psi}_1\gamma^\mu\psi_1 - i\,(\bar{V}_1^{\mu\nu}V_{1\nu}-\bar{V}_{1\nu}V_1^{\mu\nu})].$$

This operator generates meson states of spin-parity 1^-, as represented by phenomenological fields U_{11}^μ. Thus, one could attempt to describe all linear electromagnetic interactions as proceeding through the fields of 1^- mesons with suitable quantum numbers. The least massive of these, ω and ϱ^0, might be of major importance in this description. Similarly, the vector and pseudovector currents that are coupled to the Z field can be represented approximately by the phenomenological fields of known 1^- and 0^- mesons. These couplings would describe all such weak interactions.* The so called Goldberger — Treiman relations are an immediate consequence of this point of view which is, in a sense, a return to the original idea of Yukawa.

The non-leptonic weak interaction (3), as clarified by (4) and (5), can be represented by suitable components of the phenomenological fields associated with 0^- and 0^+ mesons. Particles of the latter type have not been identified with certainty **, but these excitations will exist somewhere in the mass spectrum. Such scalar fields will also represent the part of the perturbation (2) that produces $\Delta T = 1$ electromagnetic mass displacements, as well as the portion of (1) that generates the mass splittings of broken SU_3 symmetry***. Scalar fields can also be used to describe the breakdown of W_3 symmetry.

This is an outline of a phenomenological field theory, which gives quantitative expression to the general ideas that have been expressed here. It will be developed in another publication.

* Indeed, the existence of these processes seems to involve the breakdown of SU_3 symmetry, beyond the mere role of supplying the necessary energy. Thus the weak parity-violating coupling between K^* and π that accounts for $K_1^0 \to \pi + \pi$ (see Ref. 2) would vanish if the K and π masses were equal, owing to the transversality of the four-vector K^* field. A similar remark applies to baryon decays. As we shall discuss elsewhere, simple models of parity-conserving decays depend upon coupling constant differences that would be zero were SU_3 symmetry not brokden.

REFERENCES

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* This idea is used to compute the absolute rate for the $\Delta T = 3/2$ process $K^+ \to \pi^+ + \pi^0$ in [2].

** Some possibilities are dicussed in [3].

*** A perturbation that is a numerical multiple of a phenomenological boson field generates a displacement of that field, which is its non-vanishing vacuum expectation value. We propose the word vacuon for this property. (Despite its hybrid etymology, such a terminology seems preferable to the use of a biologicographic argot). The vacuon concept is not new [J. Schwinger, Phys. Rev., **104**, 1164(1956), Ann. Phys., **2**, 407 (1957). A. Salam and J. Ward, Phys. Rev. Letters **5**, 390 (1960)], but its most effective application is that of Reference 3. We have now connected this phenomenological procedure with a fundamental dynamical theory.

DISCUSSION

R. E. Marshak.

Schwinger has noted the analogy between the lepton triplet and his hadron triplets. I should like to point out that the «Kiev» symmetry about which I spoke in 1959 can be restated in terms of a symmetry of the lepton triplet and the fundamental triplet of Dirac fields in our present theory. From this analogy, one would argue that the muon mass has a similar origin to the $(\Lambda - N)$ mass difference and the four-component neutrino (consisting of the two components ν_e and $\bar{\nu}_\mu$) is the remnant of the parity degeneracy for the original bare mass «Lagrangian».

Ya. A. Smorodinskii

What follows from the ω_3 scheme with regard to the nine-component vector meson ($\omega-\phi$ problem) ?

J. Schwinger.

The proolem of mesons (vector or preudoscalar) has very little to do with W_3 since $\bar{\psi}_a\psi_b$ refer to only one U_3 group, not two. The main formula for 1^- and 0^- mesons is obtained by assuming a singlet-octet splitting and an SU_3 symmetry breaking interactions.

Ya.B. Zel'dovich

What are the properties of the fundamental poles of the different components of ψ? Is the concept of strangeness included a priori ?

J. Schwinger.

The three components of ψ are characterized by electrical charge $Q = -1, 0, 0$, hypercharge $Y = -1, -1, 0$ and isotopic spin $T_3 = -\frac{1}{2}, \frac{1}{2}, 0$. The concept of hypercharge (i. e. «strangeness») is implicit in the U_3 transformations.