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What Comes Beyond the Standard Models

Bled, July 11–19, 2016

Edited by

Norma Susana Mankoč Borštnik
Holger Bech Nielsen
Dragan Lukman

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Preface

The series of workshops on ”What Comes Beyond the Standard Models?” started in 1998 with the idea of Norma and Holger for organizing a real workshop, in which participants would spend most of the time in discussions, confronting different approaches and ideas. It is the nineteenth workshop which took place this year in the picturesque town of Bled by the lake of the same name, surrounded by beautiful mountains and offering pleasant walks and mountaineering.

In our very open minded, friendly, cooperative, long, tough and demanding discussions several physicists and even some mathematicians have contributed. Most of topics presented and discussed in our Bled workshops concern the proposals how to explain physics beyond the so far accepted and experimentally confirmed both standard models - in elementary particle physics and cosmology. Although most of participants are theoretical physicists, many of them with their own suggestions how to make the next step beyond the accepted models and theories, experts from experimental laboratories were very appreciated, helping a lot to understand what do measurements really tell and which kinds of predictions can best be tested.

The (long) presentations (with breaks and continuations over several days), followed by very detailed discussions, have been extremely useful, at least for the organizers. We hope and believe, however, that this is the case also for most of participants, including students. Many a time, namely, talks turned into very pedagogical presentations in order to clarify the assumptions and the detailed steps, analyzing the ideas, statements, proofs of statements and possible predictions, confronting participants' proposals with the proposals in the literature or with proposals of the other participants, so that all possible weak points of the proposals showed up very clearly. The ideas therefore seem to develop in these years considerably much faster than they would without our workshops.

This year experiments have not brought much new, although a lot of work and effort has been put in, but the news will come when the analyses of the data gathered with 13 TeV on the LHC will be done. The analyses might show whether there are the new family and the new scalar fields (both predicted by the spin-charge-family theory and discussed in this proceedings), as well as whether the two events, the $\approx$ 750 GeV resonance decaying into two photons (predicted by Multiple-Point-Principle) and the $\approx$ 1.8 TeV resonance decaying into several products, are nontheless real, manifesting new scalar fields. They may see the dark matter constituents, or rather not since they are much heavier neutral baryons.

The new data might answer the question, whether laws of nature are elegant (as predicted by the spin-charge-family theory and also — up to the families — other Kaluza-Klein-like theories and the string theories) or she is "just using gauge groups when needed" (what many models do).

While the spin-charge-family theory predicts the fourth family members as well as several new scalar fields which determine the higgs and the Yukawa couplings, the high energy physicists do not expect the existence of the fourth family members at all. Their expectation relies on the analyses of the experimental data grounded on the standard model assumptions with one scalar and the perturbativity, or on slight extensions of the standard model assumptions. These analyzes manifest that the results when only three families are included are in better agreement with the experimental data than those, which include also the fourth family.

The fact that the spin-charge-family theory offers the explanation for all the assumptions of the standard model, explaining also other phenomenas, like the dark matter existence and the matter/antimatter asymmetry (even "miraculous" cancellation of the triangle anomaly in the standard model seems natural in the spin-charge-family theory as presented in this proceedings), and that the standard model assumptions are in this theory not acceptable for events which are not in the area of low enough energies, it might very well be that there is the fourth family.

New data on mixing matrices of quarks and leptons, when accurate enough, will help to determine in which interval can masses of the fourth family members be expected. There are several papers in this proceedings manifesting that the more work is put into the spin-charge-family theory the more explanations for the observed phenomena and the better theoretical grounds for this theory offers.

The Multiple Point Principle was able to predict the mass of the scalar higgs, while assuming the existence of several degenerated vacuua (which all have the same energy densities, that is the same cosmological constants) and the validity of the standard model almost up to the Planck scale. The spin-charge-family theory, which explains all the assumptions of the standard model, predicts on the other hand several steps at least up to the scale of unification of the standard model gauge fields - 10^{16} GeV. Although the two theories seems almost in contradiction, they still might both be right within some accuracy. Trying to understand why some predictions of both models might agree, can help to better understand why the standard model works so well so far and how will the next step beyond the standard model, suggested by the spin-charge-family theory, manifest in experiments.

The old idea, that quarks and leptons are made up of (massless) constituents, is back, suggesting in this proceedings that quarks and leptons are composites, carrying only the $SU(3)$ and $U(1)$ charges and that the higgs is the resonance of composites, while the weak vector bosons as well as gravity origin in these two gauge fields.

However, while the spin-charge-family theory, assuming that fermions, carrying only spin (describable with both kinds of the Clifford algebra objects), should explain — to be accepted as an "elegant" theory — "how has nature decided" to come from ∞ (or from 0) to $d = (13+1)$ and to the standard model stage through the phase transitions, must the theory, assuming constituents of quarks

and leptons, explain, “why nature started with these constituents” carrying besides the spin the $SU(3)$ and $U(1)$ charges.
That the Clifford algebra manifests also in the by computers used binary code and that one can built as a computer expert, recognizing degrees of freedom from the standard model, a parallel model is also seen in the proceedings.
How much and in which direction will future cosmological experiments help to understand what are elementary constituents of matter? The DAMA/Libra experiment, running for a decade in so far in three phases, each next phase more accurate than the previous one, reports that (of what ever origin are signals which manifest the dependence of the signal on the position of the earth with respect to the sun and of the sun position in the galaxy) their measurements are more and more accurate, and the only so far acceptable explanation for the origin of the signals, which are they measuring for so many years, is the dark matter. The discussions at Bled lead to the conclusions that also other groups searching for the dark matter particles, will see these signals or some other types of signals, if the dark matter is composed of different particles.
There is the contribution about how much will the theoretical models of the elementary fermions and bosons, as well as of the cosmology influence the cosmological measurements and how might the measurements make a choice of the so far proposed models, or help to find new.
Finally the gravitational waves signal was measured, what is opening the door to further gravitational wave astronomy. This will help to understand better several phase transitions which have brought our universe, after breaking the starting (whatever) symmetry to the today stage.
As every year also this year there has been not enough time to mature the very discerning and inovative discussions, for which we have spent a lot of time, into the written contributions.
Since the time to prepare the proceedings is indeed very short, less than two months, authors did not have a time to polish their contributions carefully enough, but this is compensate by the fresh content of the contributions.
Questions and answers as well as lectures enabled by M.Yu. Khlopov via Virtual Institute of Astroparticle Physics (viavca.in2p3.fr/site.html) of APC have in ample discussions helped to resolve many dilemmas.
The reader can find the records of all the talks delivered by cosmovia since Bled 2009 on viavca.in2p3.fr/site.html in Previous - Conferences. The three talks delivered by: Holger Bech Nielsen (New Resonances/ Fluctuations? at LHC Bound states of tops and anti tops), Norma Mankoc-Borstnik (Do no observations so far of the fourth family quarks speak against the spin-charge-family theory?) and Maxim Khlopov (Nonstandard cosmologies from BSM physics), can be accessed directly at

http://viavca.in2p3.fr/what_comes_beyond_the_standard_model_xix.html

Most of the talks can be found on the workshop homepage

http://bsm.fmf.uni-lj.si/.

Bled Workshops owe their success to participants who have at Bled in the heart of Slovene Julian Alps enabled friendly and active sharing of information and ideas, yet their success was boosted by vidoeconferences.
Let us conclude this preface by thanking cordially and warmly to all the participants, present personally or through the teleconferences at the Bled workshop, for their excellent presentations and in particular for really fruitful discussions and the good and friendly working atmosphere.
The workshops take place in the house gifted to the Society of Mathematicians, Physicists and Astronomers of Slovenia by the Slovenian mathematician Josip Plemelj, well known to the participants by his work in complex algebra.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(the Organizing comittee)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(the Editors)

Ljubljana, December 2016

1 Predgovor (Preface in Slovenian Language)

Serija delavnic „Kako preseči oba standardna modela, kozmološkega in elektrošibkega" ("What Comes Beyond the Standard Models?") se je začela leta 1998 z idejo Norme in Holgerja, da bi organizirali delavnice, v katerih bi udeleženci v izčrpnih diskusijah kritično soočili različne ideje in teorije. Letos smo imeli devetnajsto delavnico na Bledu ob slikovitem jezeru, kjer prijetni sprehodi in pohodi na čudovite gore, ki kipijo nad mestom, ponujajo priložnosti in vzpodbudo za diskusije.
K našim zelo odprtim, prijateljskim, dolgim in zahtevnim diskusijam, polnim iskrivega sodelovanja, je prispevalo veliko fizikov in celo nekaj matematikov. Večina predlogov teorij in modelov, predstavljenih in diskutiranih na naših Blejskih delavnicah, išče odgovore na vprašanja, ki jih v fizikalni skupnosti sprejeta in s številnimi poskusi potrjena standardni model osnovnih fermionskih in bozonskih polj ter kozmološki standardni model puščata odprta. Čeprav je večina udeležencev teoretičnih fizikov, mnogi z lastnimi idejami kako narediti naslednji korak onkraj sprejetih modelov in teorij, so še posebej dobrodošli predstavniki eksperimentalnih laboratorijev, ki nam pomagajo v odprtih diskusijah razjasniti resnično sporočilo meritev in kakšne napovedi so potrebne, da jih lahko s poskusi dovolj zanesljivo preverijo.
Organizatorji moramo priznati, da smo se na blejskih delavnicah v (dolgih) predstavitvah (z odmori in nadaljevanji čez več dni), ki so jim sledile zelo podrobne diskusije, naučili veliko, morda več kot večina udeležencev. Upamo in verjamemo, da so veliko odnesli tudi študentje in večina udeležencev. Velikokrat so se predavanja spremenila v zelo pedagoške predstavitve, ki so pojasnile predpostavke in podrobne korake, soočile predstavljene predloge s predlogi v literaturi ali s predlogi ostalih udeležencev ter jasno pokazale, kje utegnejo tičati šibke točke predlogov. Zdi se, da so se ideje v teh letih razvijale bistveno hitreje, zahvaljujoč prav tem delavnicam.
To leto poskusi niso prinesli veliko novega, četudi je bilo v eksperimente vloženega ogromno dela, idej in truda. Nove reultate in z njimi nova spoznanja je pričakovati šele, ko bodo narejene analize podatkov, pridobljenih na posodobljenem trkalniku (the Large Hadron Collider) pri 13 TeV. Tedaj bomo morda izvedeli ali obstajajo nova družina in nova skalarna polja (kar napoveduje teorija spinov-nabojev-družin v zborničnem prispevku). Izvedeli bodo tudi, ali sta dva dogodka, eden pri ≈ 750 GeV, ki razpade v dva fotona (kot napove 'Multiple-Point-Principle') in drugi pri ≈ 1.8 TeV, ki razpade v več produktov, resnična in ju povzroča razpad vezanega stanja šestih kvarkov in šestih antikvarkov ali pa nova skalarna polja. Morda bodo izmerili tudi delce temne snovi, kar pa ni prav verjetno, če temno snov gradijo mnogo težji nevtralni barioni.

Novi podatki bodo morda dali odgovor na vprašanje, ali so zakoni narave preprosti (kot napove teorija spinov-nabojev-družin kakor tudi — razen družin — ostale teorije Kaluza-Kleinovega tipa, pa tudi teorije strun), ali pa narava preprosto "uporabi umeritvene grupe, kadar jih potrebuje" (kar počne veliko modelov).
Teorija spinov-nabojev-družin napove člane četrte družine in več novih skalarnih polj, ki določajo higgsov skalar in Yukawine sklopitve. Vendar večina fizikov, ki so aktivni na tem področju, meni, da dosedanji poskusi četrte družine ne dopuščajo. Četudi je res, da se analize eksperimentalnih podatkov, ki vključujejo četrto družino, slabše ujemajo z meritvami kot tiste, ki je ne, je res tudi, da analize temeljijo na privzetkih standardnega modela, da je skalarno polje (higgs) eno samo in da so privzetki perturbativnosti sprejemljivi, ali pa na majhnih razširitvah predpostavk standardnega modela.
Ker ponuja teorija spinov-nabojev-družin, zgrajena na preprosti začetni akciji, razlago za vse privzetke standardnega modela, ponuja pa še mnogo več, denimo pojasnilo za izvor temne snovi in za asimetrijo med snovjo in antisnovjo v opazljivem delu vesolja (celo „čudežno" krajšanje trikotniške anomalije v standardnem modelu se zdi naravno v teoriji spinov-nabojev-družin, kot je prikazano v tem zborniku) in ker predpostavkam standardnega modela ta teorija ne pritrjuje, kadar gre za dogodke, ki niso v območju dovolj nizkih energij, se zde napovedi, da četrte družine ni, preuranjena.
Novi podatki o mešalnih matrikah kvarkov in leptonov bodo, če bodo dovolj natančni, pomagali določiti interval pričakovanih mas za člane četrte družine. V tem zborniku je nekaj prispevkov, ki kažejo, da z več vloženega dela v teorijo spinov-nabojev-družin le ta ponudi več razlag za opažene pojave in boljše teoretične temelje teorije.
Načelo večkratnosti točk („Multiple Point Principle") je omogočilo napoved mase higgsovega skalarja, ob predpostavki, da obstoja več degeneriranih vakuumskih stanj (ki imajo vsa enako energijsko gostoto, to je enako kozmološko konstanto) in da velja standardni model skoraj do Planckove skale. Nasprotno pa napove teorija spinov-nabojev-družin, več stopenj vsaj do skale poenotenja umeritvenih polj standardnega modela - 10^{16} GeV. Čeprav se ti dve teoriji zdita v nasprotju, sta vendarle lahko v znotraj določene natančnosti celo skladni. Razumevanje morebitne skladnosti napovedi obeh modelov lahko pomaga bolje razumeti, zakaj standardni model dosedaj deluje tako dobro in kako se bodo naslednji koraki onkraj standardnega modela, ki jih predlaga teorija spinov-nabojev-družin, kazali v poskusih.
Stara ideja, da so kvarki in leptoni sestavljeni iz (brezmasnih) delcev, se vrača. Prispevek v tem zborniku predlaga, da so kvarki in leptoni sestavljeni delci, ki imajo samo naboje grup $SU(3)$ in $U(1)$, da je higgs resonanca teh sestavnih delcev, šibki bozoni in gravitacija pa so porojeni iz umeritvenih polj teh dveh vrst nabojev. Medtem, ko mora teorija spinov-nabojev-družin, ki predpostavi, da fermioni nosijo samo spin (ki ga opišeta obe vrsti Cliffordovih objektov) pojasniti — če naj velja kot "elegantna" teorija — kako se je „narava odločila" da gre iz ∞ (ali iz 0) v $d = (13 + 1)$ ter do današnjega stanja preko faznih prehodov, v katerih se je začetna simetrija zlomila do današnje, mora teorija, ki privzame, da so kvarki

in leptoni sestavljeni delci, pojasniti „zakaj je narava začela s temi poddelci", ki nosijo razen spina še naboje grup $SU(3)$ in $U(1)$ naboje.
V zborniku je prispevek, ki kaže, da imajo Cliffordove algebre realizacijo tudi v binarni kodi, ki jo uporabljajo računalniki, in da lahko računalniški strokovnjak v njej prepozna prostostne stopnje standardnega modela in razvije vzporedni model za delce.
V kateri smeri in kako bodo bodoči kozmološki poskusi pomagali razumeti, kaj so osnovi delci snovi? Eksperiment DAMA/Libra, ki meri časovno odvisnost intenzitete signalov od lege Zemlje glede na Sonce in od lege Sonca v Galaksiji, teče že desetletje. Zdaj teče že tretja faza, vsaka naslednja faza je bolj natančna od prejšnje. Meritve povedo, da je časovna odvisnost intenzitete signala izrazita in nedvoumna ne glede na to, kakšen je izvor signalov. Edina doslej sprejemljiva razlaga je, da je temna snov tista, ki proži izmerjene signale. Diskusije na delavnici vodijo k zaključku, da bodo slej ko prej tudi ostale skupine, ki iščejo delce temne snovi, zaznale te signale ali kakšne druge tipe signalov, če temno snov sestavlja več komponent.
V zborniku je tudi prispevek na temo, kakšen utegne biti vpliv teoretičnih modelov za opis osnovnih fermionov in bozonov ter vpliv kozmoloških modelov na kozmološke meritve in kako lahko meritve podprejo ali izločijo predlagane modele, ali pomagajo najti drugačno pot in razumevanje vesolja.
Končno je uspelo izmeriti gravitacijske valove, kar bo prav gotovo vzpodbudilo nove eksperimente, ki bodo z merjenjem gravitacijskih valov pomagali bolje razumeti fazne prehode v zgodovini vesolja, ki so vodili do zlomitev (katerekoli že) začetetne simetrije in pripeljali vesolje v stanje, ki ga danes opazujemo.
Kot vsako leto nam tudi letos ni uspelo predstaviti v zborniku kar nekaj zelo obetavnih diskusij, ki so tekle na delavnici in za katere smo porabili veliko časa.
Premalo je bilo časa do zaključka redakcije, manj kot dva meseca, zato avtorji niso mogli povsem izpiliti prispevkov, vendar upamo, da to nadomesti svežina prispevkov.
Četudi so k uspehu „Blejskih delavnic" največ prispevali udeleženci, ki so na Bledu omogočili prijateljsko in aktivno izmenjavo mnenj v osrčju slovenskih Julijcev, so k uspehu prispevale tudi videokonference, ki so povezale delavnice z laboratoriji po svetu. Vprašanja in odgovori ter tudi predavanja, ki jih je v zadnjih letih omogočil M.Yu. Khlopov preko Virtual Institute of Astroparticle Physics (viavca.in2p3.fr/site.html, APC, Pariz), so v izčrpnih diskusijah pomagali razčistiti marsikatero dilemo.
Bralec najde zapise vseh predavanj, objavljenih preko "cosmovia" od leta 2009, na viavca.in2p3.fr/site.html v povezavi Previous - Conferences. Troje letošnjih predavanj,
Holger Bech Nielsen (New Resonances/ Fluctuations? at LHC Bound states of tops and anti tops), Norma Mankoc-Borstnik (Do no observations so far of the fourth family quarks speak against the spin-charge-family theory?) in Maxim Khlopov (Nonstandard cosmologies from BSM physics), je dostopnih na

http://viavca.in2p3.fr/what_comes_beyond_the_standard_model_xix.html

Večino predavanj najde bralec na spletni strani delavnice na

http://bsm.fmf.uni-lj.si/.

Naj zaključimo ta predgovor s prisrčno in toplo zahvalo vsem udeležencem, prisotnim na Bledu osebno ali preko videokonferenc, za njihova predavanja in še posebno za zelo plodne diskusije in odlično vzdušje.
Delavnica poteka v hiši, ki jo je Društvu matematikov, fizikov in astronomov Slovenije zapustil v last slovenski matematik Josip Plemelj, udeležencem delavnic, ki prihajajo iz različnih koncev sveta, dobro poznan po svojem delu v kompleksni algebri.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(Organizacijski odbor)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(uredniki)

Ljubljana, grudna (decembra) 2016

Talk Section

All talk contributions are arranged alphabetically with respect to the authors' names.

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1 DAMA/LIBRA Results and Perspectives

R. Bernabei[1], P. Belli[1], S. d'Angelo[1], A. Di Marco[1], F. Montecchia[1 †],
F. Cappella[1], A. d'Angelo[1], A. Incicchitti[1],
V. Caracciolo[3 ‡], R. Cerulli[3],
C.J. Dai[4], H.L. He[4], X.H. Ma[4], X.D. Sheng[4], R.G. Wang[4], Z.P. Ye[4 §]

[1]Dip. di Fisica, Univ. Tor Vergata
and INFN-Roma Tor Vergata, I-00133 Rome, Italy
[2]Dip. di Fisica, Univ. di Roma La Sapienza
and INFN-Roma, I-00185 Rome, Italy
[3]Laboratori Nazionali del Gran Sasso, I.N.F.N., Assergi, Italy
[4]Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918/3, Beijing 100049, China

Abstract. The DAMA/LIBRA experiment (∼ 250 kg of highly radio-pure NaI(Tl)) is running deep underground at the Gran Sasso National Laboratory (LNGS) of the I.N.F.N. Here we briefly recall the results obtained in its first phase of measurements (DAMA/LIBRA–phase1; total exposure: 1.04 ton $\times$ yr). DAMA/LIBRA–phase1 and the former DAMA/NaI (cumulative exposure: 1.33 ton $\times$ yr) give evidence at 9.3 σ C.L. for the presence of DM particles in the galactic halo by exploiting the model-independent DM annual modulation signature. No systematic or side reaction able to mimic the exploited DM signature has been found or suggested by anyone over more than a decade. At present DAMA/LIBRA–phase2 is running with increased sensitivity.

Povzetek. Poskus DAMA/LIBRA, ki vsebuje ∼ 250 kg visoko radio čistegaNaI(Tl), poteka globoko v podzemlju v Gran Sasso National Laboratory (LNGS) v okviru inštitutov I.N.F.N. Avtorji na kratko predstavijo rezultate prve in druge skupine meritev, ki kažeta letno modulacijo signala, (DAMA/LIBRA–faza 1; skupna ekspozicija: 1.04 ton $\times$ let). DAMA/LIBRA–faza 1 je skupaj s predhodnim poskusom DAMA/NaI (skupna ekspozicija 1.33 ton $\times$ let) potrdila prisotnost delcev v naši galaksiji, ki utegnejo biti temna snov, z zanesljivostjo 9.3 σ. V obdobju več kot desetih let ni njihova ali katerakoli druga skupina uspela najti nobene druge razlage za reakcijo, ki bi povzročila letno odvisnost signala. Trenutno poteka meritev s povečano občutljivostjo, DAMA/LIBRA–faza 2.

1.1 Introduction

The DAMA project is based on the development and use of low background scintillators. In particular, the second generation DAMA/LIBRA apparatus [1–21], as the former DAMA/NaI (see for example Ref. [8,22,23] and references

† also Dip. di Ingegneria Civile e Ingegneria Informatica, Università di Roma "Tor Vergata", I-00133 Rome, Italy
‡ e-mail: vincenzo.caracciolo@lngs.infn.it
§ also University of Jing Gangshan, Ji'an, Jiangxi, China

therein), is further investigating the presence of DM particles in the galactic halo by exploiting the model independent DM annual modulation signature, originally suggested in the mid 80's[24]. At present DAMA/LIBRA is running in its phase2 with increased sensitivity. The detailed description of the DAMA/LIBRA set-up during the phase1 has been discussed in details in Ref. [1–4,8,17–21].

The signature exploited by DAMA/LIBRA (the model independent DM annual modulation) is a consequence of the Earth's revolution around the Sun; in fact, the Earth should be crossed by a larger flux of DM particles around $\simeq 2$ June (when the projection of the Earth orbital velocity on the Sun velocity with respect to the Galaxy is maximum) and by a smaller one around $\simeq 2$ December (when the two velocities are opposite). This DM annual modulation signature is very effective since the effect induced by DM particles must simultaneously satisfy many requirements: the rate must contain a component modulated according to a cosine function (1) with one year period (2) and a phase peaked roughly $\simeq 2$ June (3); this modulation must only be found in a well-defined low energy range, where DM particle induced events can be present (4); it must apply only to those events in which just one detector of many actually "fires" (*single-hit* events), since the DM particle multi-interaction probability is negligible (5); the modulation amplitude in the region of maximal sensitivity must be $\simeq 7\%$ for usually adopted halo distributions (6), but it can be larger (even up to $\simeq 30\%$) in case of some possible scenarios such as e.g. those in Ref. [25,26]. Thus this signature is model independent, very discriminating and, in addition, it allows the test of a large range of cross sections and of halo densities. This DM signature might be mimicked only by systematic effects or side reactions able to account for the whole observed modulation amplitude and to simultaneously satisfy all the requirements given above. No one is available [1–4,7,8,14,27,22,23,12,13,15,16,19,21].

1.2 The results of DAMA/LIBRA–phase1 and DAMA/NaI

The total exposure of DAMA/LIBRA–phase1 is 1.04 ton $\times$ yr in seven annual cycles; when including also that of the first generation DAMA/NaI experiment it is 1.33 ton $\times$ yr, corresponding to 14 annual cycles [2–4,8].

To point out the presence of the signal the time behaviour of the experimental residual rates of the *single-hit* scintillation events for DAMA/NaI and DAMA/LIBRA–phase1 in the (2–6) keV energy interval is plotted in Fig. 1.1. The χ^2 test excludes the hypothesis of absence of modulation in the data: χ^2/d.o.f. = 83.1/50 for the (2–6) keV energy interval (P-value = 2.2×10^{-3}). When fitting the *single-hit* residual rate of DAMA/LIBRA–phase1 together with the DAMA/NaI ones, with the function: $A \cos \omega(t - t_0)$, considering a period $T = \frac{2\pi}{\omega} = 1$ yr and a phase $t_0 = 152.5$ day (June 2^{nd}) as expected by the DM annual modulation signature, the following modulation amplitude is obtained: $A = (0.0110 \pm 0.0012)$ cpd/kg/keV corresponding to 9.2 σ C.L.. When the period, and the phase are kept free in the fitting procedure, the modulation amplitude is (0.0112 ± 0.0012) cpd/kg/keV (9.3 σ C.L.), the period $T = (0.998 \pm 0.002)$ year and the phase $t_0 = (144 \pm 7)$ day, values well in agreement with expectations for a DM annual

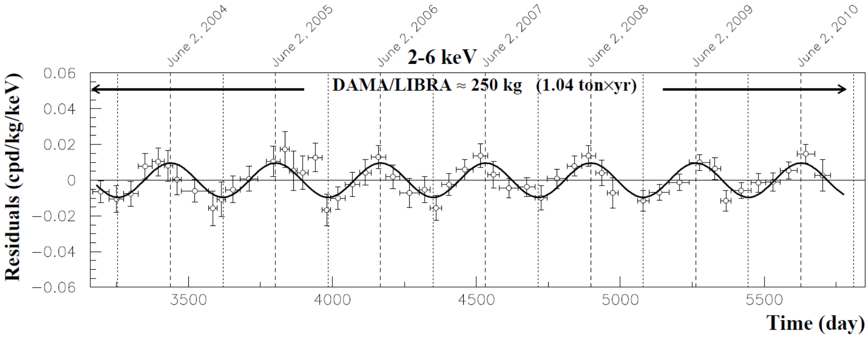

Fig. 1.1. Experimental residual rate of the *single-hit* scintillation events measured by DAMA/LIBRA–phase1 in the (2–6) keV energy interval as a function of the time. The data points present the experimental errors as vertical bars and the associated time bin width as horizontal bars. The superimposed curves are the cosinusoidal functions behaviors $A\cos\omega(t-t_0)$ with a period $T=\frac{2\pi}{\omega}=1$ yr, a phase $t_0=152.5$ day (June 2^{nd}) and modulation amplitudes, A, equal to the central values obtained by best fit on these data points and those of DAMA/NaI. The dashed vertical lines correspond to the maximum expected for the DM signal (June 2^{nd}), while the dotted vertical lines correspond to the minimum.

modulation signal. In particular, the phase is consistent with about June 2^{nd} and is fully consistent with the value independently determined by Maximum Likelihood analysis [4][1]. The run test and the χ^2 test on the data have shown that the modulation amplitudes singularly calculated for each annual cycle of DAMA/NaI and DAMA/LIBRA–phase1 are normally fluctuating around their best fit values [2–4,8].

We have also performed a power spectrum analysis of the *single-hit* residuals of DAMA/LIBRA–phase1 and DAMA/NaI [8], obtaining a clear principal mode in the (2–6) keV energy interval at a frequency of 2.737×10^{-3} d^{-1}, corresponding to a period of $\simeq 1$ year, while only aliasing peaks are present just above.

Absence of any significant background modulation in the energy spectrum has been verified in energy regions not of interest for DM [4]; it is worth noting that the obtained results account of whatever kind of background and, in addition, no background process able to mimic the DM annual modulation signature (that is able to simultaneously satisfy all the peculiarities of the signature and to account

[1] For completeness, we recall that a slight energy dependence of the phase could be expected in case of possible contributions of non-thermalized DM components to the galactic halo, such as e.g. the SagDEG stream [28,30,31] and the caustics [32].

for the whole measured modulation amplitude) is available (see also discussions e.g. in Ref. [1–4,7,8,14,15]).

A further relevant investigation in the DAMA/LIBRA–phase1 data has been performed by applying the same hardware and software procedures, used to acquire and to analyse the *single-hit* residual rate, to the *multiple-hit* one. In fact, since the probability that a DM particle interacts in more than one detector is negligible, a DM signal can be present just in the *single-hit* residual rate. Thus, the comparison of the results of the *single-hit* events with those of the *multiple-hit* ones corresponds practically to compare between them the cases of DM particles beam-on and beam-off. This procedure also allows an additional test of the background behaviour in the same energy interval where the positive effect is observed. In particular, the residual rates of the *single-hit* events measured in the (2–6) keV energy interval over the DAMA/LIBRA–phase1 annual cycles, as collected in a single cycle, are reported in Ref. [4] together with the residual rates of the *multiple-hit* events in the same energy interval. A clear modulation satisfying all the peculiarities of the DM annual modulation signature is present in the *single-hit* events, while the fitted modulation amplitude for the *multiple-hit* residual rate is well compatible with zero: $-(0.0005 \pm 0.0004)$ cpd/kg/keV in the same energy region (2–6) keV. Thus, again evidence of annual modulation with the features required by the DM annual modulation signature is present in the *single-hit* residuals (events class to which the DM particle induced events belong), while it is absent in the *multiple-hit* residual rate (event class to which only background events belong). Similar results were also obtained for the last two annual cycles of the DAMA/NaI experiment [23]. Since the same identical hardware and the same identical software procedures have been used to analyse the two classes of events, the obtained result offers an additional strong support for the presence of a DM particle component in the galactic halo.

By performing a maximum-likelihood analysis of the *single-hit* scintillation events, it is possible to extract from the data the modulation amplitude, S_m, as a function of the energy considering T =1 yr and $t_0 = 152.5$ day. Again the results have shown that positive signal is present in the (2–6) keV energy interval, while S_m values compatible with zero are present just above; for details see Ref. [4]. Moreover, as described in Ref. [2–4,8], the observed annual modulation effect is well distributed in all the 25 detectors, the annual cycles and the energy bins at 95% C.L. Further analyses have been performed. All of them confirm the evidence for the presence of an annual modulation in the data satisfying all the requirements for a DM signal.

Sometimes naive statements were put forwards as the fact that in nature several phenomena may show some kind of periodicity. The point is whether they might mimic the annual modulation signature in DAMA/LIBRA (and former DAMA/NaI), i.e. whether they might be not only quantitatively able to account for the observed modulation amplitude but also able to contemporaneously satisfy all the requirements of the DM annual modulation signature. The same is also for side reactions. This has already been deeply investigated in Ref. [1–4] and references therein; the arguments and the quantitative conclusions, presented there, also apply to the entire DAMA/LIBRA–phase1 data. Additional arguments can be found in Ref. [7,8,14,15].

No modulation has been found in any possible source of systematics or side reactions; thus, cautious upper limits on possible contributions to the DAMA/LIBRA measured modulation amplitude are summarized in Ref. [2–4]. It is worth noting that they do not quantitatively account for the measured modulation amplitudes, and also are not able to simultaneously satisfy all the many requirements of the signature. Similar analyses have also been done for the DAMA/NaI data [22,23]. In particular, in Ref. [15] it is shone that, the muons and the solar neutrinos cannot give any significant contribution to the DAMA annual modulation results.

In conclusion, DAMA give model-independent evidence (at 9.3σ C.L. over 14 independent annual cycles) for the presence of DM particles in the galactic halo.

As regards comparisons, we recall that no direct model independent comparison is possible in the field when different target materials and/or approaches are used; the same is for the strongly model dependent indirect searches. In particular, the DAMA model independent evidence is compatible with a wide set of scenarios regarding the nature of the DM candidate and related astrophysical, nuclear and particle Physics; as examples some given scenarios and parameters are discussed e.g. in Ref. [22,2,8] and references therein. Further large literature is available on the topics. In conclusion, both negative results and possible positive hints are compatible with the DAMA model-independent DM annual modulation results in various scenarios considering also the existing experimental and theoretical uncertainties; the same holds for the strongly model dependent indirect approaches (see e.g. arguments in Ref. [8] and references therein).

The *single-hit* low energy scintillation events collected by DAMA/LIBRA–phase1 have also been investigated in terms of possible diurnal effects[14]. In particular, a diurnal effect with the sidereal time is expected for DM because of Earth rotation; this DM second-order effect is model-independent and has several peculiar requirements as the DM annual modulation effect has. At the present level of sensitivity the presence of any significant diurnal variation and of diurnal time structures in the data can be excluded for both the cases of solar and sidereal time; in particular, the DM diurnal modulation amplitude expected, because of the Earth diurnal motion, on the basis of the DAMA DM annual modulation results is below the present sensitivity [14]. It will be possible to investigate such a diurnal effect with adequate sensitivity only when a much larger exposure will be available; moreover better sensitivities can also be achieved by lowering the software energy threshold as in the presently running DAMA/LIBRA–phase2.

The data of DAMA/LIBRA–phase1 have also been used to investigate the so-called Earth Shadow Effect which could be expected for DM candidate particles inducing nuclear recoils; this effect would be induced by the variation during the day of the Earth thickness crossed by the DM particle in order to reach the experimental set-up. It is worth noting that a similar effect can be pointed out only for candidates with high cross-section with ordinary matter, which implies low DM local density in order to fulfill the DAMA/LIBRA DM annual modulation results. Such DM candidates could get trapped in substantial quantities in the Earths core; in this case they could annihilate and produce secondary particles (e.g.

neutrinos) and/or they could carry thermal energy away from the core, giving potentiality to further investigate them. The results, obtained by analysing in the framework of the Earth Shadow Effect the DAMA/LIBRA–phase1 (total exposure 1.04 ton$\times$yr) data are reported in Ref. [20].

For completeness we recall that other rare processes have also been searched for by DAMA/LIBRA-phase1; see for details Refs. [9–11].

1.3 DAMA/LIBRA–phase2 and perspectives

An important upgrade has started at end of 2010 replacing all the PMTs with new ones having higher Quantum Efficiency; details on the developments and on the reached performances in the operative conditions are reported in Ref. [6]. They have allowed us to lower the software energy threshold of the experiment to 1 keV and to improve also other features as e.g. the energy resolution [6].

Since the fulfillment of this upgrade and after some optimization periods, DAMA/LIBRA–phase2 is continuously running in order e.g.:

1. to increase the experimental sensitivity thanks to the lower software energy threshold;
2. to improve the corollary investigation on the nature of the DM particle and related astrophysical, nuclear and particle physics arguments;
3. to investigate other signal features and second order effects. This requires long and dedicated work for reliable collection and analysis of very large exposures.

In the future DAMA/LIBRA will also continue its study on several other rare processes as also the former DAMA/NaI apparatus did.

Moreover, the possibility of a pioneering experiment with anisotropic $ZnWO_4$ detectors to further investigate, with the directionality approach, those DM candidates that scatter off target nuclei is in progress [29].

Finally, future improvements of the DAMA/LIBRA set-up to further increase the sensitivity (possible DAMA/LIBRA-phase3) and the developments towards the possible DAMA/1ton (1 ton full sensitive mass on the contrary of other kind of detectors), we proposed in 1996, are considered at some extent. For the first case developments of new further radiopurer PMTs with high quantum efficiency are progressed, while in the second case it would be necessary to overcome the present problems regarding: i) the supplying, selection and purifications of a large number of high quality NaI and, mainly, TlI powders; ii) the availability of equipments and competence for reliable measurements of small trace contaminants in ppt or lower region; iii) the creation of updated protocols for growing, handling and maintaining the crystals; iv) the availability of large Kyropoulos equipments with suitable platinum crucibles; v) etc.. At present, due to the change of rules for provisions of strategical materials, the large costs and the lost of some equipments and competence also at industry level, new developments of ultra-low-background NaI(Tl) detectors appear to be quite difficult. On the other hand, generally larger masses do not imply a priori larger sensitivity; in case the DM annual modulation signature is exploited, the improvement of other parameters of the experimental set-up (as e.g. the energy threshold, the running time,...) plays an important role as well.

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2 Experience in Modeling Properties of Fundamental Particles Using Binary Codes

E.G. Dmitrieff ⋆

29 88 Postyshev bvd., Irkutsk, Russia

Abstract. This work summarizes our study in fundamental particles modeling, based on the amount of information behind particular property. We had built our models firstly using linear binary sequences, then continued with cyclic and spatial arranged binary codes, and then came to space tessellation, isomorphic with these codes.

We show that properties of particles and vacuum can be effectively represented in these models, and some predictions can be made. In particular, we imply existence and predict structure of new fundamental massless scalar boson, that forms vacuum condensate.

Povzetek. Avtor v prispevku predstavi svoj model za opis lastnosti osnovnih delcev, ki temelji na količini informacij, ki je potrebna za opis določenih lastnosti. Uporabi binarna zaporedja, nadaljuje s cikličnimi in prostorsko razporejenimi binarnimi kodami in zaključi s teselacijami prostora, ki so izomorfne tem kodam.

Pokaže, da lahko s tem modelom preprosto opiše lastnosti osnovnih delcev in vakuuma in ponudi nekaj napovedi. Napove obstoj in strukturo novega fundamentalnega skalarnega bozona, ki tvori vakuumski kondenzat.

2.1 Introduction

There are several approaches in building fundamental particle models. The mainstream one is based on the Quantun Field Theory [1].

In contrast, as structural elements for building our models, we chose Boolean algebra objects, namely spatial binary codes, or bit graphs. They are extensions of bit sequences used in computers (see Appendix 2.8). We needed to extend them to reflect the multi-valence – in the first place, of the color. Having counted the number of possible values for particular property, we chose the minimal sufficient number of bits [2]. Then, we tried to extract from the codes as much data as possible.

We shall demonstrate how one can start, and how far one can go using codes. Also, we point out when the codes should be replaced with more relevant building blocks.

Since the Boolean codes, that we used, are the special case of Clifford algebra objects [3], our models can be compared to other models based on the Clifford algebra (see [5] and other references appearing in this paper).

⋆ eliadmitrieff@gmail.com

2.2 Models overview

Our work is an application of the approach mentioned above to the domain of fundamental particles' properties - first of all, to their quantum numbers. The result is a family of models, each representing a set of particle's properties. They are shown in Table 2.1.

Properties	Binary code models 3-bit	4-bit	6-bit	8-bit	8.1
Electric charge Q	$\Downarrow$	+	+	+	+
Dirac neutrinos	$\Downarrow$	+	+	+	+
Matter/antimatter	+	+	-	-	-
Baryon number B	+	-	-	-	-
Lepton number L	+	-	-	-	-
Weak isospin T_3	+	$\Downarrow$	$\Downarrow$	+	+
Weak hypercharge Y_W	+	+	+	+	+
Color charge	-	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
Fermion flavor	-	-	-	$\Downarrow$	$\Downarrow$
Spin states	-	-	-	$\pm$	$\pm$
Handedness	-	-	-	-	$\pm$
Mass	-	-	-	-	$\pm$
Interactions	$\pm$	$\pm$	-	-	-
Condensates	-	-	-	-	$\pm$
Applied to fermions	+	+	+	+	+
Applied to bosons	-	+	+	+	+
Applied to vacuum	-	$\pm$	+	+	+

Table 2.1. Family of models representing particular properties. Arrow sign ($\Downarrow$) means that the model is based on known values of this property; plus sign (+) means that values of this property are derived from the model; $\pm$ sign means that in this model some approximation is discussed.

Models inherit derived properties from their ascenders, so they can skip deriving some of them. properties derived in it., but, being based on other assumptions, does not inherit the added "by hand" basement of it. Instead, it introduces its own basement, allowing to derive the values, that have been used to build the ascending model.This mechanism is also used to avoid direct deriving of all the data in each model. So all the models are stand-alone, but 'help' each other as if they were 'family members'.

In the first, simplest one, we allocate 3 bits for storing information about electrical charge of each particle, and then express several quantum numbers (B, L, T_3, Y_W) as combination (mostly linear) of these bits. Also, we consider some bitwise operations as symmetry and interaction representations.

To represent more properties, including color, flavor and spin states, we redesigned the model several times appended more bits when necessary, until we came to 8-bit model.

In advanced 8-bit model (8.1), the vacuum is considered as a condensed state, or the tessellation, of the multiple copies of new scalar particle. The code of this particle in this model was reserved for the singlet, or longitudinal, photon. The vacuum particle is considered the same way as others. Also, other particles are considered regarding this background condensate – as a set of one or several *structure defects* of it. This approach also allows to obtain the first approximation of particles' masses, based on the count of structure defects.

The following models are not included here in details since they are not built using codes.

Since we needed to take in account the interface between neighboring bits, in 3D model, instead of bits, we used electrically charged polyhedra. We supposed that they are organized into the Weaire-Phelan honeycomb structure [6] due to its minimal surface square among the structures formed by polyhedra of equal volume. We assume that the default alternation of positive- and negative-charged polyhedra follows our 8.1 model of vacuum condensate, while the anti-structure defects of this alternation represent particles. The square of the walls between oppositely-charged polyhedra, that is geometrically dependent on the defects' structure, is used as second approximation of particles' masses.

The *distortion* of the structure, native to the vacuum condensate and caused by defects, is taken in account in 3D.1 model, allowing to consider the relative small influence of the curvature on the surface energy.

2.3 Electric charge: 3-bit one's complement code model

2.3.1 Charge data analysis

The first property, that attracted our attention, was the particle's electric charge Q. Among 61 known fundamental particles [8], [9] there are only 7 different values of electric charge (we do not take now any assumptions about the matter type, counting both particles and antiparticles). The charges are symmetrically distributed from -1 through 0 to $+1$ keeping the interval $\Delta Q = 1/3$ (see Table 2.2).

		Count of particles with electrical charge Q of						
Set of particles	total	-1	$-2/3$	$-1/3$	0	$+1/3$	$+2/3$	$+1$
All particles	61	4	9	9	17	9	9	4
All fermions	48	3	9	9	6	9	9	3
Fermions of one family	12	1	3	3	2	3	3	1
All bosons	13	1	-	-	11	-	-	1
Vector bosons	12	1	-	-	10	-	-	1
Scalar boson	1	-	-	-	1	-	-	-

Table 2.2. Distribution of known electrical charge values in different groups of fundamental particles. Each quark color state, and also particles with their antiparticles, including neutrino, are counted separately. In contrast, different spin- and handedness states are counted as one particle. The 8 gluons, counted among 13 bosons, are the QCD combinations [7].

Following the approach of counting minimal information needed to represent the experimental data, we should use at least 3 bits to represent seven values [2].

$$N = \log_2 7 \approx 2.807 \tag{2.1}$$

Moreover, all 7 values of Q are peculiar to fermions only. The known fundamental bosons can have just 3 integer values of electrical charge: -1, 0 and $+1$. Also, each fermion family has the same distribution of Q values. So, the model of electrical charge for one fermion family members, is the most complex case. Therefore, it should be appropriate for all the fundamental particles' charge, and we can keep focusing on one fermion family only.

Additionally, we assumed, that the neutrino and anti-neutrino are not just only different, Dirac particles, but their zero electric charges are also not the same in their nature — but just coinciding. This assumption looks natural while taking into account, that the electrical charge can be represented as the sum of weak isospin and the half of weak hyper-charge, and they have different signs for the particle and anti-particle:

$$Q = T_3 + \frac{Y_W}{2}; Q_{\nu_L} = \frac{1}{2} - \frac{1}{2}; Q_{\tilde{\nu_R}} = -\frac{1}{2} + \frac{1}{2}. \tag{2.2}$$

So we consider they have different origins, but degenerated values, assuming +0 for neutrino and -0 for anti-neutrino.

2.3.2 Charge coding

After these assumptions, we have 8 different electrical charges (with two of them degenerated), even and symmetrically distributed around zero. The number of required bits become exactly 3 because 8 is the integer power of two:

$$N = \log_2 8 = 3. \tag{2.3}$$

To get the integer numbers, we multiplied it by 3:

$$3Q \in \{-3, -2, -1, -0, +0, 1, 2, 3\} \tag{2.4}$$

We used then the code of $N = 3$ bits with *ones' complement* convention (see Appendix 2.9 for details) to obtain binary representations for charge values.

We chose this convention because it has exactly the same data range $[-(2^{N-1} - 1) \dots 2^{N-1} - 1]$ as the data to be coded (2.4), so all the possible code combinations map to existing particle charges without exceptions (see Table 2.3).

Having three bits q_i of the 3Q representation, we always can retrieve the charge value back (2.93):

$$3Q = -3q_2 + 2q_1 + q_0 \tag{2.5}$$

or

$$Q = \frac{2q_1 + q_0}{3} - q_2. \tag{2.6}$$

Electrical charge Q	-1	-2/3	-1/3	-0	+0	+1/3	+2/3	+1
Fermions	l^-	$\tilde{u}$	d	$\tilde{\nu}$	ν	$\tilde{d}$	u	l^+
Integer charge 3Q	-3	-2	-1	-0	+0	1	2	3
Charge code $q_2q_1q_0$	100	101	110	111	000	001	010	011
The least significant bit q_0	0	1	0	1	0	1	0	1
Ordinal matter	+	-	+	-	+	-	+	-
Anti-matter	-	+	-	+	-	+	-	+
Two lower bits q_1q_0	00	01	10	11	00	01	10	11
Kind of fermions								
...leptons	00	-	-	-	00	-	-	-
...anti-leptons	-	-	-	11	-	-	-	11
...quarks	-	-	10	-	-	-	10	
...anti-quarks	-	01	-	-	-	01	-	-
Lepton number								
$L = 1 - q_1 - q_0$	1	0	0	-1	1	0	0	-1
Baryon number								
$B = (q_1 - q_0)/3$	0	-1/3	1/3	0	0	-1/3	1/3	0
Weak hypercharge:								
$Y_{wL} = \frac{4}{3}q_1 + q_0(\frac{5}{3} - 2q_2) - 1$	-1	-4/3	1/3	(0)	-1	2/3	1/3	2
$Y_{wR} = \frac{4q_1 - q_0}{3} - 2q_2(1 - q_0)$	-2	-1/3	-2/3	1	(0)	-1/3	4/3	1
$Y_w = \frac{4}{3}q_1 + \frac{2}{3}q_0 - 1$	-1	-1/3	1/3	(1)	-1	-1/3	1/3	1
$Y_w^* = \frac{4}{3}q_1 + \frac{2}{3}q_0 - 2q_2$	-2	-4/3	-2/3	0	(0)	2/3	4/3	2
The eldest bit q_2	1	1	1	1	0	0	0	0
Weak isospin:								
$T_{3L} = (1 - q_0)(\frac{1}{2} - q_2)$	-1/2	0	-1/2	(0)	1/2	0	1/2	0
$T_{3R} = q_0(\frac{1}{2} - q_2)$	0	-1/2	0	-1/2	(0)	1/2	0	1/2
$T_3 = \frac{1}{2} - q_2$	-1/2	-1/2	-1/2	(-1/2)	1/2	1/2	1/2	1/2
$T_3^* = 0$	0	0	0	0	(0)	0	0	0
Bosons	W^-			Z^0, H^0 | g?	γ			W^+
Scalar / Singlet (S = 0)								
$Y_w = \frac{4}{3}q_1 + \frac{2}{3}q_0 - 1$	(-1)			1	(-1)			(1)
$T_3 = \frac{1}{2} - q_2$	(-1/2)			-1/2	(1/2)			(1/2)
Vector/ Triplet (S = ±1)								
$Y_w^* = 0$	0			0	0			0
$T_3^* = \frac{2}{3}q_1 + \frac{1}{3}q_0 - q_2$	-1			0	0			1

Table 2.3. Electrical charge distribution, 3-bit charge codes and fermions' quantum numbers as bit combinations. Symbols u, d, l, ν mean u- and d-type quarks, charged leptons and neutrinos regardless of their membership in families. The estimated values

2.3.3 Symmetries as bitwise operations

Two operations on bits q_i appeared to be isomorphic with two particles' symmetries: isotopic symmetry and charge inversion.

- Since the bitwise inversion is the negation of value, it acts as charge symmetry operation C over the representations, transforming code of any anti-particle into the code of corresponding particle, and vice versa:

$$\neg[\tilde{u}] = \neg 101 = 010 = [u] \tag{2.7}$$

$$\neg[l^-] = \neg\mathtt{100} = \mathtt{011} = [l^+] \tag{2.8}$$

(square brackets here mean obtaining the ones' complement code, in form of $\mathtt{q_2q_1q_0}$, that is shown in typewriter font).

- The q_2-only inversion (i.e. bitwise *exclusive or* operation $\oplus$ between charge code and $\mathtt{100}$) corresponds to the isotopic symmetry operation (marked here as $'$) between up and down particles:

$$[\tilde{u}'] = [\tilde{u}] \oplus \mathtt{100} = \mathtt{101} \oplus \mathtt{100} = \mathtt{001} = [\tilde{d}], \tag{2.9}$$

$$[\nu'] = [\nu] \oplus \mathtt{100} = \mathtt{000} \oplus \mathtt{100} = \mathtt{100} = [l^-] \tag{2.10}$$

(we assume that antiparticles have reversed positions in isotopic doublets: anti-up quark is *anti-up*, i.e. *down*, and anti-down is *up*).
Following (2.5), (2.90), for any particle with charge Q:

$$3Q' = 2q_1 + q_0 - 3(\neg q_2) = 2q_1 + q_0 + 3(q_2 - 1) = \tag{2.11}$$
$$= (2q_1 + q_0 - 3q_2) + 6q_2 - 3 = 3Q + 3(2q_2 - 1),$$

so

$$3Q' = 3Q - 3 \text{ in case } q_2 = 0 \text{ (up particles);} \tag{2.12}$$

$$3Q' = 3Q + 3 \text{ in case } q_2 = 1 \text{ (down particles).} \tag{2.13}$$

This allows to use *addition*[1] operator $+$ instead of *exclusive or* $\oplus$:

$$[\tilde{u}'] = [\tilde{u}] + 3 = \mathtt{101} + \mathtt{011} = -2 + 3 = 1 = \mathtt{001} = [\tilde{d}] \tag{2.14}$$

$$[\nu'] = [\nu] - 3 = \mathtt{000} + \mathtt{100} = +0 + (-3) = -3 = \mathtt{100} = [l^-]. \tag{2.15}$$

Note that $+3$ and -3 coincide with charged weak bosons' 3Q values:

$$+3 = \mathtt{011} = [W^+], -3 = \mathtt{100} = [W^-], \tag{2.16}$$

so weak interactions can be expressed in code form, as charge conserving equation. For instance:

$$u \to d + W^+;$$
$$\frac{2}{3} = -\frac{1}{3} + 1;$$
$$\mathtt{010} = \mathtt{110} + \mathtt{011};$$
$$[u] = [d] + [W^+]. \tag{2.17}$$

- The same way, we can express weak neutral-current, electromagnetic and also strong interactions, since $[Z^0], [\gamma], [g] \in \{\mathtt{000}; \mathtt{111}\}$:

$$[l^-] = [l^-] + [\gamma], \tag{2.18}$$

$$[d^r] + [g^{\bar{r}g}] = [d^g] \tag{2.19}$$

(Color state is shown as upper index).
In 3-bit model, all these interactions are considered as identity operation $\mathtt{E}$, turning particles' codes into themselves: up into ups', downs' into downs', quarks' into quarks', leptons' into leptons'. But, since codes are multivalent, color state or family membership may change.

[1] Addition is performed according to rules for ones' complement code operations [14].

2.3.4 Quantum numbers as bit combinations

Filling the constructed codes of Q in a tabular form, we found that several linear combinations of bits produce values, equal to fermion's quantum numbers (see table 2.3).

- The least significant bit q_0 is equal to 1 for antiparticles and 0 for particles.
- Two lower bits $q_1 q_0$ form lepton number L, baryon number B and weak hypercharge Y_w:

$$L = 1 - q_1 - q_0, \tag{2.20}$$

$$B = \frac{q_1 - q_0}{3}; \tag{2.21}$$

for left-handed particles ($q_0 = 0$) and right-handed anti-particles ($q_0 = 1$):

$$Y_w = B - L = \frac{4}{3} q_1 + \frac{2}{3} q_0 - 1, \text{ or} \tag{2.22}$$

$$\frac{Y_w}{2} = \frac{2q_1 + q_0}{3} - \frac{1}{2}. \tag{2.23}$$

- The eldest bit q_2 defines the weak isospin of left-handed particles and right-handed anti-particles:

$$T_3 = \frac{1}{2} - q_2. \tag{2.24}$$

Since for right-handed particles and left-handed anti-particles

$$\frac{Y_w^*}{2} = \frac{2q_1 + q_0}{3} - q_2 = Q, \tag{2.25}$$

$$T_3^* = 0, \tag{2.26}$$

we can express Y_{wL} and Y_{wR} separately, for both particles and anti-particles, as quadratic combinations:

$$Y_{wL} = \frac{4}{3} q_1 + q_0 (\frac{5}{3} - 2q_2) - 1 \tag{2.27}$$

$$Y_{wR} = \frac{4q_1 - q_0}{3} - 2q_2(1 - q_0) \tag{2.28}$$

$$T_{3L} = (1 - q_0)(\frac{1}{2} - q_2) \tag{2.29}$$

$$T_{3R} = q_0(\frac{1}{2} - q_2) \tag{2.30}$$

For vector bosons, equations for weak isospin and weak hypercharge are reversed, comparing to right-handed fermions:

$$T_3^* = \frac{2q_1 + q_0}{3} - q_2 = Q, \tag{2.31}$$

$$\frac{Y_w^*}{2} = 0, \tag{2.32}$$

and scalar Higgs boson has the same equations as (2.24, 2.22)

Basing on the least significant bit q_0, it is possible to give the term definitions for *ordinal matter* and *anti-matter*, that do not rely on their cosmological distribution. Instead, they depend on the choice of positive or negative sign of electrical charge. In case the electron is considered negative-charged, as usual, the definitions are the following:

The ordinal matter consist of one or more fundamental fermions having the value of 0 in the least significant bit of ones' complement codes of their electrical charges, expressed in units of $\frac{1}{3}|e|$;

The anti-matter consist of fermions having the value of 1 in this bit.

The mixed matter consist of fermons of both types noted above (for instance, mesons and the positronium).

2.4 Intermediate 4- and 6-bit code models

2.4.1 Fermions: 4-bit model

Color coding The 3-bit model, shown above, represents several properties of fundamental particles, that do not depend on their color state. We could not suggest any q_i combination, that would reasonably express colors.

But, we notice the following facts:

- the color states of quarks (colors) differs from color states of anti-quarks (anti-colors) and also from colorless color states of leptons, so the count of different color states is at least 7;
- there is neither red nor colorless anti-quark, neither colorless nor magenta quark, neither green nor yellow lepton, so the number of color states for the particular kind of fermions is limited to 3 (quarks) or 1 (leptons);
- the numerator in the expression (2.6) of electrical charge Q

$$N_1 = 2q_1 + q_0 \tag{2.33}$$

  has a form of 2-bit decomposition into powers of two (2.86) of some positive integer number. Its value is 0 for leptons, 1 for anti-quarks, 2 for quarks and 3 for anti-leptons (see table 2.4);
- for *any* code of $N = 3$ bits, containing N_1 digits `1` (or `0`, symmetrically), the count of possible combinations is $C_3^{N_1}$, i.e. 1, 3, 3, and 1, respectively. In case all three bits have the same value, there is only one combination; but when they are different, the number of variants rises up to three;
- these counts of combinations coincide with the count of different color states.

Supposing that the 'colorless' color of leptons also differs from the 'colorless' color of anti-leptons (like we did with neutrino's and anti-neutrino's charge), the number of known colors becomes 8, so the color requires 3 bits to be coded.

Following facts listed above, we allocated 3 bits to be the color representation, and assigned the combinations to fermion kinds accordingly to number of digit `1` they contain (table 2.4) .

Particles	Leptons l^-, ν	Anti-leptons $l^+, \tilde{\nu}$	Quarks u, d	Anti-quarks $\tilde{u}$, $\tilde{d}$
Count of color states = $C_3^{N_1}$	1	1	3	3
Bits $q_1 q_0$	`00`	`11`	`10`	`01`
$N_1 = 2q_1 + q_0$	0	3	2	1
3-bit codes with N_1 digits `1`	$\begin{smallmatrix} 0 & \\ 0 & 0 \end{smallmatrix}$	$\begin{smallmatrix} 1 & \\ 1 & 1 \end{smallmatrix}$	$\begin{smallmatrix} 0 & \\ 1 & 1 \end{smallmatrix}$	$\begin{smallmatrix} 1 & \\ 0 & 0 \end{smallmatrix}$
Sequences $c_2 c_1 c_0$	`000`	`111`	`011, 101, 110` red, green, blue	`100, 010, 001` cyan, magenta, yellow

Table 2.4. Count of color states and 3-bit color codes.

Color code geometry Besides determination of the bit count, and the count of particular digits `1` and `0` in it, we should also postulate the *coding convention*, i. e. the way to obtain concrete color values from the particle's color code.

Nevertheless, it is not obvious that color states of quarks and anti-quarks can really have particular values.

Comparing the color to the electrical charge Q, we see the following:

- while the electrical charge can be easily measured, the color state of particular quark is not observable due to confinement. The color seems like just internal degree of freedom, allowing co-existence of three quarks in a baryon, which would appear otherwise in the same quantum state [7]. So all we know about three color states of quarks being confined in baryons, is that they are *different*, but their exact values may be on principle inaccessible. Also, quark and anti-quark, composing a meson, are in *opposite* color states, but what color do they have exactly, seems also unknown on principle;
- Electrical charge is *a number*. Charges can be added, multiplied by other numbers, arranged by value in descending or ascending order and so on. In contrast, colors can not be ordered this way because the question "What is *greater* – green or blue?" seems to have no meaning. Also colors can be neither added nor multiplied with reasonable sense. It means that color charge is *not a number*, so no coding convention used for numbers should be applied. Moreover, representing color as a linear bit sequence (that is isomorphic to a number), we can not avoid distorting of sense, because numbers have some additional information of order, but colors do not have it.
- Nevertheless, in mesons each particular color of quark points to the concrete opposite anti-color of anti-quark. It means that the set of colors and the set of anti-colors are ordered in the same way, so answers for questions "Is green *next* to red?" and "Is anti-green *next* to anti-red" should be either *yes* for both, or *no* also for both. This kind of order is not linear, but cyclic, and cycles "– red – green – blue – " and "– anti-red – anti-green – anti-blue –" are synchronized and have the same direction.

Taking noted above in account, we chose a 3-bit *loop* code, meaning there is neither dedicated lowest, nor the eldest, nor the middle bit. These codes do not look like $c_2c_1c_0$ (that would assume some integer's binary decomposition), but rather like ${}^{c_i}_{c_j}c_k$, where $i, j, k \in \{0; 1; 2\}$ and $i \neq j \neq k$. Writing bits this way, we emphasize that all three bits are peers, so their positions are as equivalent as vertices in equilateral triangle.

In order to read this code as a bit sequence (or as an integer number, that is isomorphic), it is necessary to choose the bit that will be accessed first, and then the clockwise or counterclockwise direction[2].

This act of selection, in fact, puts additional information to the result and breaks the triangular symmetry.

In case of random selection, each color code of quark or anti-quark, containing bits of both values, would produce three different sequences.

In contrast with quarks, lepton's and anti-leptons' color codes, containing equal bits, always produce the same fixed sequence regardless of starting point.

To get compatible with QCD color names, we associated them with serialized color codes as shown in Table 2.4.

For equilateral triangle codes, both cyclic directions produce coinciding 3-bit sequences, but in different order. Since order is significant (for instance, to keep entanglement), the color coding convention should be *3-bit directed loop*.

Operations and properties The count of digit `1` in color code is equal to N_1:

$$N_1 = \sum_{i=0}^{2} c_i. \tag{2.34}$$

Combining q_2 with N_1, we can calculate the electric charge

$$Q = \sum_{i=0}^{2} \frac{c_i}{3} - q_2, \tag{2.35}$$

weak hypercharge

$$\frac{Y_w}{2} = \sum_{i=0}^{2} \frac{c_i}{3} - \frac{1}{2} \tag{2.36}$$

and other properties listed in section 2.3.

Additionally, now we have color representations. Codes for anti-colors of anti-quarks have the only one bit with value of `1`: anti-red or cyan ($\tilde{r}$), anti-green or magenta ($\tilde{g}$) and anti-blue or yellow ($\tilde{b}$). Codes for quark colors – red (r), green (g) and blue (b) – have two digits `1`. Following this analogue, leptons are white with color code of three zeros, and anti-leptons are black, with code of three digits `1`.

Note that we utilize each of $2^4 = 16$ code combination, and keep two of three symmetries mentioned above.

[2] Note that it is not trivial task to store non-sequential, particularly triangle-symmetrical codes in computer memory. Computer architecture is designed to store and process integer numbers as bit sequences with known dedicated (first) bit address. The triangle structure can be realized, for instance, as a loop buffer with random current pointer. Codes of entangled particles should keep sharing the same pointer.

- The all-bit inversion transforms the code of particle into the code of its anti-particle with the opposite color:

$$\neg[l^-]_4 = \neg 1\binom{0\ \ 0}{0} = 0\binom{1\ \ 1}{1} = [l^+]_4; \tag{2.37}$$

$$\neg[\tilde{u}]_4 = \neg 1\binom{1\ \ 0}{0} = 0\binom{0\ \ 1}{1} = [u]_4, \tag{2.38}$$

or, with particular color – for instance, with anti-red anti-up quark:

$$\neg[\tilde{u}^{\tilde{r}}]_4 = \neg 1(100) = 0(011) = [u^r]_4 \tag{2.39}$$

(Square brackets with index $_4$ mean obtaining the 4-bit code, in native $q_2\binom{c_i\ \ c_k}{c_j}$ or serialized $q_2(c_2 c_1 c_0)$ form).
- The q_2 inversion does not affect $q_1 q_0$, so it does not change c_i, transforming between up and down fermions with the same color.

2.4.2 Bosons: 6-bit model

Gluons Having the method of color representation, we applied it to obtain codes for gluons. They have color and anti-color, so suitable codes for them should be 6-bit combinations, containing two triangle color codes, of 3 bits each.

The problem is that gluons are not electrically-charged, but both color and anti-color codes, used in 4-bit fermion model, also carry the positive electrical charge: it is the only q_2 that has minus sign in equation (2.35) for Q. Counted for whole 6-bit code, it would be

$$\sum_{i=0}^{2} \frac{c_i^c}{3} + \sum_{i=0}^{2} \frac{c_i^{\tilde{c}}}{3} = \frac{2}{3} + \frac{1}{3} = +1 > Q_g = 0, \tag{2.40}$$

where $c^c, c^{\tilde{c}}$ are codes of gluon's color and anti-color.

Nevertheless, we noticed that this problem does not arise in case we also include terms $-1/2$ in the calculation, as we did with $\frac{Y_{wL}}{2}$ (2.36). These terms shift the charge value of each bit down, so shifted values are symmetrical regarding 0, and therefore they can produce both positive and negative sums. It can mean that we should use more appropriate and more symmetrical values than 0 and 1.

Following this, we introduced new bit-like[3] symbols b_i that are just scaled and shifted c_i:

$$b_i = \frac{c_i}{3} - \frac{1}{6}, i \in \{0, 1, 2\}, b_i \in \{-\frac{1}{6}; \frac{1}{6}\} = \{⓪;\ ①\}. \tag{2.41}$$

This substitutions puts the denominator $1/3$ and also the shift $-1/6$ inside bits b_i, so their electrical charge values become symmetrical $\pm 1/6$. Further we use symbols ⓪ $= -\frac{1}{6}$ and ① $= +\frac{1}{6}$ as short form for values of b_i.

[3] Symbols b_i are not true bits, i.e. binary digits. They are although binary (can be of one of two values), but not digits since they are shifted and scaled (while digits supposed to have values in {0; 1}). But they are isomorphic with c_i that are digits. So we still use word 'bit' for b_i, keeping that in mind.

The gluon electrical charge expressed through b_i is

$$Q_g = \sum_{i=0}^{2} b_i^c + \sum_{i=0}^{2} b_i^{\tilde{c}} = \sum {}^{\textcircled{1}}_{\textcircled{0}}\textcircled{1} + \sum {}^{\textcircled{0}}_{\textcircled{0}}\textcircled{1} = \frac{1}{6} - \frac{1}{6} = 0, \tag{2.42}$$

as supposed. The 'anti-gluons', i.e. combinations of anti-color and color in reversed order also can be coded this way with triangles exchanged. So these 18 codes can be representations of gluon states ($g^{r\tilde{b}}$ and so on) forming the basis for combination, that produces eight QCD gluons.

Electroweak bosons Extending this approach to colorless color-anticolor pairs, i.e. white-black and black-white, we get just *two* possible codes for colorless zero-charged bosons: ${}^{\textcircled{1}}_{\textcircled{1}}\textcircled{1}\,{}^{\textcircled{0}}_{\textcircled{0}}\textcircled{0}$ and ${}^{\textcircled{0}}_{\textcircled{0}}\textcircled{0}\,{}^{\textcircled{1}}_{\textcircled{1}}\textcircled{1}$.

In Standard Model there are *three* known particles with this set of properties:

- Z^0, as usual boson, supposed to be either in singlet ($S = 0$) or in triplet ($S = \pm 1$) spin state;
- photon γ that is transversal, i.e. triplet-only ($S = \pm 1$);
- scalar Higgs boson H^0 with just one singlet spin state ($S = 0$).

So photon and Higgs boson together carry the same amount of information about spin – as would usual vector boson do – and may share one 6-bit double-color code[4].

In this model it is still not clear, which of two codes should be assigned to γ together with H^0 and to Z^0:

$$Q_\gamma = Q_{H^0} = Q_{Z^0} = \sum {}^{\textcircled{1}}_{\textcircled{1}}\textcircled{1} + \sum {}^{\textcircled{0}}_{\textcircled{0}}\textcircled{0} = \frac{1}{2} - \frac{1}{2} = 0. \tag{2.43}$$

We see that the order of primary and secondary triangle color codes is significant, so the 6-bit code should be treated as ordered.

Finally, combining codes of black with black, and also of white with white, we get the only two codes with $Q = \pm 1$, which should represent W^+ (${}^{\textcircled{1}}_{\textcircled{1}}\textcircled{1}\,{}^{\textcircled{1}}_{\textcircled{1}}\textcircled{1}$) and W^- (${}^{\textcircled{0}}_{\textcircled{0}}\textcircled{0}\,{}^{\textcircled{0}}_{\textcircled{0}}\textcircled{0}$).

2.4.3 6-bit code model for fermions

After completing two-color 6-bit codes for bosons, we get back to fermions. The idea is to express their 4-bit codes, i.e. one q_2 and three c_i bits, also through b_i symbols, unifying them with bosons.

As we had scaled c_i bits and shifted them *down* (2.41), we scale the q_2 bit and shifted it *up*. Additionally, we *split* it into three symbols $b_2 b_1 b_0$, repeating the

[4] Positioning the Higgs boson in one cell with the photon is poorly correlated with known values of its weak hyper-charge and weak isospin (see Table 2.3). We solve this problem in Advanced 8.1 model (section 2.6).

		White (1) ⓪⓪⓪	Anti-colors (3) ⓪⓪①	Colors (3) ⓪①①	Black (1) ①①①
Black (1)	①①①	γ/H^0			W^+
Colors (3)	⓪①①		$g^{c\bar{c}}$		
Anti-colors (3)	⓪⓪①			$g^{\bar{c}c}$	
White (1)	⓪⓪⓪	W^-			Z^0

Table 2.5. 6-bit codes for bosons

same value three times. It is necessary because effective absolute value of q_2 is 1, that is three times greater than effective value 1/3 of c_i (2.35), so

$$b_i^{T_3} = \frac{1}{6} - \frac{q_2}{3}, i \in \{0; 1; 2\}. \tag{2.44}$$

These symbols $b_i^{T_3}$ have the same domain as b_i^c and $b_i^{\bar{c}}$ (2.41): $b_i \in \{$⓪; ①$\}$. Being of the same value, three $b_i^{T_3}$ always produce the same sequence regardless of their symmetry.

Electroweak quantum numbers are expressed through b_i in a more simple way than through q_i in (2.24, 2.23, 2.6), without additional constants:

$$T_3 = \frac{1}{2} - q_2 = \frac{1}{2} - \sum_{i=0}^{2} \frac{q_2}{3} = \sum_{i=0}^{2} (\frac{1}{6} - \frac{q_2}{3}) = \sum_{i=0}^{2} b_i^{T_3}; \tag{2.45}$$

$$\frac{Y_w}{2} = \sum_{i=0}^{2} b_i^c; \tag{2.46}$$

$$Q = \sum_{i=0}^{5} b_i \tag{2.47}$$

(in last equation all six bits are summed).

	T_{3L}	b^{T_3}	Leptons (1) ⓪⓪⓪	Anti-quarks (3) ⓪⓪①	Quarks (3) ⓪①①	Anti-leptons (1) ①①①
Up	$\frac{1}{2}$	①①①	ν	$\tilde{d}$	u	l^+
Down	$-\frac{1}{2}$	⓪⓪⓪	l^-	$\tilde{u}$	d	$\tilde{\nu}$

Table 2.6. 6-bit codes for fermions

2.5 8-bit code model extending the 6-bit one with boson spin states and fermion flavors

The unified 6-bit representations of fermions and bosons are multivalent. Three fermions of different families share the same code. Two states of bosons – singlet and triplet – do the same. Following the approach of building model with b-type symbols only, we have to keep in mind the total sum of them, that must be equal to the particle's electric charge.

Both family membership, and being in particular spin state do not affect the electric charge. So, we can extend codes, keeping total charge intact, just with *even* number of b-type bits. Combinations of odd number of them are not electrically neutral, since counts of positive and negative values in this case would never be equal:

$$\sum_{i=0}^{2n} b_i \neq 0, n \in \mathbb{Z} \tag{2.48}$$

Adding n bit pairs, we would get $C_{2n}^{n} = \frac{(2n)!}{(n!)^2} = 2, 6, 20, ...$ *zero-charged* combinations of them. We see that one additional pair is quite enough to represent *two* boson's states.

There is no suitable pair count to represent *three* fermion families with this approach. The nearest number of them, with two pairs added, would be six. So it is suitable to represent six flavors. The fermion code in this case would contain $6 + 4 = 10$ bit.

To represent three families, we could also use three-bit combinations with different bits, since $C_3^1 = C_3^2 = 3$, but we would then have extra charge of $\pm 1/6$.

We took in account, that, in contrast with bosons' 6-bit codes, fermions' ones have T_3 codes of three b_i bits. These codes utilize only two of eight possible combinations: ⓪⓪⓪ and ①①①, because just these codes have suitable values of electric charge to get $T_3 = \pm 1/2$. Six other combinations containing both ⓪ and ① are not in use since they have wrong charge values of $\pm 1/6$.

We found that these 'wrong' combinations coincide with combinations required to represent families, and their extra charge can be effectively compensated by adding one equal-valued b-bit pair of opposite charge. The fermion code would be of 8 bits, that is preferable than 10.

Thus, we added a pair of b_i^p symbols to both bosons' and fermions' codes, allocating four possible combinations of them in the following way:

- For bosons, we used two zero-charged combinations of different bits, ⓪① and ①⓪. We associated one of them with singlet and another with triplet state by arbitrary choice .
- For fermions, firstly, we replaced b^{T_3} code, that has two 'right' combinations, with 3-bit b^f code[5], having six 'wrong' ones, that carry $\pm 1/6$ instead of $\pm 1/2$. Secondly, compensating extra electrical charge of $\mp 1/3$, we applied the pair with both equal values: ①① for up fermions and ⓪⓪ for down ones, with electrical charge of $\pm 1/3$:

$$①①① \Rightarrow \begin{cases} ①①⓪ \\ ①⓪① + ①①; \\ ⓪①① \end{cases} \quad ⓪⓪⓪ \Rightarrow \begin{cases} ⓪⓪① \\ ⓪①⓪ + ⓪⓪ \\ ①⓪⓪ \end{cases} \tag{2.49}$$

[5] f means 'family' or 'flavor'

T_3 is calculated now as sum of five bits:

$$T_3 = \sum_{i=0}^{2} b_i^f + \sum_{i=0}^{1} b_i^p, \tag{2.50}$$

$$T_3^{up} = (⓪ + ① + ①) + (① + ①) = \frac{1}{2}; \tag{2.51}$$

$$T_3^{down} = (⓪ + ① + ⓪) + (⓪ + ⓪) = -\frac{1}{2}. \tag{2.52}$$

Again, we can not reasonably suppose which code of three should be assigned to the particular family. The corresponding members of different families seem to be absolutely equal in all aspects excepting their masses and mixing, but the mass is a property that in 8-bit model is disregarded. Nevertheless, family number is doubtlessly *a number*, since families are quite ordered at least by masses of their members. So the bits in family code are arranged sequentially.

Now each boson, as double-color state in one of two spin states, can be represented with 8-bit code, keeping correct electrical charge. Also, each fermion can have such a code assigned. It can be treated as a color-isospin state, being a member of one of three families or as color state in one of six flavors (see Table 2.7). In this table, we chose one of several possible assignments between codes and particles since the particular color, flavor and spin state are not strictly determined.

2.6 Advanced 8-bit code model

Focusing on the geometrical structure of the code, we adjusted the 8-bit model to represent the vacuum condensate, ordered the particles according to number of differences each of them has with the vacuum background, and considered the example of the of handedness mechanism .

2.6.1 Geometry of the 8-bit code

Choosing the most appropriate code convention for representing particular property, we also chose mutual arrangement of its bits – or, in other words, the code's geometry, structure or symmetry. It is significant because, as we have seen, it has great influence on the number of possible variants. Now, for each particle, we have a code of three parts: 3-bit color code, 3-bit code of secondary color or flavor, and 2-bit spin state code. The relative position of these parts is also significant, and should be appropriately chosen.

Comparing fermions' and bosons' codes, we found that fermion flavor code, being a bit sequence, can be easily produced from the secondary color loop code by breaking its equilateral triangle symmetry. The flavor code can remain being a triangle, but some asymmetry in its environment should make one vertex and one direction preferable.

Favoring this assumption is the fermion mixing [9], [10], [11]. It shows that although family membership is mostly conserved, under particular circumstances it is arbitrary.

			Primary color codes b^{c_1}			
			White	Anti-colors	Colors	Black
			⓪ ⓪⓪	⓪ ⓪① (3)	⓪ ①① (3)	① ①①
Pair b^p	3 bits	charge	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$
			Fermions			
Isotopic state	Flavor b^f	T_3	Leptons	Anti-quarks	Quarks	Anti-leptons
①①	①①⓪		ν_e	$\tilde{d}$	u	e^+
Up	①⓪①	$\frac{1}{2}$	ν_μ	$\tilde{s}$	c	μ^+
	⓪①①		ν_τ	$\tilde{b}$	t	τ^+
⓪⓪	⓪⓪①		e^-	$\tilde{u}$	d	$\tilde{\nu}_e$
Down	⓪①⓪	$-\frac{1}{2}$	μ^-	$\tilde{c}$	s	$\tilde{\nu}_\mu$
	①⓪⓪		τ^-	$\tilde{t}$	b	$\tilde{\nu}_\tau$
			Bosons			
Spin state	2^{nd} color b^{c_2}	charge	Electro-weak	Strong (gluons)		Electro-weak
⓪①	① ①①	$\frac{1}{2}$	H^0			$W^+_{\uparrow\downarrow}$
Singlet	⓪ ①①	$\frac{1}{6}$		$g^{c\tilde{c}}_{\uparrow\downarrow}$		
$S = 0$	⓪ ⓪①	$-\frac{1}{6}$			$g^{\tilde{c}c}_{\uparrow\downarrow}$	
	⓪ ⓪⓪	$-\frac{1}{2}$	$W^-_{\uparrow\downarrow}$			$Z^0_{\uparrow\downarrow}$
①⓪	① ①①	$\frac{1}{2}$	$\gamma_{\uparrow\uparrow}$			$W^+_{\uparrow\uparrow}$
Triplet	⓪ ①①	$\frac{1}{6}$		$g^{c\tilde{c}}_{\uparrow\uparrow}$		
$S = \pm 1$	⓪ ⓪①	$-\frac{1}{6}$			$g^{\tilde{c}c}_{\uparrow\uparrow}$	
	⓪ ⓪⓪	$-\frac{1}{2}$	$W^-_{\uparrow\uparrow}$			$Z^0_{\uparrow\uparrow}$

Table 2.7. 8-bit codes associated with fundamental particles. The code for a particle is a combination of three parts from row and column: one bit pair and two 3-bit codes, 8 bits altogether. Bit ① carries electrical charge 1/6 while ⓪ carries −1/6. Electrical charge Q is the sum of them all, and other quantum numbers can be retrieved from it using 3-bit model. Secondary color codes have the same meanings with primary ones. Color and anti-color triangle codes produce three individual sequential codes for particular colors each. It causes that there are 3 (anti)quarks of different color and $3 \times 3 = 9$ gluon color-anticolor combinations in their table cells. Since the particular color, flavor and spin state are not strictly determined, one of several possible assignments between codes and particles is chosen.

Another possibility is to consider the flavor code and secondary color code are two different codes. This would cause the bit number to be increased to 11. Taking in account the symmetry between secondary and primary color codes, and following the requirement of even total bit count, we would increase it up to 14. Keeping that in mind, in this section we are following the 8-bit model. In it, the asymmetrical flavor code appears just along with charged bit pair (two equal-valued bits). This charged pair can be produced from zero-charged one, that is inherent to bosons, by inversion of one of its bits. So this inverted bit can be the factor that breaks triangle symmetry of color code, turning it into the code of flavor. It means, that no additional information required to choose secondary color or flavor code: it is determined by the equality

$$b_0^p = b_1^p \tag{2.53}$$

(`false` means color, `true` - flavor).

Following this, we supposed the suitable structure of 8-bit code as two coaxial triangles with two bits of the pair residing at both sides from triangle face of the secondary color code.

The triangles are two bases of triangular prism (or anti-prism). The pair, in case of bosons, is aligned along the C_3 symmetry axis that connects centers of bases. Or, that is the same, there are two coaxial tetrahedrons pointing in the same direction with their b^p vertices:

$$b_1^p \left(\begin{smallmatrix} b_0^{c_2} \\ b_1^{c_2} \end{smallmatrix} b_2^{c_2} \right) \; b_0^p \left(\begin{smallmatrix} b_0^{c_1} \\ b_1^{c_1} \end{smallmatrix} b_2^{c_1} \right) \tag{2.54}$$

(at the chart above the triangles shown in parenthesis should be turned perpendicular to the paper, so b_0^p forms a regular pyramid with bits of b^{c_2}, and also b_1^p with b^{c_1}).

For instance, according to the table 2.7:

$$[W^+_{\uparrow\downarrow}]_8 = ⓪ \left(\begin{smallmatrix} ① \\ ① \end{smallmatrix} ① \right) \; ① \left(\begin{smallmatrix} ① \\ ① \end{smallmatrix} ① \right); \tag{2.55}$$

$$[\gamma_{\uparrow\uparrow}]_8 = ① \begin{smallmatrix} ① \\ ① \end{smallmatrix} ① \; ⓪ \begin{smallmatrix} ⓪ \\ ⓪ \end{smallmatrix} ⓪. \tag{2.56}$$

In bosons, both triangles are symmetrical and therefore represent colors.

In case of fermions, one bit in the pair has exchanged with one of bits in the triangle, while another triangle next to the unchanged bit keeps being a color code. We suppose that changed bit goes off the axis some way, causing asymmetry that turns the triangle into the code of flavor, and the whole boson particle into asymmetrical fermion:

$$[e^-]_8 = ⓪ \begin{smallmatrix} ① \\ ⓪ \end{smallmatrix} ⓪ \; ⓪ \begin{smallmatrix} ⓪ \\ ⓪ \end{smallmatrix} ⓪; \tag{2.57}$$

$$[\mu^-]_8 = ⓪ \begin{smallmatrix} ⓪ \\ ① \end{smallmatrix} ⓪ \; ⓪ \begin{smallmatrix} ⓪ \\ ⓪ \end{smallmatrix} ⓪; \tag{2.58}$$

$$[\tau^-]_8 = ⓪ \begin{smallmatrix} ⓪ \\ ⓪ \end{smallmatrix} ① \; ⓪ \begin{smallmatrix} ⓪ \\ ⓪ \end{smallmatrix} ⓪; \tag{2.59}$$

$$[c]_8 = ① \begin{smallmatrix} ① \\ ⓪ \end{smallmatrix} ① \; ① \begin{smallmatrix} ① \\ ⓪ \end{smallmatrix} ①. \tag{2.60}$$

The distances between exchanged bits are different for different flavors due to asymmetry.

2.6.2 Simple linear condensate model

The structure of particle code (2.54) allows to represent not just lonely particles, but also the condensed state of them.

In the simplest case, we can join two or more identical codes together in one dimension, along the line of symmetry axis from left to right. Some of the bits at the right side of the left particle code are in close proximity to bits at the left side of the right particle codeIt causes ambivalence since we could treat right side of left code together with left side of right code also as a code of some particle.

Here we consider electrically neutral, colorless boson condensate codes, that can be formed by codes of H^0, Z^0 and γ.

The code assigned to the Higgs boson from Table 2.7, being repeated, forms the following condensate code:

$$[\dots H^0 \dots]_8 = \dots \textcircled{0} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{1} \substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}\ \textcircled{0} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{1} \substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}\ \textcircled{0} \dots \tag{2.61}$$

It has an asymmetry: $\textcircled{0}$ is to the left from $\substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}$.

The photon condensate code is:

$$[\dots \gamma \dots]_8 = \dots \textcircled{1} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{0} \substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}\ \textcircled{1} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{0} \substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}\ \textcircled{1} \dots \tag{2.62}$$

This condensate has $\textcircled{0}$ to the right of $\substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}$. In it, compared to $[\dots H^0 \dots]$, bits in pairs are exchanged – or, globally shifted half-code size along the axis.

It looks like γ condensate code can be produced from (2.61) by *axis-reversing* and *mirror-reflection*. After reversing, to get the same results as before, the triangles should be processed in backward direction[6]. To keep the direction, one should perform mirroring after axis-reversing.

Z^0 condensate codes are the following:

$$[\dots Z^0_{\uparrow\uparrow} \dots]_8 = \dots \textcircled{1} \substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}\ \textcircled{0} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{1} \substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}\ \textcircled{0} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{1} \dots; \tag{2.63}$$

$$[\dots Z^0_{\uparrow\downarrow} \dots]_8 = \dots \textcircled{0} \substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}\ \textcircled{1} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{0} \substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}\ \textcircled{1} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{0} \dots. \tag{2.64}$$

They can be produced from (2.61) and (2.62) with all-bit inversion C.

We note that in case both primary and secondary code have the same cyclic direction, the same result is produced by the *shift* on half-translation-unit, i.e. four bits.

In case the directions are opposite, one should additionally perform the mirror-reflection P after shift to change back the direction. Therefore, sequential C and P applied to any whole condensate code would produce almost original but shifted state.

The same way we can write simple linear gluon condensate code. For instance, a chain of *diagonal* gluon codes, joined together, should be the following:

$$[\dots g^{c\bar{c}} \dots]_8 = \dots \textcircled{0} \substack{\textcircled{1}\\ \textcircled{0}} \textcircled{1}\ \textcircled{1} \substack{\textcircled{0}\\ \textcircled{1}} \textcircled{0}\ \textcircled{0} \substack{\textcircled{1}\\ \textcircled{0}} \textcircled{1}\ \textcircled{1} \substack{\textcircled{0}\\ \textcircled{1}} \textcircled{0}\ \textcircled{0} \dots \tag{2.65}$$

It is almost colorless, since each color triangle has two anti-color neighbors, and vice versa. On both ends of this chain there are 'bare' color and anti-color triangles. The chain of 'anti-gluons' looks like shifted gluon chain, excepting color and anti-color on ends are reversed:

$$[\dots g^{\bar{c}c} \dots]_8 = \dots \textcircled{1} \substack{\textcircled{0}\\ \textcircled{1}} \textcircled{0}\ \textcircled{0} \substack{\textcircled{1}\\ \textcircled{0}} \textcircled{1}\ \textcircled{1} \substack{\textcircled{0}\\ \textcircled{1}} \textcircled{0}\ \textcircled{0} \substack{\textcircled{1}\\ \textcircled{0}} \textcircled{1}\ \textcircled{0} \dots \tag{2.66}$$

[6] The triangle color codes are directed, since colors have strict-defined operation *next* (2.4.1)

We suppose that these gluon code chains, joined to quark and anti-quark codes with opposite colors at ends, could be used to represent mesons.

We also suppose that non-diagonal gluon codes could produce more complicated (non-linear) condensate codes, useful in hadron modeling.

One of the possible ways to build the spatial condensate code is to join the linear condensate codes as threads into bunches, producing the lattice. Thus, considering the linear condensate code as a thread among similar threads, we could take in account the neighboring bits in other threads.

2.6.3 Particles as differences with background condensate and an example of mass representation at electroweak scale

We found that vacuum condensate can be represented by any of four colorless codes shown above (2.61 – 2.64), so one of them should be chosen.

The particle forming the condensate 'vanishes' because such a particle does not differ from the background[7]. One of three remaining zero-charged colorless particle codes differs from vacuum code by 2 bits (one bit pair); the second one differs from it by both triangles, i.e. 6 bits, and last one differs by all 8 bits.

We noted that ratio of different bits for two latter particles, $6/8 = 0.75$ is just slightly $(2,88\%)$ higher than 0.729, that is experimental value of m_Z/m_H [12], [13].

Supposing that masses of particles somehow depend on the number of differences their codes have with the code of massless vacuum, we also checked the $W^{\pm}_{\uparrow\uparrow}$ code (2.55).

Comparing to the code of 'vanished' longitudinal photon, that we used to represent H^0, $[W^{\pm}_{\uparrow\uparrow}]_8$ has 5 different bits (three in triangle, two in pair) so this ratio $5/8 = 0.625$ is slightly less $(2,74\%)$ than experimental $m_W/m_H \approx 0.6426$.

We supposed that these differences in ratios are connected to the different contribution in mass, caused by the change in the pair and in the triangle. Using the experimental values of m_Z and m_W as defining points, we estimated these contributions for the pair (m_2) and the triangle (m_3).

Dividing experimental m_Z by two, since its code supposed to consist of two changed triangles, we get the estimated mass contribution of one changed triangle:

$$m_3 = \frac{m_Z}{2} = \frac{91.1876}{2} \approx 45.6(\text{GeV}). \tag{2.67}$$

Subtracting it from W mass, we get the estimated mass contribution of the changed pair [8]:

$$m_2 = m_W - m_3 = 80.385 - 45.6 \approx 34.8(\text{GeV}). \tag{2.68}$$

So the mass of particle with the 8-bit-changed code should be

$$m_8 = m_3 + m_3 + m_2 \approx \frac{m_Z}{2} + m_W \approx 126.0(\text{GeV}), \tag{2.69}$$

[7] The vacuum condensate particles are supposed to be observable individually on the background of *other* condensates, for instance, gluon condensate in hadrons.

[8] Note that *one* changed bit in pair and *one* changed bit in triangle appear to have similar but slightly different mass effects, about 17.4 and 15.2 GeV, respectively.

that is 0.7% higher than experimental mass of Higgs boson.

The photon with 2 changed bits, following the assumptions above, also would be massive, with $m_\gamma = m_2$. However, there is difference: in case of 2 changes, both changed bits are isolated, having no changed neighbors, while in Z^0 and $W^\pm$ codes the changed bits of pair touch other changed bits of triangle(s). So the mass is probably dependent not on the number of changed bits directly, but on *differences in interface* between bits caused by these changes[9].

Applying this approach to the code of the last known particle in the same mass scale, t quark, with experimental mass of 173.07 GeV, we found that a particle code with 11 changed bits should produce such a mass:

$$m_{11b} = m_3 + m_3 + m_3 + m_2 = 171.6(\text{GeV}), \tag{2.70}$$

or, taking in account that one of triangles, representing color, in the quark should have one or two changed bits,

$$m_{11f} = \frac{2}{3}m_3 + m_3 + m_3 + \frac{3}{2}m_2 = 173.8(\text{GeV}). \tag{2.71}$$

To keep total electrical charge of 2/3, there should be one changed bit more since the number of changes must be even. This 12^{th} bit should be isolated to have no mass contribution.

Examples shown above lead to the conclusion that the 8-bit model, allowing to explain mass ratios of heavy bosons, but predicting also heavy fermion due to their flavor codes, needs to be extended further to explain the existing mass hierarchy. We make some hypothesis about how in can be done:

- One way to reduce mass is connected with isolating of changed bits, especially in flavor code. These bits can be distributed, for instance, among bits in the neighboring vacuum condensate threads. It might be not applicable to color codes since color triangles supposed to be strict equilateral.
- exchanging of charge between neighboring bits should not affect interface between them, since they remain different after exchange is performed.
- T-quark is the only quark that was explored non-confined, so other quarks might also have masses in compatible range. The confinement, considered as joining with gluon condensate, might be a way to reduce masses due to 'optimizing interfaces' by touching the 'bare' anti-color code of gluon chain with the 'bare' color code of the quark.

2.6.4 Upgrade of 8-bit model

So we concluded that codes with 2, 6, and 8 changed bits should be assigned, respectively, to the photon, triplet Z^0 boson, and, at least, Higgs boson together with singlet Z^0, sharing the same code.

[9] Studying Weaire-Phelan structure we found out that among 12 close bit's neighbors, 6 of them are of the same charge and 6 ones of another. So there are no changes in interface after single inversion, since the inverted bit still has 6 neighbors of another charge and 6 ones of the same. Also there is no changes in the bit's interface when all 12 neighbors are inverted.

Remembering that in simple 8-bit model we had assigned the codes of spin states, and also the codes for H^0, γ, and Z^0 by arbitrary decision we adjusted the model in the following way:

- We left the code for the photon intact.
- So the vacuum condensate code should be the code reserved for longitudinal photon. We marked it with symbol V^0. As a singlet, or scalar particle, it is supposed to follow the equations (2.24, 2.22), having

$$T_3 = 1/2, Y_w/2 = -1/2. \tag{2.72}$$

- We assigned the code, produced from code of vacuum condensate with all-8-bit inversion, to the Higgs boson H^0. It might be the same particle with the singlet Z^0.
- We also adjusted the meaning of bit pair: it represents singlet state when it is *opposite* to the charge of triangles.
- The last code left we assigned to the triplet state of Z^0.
- We assumed that cyclic directions of primary and secondary color codes are opposite, so $\mathsf{CP} = \mathtt{shift}$;

The result is shown in the Table 2.8.

2.6.5 Handedness

Both 'wrong' right-handed fermions and left-handed anti-fermions are known to have $T_3^* = 0$, so $Y_w^*/2 = Q$.

Combining odd number of b bits, it is not possible to get integer value (2.48), so codes for them should have even bit count.

As we have shown (2.25, 2.26), the correct values can be produced by explicitly changed formulas. Using b bits, it is done by including all the 8 bits in the equation for $Y_w^*/2$ and none for T_3.

The known values for triplet states of vector bosons are produced in reversed way: all the 8 bits are summed in equation for T_3 and none for $Y_w^*/2$.

Since the effect of handedness on T_3 and $Y_w/2$ is always of $\pm 1/2$, the same result can be obtained by adding or subtracting the corresponding parts of the scalar vacuum particle (2.72), also making bit count even.

In the simple linear condensate model this can be shown, for instance, as shift of the particle code on a half-length, so both T_3 codes of vacuum and particle joins together, and both $Y_w/2$ codes do the same.

We consider the particle, for instance, electron e^-, surrounded by vacuum condensate, with the following code:

$$[\ldots V^0 e^- V^0 \ldots]_{8.1} = \ldots \underbrace{\underbrace{\textcircled{0} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{1}}_{T_3=+\frac{1}{2}}\ \underbrace{\substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}}_{\frac{Y_w}{2}=-\frac{1}{2}}}_{V^0}\ \underbrace{\underbrace{\{\textcircled{0} \substack{\textcircled{1}\\ \textcircled{0}} \dot{\textcircled{0}}\ \dot{\textcircled{0}}}_{T_3=-\frac{1}{2}}\ \underbrace{\substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0}\}}_{\frac{Y_w}{2}=-\frac{1}{2}}}_{e_L^-}\ \textcircled{0} \substack{\textcircled{1}\\ \textcircled{1}} \textcircled{1}\ \textcircled{1} \substack{\textcircled{0}\\ \textcircled{0}} \textcircled{0} \ldots \tag{2.73}$$

Both vacuum and electron codes represent certain values of weak isospin and weak hyper-charge. This electron code is left-handed, having $T_3 = -1/2$ and $Y_w = -1$. This code differs from the vacuum code with three changed bits, marked with dots.

When we shift the electron code to the left (or right) by *half-unit* length, the T_3 code of electron overlaps with the T_3 code of vacuum, and codes of Y_w do the same:

$$\ldots \underbrace{\underbrace{\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}\{\dot{\textcircled{0}}\genfrac{}{}{0pt}{}{\dot{\textcircled{1}}}{\textcircled{0}}\textcircled{0}}_{T_3=0}\;\underbrace{\textcircled{0}\genfrac{}{}{0pt}{}{\dot{\textcircled{0}}}{\dot{\textcircled{0}}}\dot{\textcircled{0}}\}\textcircled{1}\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}}_{\frac{Y_w}{2}=-1}}_{e_R^-}\;\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}\;\textcircled{1}\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}\ldots \tag{2.74}$$

(two bits of the pair supposed to participate not in Y_w but in T_3 code, however it does not matter here since they are mutual compensated).

This shifted code has five changed (dotted) bits. Combined with parts of vacuum condensate code to the left and to the right, this shifted electron code has $T_3 = 0$ and $Y_w = -2$, so it can be treated as code of right-handed electron.

Replacing the code of electron with d-quark code, we have

$$[\ldots V^0 d V^0 \ldots]_{8.1} = \ldots \underbrace{\underbrace{\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}\;\textcircled{1}}_{T_3=+\frac{1}{2}}\;\underbrace{\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}}_{\frac{Y_w}{2}=-\frac{1}{2}}}_{V^0}\;\underbrace{\underbrace{\{\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\dot{\textcircled{0}}}\dot{\textcircled{0}}\;\dot{\textcircled{0}}}_{T_3=-\frac{1}{2}}\;\underbrace{\genfrac{}{}{0pt}{}{\textcircled{0}}{\dot{\textcircled{1}}}\dot{\textcircled{1}}\}}_{\frac{Y_w}{2}=+\frac{1}{6}}}_{d_L}\;\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}\;\textcircled{1}\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}\ldots, \tag{2.75}$$

and shifted

$$\ldots \underbrace{\underbrace{\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}\{\dot{\textcircled{0}}\genfrac{}{}{0pt}{}{\dot{\textcircled{1}}}{\textcircled{0}}\textcircled{0}}_{T_3=0}\;\underbrace{\textcircled{0}\genfrac{}{}{0pt}{}{\dot{\textcircled{0}}}{\textcircled{1}}\textcircled{1}\}\textcircled{1}\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}}_{\frac{Y_w}{2}=-\frac{1}{6}}}_{d_R}\;\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}\;\textcircled{1}\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}\ldots. \tag{2.76}$$

Considering positive-charged particles ($b_1^p = \textcircled{0}$), for instance u-quark, we should *subtract* vacuum codes from particle's ones (i.e. add to inverted code) to get the correct result. We suppose that the necessity of this correction is caused by incomplete relevance of linear condensate model.

The original and shifted state of u-quark are

$$[\ldots V^0 \nu_e V^0 \ldots]_{8.1} = \ldots \underbrace{\underbrace{\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}\;\textcircled{1}}_{T_3=+\frac{1}{2}}\;\underbrace{\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}}_{\frac{Y_w}{2}=-\frac{1}{2}}}_{V^0}\;\underbrace{\underbrace{\{\dot{\textcircled{1}}\genfrac{}{}{0pt}{}{\dot{\textcircled{0}}}{\textcircled{1}}\textcircled{1}\;\textcircled{1}}_{T_3=+\frac{1}{2}}\;\underbrace{\genfrac{}{}{0pt}{}{\textcircled{0}}{\dot{\textcircled{1}}}\dot{\textcircled{1}}\}}_{\frac{Y_w}{2}=+\frac{1}{6}}}_{u_L}\;\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}\;\textcircled{1}\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}\ldots \tag{2.77}$$

and

$$\ldots \underbrace{\underbrace{\textcircled{1}\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}\{\dot{\textcircled{1}}\genfrac{}{}{0pt}{}{\dot{\textcircled{0}}}{\textcircled{1}}\textcircled{1}}_{T_3=0}\;\underbrace{\textcircled{1}\genfrac{}{}{0pt}{}{\textcircled{0}}{\dot{\textcircled{1}}}\dot{\textcircled{1}}\}\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}}_{\frac{Y_w}{2}=+\frac{2}{3}}}_{u_R}\;\textcircled{0}\genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}}\textcircled{1}\;\textcircled{1}\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}}\textcircled{0}\ldots \tag{2.78}$$

(here the parts of vacuum codes outside braces intended to be subtracted are inverted, or, what is the same, exchanged).

They both have 4 changed bits. Note that u-type quarks and ũ-type anti-quarks are the only fermions having similar color and flavor codes with the same counts of `1` and `0`. So, the fluctuation between them would be the most easy.

The left-handed neutrino has only two changed bits, like the photon:

$$[\dots V^0 \nu_e V^0 \dots]_{8.1} = \dots \underbrace{\underbrace{\textcircled{0} \genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}} \textcircled{1}\, \textcircled{1}}_{T_3=+\frac{1}{2}} \; \underbrace{\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}} \textcircled{0}}_{\frac{Y_W}{2}=-\frac{1}{2}}}_{V^0} \; \underbrace{\{\underbrace{\dot{\textcircled{1}} \genfrac{}{}{0pt}{}{\dot{\textcircled{0}}}{\textcircled{1}} \textcircled{1}\, \textcircled{1}}_{T_3=+\frac{1}{2}} \; \underbrace{\genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}} \textcircled{0}\}}_{\frac{Y_W}{2}=-\frac{1}{2}}}_{\nu_L} \; \textcircled{0} \genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}} \textcircled{1}\, \textcircled{1} \genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}} \textcircled{0} \dots \tag{2.79}$$

In contrast, the right-handed neutrino would have 6 changed bits and zero value of both weak hyper-charge and isospin:

$$\dots \underbrace{\underbrace{\textcircled{1} \genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}} \textcircled{0} \{\textcircled{1} \genfrac{}{}{0pt}{}{\textcircled{0}}{\dot{\textcircled{1}}} \dot{\textcircled{1}}}_{T_3=0} \; \underbrace{\dot{\textcircled{1}} \genfrac{}{}{0pt}{}{\dot{\textcircled{0}}}{\dot{\textcircled{0}}} \dot{\textcircled{0}}\} \, \textcircled{0} \genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}} \textcircled{1}}_{\frac{Y_W}{2}=0}}_{\nu_{eR}} \; \textcircled{0} \genfrac{}{}{0pt}{}{\textcircled{1}}{\textcircled{1}} \textcircled{1}\, \textcircled{1} \genfrac{}{}{0pt}{}{\textcircled{0}}{\textcircled{0}} \textcircled{0} \dots \tag{2.80}$$

This large number of changed bits, as we have seen, may cause mass of the right neutrino in the electroweak mass scale.

The model of the particle surrounded by the condensate, has another degree of freedom: after shifting, the right and left parts of code could be exchanged. After that, the particle code has the same changed bit count, as before. So it can be used as representation of flipping the spin.

All the $2^8 = 256$ codes possible in the 8.1 model are listed in the Table 2.8. Some of them are assigned to particles or particle components; others are vacant. Several vacant codes may be really in use because of shifting. Anyway, some of them can correspond to new particles, so they should be explored.

2.7 Summary

We considered several models of fundamental particles and condensates, based on bit graphs. We managed to represent all the particles of Standard Model. The bit code approach seems to be relevant and efficient for modeling discrete properties. Nevertheless, such a properties like masses, require some improvements to take in account interfaces or relationships between neighboring bits.

2.8 Appendix: Bit graphs

Natural number a is usually represented in computers as a sequence of bits, that are factors in the decomposition of a into powers of two:

$$a = \sum_{i=0}^{N-1} 2^i a_i, \tag{2.81}$$

	$b_0^p \rightarrow$	0								1							
b_1^p	$b^{c_2} \backslash b^{c_1}$	000	100	010	001	011	101	110	111	000	100	010	001	011	101	110	111
		$(-1\frac{1}{3})$ 4			(-1) 5			$(-\frac{2}{3})$ 6	$(-\frac{1}{3})$ 7	(-1) 3			$(-\frac{2}{3})$ 4			$(-\frac{1}{3})$ 5	(0) 6
	000		e_R^-	μ_R^-	τ_R^-					$W^-_{\uparrow\downarrow}$							$Z^0 \upuparrows$
		(-1) 3			$(-\frac{2}{3})$ 4			$(-\frac{1}{3})$ 5	(0) 6	$(-\frac{2}{3})$ 2			$(-\frac{1}{3})$ 3			(0) 4	$(\frac{1}{3})$ 5
	100	e_L^-	$\tilde{u}_R^{\tilde{r}}, \tilde{u}_L^{\tilde{r}}$	$\tilde{c}_R^{\tilde{r}}, \tilde{u}_L^{\tilde{g}}$	$\tilde{t}_R^{\tilde{r}}, \tilde{u}_L^{\tilde{b}}$	d_L^r	d_L^g	d_L^b	$\tilde{\nu}_{eL}$					$g^{\tilde{r}r}_{\uparrow\downarrow}$	$g^{\tilde{r}g}_{\uparrow\downarrow}$	$g^{\tilde{r}b}_{\uparrow\downarrow}$	
	010	μ_L^-	$\tilde{u}_R^{\tilde{g}}, \tilde{c}_L^{\tilde{r}}$	$\tilde{c}_R^{\tilde{g}}, \tilde{c}_L^{\tilde{g}}$	$\tilde{t}_R^{\tilde{g}}, \tilde{c}_L^{\tilde{b}}$	s_L^r	s_L^g	s_L^b	$\tilde{\nu}_{\mu L}$					$g^{\tilde{g}r}_{\uparrow\downarrow}$	$g^{\tilde{g}g}_{\uparrow\downarrow}$	$g^{\tilde{g}b}_{\uparrow\downarrow}$	
0	001	τ_L^-	$\tilde{u}_R^{\tilde{b}}, \tilde{t}_L^{\tilde{r}}$	$\tilde{c}_R^{\tilde{b}}, \tilde{t}_L^{\tilde{g}}$	$\tilde{t}_R^{\tilde{b}}, \tilde{t}_L^{\tilde{b}}$	b_L^r	b_L^g	b_L^b	$\tilde{\nu}_{\tau L}$					$g^{\tilde{b}r}_{\uparrow\downarrow}$	$g^{bg}_{\uparrow\downarrow}$	$g^{\tilde{b}b}_{\uparrow\downarrow}$	
		$(-\frac{2}{3})$ 2			$(-\frac{1}{3})$ 3			(0) 4	$(\frac{1}{3})$ 5	$(-\frac{1}{3})$ 1			(0) 2			$(\frac{1}{3})$ 3	$(\frac{2}{3})$ 4
	011		d_R^r	s_R^r	b_R^r						$g^{r\tilde{r}}_{\uparrow\downarrow}$	$g^{r\tilde{g}}_{\uparrow\downarrow}$	$g^{r\tilde{b}}_{\uparrow\downarrow}$				
	101		d_R^g	s_R^g	b_R^g						$g^{g\tilde{r}}_{\uparrow\downarrow}$	$g^{g\tilde{g}}_{\uparrow\downarrow}$	$g^{g\tilde{b}}_{\uparrow\downarrow}$				
	110		d_R^b	s_R^b	b_R^b						$g^{b\tilde{r}}_{\upuparrows}$	$g^{b\tilde{g}}_{\upuparrows}$	$g^{b\tilde{b}}_{\upuparrows}$				
		$(-\frac{1}{3})$ 1			(0) 2			$(\frac{1}{3})$ 3	$(\frac{2}{3})$ 4	(0) 0			$(\frac{1}{3})$ 1			$(\frac{2}{3})$ 2	(1) 3
	111		$\tilde{\nu}_{eR}$	$\tilde{\nu}_{\mu R}$	$\tilde{\nu}_{\tau R}$					$V^0(\gamma_{\uparrow\downarrow})$							$W^+_{\uparrow\downarrow}$
		(-1) 5			$(-\frac{2}{3})$ 6			$(-\frac{1}{3})$ 7	(0) 8	$(-\frac{2}{3})$ 4			$(-\frac{1}{3})$ 5			(0) 6	$(\frac{1}{3})$ 7
	000	$W^-_{\upuparrows}$							$H^0, Z^0_{\uparrow\downarrow}$					ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	
		$(-\frac{2}{3})$ 4			$(-\frac{1}{3})$ 5			(0) 6	$(\frac{1}{3})$ 7	$(-\frac{1}{3})$ 3			(0) 4			$(\frac{1}{3})$ 5	$(\frac{2}{3})$ 6
	100					$g^{\tilde{r}r}_{\upuparrows}$	$g^{\tilde{r}g}_{\upuparrows}$	$g^{\tilde{r}b}_{\upuparrows}$						$\tilde{d}_R^{\tilde{r}}$	$\tilde{s}_R^{\tilde{r}}$	$\tilde{b}_R^{\tilde{r}}$	
	010					$g^{\tilde{g}r}_{\upuparrows}$	$g^{\tilde{g}g}_{\upuparrows}$	$g^{\tilde{g}b}_{\upuparrows}$						$\tilde{d}_R^{\tilde{g}}$	$\tilde{s}_R^{\tilde{g}}$	$\tilde{b}_R^{\tilde{g}}$	
1	001					$g^{\tilde{b}r}_{\upuparrows}$	$g^{\tilde{b}g}_{\upuparrows}$	$g^{\tilde{b}b}_{\upuparrows}$						$\tilde{d}_R^{\tilde{b}}$	$\tilde{s}_R^{\tilde{b}}$	$\tilde{b}_R^{\tilde{b}}$	
		$(-\frac{1}{3})$ 3			(0) 4			$(\frac{1}{3})$ 5	$(\frac{2}{3})$ 6	(0) 2			$(\frac{1}{3})$ 3			$(\frac{2}{3})$ 4	(1) 5
	011		$g^{r\tilde{r}}_{\uparrow\downarrow}$	$g^{r\tilde{g}}_{\uparrow\downarrow}$	$g^{r\tilde{b}}_{\uparrow\downarrow}$					ν_{eL}	$\tilde{d}_L^{\tilde{r}}$	$\tilde{d}_L^{\tilde{g}}$	$\tilde{d}_L^{\tilde{b}}$	u_R^r, u_L^r	c_R^r, u_L^g	t_R^r, u_L^b	e_L^+
	101		$g^{g\tilde{r}}_{\uparrow\downarrow}$	$g^{g\tilde{g}}_{\uparrow\downarrow}$	$g^{g\tilde{b}}_{\uparrow\downarrow}$					$\nu_{\mu L}$	$\tilde{s}_L^{\tilde{r}}$	$\tilde{s}_L^{\tilde{g}}$	$\tilde{s}_L^{\tilde{b}}$	u_R^g, c_L^r	c_R^g, c_L^g	t_R^g, c_L^b	μ_L^+
	110		$g^{b\tilde{r}}_{\uparrow\downarrow}$	$g^{b\tilde{g}}_{\uparrow\downarrow}$	$g^{b\tilde{b}}_{\uparrow\downarrow}$					$\nu_{\tau L}$	$\tilde{b}_L^{\tilde{r}}$	$\tilde{b}_L^{\tilde{g}}$	$\tilde{b}_L^{\tilde{b}}$	u_R^b, t_L^r	c_R^b, t_L^g	t_R^b, t_L^b	τ_L^+
		(0) 2			$(\frac{1}{3})$ 3			$(\frac{2}{3})$ 4	(1) 5	$(\frac{1}{3})$ 1			$(\frac{2}{3})$ 2			(1) 3	$(1\frac{1}{3})$ 4
	111	$\gamma_{\upuparrows}$							$W^+_{\upuparrows}$					e_R^+	μ_R^+	τ_R^+	

Table 2.8. Code combinations assigned to particles in 8.1 model. The right-handed codes are the same with left-handed, but they are placed in shifted position in relation to the condensate. In top-right corner of cells there are the electrical charge (Q) and the number of changed bits.

where N is a number of bits required, determined by magnitude of a:

$$N \geqslant \log_2 a, N \in \mathbb{Z}. \tag{2.82}$$

In order to stop the processing properly, N should be known just after operating on the last bit in a sequence.

In fact, due to logarithmic dependence, N can be prefixed, by choosing it large enough, exceeding the logarithm of any value from the data domain.

The common way to express these sequences in paper writing is to write a text line of symbols `0` and `1`, delimited with white spaces aside, implicitly assuming that the rightmost digit represents the least significant bit, i.e. a factor a_0 of 2^0. The leftmost bit is the highest-power non-zero (i.e. one) factor, so all bits to the left are known to be `0` and therefore can be omitted. N in this case is the total count of digits `0` and `1`, and i is the zero-based right-to-left index of digit[10].

To represent these sequences explicitly, we use non-weighted directed graphs with bits in vertices. Each vertex in this case has the only outgoing directed edge, pointing to the next bit's vertex: $a_{i+1} \leftarrow a_i$. For instance, with prefixed $N = 3$

$$\dashleftarrow \mathtt{1} \leftarrow \mathtt{1} \leftarrow \mathtt{0} \Rightarrow \mathtt{110}_2 \tag{2.83}$$

(the leftmost bit points to nothing); or, with self-determined N:

$$\hookrightarrow \mathtt{0} \leftarrow \mathtt{1} \leftarrow \mathtt{1} \leftarrow \mathtt{0} \Rightarrow \mathtt{110}_2 \tag{2.84}$$

(zero next to the rightmost `1` is pointing to itself, representing the infinite sequences of zero factors of 2 in higher powers, and also, in fact, terminates the sequence).

To read the value, the graph is processed starting with the dedicated vertex (it is the leftmost one, that is *not next* to any vertex),then following with the next one, repeating until either the prefixed count elapses, or nothing-pointed or self-pointing vertex is detected.

In case the vertices in graph are partially rearranged, the result value of this graph processing is changed. While the graph remains an open, non-circular sequence, values remain strict-defined:

$$\mathtt{1} \leftarrow \mathtt{1} \leftarrow \mathtt{0} \Rightarrow \mathtt{101}_2 = 5. \tag{2.85}$$

When the sequence is closed into a loop, the value becomes arbitrary since there is no more dedicated starting point:

$$\left({}_{\mathtt{1}} \nearrow \mathtt{1} \searrow {}_{\mathtt{0}} \ \ {}_{\mathtt{1} \longleftarrow \mathtt{0}} \right) \Rightarrow \{\mathtt{110}_2, \mathtt{011}_2, \mathtt{101}_2\} = \{6, 3, 5\}$$

Some loop codes (namely, containing all the same digits) represent strictly defined values:

$$\left({}_{\mathtt{1}} \nearrow \mathtt{1} \searrow {}_{\mathtt{1}} \ \ {}_{\mathtt{1} \longleftarrow \mathtt{1}} \right) \Rightarrow \mathtt{111}_2 = 7.$$

[10] This is the same way, as usual decimal numbers are represented in writing, excepting the base of power, the base of logarithm, and the count of different digits, are equal to 2 instead of 10.

As we have seen, the linear graphs (sequences) can be written in text *lines* with implicit (omitted) edges, since they do not contain loops. The same way, some containing loops graphs also can be shown with implicit edges, using *flat* tessellations, for instance pie charts or square- or hexagonal matrices.

The more vertices contained in the graph, the more dimensions is required to represent its edges implicitly. For instance, 4-bit graph with vertices, coupled equally each-to-each, contains 4 loops, and its 6 equal edges can be implicitly expressed by placing bits in vertices of tetrahedron. 5 bits would require 4D space and placement in vertices of the simplex, and so on. The count of different combinations for these each-to-each coupled graphs is $C_{N_1+N_0}^{N_1}$ (where N_0 and N_1 are counts of digits `0` and `1` in the graph).

Graphs with non-equal and/or not each-to-each coupling edges also can be isomorphic to geometrical shapes: for instance, 8-bit graph with each-to-each coupled vertices having 12 edges of the first type, 12 of the second type, and 4 of the third type, can be implicitly represented as 3-dimensional cube, since it has 12 edges, 12 face diagonals and 4 own diagonals.

Also, bit graps, implicit or explicit, can be continuous, (quasi)periodical, repeating the distribution of vertices and edges. They can be used, for instance, to represent fields.

Bit graphs also can be nesting, meaning that vertices may contain either bits or bit graphs. The edges also can be nested the same way. In this case, inner graphs can require more dimensions then outer ones to be implicitly coupled. So nested graphs can be a model for folded dimensions.

2.9 Appendix: Ones' complement code convention

Briefly, in one's complement coding convention, the negation is bitwise inversion, the eldest bit represents the sign, and there are two different zero representations, positive and negative [14].

The non-negative number a_+ is coded in the general way: all binary digits a_i (`0` or `1`) excepting the eldest a_{N-1} (which is equal to `0`) are factors in the decomposition of a_+ into powers of two:

$$a_+ = \sum_{i=0}^{N-2} 2^i a_i. \tag{2.86}$$

Bits a_i could be obtained as remaining from sequential division by two:

$$d_{-1} = a_+; \tag{2.87}$$

$$d_i = \left[\frac{d_{i-1}}{2}\right]; a_i = d_{i-1} - 2d_i \tag{2.88}$$

(square brackets here mean the integer division).

The representation of negative number is produced from representation of its absolute value by inversion of all their bits. Thus, the eldest bit a_{N-1} takes the value of 1. This bit does not act as a factor in the decomposition, but when it has the value of 1, it changes the composition formula, so it takes inverted binary digits a_i:

$$a_- = -\sum_{i=0}^{N-2} 2^i(\neg a_i). \tag{2.89}$$

Taking into account

$$\neg a_i = 1 - a_i, \tag{2.90}$$

we have

$$a_- = \sum_{i=0}^{N-2} 2^i(a_i - 1) = \sum_{i=0}^{N-2} 2^i a_i - \sum_{i=0}^{N-2} 2^i = a_+ - (2^{N-1} - 1), \tag{2.91}$$

so a_{N-1} is the factor of an extra negative term $(1 - 2^{N-1})$ in the decomposition:

$$a = (1 - 2^{N-1})a_{N-1} + \sum_{i=0}^{N-2} 2^i a_i. \tag{2.92}$$

For code of $N = 3$ bits, the value of a is expressed through them as

$$a = -3a_2 + 2a_1 + a_0. \tag{2.93}$$

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3 Quark and Lepton Masses and Mixing From a Gauged $SU(3)_F$ Family Symmetry With a Light $\mathcal{O}(eV)$ Sterile Dirac Neutrino

A. Hernández-Galeana **, ***

APC laboratory, Université Paris Diderot, Bâtiment Condorcet,
10, rue Alice Domon et Léonie Duquet 75205 Paris cedex 13, France

Abstract. In the framework of a complete vector-like and universal gauged $SU(3)_F$ family symmetry, we report a global region in the parameter space where this approach can account for a realistic spectrum of quark masses and mixing in a 4×4 non-unitary V_{CKM}, as well as for the known charged lepton masses and the squared neutrino mass differences reported from neutrino oscillation experiments. The $SU(3)_F$ family symmetry is broken spontaneously in two stages by heavy SM singlet scalars, whose hierarchy of scales yield and approximate $SU(2)_F$ global symmetry associated to the almost degenerate boson masses of the order of the lower scale of the $SU(3)_F$ SSB. The gauge symmetry, the fermion content, and the transformation of the scalar fields, all together, avoid Yukawa couplings between SM fermions. Therefore, in this scenario ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level from Dirac See-saw mechanisms, while light fermions, including light neutrinos, obtain masses from radiative corrections mediated by the massive gauge bosons of the $SU(3)_F$ family symmetry. The displayed fit parameter space region solution for fermion masses and mixing yield the vector-like fermion masses: $M_D \approx 3.2\,\text{TeV}$, $M_U \approx 6.9\,\text{TeV}$, $M_E \approx 21.6\,\text{TeV}$, $SU(2)_F$ family gauge boson masses of $\mathcal{O}(2\,\text{TeV})$, and the squared neutrino mass differences: $m_2^2 - m_1^2 \approx 7.5 \times 10^{-5}\ eV^2$, $m_3^2 - m_2^2 \approx 2.2 \times 10^{-3}\ eV^2$, $m_4^2 - m_1^2 \approx 0.81\ eV^2$.

Povzetek. Avtor ponudi razširitev *standardnega modela*, ki k poznanim grupam doda še družinsko (umeritveno) grupo $SU(3)_F$. Poišče območje v prostoru parametrov, ki ponudi eksperimentalno sprejemljive lastnosti kvarkov in leptonov. Družinsko simetrijo zlomi v dveh korakih. Težki fermioni - kvarka t in b ter lepton tau - postanejo masivni na drevesnem nivoju (z mehanizmom gugalnice, see-saw), ostali pa s popravki višjih redov. Mase fermionov, pri katerih so kvantna števila levoročnih in desnoročnih fermionov enaka, so nekaj TeV ali več: $M_D \approx 3.2\,\text{TeV}$, $M_U \approx 6.9\,\text{TeV}$, $M_E \approx 21.6\,\text{TeV}$. Mase bozonov z družinskimi kvantnimi števili so $\mathcal{O}(2\,\text{TeV})$, razliki kvadratov nevtrinskih mass pa so:$m_2^2 - m_1^2 \approx 7.5 \times 10^{-5}\ eV^2$, $m_3^2 - m_2^2 \approx 2.2 \times 10^{-3}\ eV^2$, $m_4^2 - m_1^2 \approx 0.81\ eV^2$.

** On sabbatical leave from:
Departamento de Física, ESFM - Instituto Politécnico Nacional
U. P. "Adolfo López Mateos". C. P. 07738, Ciudad de México, México.
*** e-mail: albino@esfm.ipn.mx

3.1 Introduction

The origen of the hierarchy of fermion masses and mixing is one of the most important open problems in particle physics. Any attempt to account for this hierarchy introduce a mass generation mechanism which distinguish among the different Standard Model (SM) quarks and leptons.

After the discovery of the scalar Higgs boson on 2012, LHC has not found a conclusive evidence of new physics. However, there are theoretical motivations to look for new particles in order to answer some open questions like; neutrino ossccillations, dark matter, stability of the Higgs mass against radiative corrections,,etc.

In this article, we address the problem of charged fermion masses and quark mixing within the framework of an extension of the SM introduced by the author in [1]. This Beyond Standard Model (BSM) proposal include a vector gauged $SU(3)_F$ family symmetry[1] commuting with the SM group and introduce a hierarchical massgeneration mechanism in which the light fermions obtain masses through loop radiative corrections, mediated by the massive bosons associated to the $SU(3)_F$ family symmetry that is spontaneously broken, while the masses of the top and bottom quarks as well as for the tau lepton, are generated at tree level from "Dirac See-saw"[3] mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlets U, D, E and N vector-like fermions. Due to the fact that these vector-like quarks do not couple to the W boson, the mixing of U and D vector-like quarks with the SM quarks gives rise to and extended 4×4 non-unitary CKM quark mixing matrix [4].

3.2 Model with $SU(3)_F$ flavor symmetry

3.2.1 Fermion content

We define the gauge symmetry group

$$G \equiv SU(3)_F \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \tag{3.1}$$

where $SU(3)_F$ is a completely vector-like and universal gauged family symmetry. That is, the corresponding gauge bosons couple equally to Left and Right Handed ordinary Quarks and Leptons, including right handed neutrinos. $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the "Standard Model" (SM) gauge group, with g_H, g_s, g and g' the coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under G as:

Ordinary Fermions: $q^o_{iL} = \begin{pmatrix} u^o_{iL} \\ d^o_{iL} \end{pmatrix}$, $l^o_{iL} = \begin{pmatrix} \nu^o_{iL} \\ e^o_{iL} \end{pmatrix}$, $Q = T_{3L} + \frac{1}{2}Y$

$$\Psi^o_q = (3, 3, 2, \frac{1}{3})_L = \begin{pmatrix} q^o_{1L} \\ q^o_{2L} \\ q^o_{3L} \end{pmatrix} \quad , \quad \Psi^o_l = (3, 1, 2, -1)_L = \begin{pmatrix} l^o_{1L} \\ l^o_{2L} \\ l^o_{3L} \end{pmatrix}$$

[1] See [1,2] and references therein for some other $SU(3)_F$ family symmetry model proposals.

$$\Psi_u^o = (3,3,1,\frac{4}{3})_R = \begin{pmatrix} u_R^o \\ c_R^o \\ t_R^o \end{pmatrix} \quad , \quad \Psi_d^o = (3,3,1,-\frac{2}{3})_R = \begin{pmatrix} d_R^o \\ s_R^o \\ b_R^o \end{pmatrix}$$

$$\Psi_e^o = (3,1,1,-2)_R = \begin{pmatrix} e_R^o \\ \mu_R^o \\ \tau_R^o \end{pmatrix}$$

where the last entry corresponds to the hypercharge Y, and the electric charge is defined by $Q = T_{3L} + \frac{1}{2}Y$. The model also includes two types of extra fermions:

Right Handed Neutrinos: $\Psi_{\nu_R}^o = (3,1,1,0)_R = \begin{pmatrix} \nu_{e_R} \\ \nu_{\mu_R} \\ \nu_{\tau_R} \end{pmatrix}$,

and the $SU(2)_L$ weak singlet vector-like fermions

Sterile Neutrinos: $N_L^o, N_R^o = (1,1,1,0)$,

The Vector Like quarks:

$$U_L^o, U_R^o = (1,3,1,\frac{4}{3}) \quad , \quad D_L^o, D_R^o = (1,3,1,-\frac{2}{3}) \tag{3.2}$$

and

The Vector Like electrons: $E_L^o, E_R^o = (1,1,1,-2)$

The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U\, \bar{U}_L^o\, U_R^o \,+\, M_D\, \bar{D}_L^o\, D_R^o \,+\, M_E\, \bar{E}_L^o\, E_R^o + h.c.\,, \tag{3.3}$$

and

$$m_D\, \bar{N}_L^o\, N_R^o \,+\, m_L\, \bar{N}_L^o\, (N_L^o)^c \,+\, m_R\, \bar{N}_R^o\, (N_R^o)^c \,+\, h.c \tag{3.4}$$

The above fermion content make the model anomaly free. After the definition of the gauge symmetry group and the assignment of the ordinary fermions in the usual form under the standard model group and in the fundamental 3-representation under the $SU(3)_F$ family symmetry, the introduction of the right-handed neutrinos is required to cancel anomalies[5]. The $SU(2)_L$ weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms. These vector like fermions play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses, together with the radiative corrections.

3.3 $SU(3)_F$ family symmetry breaking

We need to be consistent with low energy Standard Model (SM) and simultaneously we would like to generate and account for the hierarchy of fermion masses and mixing after spontaneously symmetry breaking (SSB) down to $SU(3)_C \times U(1)_Q$. Previous basic assumptions of this BSM define the required scalar fields and V.E.V's to achieve the desired symmetry breaking.

To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of $SU(3)_F$, we introduce the flavon scalar fields: $\eta_i,\ i = 2, 3,$

$$\eta_i = (3, 1, 1, 0) = \begin{pmatrix} \eta^o_{i1} \\ \eta^o_{i2} \\ \eta^o_{i3} \end{pmatrix} \quad , \quad i = 2, 3$$

with the "Vacuum ExpectationValues" (VEV's):

$$\langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3)\,. \tag{3.5}$$

The above scalar fields and VEV's break completely the $SU(3)_F$ flavor symmetry. The corresponding $SU(3)_F$ gauge bosons are defined in Eq.(3.17) through their couplings to fermions. Thus, the contribution to the horizontal gauge boson masses from Eq.(3.5) read

- $\langle \eta_2 \rangle$: $\frac{g^2_{H_2} \Lambda^2_2}{2} (Y^+_1 Y^-_1 + Y^+_3 Y^-_3) + \frac{g^2_{H_2} \Lambda^2_2}{4} (Z^2_1 + \frac{Z^2_2}{3} - 2 Z_1 \frac{Z_2}{\sqrt{3}})$
- $\langle \eta_3 \rangle$: $\frac{g^2_{H_3} \Lambda^2_3}{2} (Y^+_2 Y^-_2 + Y^+_3 Y^-_3) + g^2_{H_3} \Lambda^2_3 \frac{Z^2_2}{3}$

These two scalars in the fundamental representation is the minimal set of scalars to break down completely the $SU(3)_F$ family symmetry.

$$SU(3)_F \times G_{SM} \xrightarrow{\langle \eta_3 \rangle, \langle \eta_2 \rangle} SU(2)_F \,?\, \times G_{SM} \xrightarrow{\langle \eta_2 \rangle, \langle \eta_3 \rangle} G_{SM}$$

FCNC ?

$\Lambda_3(\Lambda_2)$: 5 very heavy boson masses ($\geq 100\,\mathrm{TeV}'s$)

$\Lambda_2(\Lambda_3)$: 3 heavy boson masses (a few $\mathrm{TeV}'s$).

Notice that the hierarchy of scales $\Lambda_3 \gg \Lambda_2$ define an "approximate $SU(2)_F$ global symmetry" in the spectrum of $SU(3)_F$ gauge boson masses. To suppress properly the FCNC like, for instance PDG 2016 [9] : $\mu \to e\gamma$ ($Br < 5.7 \times 10^{-13}$), $\mu \to e\,e\,e$ ($Br < 1 \times 10^{-12}$), $K^o - \bar{K^o}$, it is relevant which gauge bosons become massive at the lower scale of the $SU(3)_F$ symmetry breaking.

Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms.

$$M^2_2\, Y^+_1 Y^-_1 + M^2_3\, Y^+_2 Y^-_2 + (M^2_2 + M^2_3)\, Y^+_3 Y^-_3 + \frac{1}{2} M^2_2\, Z^2_1 + \frac{1}{2} \frac{M^2_2 + 4M^2_3}{3}\, Z^2_2 - \frac{1}{2}(M^2_2) \frac{2}{\sqrt{3}}\, Z_1\, Z_2 \tag{3.6}$$

$$M_2^2 = \frac{g_H^2 \Lambda_2^2}{2} \quad , \quad M_3^2 = \frac{g_H^2 \Lambda_3^2}{2} \quad , \quad y \equiv \frac{M_3}{M_2} = \frac{\Lambda_3}{\Lambda_2} \tag{3.7}$$

	Z_1	Z_2
Z_1	M_2^2	$-\frac{M_2^2}{\sqrt{3}}$
Z_2	$-\frac{M_2^2}{\sqrt{3}}$	$\frac{M_2^2+4M_3^2}{3}$

Table 3.1. $Z_1 - Z_2$ mixing mass matrix

Diagonalization of the $Z_1 - Z_2$ squared mass matrix yield the eigenvalues

$$M_-^2 = \frac{2}{3}\left(M_2^2 + M_3^2 - \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2}\right)_- \tag{3.8}$$

$$M_+^2 = \frac{2}{3}\left(M_2^2 + M_3^2 + \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2}\right)_+ \tag{3.9}$$

$$M_2^2\, Y_1^+ Y_1^- + M_3^2\, Y_2^+ Y_2^- + (M_2^2 + M_3^2)\, Y_3^+ Y_3^- + M_-^2\, \frac{Z_-^2}{2} + M_+^2\, \frac{Z_+^2}{2} \tag{3.10}$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \tag{3.11}$$

$$\cos\phi\ \sin\phi = \frac{\sqrt{3}}{4}\, \frac{M_2^2}{\sqrt{M_2^4 + M_3^2(M_3^2 - M_2^2)}}$$

Due to the $Z_1 - Z_2$ mixing, we diagonalize the propagators involving Z_1 and Z_2 gauge bosons according to Eq.(3.11)

$$Z_1 = \cos\phi\ Z_- - \sin\phi\ Z_+ \quad , \quad Z_2 = \sin\phi\ Z_- + \cos\phi\ Z_+$$

3.4 Electroweak symmetry breaking

Recently ATLAS[6] and CMS[7] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to a BSM theory. For electroweak symmetry breaking we introduction two triplets of $SU(2)_L$ Higgs doublets, namely;

$$\Phi^u = (3,1,2,-1) = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_1^u \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_2^u \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_3^u \end{pmatrix} \quad , \qquad \Phi^d = (3,1,2,+1) = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_1^d \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_2^d \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_3^d \end{pmatrix} ,$$

and the VEV?s

$$\Phi^u\rangle = \begin{pmatrix} \langle \Phi_1^u \rangle \\ \langle \Phi_2^u \rangle \\ \langle \Phi_3^u \rangle \end{pmatrix} \quad , \quad \langle \Phi^d \rangle = \begin{pmatrix} \langle \Phi_1^d \rangle \\ \langle \Phi_2^d \rangle \\ \langle \Phi_3^d \rangle \end{pmatrix} ,$$

where

$$\Phi_i^u\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{ui} \\ 0 \end{pmatrix} \quad , \quad \langle \Phi_i^d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{di} \end{pmatrix} .$$

The contributions from $\langle \Phi^u \rangle$ and $\langle \Phi^d \rangle$ yield the W and Z gauge boson masses and mixing with the $SU(3)_F$ gauge bosons

$$\frac{g^2}{4} (v_u^2 + v_d^2) W^+ W^- + \frac{(g^2 + g'^2)}{8} (v_u^2 + v_d^2) Z_o^2$$

$$+\tfrac{1}{4}\sqrt{g^2 + g'^2}\, g_H\, Z_o \, \Big[(v_{1u}^2 - v_{2u}^2 - v_{1d}^2 + v_{2d}^2) Z_1$$

$$+(v_{1u}^2 + v_{2u}^2 - 2v_{3u}^2 - v_{1d}^2 - v_{2d}^2 + 2v_{3d}^2) \tfrac{Z_2}{\sqrt{3}}$$

$$+2\,(v_{1u}v_{2u} - v_{1d}v_{2d}) \tfrac{Y_1^+ + Y_1^-}{\sqrt{2}} + 2\,(v_{1u}v_{3u} - v_{1d}v_{3d}) \tfrac{Y_2^+ + Y_2^-}{\sqrt{2}}$$

$$+2\,(v_{2u}v_{3u} - v_{2d}v_{3d}) \tfrac{Y_3^+ + Y_3^-}{\sqrt{2}} \Big]$$

$$+ \text{ tiny contributions to the } SU(3) \text{ gauge boson masses} \ ,$$

$v_u^2 = v_{1u}^2 + v_{2u}^2 + v_{3u}^2$, $v_d^2 = v_{1d}^2 + v_{2d}^2 + v_{3d}^2$. Hence, if we define as usual $M_W = \frac{1}{2} g v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

$$Y_j^1 = \frac{Y_j^+ + Y_j^-}{\sqrt{2}} \quad , \quad Y_j^\pm = \frac{Y_j^1 \mp i Y_j^2}{\sqrt{2}} \tag{3.12}$$

The mixing of Z_o neutral gauge boson with the $SU(3)_F$ gauge bosons modify the couplings of the standard model Z boson with the ordinary quarks and leptons

3.5 Fermion masses

3.5.1 Dirac See-saw mechanisms

Now we describe briefly the procedure to get the masses for fermions. The analysis is presented explicitly for the charged lepton sector, with a completely analogous procedure for the u and d quarks and Dirac neutrinos. With the fields of particles introduced in the model, we may write the gauge invariant Yukawa couplings, as

$$h\,\bar{\psi}_l^o\,\Phi^d\,E_R^o \;+\; h_2\,\bar{\psi}_e^o\,\eta_2\,E_L^o \;+\; h_3\,\bar{\psi}_e^o\,\eta_3\,E_L^o \;+\; M\,\bar{E}_L^o\,E_R^o \;+ h.c \tag{3.13}$$

where M is a free mass parameter (because its mass term is gauge invariant) and h, h_2 and h_3 are Yukawa coupling constants. When the involved scalar fields acquire VEV's we get, in the gauge basis ${\psi_{L,R}^o}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms $\bar{\psi}_L^o \mathcal{M}^o \psi_R^o + h.c$, where

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & h\,v_1 \\ 0 & 0 & 0 & h\,v_2 \\ 0 & 0 & 0 & h\,v_3 \\ 0 & h_2\Lambda_2 & h_3\Lambda_3 & M \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & M \end{pmatrix} . \tag{3.14}$$

Notice that $\mathcal{M}^o$ has the same structure of a See-saw mass matrix, here for Dirac fermion masses. So, we call $\mathcal{M}^o$ a **"Dirac See-saw"** mass matrix. $\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi_{L,R}^o = V_{L,R}^o\,\chi_{L,R}$. The orthogonal matrices V_L^o and V_R^o are obtained explicitly in the Appendix A. From V_L^o and V_R^o, and using the relationships defined in this Appendix, one computes

$${V_L^o}^T \mathcal{M}^o\,V_R^o = Diag(0, 0, -\lambda_3, \lambda_4) \tag{3.15}$$

$${V_L^o}^T \mathcal{M}^o {\mathcal{M}^o}^T\,V_L^o = {V_R^o}^T {\mathcal{M}^o}^T \mathcal{M}^o\,V_R^o = Diag(0, 0, \lambda_3^2, \lambda_4^2)\,. \tag{3.16}$$

where λ_3^2 and λ_4^2 are the nonzero eigenvalues defined in Eqs.(3.50-3.51), λ_4 being the fourth heavy fermion mass, and λ_3 of the order of the top, bottom and tau mass for u, d and e fermions, respectively. We see from Eqs.(3.15,3.16) that at tree level the See-saw mechanism yields two massless eigenvalues associated to the light fermions:

3.6 One loop contribution to fermion masses

Subsequently, the masses for the light fermions arise through one loop radiative corrections. After the breakdown of the electroweak symmetry we can construct the generic one loop mass diagram of Fig. 1. Internal fermion line in this diagram represent the Dirac see-saw mechanism implemented by the couplings in Eq.(3.13). The vertices read from the $SU(3)_F$ flavor symmetry interaction Lagrangian

$$
\begin{aligned}
i\mathcal{L}_{int} &= \frac{g_H}{\sqrt{2}}\left(\bar{e^o}\gamma_\mu\mu^o Y_1^+ + \bar{e^o}\gamma_\mu\tau^o Y_2^+ + \bar{\mu^o}\gamma_\mu\tau^o Y_3^+ + h.c.\right) \\
&+ \frac{g_H}{2}\left(\bar{e^o}\gamma_\mu e^o - \bar{\mu^o}\gamma_\mu\mu^o\right) Z_1^\mu + \frac{g_H}{2\sqrt{3}}\left(\bar{e^o}\gamma_\mu e^o + \bar{\mu^o}\gamma_\mu\mu^o - 2\bar{\tau^o}\gamma_\mu\tau^o\right) Z_2^\mu
\end{aligned} \tag{3.17}
$$

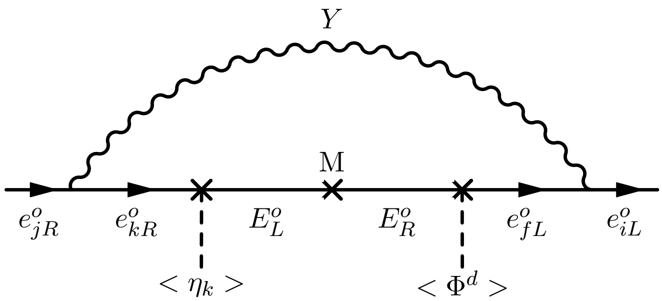

Fig. 3.1. Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

where g_H is the $SU(3)_F$ coupling constant, Z_1, Z_2 and Y_i^j $, i = 1,2,3$ $, j = 1,2$ are the eight gauge bosons. The crosses in the internal fermion line mean tree level mixing, and the mass M generated by the Yukawa couplings in Eq.(3.13) after the scalar fields get VEV's. The one loop diagram of Fig. 1 gives the generic contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

$$
c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o\ (V_L^o)_{ik}(V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi} \tag{3.18}
$$

where M_Y is the gauge boson mass, c_Y is a factor coupling constant, Eq.(3.17), $m_3^o = -\sqrt{\lambda_3^2}$ and $m_4^o = \lambda_4$ are the See-saw mass eigenvalues, Eq.(3.15), and $f(x,y) = \frac{x^2}{x^2-y^2} \ln \frac{x^2}{y^2}$. Using the results of Appendix A, we compute

$$
\sum_{k=3,4} m_k^o\ (V_L^o)_{ik}(V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i\ b_j\ M}{\lambda_4^2 - \lambda_3^2} F(M_Y)\,, \tag{3.19}
$$

$i = 1,2,3\,, j = 2,3$, and $F(M_Y) \equiv \frac{M_Y^2}{M_Y^2-\lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2-\lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$. Adding up all the one loop $SU(3)_F$ gauge boson contributions, we get the mass terms $\bar{\psi_L^o}\mathcal{M}_1^o\ \psi_R^o +$ h.c.,

$$
\mathcal{M}_1^o = \begin{pmatrix} D_{11} & D_{12} & D_{13} & 0 \\ 0 & D_{22} & D_{23} & 0 \\ 0 & D_{32} & D_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi}\,, \tag{3.20}
$$

$$
\begin{aligned}
D_{11} &= \mu_{11}(\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12} + F_m) + \frac{1}{2}(\mu_{22}F_1 + \mu_{33}F_2) \\
D_{12} &= \mu_{12}(-\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12}) \\
D_{13} &= -\mu_{13}(\frac{F_{Z_2}}{6} + F_m) \\
D_{22} &= \mu_{22}(\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12} - F_m) + \frac{1}{2}(\mu_{11}F_1 + \mu_{33}F_3) \\
D_{23} &= -\mu_{23}(\frac{F_{Z_2}}{6} - F_m) \\
D_{32} &= -\mu_{32}(\frac{F_{Z_2}}{6} - F_m) \\
D_{33} &= \mu_{33}\frac{F_{Z_2}}{3} + \frac{1}{2}(\mu_{11}F_2 + \mu_{22}F_3) \,,
\end{aligned}
$$

Here,

$$F_1 \equiv F(M_{Y_1}) \quad , \quad F_2 \equiv F(M_{Y_2}) \quad , \quad F_3 \equiv F(M_{Y_3})$$

$$F_{Z_1} = \cos^2\phi \, F(M_-) + \sin^2\phi \, F(M_+) \; , \; F_{Z_2} = \sin^2\phi \, F(M_-) + \cos^2\phi \, F(M_+)$$

$$M_{Y_1}^2 = M_2^2 \quad , \quad M_{Y_2}^2 = M_3^2 \quad , \quad M_{Y_3}^2 = M_2^2 + M_3^2$$

$$F_m = \frac{\cos\phi\sin\phi}{2\sqrt{3}} \left[F(M_-) - F(M_+) \right]$$

with M_2, M_3, M_-, M_+ the horizontal boson masses, Eqs.(3.7-3.9),

$$\mu_{ij} = \frac{a_i \, b_j \, M}{\lambda_4^2 - \lambda_3^2} = \frac{a_i \, b_j}{a \, b} \lambda_3 \, c_\alpha \, c_\beta \,, \tag{3.21}$$

and $c_\alpha \equiv \cos\alpha$, $c_\beta \equiv \cos\beta$, $s_\alpha \equiv \sin\alpha$, $s_\beta \equiv \sin\beta$, as defined in the Appendix, Eq.(3.52). Therefore, up to one loop corrections we obtain the fermion masses

$$\bar{\psi}_L^o \mathcal{M}^o \, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \, \psi_R^o = \bar{\chi}_L \, \mathcal{M} \, \chi_R \,, \tag{3.22}$$

with $\mathcal{M} \equiv \left[Diag(0, 0, -\lambda_3, \lambda_4) + V_L^{o\,T} \mathcal{M}_1^o \, V_R^o \right]$.

Using V_L^o, V_R^o from Eqs.(3.45-3.46) we get the mass matrix up to one loop radiative corrections:

$$
\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta\, m_{13} & s_\beta\, m_{13} \\ m_{21} & m_{22} & c_\beta\, m_{23} & s_\beta\, m_{23} \\ c_\alpha\, m_{31} & c_\alpha\, m_{32} & (-\lambda_3 + c_\alpha c_\beta\, m_{33}) & c_\alpha s_\beta\, m_{33} \\ s_\alpha\, m_{31} & s_\alpha\, m_{32} & s_\alpha c_\beta\, m_{33} & (\lambda_4 + s_\alpha s_\beta\, m_{33}) \end{pmatrix} , \qquad (3.23)
$$

where

$$
m_{11} = \delta\, c_1\, \pi_1 \quad , \quad m_{21} = -\delta\, s_1\, s_2\, \pi_1 \quad , \quad m_{31} = \delta\, c_2\, s_1\, \pi_1
$$

$$
m_{12} = \delta\, s_1\, s_r\, (c_1\, c_2\, c_r\, \Delta + \pi_3)
$$

$$
m_{13} = -\delta\, s_1\, (c_1\, c_2\, Fm - c_1\, c_2\, s_r^2\, \Delta + c_r\, \pi_3)
$$

$$
m_{22} = \delta(-3\, c_2\, c_r\, s_2\, s_r\, F_m + c_1^2\, c_2\, c_r\, s_2\, s_r\, \Delta + c_2\, c_r\, \pi_2 + c_1\, s_2\, s_r\, \pi_3)
$$

$$
m_{23} = \delta(c_2\, s_2\, (1 + s_1^2 - 3s_r^2)\, F_m + c_1^2\, c_2\, s_2\, s_r^2\, \Delta + c_2\, s_r\, \pi_2 - c_1\, c_r\, s_2\, \pi_3)
$$

$$
m_{32} = \delta(-c_r\, s_r\, (-1 + 3s_2^2)\, F_m + c_r\, (c_2^2 s_1^2 + s_2^2)\, s_r\, \Delta + c_r\, s_2\, \pi_2
$$
$$
-c_1\, c_2\, s_r\, \pi_3)
$$

$$
m_{33} = \delta(-\tfrac{F_{Z_2}}{6} - (c_2^2\, s_1^2 - s_2^2 - s_r^2 + 3s_2^2\, s_r^2)\, F_m + (c_2^2\, s_1^2 + s_2^2)\, s_r^2\, \Delta
$$
$$
+s_2\, s_r\, \pi_2 + c_1\, c_2\, c_r\, \pi_3).
$$

$s_1, s_2, s_r, s_\alpha, s_\beta, \lambda_3, \lambda_4$ come from the diagonalization of the tree level mass matrix $\mathcal{M}^o$, Eq. (3.14), are defined in Appendix 3.9.

$$
\delta = \frac{\alpha_H}{\pi}\, c_\alpha\, c_\beta\, \lambda_3 \ , \ \Delta = \tfrac{1}{4}(F_{Z_2} - F_{Z_1})\, , \, \pi_1 = \frac{1}{2}(c_1\, c_2\, c_r\, F_2 + F_1\, s_2\, s_r)
$$

$$
\pi_2 = \frac{1}{2}(c_1\, c_2\, c_r\, F_3 + F_{Z_1}\, s_2\, s_r)\ , \ \pi_3 = \tfrac{1}{2}(c_1\, c_2\, c_r\, F_{Z_2} + F_3\, s_2\, s_r)
$$

The diagonalization of $\mathcal{M}$, Eq.(3.23) gives the physical masses for u and d quarks, e charged leptons and ν Dirac neutrino masses.

Using a new biunitary transformation $\chi_{L,R} = V_{L,R}^{(1)}\ \Psi_{L,R}$; $\bar{\chi}_L\ \mathcal{M}\ \chi_R = \bar{\Psi}_L\ {V_L^{(1)}}^T \mathcal{M}\ V_R^{(1)}\ \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$V_L^{(1)^T} \mathcal{M}\, \mathcal{M}^T \, V_L^{(1)} = V_R^{(1)^T} \mathcal{M}^T \, \mathcal{M} \, V_R^{(1)} = Diag(m_1^2, m_2^2, m_3^2, M_F^2) \,, \tag{3.24}$$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$\psi_L^o = V_L^o \, V_L^{(1)} \, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o \, V_R^{(1)} \, \Psi_R \tag{3.25}$$

It is worth to comment here that neutrinos may also obtain left-handed and right-handed Majorana masses both from tree level and radiative corrections.

3.6.1 Quark $(V_{CKM})_{4\times4}$ and Lepton $(U_{PMNS})_{4\times8}$ mixing matrices

Within this $SU(3)_F$ family symmetry model, the transformation from massless to physical mass fermion eigenfields for quarks and charged leptons is

$$\psi_L^o = V_L^o \, V_L^{(1)} \, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o \, V_R^{(1)} \, \Psi_R \,,$$

Recall now that vector like quarks, Eq.(3.2), are $SU(2)_L$ weak singlets, and hence, they do not couple to *W* boson in the interaction basis. In this way, the interaction of L-handed up and down quarks; ${f_{uL}^o}^T = (u^o, c^o, t^o)_L$ and ${f_{dL}^o}^T = (d^o, s^o, b^o)_L$, to the *W* charged gauge boson is

$$\frac{g}{\sqrt{2}} \bar{f^o}_{uL} \gamma_\mu f_{dL}^o W^{+\mu} =$$

$$\frac{g}{\sqrt{2}} \bar{\Psi}_{uL} \, [(V_{uL}^o \, V_{uL}^{(1)})_{3\times4}]^T \, (V_{dL}^o \, V_{dL}^{(1)})_{3\times4} \, \gamma_\mu \Psi_{dL} \, W^{+\mu} \,, \tag{3.26}$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} = [(V_{uL}^o \, V_{uL}^{(1)})_{3\times4}]^T \, (V_{dL}^o \, V_{dL}^{(1)})_{3\times4} \tag{3.27}$$

3.7 Numerical results

To illustrate the spectrum of masses and mixing from this scenario, let us consider the following fit of space parameters at the M_Z *scale [8]*

Using the input values for the $SU(3)_F$ family symmetry:

$$M_2 = 2\,\text{TeV} \quad , \quad M_3 = 2000\,\text{TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.2 \tag{3.28}$$

with M_2, M_3 horizontal boson masses, Eq.(3.7), and the coupling constant, respectively, and the tree level mixing angles

$$s_{1d} = s_{1e} = 0.6 \qquad s_{2d} = s_{2e} = 0.1047$$
$$s_{1u} = s_{1\nu} = 0.575341 \qquad s_{2u} = s_{2\nu} = 0.0925127$$

we obtain the following tree level $\mathcal{M}_f^o$, one loop $\mathcal{M}_f$, $f = u, d, e, \nu$ mass matrices, mixing and mass eigenvalues:

3.7.1 Quark masses and $(V_{CKM})_{4\times4}$ mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 108921. \\ 0 & 0 & 0 & 17589.5 \\ 0 & 0 & 0 & 154844. \\ 0 & -6.42288 \times 10^6 & 462459. & 2.5111 \times 10^6 \end{pmatrix} \text{MeV}, \tag{3.29}$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} 7.19764 & -626.533 & -1479.88 & -3792.15 \\ -0.468392 & -81.7707 & -197.807 & -506.875 \\ 5.04103 & 1502.25 & -172425. & 12057.2 \\ 0.0504129 & 15.0233 & 47.0554 & 6.91226 \times 10^6 \end{pmatrix} \text{MeV} \tag{3.30}$$

and the u-quark masses

$$(\, m_u\, ,\, m_c\, ,\, m_t\, ,\, M_U\,) = (\, 1.396\, ,\, 644.835\, ,\, 172438\, ,\, 6.912 \times 10^6\,)\ \text{MeV} \tag{3.31}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 2860.87 \\ 0 & 0 & 0 & 501.98 \\ 0 & 0 & 0 & 3814.49 \\ 0 & -2.3645 \times 10^6 & 323661. & 2.17117 \times 10^6 \end{pmatrix} \text{MeV} \tag{3.32}$$

$$\mathcal{M}_d = \begin{pmatrix} -4.22954 & 3.26664 & 26.4239 & 29.045 \\ 19.9418 & -41.6 & -57.7027 & -63.4265 \\ -2.27726 & -31.0285 & -2859.26 & 755.343 \\ -0.002277 & -0.031028 & 0.687179 & 3.2264 \times 10^6 \end{pmatrix} \text{MeV} \tag{3.33}$$

$$(\, m_d\, ,\, m_s\, ,\, m_b\, ,\, M_D\,) = (\, 2.501\, ,\, 45.803\, ,\, 2860.14\, ,\, 3.226 \times 10^6\,)\ \text{MeV} \tag{3.34}$$

and the quark mixing

$$(V_{CKM})_{4\times 4} = \begin{pmatrix} -0.97445 & 0.224576 & -0.003514 & -0.000021 \\ -0.224523 & -0.973562 & 0.042015 & -0.000010 \\ 0.006011 & 0.041720 & 0.999041 & -0.001233 \\ -0.000219 & -0.0011268 & -0.011702 & 0.000014 \end{pmatrix} \tag{3.35}$$

3.7.2 Lepton masses and $(U_{PMNS})_{4\times 4}$ mixing:

Charged leptons:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 129165. \\ 0 & 0 & 0 & 22663.9 \\ 0 & 0 & 0 & 172220. \\ 0 & -337398. & 32029.6 & 2.16401\times 10^7 \end{pmatrix} \text{MeV} \tag{3.36}$$

$$\mathcal{M}_e = \begin{pmatrix} 2.2376 & -66.4545 & -394.792 & -6.18239 \\ -0.175708 & -9.01417 & -57.8818 & -0.906422 \\ 1.66889 & 164.714 & -1693.76 & 26.556 \\ 0.016689 & 1.64723 & 16.9589 & 2.16438\times 10^7 \end{pmatrix} \text{MeV} \tag{3.37}$$

fit the charged lepton masses:

$$(m_e\,,\, m_\mu\,,\, m_\tau\,,\, M_E) = (0.486\,,\, 102.702\,,\, 1746.17\,,\, 2.164\times 10^7\,)\ \text{MeV} \tag{3.38}$$

Dirac neutrino masses:

$$\mathcal{M}_\nu^o = \begin{pmatrix} 0 & 0 & 0 & 0.076760 \\ 0 & 0 & 0 & 0.012395 \\ 0 & 0 & 0 & 0.109124 \\ 0 & -0.108392 & -0.264395 & 0.854133 \end{pmatrix} \text{eV} \tag{3.39}$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0.015703 & -0.004190 & 0.009713 & 0.003179 \\ -0.001021 & -0.01824 & -0.005614 & -0.001837 \\ 0.010890 & 0.004245 & -0.048705 & -0.002164 \\ 0.001539 & 0.000600 & -0.000934 & 0.909297 \end{pmatrix} \text{eV} \tag{3.40}$$

fit the light neutrino masses:

$$(m_1\,,\, m_2\,,\, m_3\,,\, m_4) = (\,0.017127\,,\, 0.0192\,,\, 0.050703\,,\, 0.909309\,)\ \text{eV} \tag{3.41}$$

the squared mass differences:

$$m_2^2 - m_1^2 = 0.000075 \text{ eV}^2 \quad , \quad m_3^2 - m_2^2 = 0.00220 \text{ eV}^2 \tag{3.42}$$

and the lepton mixing

$$(U_{PMNS})_{4\times 4} = \begin{pmatrix} 0.610887 & -0.786302 & -0.092369 & 0.003816 \\ -0.709911 & -0.595411 & 0.374805 & 0.032066 \\ -0.349473 & -0.164968 & -0.912482 & -0.133926 \\ 0.001821 & 0.000257 & 0.009733 & 0.001377 \end{pmatrix} \tag{3.43}$$

3.8 Conclusions

Within the frame work of a gauged $SU(3)_F$ family symmetry model, we have reported in section 7 a global fit region of the parameter space where this scenario can accommodate a realistic spectrum for the ordinary quark masses and mixing in a non-unitary $(V_{CKM})_{4\times 4}$, for the charged lepton masses and the squared neutrino mass differences, within allowed values reported in PDG 2016 [9].

Simultaneously, some of extra particles introduced in this scenario; horizontal gauge bosons and vector-like fermions are predicted to lie within a few TeV's region, and hence, within current LHC energies.

It is worth to comment that the gauge symmetry $G \equiv SU(3)_F \times G_{SM}$, the fermion content, and the transformation of the scalar fields, all together, avoid Yukawa couplings between SM fermions. So, the scalar fields introduced to break the symmetries in the model: η_2, η_3, Φ^u and Φ^d couple ordinary fermions with their corresponding vector-like fermion U, D, E and N, through the tree level Yukawa couplings. Therefore, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.

3.9 Appendix: Diagonalization of the generic Dirac See-saw mass matrix

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & c \end{pmatrix} \tag{3.44}$$

Using a biunitary transformation $\psi_L^o = V_L^o \, \chi_L$ and $\psi_R^o = V_R^o \, \chi_R$ to diagonalize $\mathcal{M}^o$, the orthogonal matrices V_L^o and V_R^o may be written explicitly as

$$V_L^o = \begin{pmatrix} c_1 & -s_1 \, s_2 & s_1 \, c_2 \, c_\alpha & s_1 \, c_2 \, s_\alpha \\ 0 & c_2 & s_2 \, c_\alpha & s_2 \, s_\alpha \\ -s_1 & -c_1 \, s_2 & c_1 \, c_2 \, c_\alpha & c_1 \, c_2 \, s_\alpha \\ 0 & 0 & -s_\alpha & c_\alpha \end{pmatrix} \tag{3.45}$$

$$V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_r & s_r\, c_\beta & s_r\, s_\beta \\ 0 & -s_r & c_r\, c_\beta & c_r\, s_\beta \\ 0 & 0 & -s_\beta & c_\beta \end{pmatrix} \tag{3.46}$$

$$s_1 = \frac{a_1}{a_n} \quad , \quad c_1 = \frac{a_3}{a_n} \quad , \quad s_2 = \frac{a_2}{a} \quad , \quad c_2 = \frac{a_n}{a} \quad , \quad s_r = \frac{b_2}{b} \quad , \quad c_r = \frac{b_3}{b} \tag{3.47}$$

$$a_n = \sqrt{a_1^2 + a_3^2} \quad , \quad a = \sqrt{a_1^2 + a_2^2 + a_3^2} \quad , \quad b = \sqrt{b_2^2 + b_3^2} \tag{3.48}$$

$$a_1 = a\, s_1\, c_2 \quad , \quad a_2 = a\, s_2 \quad , \quad a_3 = a\, c_1\, c_2 \quad , \quad b_3 = b\, c_r \quad , \quad b_2 = b\, s_r \tag{3.49}$$

$$\lambda_3^2 = \frac{1}{2}\left(B - \sqrt{B^2 - 4D}\right) \quad , \quad \lambda_4^2 = \frac{1}{2}\left(B + \sqrt{B^2 - 4D}\right) \tag{3.50}$$

are the nonzero eigenvalues of $\mathcal{M}^o \mathcal{M}^{oT}$ ($\mathcal{M}^{oT} \mathcal{M}^o$), and

$$B = a^2 + b^2 + c^2 = \lambda_3^2 + \lambda_4^2 \quad , \quad D = a^2 b^2 = \lambda_3^2 \lambda_4^2 \; , \tag{3.51}$$

$$\begin{aligned} \cos\alpha &= \sqrt{\frac{\lambda_4^2 - a^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\alpha = \sqrt{\frac{a^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \, , \\ \cos\beta &= \sqrt{\frac{\lambda_4^2 - b^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\beta = \sqrt{\frac{b^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \, . \end{aligned} \tag{3.52}$$

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4 Nonstandard Cosmologies From Physics Beyond the Standard Model

M.Yu. Khlopov[1,2] ⋆

[1]National Research Nuclear University "MEPHI" (Moscow Engineering Physics Institute), 115409 Moscow, Russia
Centre for Cosmoparticle Physics "Cosmion" 115409 Moscow, Russia
[2]APC laboratory 10, rue Alice Domon et Lonie Duquet 75205 Paris Cedex 13, France

Abstract. The modern cosmology is based on inflationary models with baryosynthesis and dark matter/energy. It implies extension of particle symmetry beyond the Standard model. Studies of physical basis of the modern cosmology combine direct searches for new physics at accelerators with its indirect non-accelerator probes, in which cosmological consequences of particle models play important role. The cosmological consequences of particle models inevitably go beyond the 'standard' cosmological ΛCDM model and some possible feature of such 'nonstandard'cosmological scenarios is the subject of the present brief review.

Povzetek. Kozmološki modeli gradijo na inflacijskih modelih, na bariogenezi in na obstoju temne snovi in temne energije. Vse to zahteva več kot pa ponudi standardni model osnovnih delcev. Kozmološke meritve in meritve na pospeševalnikih skupaj preverjajo kozmološke modele in modele osnovnih delcev. V kratkem pregledu se avtor omeji na nekatere značilnosti 'nestandardnih' kozmoloških modelov.

4.1 Introduction

The now standard ΛCDM cosmological model involving inflation, baryosynthesis and dark matter/energy implies physics beyond the Standard model (BSM) of elementary particles. However, particle models, predicting new physics, can hardly reduce their cosmological consequences to these basic elements of the Standard cosmological model. One can expect that they can give rise to additional model dependent signatures of new physics and to corresponding non-Standard features of the cosmological scenario. Therefore any model that pretends to be a physical basis for the modern cosmology should be studied in more details in order to reveal such non-Standard cosmological features and we discuss in the present brief review some possible signatures for non-Standard cosmological scenarios.

The extension of the Standard model can be developed in either up-down or bottom-up direction. In the first case (as e.g. in the approach of [1]) the overwhelming framework is proposed, which embeds the Standard model and involves new physics beyond it. To be realistic this new physics should provide inflation,

⋆ khlopov@apc.univ-paris7.fr

baryosynthesis and dark matter. To be true it should be falsifiable, predicting model dependent signatures for its probe.

The bottom-up approach, motivated by internal problems and incompleteness of the Standard model, generally involves smaller number of parameters of new physics, than it inevitably takes place in the up-down case. It makes possible to study in more details such models and, in particular, their cosmological and astrophysical impact.

We'd like to consider physical motivations and effects of multi-component dark matter (Section 4.2), as well as of primordial nonlinear structures from phase transitions in early Universe and from nonhomogeneous baryosynthesis, reflected in its extreme form in the existence of antimatter domains and antimatter objects in the baryon asymmetric Universe (Section 4.3). The importance of the account for non-standard cosmological features of physics beyond the Standard model is stressed in the conclusive Section 4.4.

4.2 Multi-component dark matter

Stability of elementary constituents of matter reflects the fundamental symmetry of microworld, which prevents decays of the lightest particles that possess this symmetry. So atoms are stable because electrons cannot decay owing to electric charge conservation, while protons that can in principle decay are very longliving due to the conservation of the baryon number. Physics beyond the Standard model, extending particle symmetry, involves new conservation laws that leads to stability of new forms of matter that can play the role of dark matter candidates.

In the simplest case extension of particle symmetry can lead to only one new conservation law corresponding to a single dark matter candidate. Most popular scenarios assume that this candidate is a Weakly Interacting Massive Particle (WIMP) having strong motivation in the so called "WIMP miracle": the calculated frozen out abundance of WIMPs with mass around several tens - several hundreds GeV provides their contribution in the total density of the Universe that can explain observed dark matter density. The simplest WIMP scenario has an advantage to be checked in the combination of cosmological, astrophysical and physical effects. WIMP annihilation to ordinary particles not only determines their frozen out concentration but also should lead to contribution of energetic products of this annihilation to the fluxes of cosmic rays and cosmic gamma radiation. It supports indirect searches for dark matter following the original idea of [2]. The same process viewed in t-channel corresponds to WIMP scattering on ordinary matter, and viewed from the opposite side to creation of WIMPs in collisions of matter particles. The former motivates direct searches for dark matter, while the latter challenges WIMP searches at accelerators and colliders. WIMP paradigm found physical motivation in supersymmetric models that were considered as the mainstream in studies of physics beyond the Standard model.

However, direct searches for dark matter have controversial results and though their interpretation in the terms of WIMPs is still not ruled out [3], a more general approach to a possible solution of the dark matter problem is appealing. Here we'll discuss some possible nontrivial forms of cosmological dark matter that naturally follow from physics beyond the Standard Model. These examples reflect various features of multi-component dark matter either by compositeness of its species or by co-existence of various dark matter candidates.

4.2.1 Composite dark matter and OHe cosmology

In the same way as the ordinary matter is composed by atoms, which consist of electrically charged electrons and nuclei, bound by Coulomb forces, new electrically charged stable particles may be bound by ordinary Coulomb field in the dark atoms of the dark matter. The electrically charged constituents of dark atoms may be not only elementary particles, but can be composite objects, as are ordinary nuclei and nucleons. The problem of stable electrically charged particles is that bound with electrons, such particles with charges +1 and +2 form anomalous isotopes of hydrogen and helium. In particular, the idea of stable charged particles bound in neutral dark atoms put forward by Sheldon Glashow in his sinister model [4] found the unrecoverable problem of anomalous istotope overproduction, revealed in [5]. It is impossible to realize the dark atom scenario in any model predicting stable +1 and -1 charged species. The former inevitably bind with electrons, forming anomalous hydrogen directly, while the latter bind first with primordial helium in +1 charged ions, which in turn form anomalous hydrogen.

Starting from 2006 the solutions of dark atom scenario were proposed [6–13], in which stable -2 charged species are bound with primordial helium in neutral OHe atoms, which play important catalyzing role in reduction of all the undesirable positively charged heavy species that can give rise to anomalous isotopes. Moreover OHe atoms can be a candidate for composite dark matter, dominating in the matter density of the Universe. Such candidates for dark matter should consist of negatively doubly-charged heavy (with the mass $\sim$ 1 TeV) particles, which are called O^{--}, coupled to primordial helium. Lepton-like technibaryons, technileptons, AC-leptons or clusters of three heavy anti-U-quarks of 4th generation with strongly suppressed hadronic interactions are examples of such O^{--} particles (see [6–8,10–13] for a review and for references). Another direction of composite dark matter scenario is to consider neutral stable heavy quark clusters as it is proposed in the approach of [1]. However, even in this approach heavy stable -2 charged clusters ($\bar{u}_5\bar{u}_5\bar{u}_5$) of stable antiquarks $\bar{u}_5$ of 5th generation can also find their physical basis [9].

As it was qualitatively shown earlier (see [14,15] for the latest review), the transfer function of density perturbations of OHe dark matter has specific Warmer-than-Cold features, reflecting its composite nature and the nuclear cross sections for OHe elastic collisions with nuclei.

The cosmological and astrophysical effects of such composite dark matter (dark atoms of OHe) are dominantly related to the helium shell of OHe and involve only one parameter of new physics $-$ the mass of O^{--}.

If dark matter can bind to normal matter, the observations could come from radiative capture of thermalized OHe and could depend on the detector composition and temperature. In the matter of the underground detector local concentration of OHe is determined by the equilibrium between the infalling cosmic OHe flux and its diffusion towards the center of Earth. Since the infalling flux experiences annual changes due to Earth's rotation around Sun, this local OHe concentration possess annual modulations.

The positive results of the DAMA/NaI and DAMA/LIBRA experiments are then explained by the annual modulations of the rate of radiative capture of OHe by sodium nuclei. Such radiative capture to a low energy OHe-nucleus bound state is possible only for intermediate-mass nuclei: this explains the negative results of the XENON100 and LUX experiments. The rate of this capture can be calculated by the analogy with radiative capture of neutron by proton, taking into account the scalar and isoscalar nature of He nucleus, what makes possible only E1 transition with isospin violation in this process. In the result this rate is proportional to the temperature (to the square of relative velocity in the absence of local thermal equilibrium): this leads to a suppression of this effect in cryogenic detectors, such as CDMS.

The timescale of OHe collisions in the Galaxy exceeds the age of the Universe, what proves that the OHe gas is collisionless. However the rate of such collisions is nonzero and grows in the regions of higher OHe density, particularly in the central part of the Galaxy, where these collisions lead to OHe excitations. De-excitations of OHe with pair production in E0 transitions can explain the excess of the positron-annihilation line, observed by INTEGRAL in the galactic bulge [12,13,16–20]. The calculated rate of collisions and OHe excitation in them strongly depends on OHe density and relative velocity and the explanation of positron excess was found to be very sensitive to the dark matter density in the central part of Galaxy, where baryonic matter dominates and theoretical estimations are very uncertain. The latest analysis of dark matter distribution favors more modest values of dark matter central density, what fixes the explanation of the excess of the positron-annihilation line by OHe collisions and de-excitation in a very narrow range of the mass of O^{--} near 1.25 TeV.

In a two-component dark atom model, based on Walking Technicolor, a sparse WIMP-like component of atom-like state, made of positive and negative doubly charged techniparticles, is present together with the dominant OHe dark atom and the decays of doubly positive charged techniparticles to pairs of same-sign leptons can explain the excess of high-energy cosmic-ray positrons, found in PAMELA and AMS02 experiments [21]. This explanation is possible for the mass of decaying +2 charged particle below 1 TeV and depends on the branching ratios of leptonic channels. Since even pure lepton decay channels are inevitably accompanied by gamma radiation the important constraint on this model follows from the measurement of cosmic gamma ray background in FERMI/LAT experiment. It may be shown that the constraints on this background may be satisfied if the decaying component is distributed in disc and not in halo, what implies more sophisticated self-interacting nature of this component. In fact, a serious problem for any source of cosmic poitrons distributed in halo and not concentrated in the disc.

The crucial problem of OHe scenario is the existence of a dipole barrier in OHe nuclear interaction. The scenario in which such a barrier does not appear was considered in [22] and The over-abundance of anomalous isotopes in terrestrial matter seems to be unavoidable in this case..

This makes the full solution of OHe nuclear physics, started in [23], vital. The answer to the possibility of the creation of a dipole Coulomb barrier in OHe interaction with nuclei is crucial. Indeed, the model cannot work if no repulsive interaction appears at some distance between

The problem of the Earth's shadowing represents another potential problem for OHe scenario. The terrestrial matter is opaque for OHe, what should inevitably lead to an effect of Earth matter shadowing for the OHe flux and corresponding diurnal modulation, constrained in DAMA/LIBRA experiment [24]. The OHe model involves only one parameter of new physics - mass of O^{--} and its cosmological effects are related to nuclear and atomic physics, being within the Standard model, but even in this case many principal features of cosmological consequences remain unclear.

4.2.2 Mirror atoms

Mirror particles, first proposed by T. D. Lee and C. N. Yang in Ref. [26] to restore equivalence of left- and right-handed co-ordinate systems in the presence of P- and C- violation in weak interactions, should be strictly symmetric by their properties to their ordinary twins. After discovery of CP-violation it was shown by I. Yu. Kobzarev, L. B. Okun and I. Ya. Pomeranchuk in Ref. [27] that mirror partners cannot be associated with antiparticles and should represent a new set of symmetric partners for ordinary quarks and leptons with their own strong, electromagnetic and weak mirror interactions. It means that there should exist mirror quarks, bound in mirror nucleons by mirror QCD forces and mirror atoms, in which mirror nuclei are bound with mirror electrons by mirror electromagnetic interaction [28,29]. If gravity is the only common interaction for ordinary and mirror particles, mirror matter can be present in the Universe in the form of elusive mirror objects, having symmetric properties with ordinary astronomical objects (gas, plasma, stars, planets...), but causing only gravitational effects on the ordinary matter [30,31].

Even in the absence of any other common interaction except for gravity, the observational data on primordial helium abundance and upper limits on the local dark matter seem to exclude mirror matter, evolving in the Universe in a fully symmetric way in parallel with the ordinary baryonic matter[32,33]. The symmetry in cosmological evolution of mirror matter can be broken either by initial conditions[34,35], or by breaking mirror symmetry in the sets of particles and their interactions as it takes place in the shadow world[36,37], arising in the heterotic string model. We refer to Refs. [38–40] for current review of mirror matter and its cosmology.

Mirror matter in its fully symmetric implementation doesn't involve new parameters of new physics, since all the parameters of mirror particles and their interactions are by construction strictly equal to the corresponding values of their ordinary partners. However, though there is no common interactions between ordinary and mirror matter except for gravity, just the presence of mirror particles in the same space-time with ordinary matter causes contradictions with observations in a strictly symmetric mirror matter cosmology.

4.2.3 Unstable particles

The next to lightest particle that possess a new conserved charge may be sufficiently longliving to retain some observable trace in the Universe.

Primordial unstable particles with the lifetime, less than the age of the Universe, $\tau < t_U$, can not survive to the present time. But, if their lifetime is sufficiently large to satisfy the condition $\tau \gg (m_{Pl}/m) \cdot (1/m)$, their existence in early Universe can lead to direct or indirect traces[41].

Weakly interacting particles, decaying to invisible modes, can influence Large Scale Structure formation. Such decays prevent formation of the structure, if they take place before the structure is formed. Invisible products of decays after the structure is formed should contribute in the cosmological dark energy. The Unstable Dark matter scenarios[42–50] implied weakly interacting particles that form the structure on the matter dominated stage and then decay to invisible modes after the structure is formed.

Cosmological flux of decay products contributing into the cosmic and gamma ray backgrounds represents the direct trace of unstable particles[41,51]. If the decay products do not survive to the present time their interaction with matter and radiation can cause indirect trace in the light element abundance[52–55] or in the fluctuations of thermal radiation[56].

If the particle lifetime is much less than 1s the multi-step indirect traces are possible, provided that particles dominate in the Universe before their decay. On the dust-like stage of their dominance black hole formation takes place, and the spectrum of such primordial black holes traces the particle properties (mass, frozen concentration, lifetime) [57–59]. The particle decay in the end of dust like stage influences the baryon asymmetry of the Universe. In any way cosmophenomeno-LOGICAL chains link the predicted properties of even unstable new particles to the effects accessible in astronomical observations. Such effects may be important in the analysis of the observational data.

4.3 Primordial cosmological structures

4.3.1 Relics of phase transitions in very early Universe

Parameters of new stable and metastable particles are also determined by a pattern of particle symmetry breaking. This pattern is reflected in a succession of phase transitions in the early Universe. First order phase transitions proceed through bubble nucleation, which can result in black hole formation (see e.g. Refs. [60] and [61] for review and references). Phase transitions of the second order can lead to formation of topological defects, such as walls, string or monopoles. The observational data put severe constraints on magnetic monopole [62] and cosmic wall production [63], as well as on the parameters of cosmic strings [64,65]. Structure of cosmological defects can be changed in succession of phase transitions. More complicated forms like walls-surrounded-by-strings can appear. Such structures can be unstable, but their existence can leave a trace in nonhomogeneous distribution of dark matter and give rise to large scale structures of nonhomogeneous dark matter like *archioles* [66–68]. This effect should be taken into account in the analysis of cosmological effects of weakly interacting slim particles (WISPs) (see Ref. [69] for current review) that can play the role of cold dark matter in spite of their small mass.

A wide class of particle models possesses a symmetry breaking pattern, which can be effectively described by pseudo-Nambu–Goldstone (PNG) field and which corresponds to formation of unstable topological defect structure in the early Universe (see Ref. [61] for review and references). The Nambu–Goldstone nature in such an effective description reflects the spontaneous breaking of global U(1) symmetry, resulting in continuous degeneracy of vacua. The explicit symmetry breaking at smaller energy scale changes this continuous degeneracy by discrete vacuum degeneracy. The character of formed structures is different for phase transitions, taking place on post-inflationary and inflationary stages.

4.3.2 Large scale correlations of axion field

At high temperatures such a symmetry breaking pattern implies the succession of second order phase transitions. In the first transition, continuous degeneracy of vacua leads, at scales exceeding the correlation length, to the formation of topological defects in the form of a string network; in the second phase transition, continuous transitions in space between degenerated vacua form surfaces: domain walls surrounded by strings. This last structure is unstable, but, as was shown in the example of the invisible axion [66–68], it is reflected in the large scale inhomogeneity of distribution of energy density of coherent PNG (axion) field oscillations. This energy density is proportional to the initial value of phase, which acquires dynamical meaning of amplitude of axion field, when axion mass is switched on in the result of the second phase transition.

The value of phase changes by 2π around string. This strong nonhomogeneity of phase leads to corresponding nonhomogeneity of energy density of coherent PNG (axion) field oscillations. Usual argument (see e.g. Ref. [70] and references therein) is essential only on scales, corresponduing to mean distance between strings. This distance is small, being of the order of the scale of cosmological horizon in the period, when PNG field oscillations start. However, since the nonhomogeneity of phase follows the pattern of axion string network this argument misses large scale correlations in the distribution of oscillations' energy density.

Indeed, numerical analysis of string network (see review in the Ref. [71]) indicates that large string loops are strongly suppressed and the fraction of about 80% of string length, corresponding to long loops, remains virtually the same in all large scales. This property is the other side of the well known scale invariant character of string network. Therefore the correlations of energy density should persist on large scales, as it was revealed in Refs. [66–68].

The large scale correlations in topological defects and their imprints in primordial inhomogeneities is the indirect effect of inflation, if phase transitions take place after reheating of the Universe. Inflation provides in this case the equal conditions of phase transition, taking place in causally disconnected regions.

4.3.3 Primordial seeds for Active Galactic Nuclei

If the phase transitions take place on inflational stage new forms of primordial large scale correlations appear. The example of global U(1) symmetry, broken spontaneously in the period of inflation and successively broken explicitly after reheating, was considered in Ref. [73]. In this model, spontaneous U(1) symmetry breaking at inflational stage is induced by the vacuum expectation value $\langle\psi\rangle = f$ of a complex scalar field $\Psi = \psi \exp(i\theta)$, having also explicit symmetry breaking term in its potential $V_{eb} = \Lambda^4(1-\cos\theta)$. The latter is negligible in the period of inflation, if $f \gg \Lambda$, so that there appears a valley relative to values of phase in the field potential in this period. Fluctuations of the phase θ along this valley, being of the order of $\Delta\theta \sim H/(2\pi f)$ (here H is the Hubble parameter at inflational stage) change in the course of inflation its initial value within the regions of smaller size. Owing to such fluctuations, for the fixed value of θ_{60} in the period of inflation with *e-folding* $N = 60$ corresponding to the part of the Universe within the modern cosmological horizon, strong deviations from this value appear at smaller scales, corresponding to later periods of inflation with $N < 60$. If $\theta_{60} < \pi$, the fluctuations can move the value of θ_N to $\theta_N > \pi$ in some regions of the Universe. After reheating, when the Universe cools down to temperature $T = \Lambda$ the phase transition to the true vacuum states, corresponding to the minima of V_{eb} takes place. For $\theta_N < \pi$ the minimum of V_{eb} is reached at $\theta_{vac} = 0$, whereas in the regions with $\theta_N > \pi$ the true vacuum state corresponds to $\theta_{vac} = 2\pi$. For $\theta_{60} < \pi$ in the bulk of the volume within the modern cosmological horizon $\theta_{vac} = 0$. However, within this volume there appear regions with $\theta_{vac} = 2\pi$. These regions are surrounded by massive domain walls, formed at the border between the two vacua. Since regions with $\theta_{vac} = 2\pi$ are confined, the domain walls are closed. After their size equals the horizon, closed walls can collapse into black holes. The minimal mass of such black hole is determined by the condition that it's Schwarzschild radius, $r_g = 2GM/c^2$ exceeds the width of the wall, $l \sim f/\Lambda^2$, and it is given by $M_{min} \sim f(m_{Pl}/\Lambda)^2$. The maximal mass is determined by the mass of the wall, corresponding to the earliest region $\theta_N > \pi$, appeared at inflational stage.

This mechanism can lead to formation of primordial black holes of a whatever large mass (up to the mass of AGNs [74,75], see for latest review Ref. [76]). Such black holes appear in the form of primordial black hole clusters, exhibiting fractal distribution in space [72,77,61]. It can shed new light on the problem of galaxy formation [61,75].

The described mechanism of massive PBH clouds formation may be of special interest for the interpretation of the recently discovered gravitational wave signals from coalescence of massive black hole (BH) binaries [79,80]. It naturally leads to formation of massive BH binaries within such a cloud, while the mass range of PBHs, determined by f and Λ can naturally cover the values of tens of Solar mass.

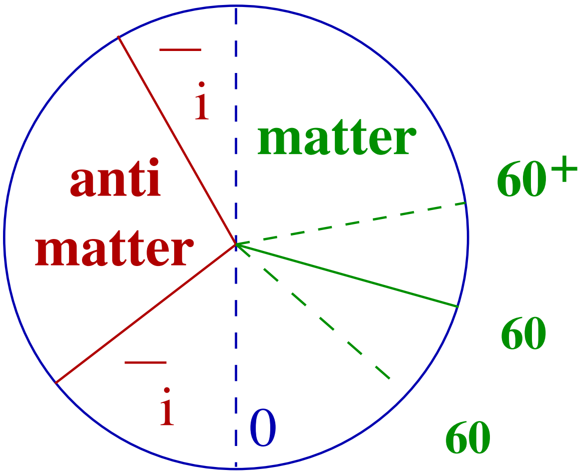

Fig. 4.1. The inflational evolution of the phase (taken from the Ref. [78]). The phase θ_{60} sits in the range $[\pi, 0]$ at the beginning of inflation and makes Brownian step $\delta\theta_{eff} = H_{infl}/(2\pi f_{eff})$ at each e–fold. The typical wavelength of the fluctuation $\delta\theta$ is equal to H_{infl}^{-1}. The whole domain H_{infl}^{-1}, containing phase θ_N gets divided, after one e–fold, into e^3 causally disconnected domains of radius H_{infl}^{-1}. Each new domain contains almost homogeneous phase value $\theta_{N-1} = \theta_N \pm \delta\theta_{eff}$. Every successive e-fold this process repeats in every domain.

4.3.4 Antimatter in Baryon asymmetric Universe?

Primordial strong inhomogeneities can also appear in the baryon charge distribution. The appearance of antibaryon domains in the baryon asymmetrical Universe, reflecting the inhomogeneity of baryosynthesis, is the profound signature of such strong inhomogeneity [81]. On the example of the model of spontaneous baryosynthesis (see Ref. [82] for review) the possibility for existence of antimatter domains, surviving to the present time in inflationary Universe with inhomogeneous baryosynthesis was revealed in [83].

The mechanism of spontaneous baryogenesis [82] implies the existence of a complex scalar field $\chi = (f/\sqrt{2}) \exp{(\theta)}$ carrying the baryonic charge. The $U(1)$ symmetry, which corresponds to the baryon charge, is broken spontaneously and explicitly. The explicit breakdown of $U(1)$ symmetry is caused by the phase-dependent term

$$V(\theta) = \Lambda^4 (1 - \cos\theta). \tag{4.1}$$

The possible baryon and lepton number violating interaction of the field χ with matter fields can have the following structure [82]

$$\mathcal{L} = g\chi\bar{Q}L + \text{h.c.}, \tag{4.2}$$

where fields Q and L represent a heavy quark and lepton, coupled to the ordinary matter fields.

In the early Universe, at a time when the friction term, induced by the Hubble constant, becomes comparable with the angular mass $m_\theta = \frac{\Lambda^2}{f}$, the phase θ starts to oscillate around the minima of the PNG potential and decays into matter fields according to (4.2). The coupling (4.2) gives rise to the following [82]: as the phase starts to roll down in the clockwise direction (Fig. 4.1), it preferentially creates excess of baryons over antibaryons, while the opposite is true as it starts to roll down in the opposite direction.

The fate of such antimatter regions depends on their size. If the physical size of some of them is larger than the critical surviving size $L_c = 8h^2$ kpc [83], they survive annihilation with surrounding matter. Evolution of sufficiently dense antimatter domains can lead to formation of antimatter globular clusters [84]. The existence of such cluster in the halo of our Galaxy should lead to the pollution of the galactic halo by antiprotons. Their annihilation can reproduce [85] the observed galactic gamma background in the range tens-hundreds MeV. The prediction of antihelium component of cosmic rays [86], accessible to future searches for cosmic ray antinuclei in PAMELA and AMS II experiments, as well as of antimatter meteorites [87] provides the direct experimental test for this hypothesis.

So the primordial strong inhomogeneities in the distribution of total, dark matter and baryon density in the Universe is the new important phenomenon of cosmological models, based on particle models with hierarchy of symmetry breaking.

4.4 Conclusions

As soon as physics beyond the Standard model involves new symmetries and mechanisms of their breaking, new model dependent non-standard features of the cosmological scenario should inevitably appear. Even rather restricted list of possible examples of such features, presented here, gives the flavor of new cosmology that can come from the new physics.

The wider is the symmetry group embedding the symmetry of the Standard model of elementary particles, the larger is the list of cosmologically viable predictions that provide various probes for the considered particle model. Entering the corresponding multi-dimensional space of parameters, we simultaneously increase the set of their probes. It makes the set of equations for these parameters over-determined and provides a complete test for however extensive theoretical model.

One can conclude that the account for non-Standard cosmological scenarios in the analysis of the data of precision cosmology extends the space of cosmological parameters and provides nontrivial test for physics beyond the Standard model.

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5 Gauge Fields With Respect to $d = (3+1)$ in the Kaluza-Klein Theories and in the *Spin-charge-family* Theory ⋆

D. Lukman and N.S. Mankoč Borštnik

Department of Physics, FMF, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

Abstract. It is shown that in the *spin-charge-family* theory, as well as in all the Kaluza-Klein like theories, vielbeins and spin connections manifest in $d = (3+1)$ space equivalent vector gauge fields, when space $d \geq 5$ manifests large enough symmetry. The authors demonstrate this equivalence in spaces with the symmetry of metric tensor $g^{\sigma\tau} = \eta^{\sigma\tau} f$, for any scalar function f of the coordinates x^σ, where x^σ denotes coordinates of space out of $d = (3+1)$. Also the connection between vielbeins and scalar fields in $d = (3+1)$ is discussed.

Povzetek. Avtorja pokažeta, da opis s tetradami in opis s spinskimi povezavami vodita v $d = (3+1)$ prostoru do ekvivalentnih vektorskih umeritvenih polj v primeru, ko ima prostor z $d \geq 5$ dovolj veliko simetrijo. To ne velja samo za teorijo *spinov-nabojev-družin*, ampak tudi za vse teorije Kaluza-Kleinovega tipa. Ekvivalenco pokažeta v prostorih, v katerih velja za metrični tenzor $g^{\sigma\tau} = \eta^{\sigma\tau} f$, kjer je f poljubna skalarna funkcija koordinat x^σ, ki sežejo v prostor z dovolj veliko simetrijo. Avtorja obravnavata tudi povezavo med tetradami in skalarnimi polji v prostoru $d = (3+1)$.

5.1 Introduction

We demonstrate in this paper that in spaces with the symmetry of metric tensor $g_{\sigma\tau} = \eta_{\sigma\tau} f^{-2}$, where (x^σ, x^τ) determine the coordinates of the (almost [10]) compactified space, $\eta_{\sigma\tau}$ is the diagonal matrix in this space and f is any scalar function of these coordinates, both procedures - the ordinary Kaluza-Klein one with the vielbeins and the one with the spin connections (related to the vielbeibs, Eq.(5.4), used in the *spin-charge-family* theory ([1–5] and the references therein) - lead in $d = (3+1)$ to the same gauge vector fields. That either the vielbeins or the spin connections represent in symmetric enough spaces in $d = (3+1)$ the same vector gauge fields, is known for a long time [6–8]. This contribution is only to clarify this equivalence.

In the starting action of the *spin-charge-family* theory [2,5,3,4] fermions interact with the vielbeins $f^{\alpha}{}_{a}$ and the two kinds of the spin-connection fields - $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ - the gauge fields of $S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$ and $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$,

⋆ This is the improved version of the 2015 Bled contribution.

respectively:

$$\mathcal{A} = \int d^d x\, E\, \frac{1}{2}\, (\bar{\psi}\, \gamma^a p_{0a} \psi) + \text{h.c.} + \int d^d x\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \tag{5.1}$$

here $p_{0a} = f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\{p_\alpha, E f^{\alpha}{}_a\}_-$, $p_{0\alpha} = p_\alpha - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}$, $R = \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\, (\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\, \omega^c{}_{b\beta})\} + \text{h.c.}$, $\tilde{R} = \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\, (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\, \tilde{\omega}^c{}_{b\beta})\} + \text{h.c.}$. The action introduces two kinds of the Clifford algebra objects, γ^a and $\tilde{\gamma}^a$,

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ \,. \tag{5.2}$$

$f^{\alpha}{}_a$ are vielbeins inverted to $e^a{}_\alpha$, Latin letters $(a, b, ..)$ denote flat indices, Greek letters $(\alpha, \beta, ..)$ are Einstein indices, $(m, n, ..)$ and $(\mu, \nu, ..)$ denote the corresponding indices in $(0, 1, 2, 3)$, $(s, t, ..)$ and $(\sigma, \tau, ..)$ denote corresponding indices in $d \geq 5$:

$$e^a{}_\alpha f^{\beta}{}_a = \delta^\beta_\alpha\,, \quad e^a{}_\alpha f^{\alpha}{}_b = \delta^a_b\,, \tag{5.3}$$

$E = \det(e^a{}_\alpha)$. The action $\mathcal{A}$ offers the explanation for the origin and all the properties of the observed fermions (of the family members and families), of the observed vector gauge fields and the scalar higgs, of the Yukawa couplings, explaining the origin of the matter/anti-matter asymmetry, the appearance of the dark matter and predicts new scalars, new families and a new gauge field ([2,1] and references therein).

The spin connection fields and the vielbeins are related fields and - if there are no spinor (fermion) sources - both kinds of the spin connection fields are expressible with the vielbeins. In Ref. [5] (Eq. (C9)) the expressions relating the spin connection fields of both kinds with the vielbeins and the spinor sources are presented.

We present below the relation among the $\omega_{ab\alpha}$ fields and the vielbeins ([8], Eq. (6.5), where the relation $e^a{}_{,\alpha} + \omega^a{}_{b\alpha}\, e^b{}_\beta - \Gamma^{\alpha'}{}_{\beta\alpha}\, e^a{}_{\alpha'} = 0$ is used), ([3], Eq. (C9)).

$$\begin{aligned}
\omega_{ab}{}^e = {} & \frac{1}{2E}\{e^e{}_\alpha\, \partial_\beta (E f^{\alpha}{}_{[a} f^{\beta}{}_{b]}) - e_{a\alpha}\, \partial_\beta (E f^{\alpha}{}_{[b} f^{\beta e]}) \\
& \qquad - e_{b\alpha} \partial_\beta (E f^{\alpha [e} f^{\beta}{}_{a]})\} \\
& + \frac{1}{4}\{\bar{\Psi}(\gamma^e\, S_{ab} - \gamma_{[a} S_{b]}{}^e)\Psi\} \\
& - \frac{1}{d-2}\{\delta^e_a [\frac{1}{E}\, e^d{}_\alpha \partial_\beta (E f^{\alpha}{}_{[d} f^{\beta}{}_{b]}) + \bar{\Psi}\gamma_d S^d{}_b\, \Psi] \\
& \qquad - \delta^e_b [\frac{1}{E} e^d{}_\alpha \partial_\beta (E f^{\alpha}{}_{[d} f^{\beta}{}_{a]}) + \bar{\Psi}\gamma_d S^d{}_a\, \Psi]\}\,.
\end{aligned} \tag{5.4}$$

When the gauge vector and scalar fields in $d = (3 + 1)$ are studied, originating in $(d - 4)$-dimensional space, the denominator $\frac{1}{d-2}$ must be replaced by $\frac{1}{(d-4)-2}$. One notices that if there are no spinor sources, carrying the spinor quantum numbers S^{ab}, then ω_{abc} is completely determined by the vielbeins (and so is $\tilde{\omega}_{abc}$). Eq. (5.4) manifests that the last terms with δ^e_a and δ^e_b do not contribute when the vector gauge fields $\omega_{st}{}^{m}$, $(s,t) = (5,6,\ldots,d)$ and $m = (0,1,2,3)$, are under consideration.

We demonstrate in this paper that in the spaces with the maximal number of the Killing vectors ([6], p. (331–340)) and with no spinor sources present, the vielbeins $f^{\sigma}{}_{m}$ and the spin connections ω_{stm} are in Kaluza-Klein theories [7,6] related. We find, Eq. 5.14: $f^{\sigma}{}_{m} = -\frac{1}{2}\, E^{\sigma}_{st}\, \omega^{st}{}_{m}(x^{\nu})$. When the vector gauge fields are superposition of the spin connection fields ($A^{Ai}_{m} = \sum_{s,t} c^{Ai}{}_{st}\, \omega^{st}{}_{m}$), the relation among the vielbeins and spin connections are correspondingly: $f^{\sigma}{}_{m} = \sum_{A} \vec{\tau}^{A\sigma}\, \vec{A}^{A}_{m}$, as presented in Eq. (5.22).

Since the vielbeins $f^{\alpha}{}_{a}$ and inverted vielbeins $e^{a}{}_{\alpha}$ (Eq. (5.3)) appear in the metric tensor as a product ($g^{\alpha\beta} = f^{\alpha}{}_{a} f^{\alpha a}, g_{\alpha\beta} = e^{a}{}_{\alpha} e_{a\alpha}$), also tensors of the vector gauge fields appear in $d = (3+1)$ in the curvature $R^{(d)}$ as it is expected (Eqs. (5.24, 5.23)): $R^{(d)} = R^{(4)} + R^{(d-4)} - \frac{1}{4} g_{\sigma\tau} E^{\sigma}{}_{st} E^{\tau}{}_{s't'}\, F^{st}{}_{\mu\nu}\, F^{s't'\mu\nu}$, that is in products.

5.2 Proof that spin connections and vielbeins lead to the same vector gauge fields in $(3+1)$-dimensional space-time

We discuss relations between the spin connections and the vielbeins when there are no spinor sources present and the space in $(d-4)$ demonstrates the desired isometry (keeping the form of the metric in $(d-4)$ space unchanged) in order to prove that both ways, either using the vielbeins or the spin connections, lead to equivalent vector gauge fields in $(3+1)$.

Let the $(d-4)$ space manifest the rotational symmetry, determined by the infinitesimal coordinate transformations of the kind

$$x'^{\mu} = x^{\mu}\,, \quad x'^{\sigma} = x^{\sigma} + \varepsilon^{st}(x^{\mu})\, E^{\sigma}_{st}(x^{\tau}) = x^{\sigma} - i\varepsilon^{st}(x^{\mu})\, M_{st}\, x^{\sigma}\,, \tag{5.5}$$

where $M^{st} = S^{st} + L^{st}$, $L^{st} = x^{s}p^{t} - x^{t}p^{s}$, S^{st} concern internal degrees of freedom of boson and fermion fields, $\{M^{st}, M^{s't'}\}_{-} = i(\eta^{st'}M^{ts'} + \eta^{ts'}M^{st'} - \eta^{ss'}M^{tt'} - \eta^{tt'}M^{ss'})$. From Eq. (5.5) it follows

$$\begin{aligned} -i\, M_{st}\, x^{\sigma} &= E^{\sigma}_{st} = x_{s}\, f^{\sigma}{}_{t} - x_{t}\, f^{\sigma}{}_{s}\,, \\ E^{\sigma}_{st} &= (e_{s\tau}\, f^{\sigma}{}_{t} - e_{t\tau}\, f^{\sigma}{}_{s})x^{\tau}\,, \\ M_{st}{}^{\sigma} &:= iE^{\sigma}_{st}\,, \end{aligned} \tag{5.6}$$

and correspondingly: $M_{st} = E^{\sigma}_{st} p_{\sigma}$. One derives, when taking into account Eq. (5.6) and the commutation relations among generators of the infinitesimal rotations, the equation for the Killing vectors E^{σ}_{st}

$$E^{\sigma}_{st} p_{\sigma} E^{\tau}_{s't'} p_{\tau} - E^{\sigma}_{s't'} p_{\sigma} E^{\tau}_{st} p_{\tau} = -i(\eta_{st'} E^{\tau}_{ts'} + \eta_{ts'} E^{\tau}_{st'} - \eta_{ss'} E^{\tau}_{tt'} - \eta_{tt'} E^{\tau}_{ss'}) p_{\tau}\,, \tag{5.7}$$

and the Killing equation

$$\begin{aligned} D_{\sigma} E_{\tau st} + D_{\tau} E_{\sigma st} &= 0\,, \\ D_{\sigma} E_{\tau st} &= \partial_{\sigma} E_{\tau st} - \Gamma^{\tau'}_{\tau\sigma} E_{\tau' st}\,. \end{aligned} \tag{5.8}$$

Let the corresponding background field ($g_{\alpha\beta} = e^{a}{}_{\alpha}\, e^{a}{}_{\beta}$) be

$$e^{a}{}_{\alpha} = \begin{pmatrix} \delta^{m}{}_{\mu} & e^{m}{}_{\sigma} = 0 \\ e^{s}{}_{\mu} & e^{s}{}_{\sigma} \end{pmatrix}, \quad f^{\alpha}{}_{a} = \begin{pmatrix} \delta^{\mu}{}_{m} & f^{\sigma}{}_{m} \\ 0 = f^{\mu}{}_{s} & f^{\sigma}{}_{s}, \end{pmatrix}, \tag{5.9}$$

so that the background field in d = (3 + 1) is flat. From $e^{a}{}_{\mu} f^{\sigma}{}_{a} = \delta^{\sigma}_{\mu} = 0$ it follows

$$e^{s}{}_{\mu} = -\delta^{m}_{\mu} e^{s}{}_{\sigma} f^{\sigma}{}_{m}\,. \tag{5.10}$$

This leads to

$$g_{\alpha\beta} = \begin{pmatrix} \eta_{mn} + f^{\sigma}{}_{m} f^{\tau}{}_{n} e^{s}{}_{\sigma} e_{s\tau} & -f^{\tau}{}_{m} e^{s}{}_{\tau} e_{s\sigma} \\ -f^{\tau}{}_{n} e^{s}{}_{\tau} e_{s\sigma} & e^{s}{}_{\sigma} e_{s\tau} \end{pmatrix}, \tag{5.11}$$

and

$$g^{\alpha\beta} = \begin{pmatrix} \eta^{mn} & f^{\sigma}{}_{m} \\ f^{\sigma}{}_{m} & f^{\sigma}{}_{s} f^{\tau s} + f^{\sigma}{}_{m} f^{\tau m} \end{pmatrix}. \tag{5.12}$$

We have: $\Gamma^{\tau'}_{\tau\sigma} = \frac{1}{2} g^{\tau'\sigma'}\, (g_{\sigma\sigma'},_{\tau} + g_{\tau\sigma'},_{\sigma} - g_{\sigma\tau},_{\sigma'})$.

One can check properties of $f^{\sigma}{}_{m} \delta^{m}_{\mu}$ under general coordinate transformations $x'^{\mu} = x'^{\mu}(x^{\nu})$, $x'^{\sigma} = x'^{\sigma}(x^{\tau})$ $\left(g'_{\alpha\beta} = \frac{\partial x^{\rho}}{\partial x'^{\alpha}} \frac{\partial x^{\delta}}{\partial x'^{\beta}} g_{\rho\delta} \right)$

$$f'^{\sigma}{}_{m} \delta^{m}_{\mu} = \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\sigma}}{\partial x^{\tau}} f^{\tau}{}_{\nu}\,. \tag{5.13}$$

Let us introduce the field $\Omega^{st}{}_{m}(x^{\nu})$, which depend only on the coordinates in d = (3 + 1), as follows

$$f^{\sigma}{}_{m} := -\frac{1}{2}\, E^{\sigma}_{st}\, \Omega^{st}{}_{m}(x^{\nu})\,, \tag{5.14}$$

with $E^{\sigma}_{st} = -i M_{st}{}^{\sigma}$ defined in Eq. (5.6). From Eqs. (5.13,5.14) follow the transformation properties of $\Omega^{st}{}_{m}$ under the coordinate transformations of Eq. (5.5).

If we look for the transformation properties of the superpositions of the fields Ω_{stm}, let say $\mathcal{A}^{Ai}{}_{m} = c^{Aist} \Omega_{stm}$ which are the gauge fields of τ^{Ai} with the commutation relations $\{\tau^{Ai}, \tau^{Bj}\}_{-} = i\delta^{A}_{B}\, f^{Aijk} \tau^{Ak}$, where $\tau^{Ai} = c^{Ai}{}_{st} M^{st}$, under the coordinate transformations of Eq. (5.5), one finds

$$\delta_0\, A^{Ai}{}_{m} = \varepsilon^{Ai}{}_{,m} + i f^{Aijk} A^{Aj}_{m}\, \varepsilon^{Ak}\,. \tag{5.15}$$

Let us make a choice of $f^{\sigma}{}_{s} = f\delta^{\sigma}_{s}$, which solves the Killing equation (5.8) if f is the scalar function of the coordinates, and has for the inverse vielbeins $e^{s}{}_{\sigma} = f^{-1} \delta^{s}{}_{\sigma}$. and let us put the expression for $f^{\sigma}{}_{m}$, Eq. (5.14), into Eq. (5.4) to see the relation among ω_{stm} and $f^{\sigma}{}_{m}$. One finds

$$\begin{aligned} \omega_{stm} = \frac{1}{2E} \{ & f^{\sigma}{}_{m}\, [e_{t\sigma} \partial_{\tau}(E f^{\tau}{}_{s}) - e_{s\sigma}\, \partial_{\tau}(E f^{\tau}{}_{t})] \\ & + e_{s\sigma} \partial_{\tau}[E(f^{\sigma}{}_{m} f^{\tau}{}_{t} - f^{\tau}{}_{m} f^{\sigma}{}_{t})] - e_{t\sigma} \partial_{\tau}[E(f^{\sigma}{}_{m} f^{\tau}{}_{s} - f^{\tau}{}_{m} f^{\sigma}{}_{s})] \}\,. \end{aligned} \tag{5.16}$$

Since we have assumed that there is no spinor sources present, the source term $\psi^{\dagger} \gamma^{0} \gamma^{m} S_{st} \psi$ is left away.

Using the inverse vielbeins $e^{s}{}_{\sigma} = f^{-1}\delta^{s}{}_{\sigma}$ and $\det(e^{s}{}_{\sigma}) = E = f^{-(d-4)}$, Eq. (5.9), and taking $\Omega_{stm} = \Omega_{stm}(x^{n})$, as assumed above, it follows (after using Eq. (5.14) and recognizing that $f^{\sigma}{}_{m} = -\frac{1}{2}\,(e_{s'\tau'}f^{\sigma}{}_{t'} - e_{t'\tau'}f^{\sigma}{}_{s'})\,x^{\tau'}\,\Omega^{s't'}{}_{m})$

$$\begin{aligned}\omega_{stm} &= \frac{1}{2}\,(\eta_{s\sigma}\,\delta^{\tau}_{t} - \eta_{t\sigma}\,\delta^{\tau}_{s})\,\partial_{\tau}f^{\sigma}_{m}\,,\\ \omega_{stm} &= \Omega_{stm}\,. \end{aligned} \tag{5.17}$$

It is therefore proven for the vielbeins $f^{\sigma}{}_{m} = -\frac{1}{2}\,E^{\sigma}_{st}\,\omega^{st}{}_{m}(x^{\nu})$, Eq. (5.14), where in $d \geq 5$ vielbeins solve the Killing equation (5.8), that the spin connections determine the gauge vector fields in $d = (3+1)$.

Statement: Let the space with $s \geq 5$ *have the symmetry allowing the infinitesimal transformations of the kind*

$$x'^{\mu} = x^{\mu}\,, \quad x'^{\sigma} = x^{\sigma} - i \sum_{A,i,s,t} \varepsilon^{Ai}(x^{\mu})\,c_{Ai}{}^{st}M_{st}\,x^{\sigma}\,, \tag{5.18}$$

then the vielbeins $f^{\sigma}{}_{m}$ in Eq. (5.9) *manifest in* $d = (3+1)$ *the vector gauge fields* $\mathcal{A}^{Ai}_{m}$

$$f^{\sigma}{}_{m} = \sum_{A} \vec{\tau}^{A\sigma}\,\vec{\mathcal{A}}^{A}_{m}\,, \tag{5.19}$$

where

$$\begin{aligned}\tau^{Ai} &= \sum_{st} c^{Ai}{}_{st}\,M^{st}\,,\\ \{\tau^{Ai},\tau^{Bj}\}_{-} &= i f^{Aijk}\tau^{Ak}\,\delta^{AB}\,,\\ \vec{\tau}^{A} &= \vec{\tau}^{A\sigma}\,p_{\sigma} = \vec{\tau}^{A\sigma}{}_{\tau}\,x^{\tau}\,p_{\sigma}\\ \tau^{Ai\sigma} &= \sum_{st} c^{Ai}{}_{st}\,M^{st\sigma} = \sum_{st} c^{Ai}{}_{st}\,(e_{s\tau}\,f^{\sigma}{}_{t} - e_{t\tau}\,f^{\sigma}{}_{s})x^{\tau}\,,\\ \mathcal{A}^{Ai}_{m} &= \sum_{st} c^{Ai}{}_{st}\,\omega^{st}{}_{m}\,. \end{aligned} \tag{5.20}$$

The relation between $\omega^{st}{}_{m}$ and vielbeins is determined by Eq. (5.16), if there are no spinor sources present.

It is not difficult to see that, if using Eq. (5.4) to find the relation between $A^{Ai}_{m} = \sum_{st} c^{Ai}{}_{st}\,\omega^{st}{}_{m}$ and $f^{\sigma}{}_{m}$ of Eq. (5.19), we end up with the relation

$$A^{Ai}_{m} = \mathcal{A}^{Ai}_{m}\,, \tag{5.21}$$

leading to the equation

$$f^{\sigma}{}_{m} = \sum_{A} \vec{\tau}^{A\sigma}\,\vec{A}_{m}\,. \tag{5.22}$$

The Lagrange function for these gauge vector fields follows from the curvature in d dimensional space $R^{\alpha}{}_{\beta\alpha\gamma}g^{\beta\gamma}$, after using Eqs. (5.12, 5.11) in the relation for $\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2}g^{\alpha\delta}\,(g_{\gamma\delta},_{\beta} + g_{\beta\delta},_{\gamma} - g_{\beta\gamma},_{\delta}\,)$ and after taking into account this relation in the Riemann tensor $R^{\alpha}{}_{\beta\gamma\delta} = \Gamma^{\alpha}{}_{\beta[\gamma},_{\delta]} + \Gamma^{\alpha}{}_{\alpha'[\gamma}\Gamma^{\alpha'}{}_{\beta\delta]}$, where $,_{\delta}$ denotes the derivative with respect to $\frac{\partial}{\partial x^\delta}$ and the parentheses require anti symmetrisation of the two indexes.

For a flat four dimensional space ($R^{(4)} = 0$) it follows for the curvature

$$
\begin{aligned}
R^{(d)} &= R^{(d-4)} - \frac{1}{4} g_{\sigma\tau} E^{\sigma}{}_{st} E^{\tau}{}_{s't'}\, F^{st}{}_{\mu\nu}\, F^{s't'\mu\nu}\,, \\
F^{st}{}_{\mu\nu} &= \partial_\mu A^{st}_\nu - \partial_\nu A^{st}_\mu - f^{(st)}{}_{(s't')(s"t")}\, A^{(s't')}_\mu\, A^{(s''t")}_\nu\,, \\
f^{\sigma}{}_m &= -\frac{1}{2} E^{\sigma}{}_{(st)}\, \omega^{st}{}_\mu\, f^{\mu}{}_m\,, \\
E^{\sigma}{}_{(st)} &= M^{st} x^\sigma = (e_{s\tau} f^{\sigma}{}_t - e_{t\tau} f^{\sigma}{}_s) x^\tau\,,
\end{aligned} \tag{5.23}
$$

where $R^{(d-4)}$ determines the curvature in the $(d-4)$ dimensional space. $f^{\sigma}{}_m$ simplifies when $f^{\sigma}{}_s = f\delta^{\sigma}_s$ and $d = (3+1)$ is a flat space to $f^{\sigma}{}_m = \omega^{\sigma}{}_{\tau m}\, x^\tau$.

When the space $(d-4)$ manifests the symmetry of Eq. (5.19)

$$
f^{\sigma}{}_m = \sum_A \vec{\tau}^{A\sigma}\, \vec{A}^A_m
$$

and $d = (3+1)$ is a flat space, the curvature $R^{(d)}$ becomes equal to [6](Eq. (10.41)) [1]

$$
\begin{aligned}
R^{(d)} &= R^{(d-4)} - \frac{1}{4} g_{\sigma\tau} E^{\sigma}{}_{Ai} E^{\tau}{}_{A'i'}\, F^{Ai}{}_{mn} F_{A'i'}{}^{mn}\,, \\
F^{Ai}{}_{mn} &= \partial_m A^{st}_n - \partial_n A^{st}_m - i f^{Aijk}\, A^{Aj}_m\, A^{Ak}_n\,, \\
A^{Ai}_m &= \sum_{st} c^{Ai}{}_{st}\, \omega^{st}{}_m\,, \\
\tau^{Ai} &= \sum_{st} c^{Aist}\, M_{st}\,.
\end{aligned} \tag{5.24}
$$

The integration of the action $\int E\, d^4x d^{(d-4)}x\, R^{(d)}$ over $(d-4)$ space (in which only even functions of the coordinates x^σ give nonzero contributions) leads to the well known effective action for the vector gauge fields in $d = (3+1)$ space: $\int E\, d^4x \{-\frac{1}{4} F^{Ai}{}_{\mu\nu}\, F^{Ai\mu\nu}\}$. All the vector gauge fields are superposition of the spin connection fields: $A^{Ai}_m = \sum_{s,t} c^{Ai}{}_{st}\, \omega^{st}{}_m$.

This completes the proof of the above statement, that the vielbeins $f^{\sigma}{}_m$, $\sigma = (5, 6, \ldots, d)$, $m = (0, 1, 2, 3)$, are expressible with the spin connection fields ω_{stm}, $(s,t) = (5, 6, \ldots, d)$: $f^{\sigma}{}_m = \sum_{A,i,s,t,} \tau^{Ai\sigma}\, c_{Ai}{}^{st}\, \omega_{stm}$.

In the subsection 5.2.1 we demonstrate the connection among the spin connection fields ω_{stm} and the vielbeins $f^{\sigma}{}_m$ when $(d-4)$ space manifests the $SU(2) \times SU(2)$ symmetry. Generalization to any symmetry in a $(d-4)$ space goes in a similar way, leading to the corresponding expressions for the vector gauge fields in $d = (3+1)$.

5.2.1 Vector gauge fields as the superposition of the spin connections

Let us demonstrate the statement that all the vector gauge fields are superposition of the spin connection fields in the case, when the space $SO(7,1)$ breaks into $SO(3,1) \times SU(2) \times SU(2)$.

[1] Ref. [13], Sect. 5.3, deriving the Lagrange function for the gauge fields by using the Clifford algebra space, allows both, the curvature R and its quadratic form R^2, Eq. (240).

One finds the coefficients ${c^{Ai}}_{st}$ for the two $SU(2)$ generators,

$$\tau^{1i} = {c^{1i}}_{st}\, M^{st} \text{ and } \tau^{2i} = {c^{2i}}_{st}\, M^{st}$$

by requiring the commutation relations $\{\tau^{Ai}, \tau^{Bj}\}_- = \delta^{AB} \varepsilon^{ijk}\, \tau^{Ak}$

$$\begin{aligned}
\vec{\tau}^1 &= \frac{1}{2}\,(M^{58} - M^{67}, M^{57} + M^{68}, M^{56} - M^{78}) \\
\vec{\tau}^2 &= \frac{1}{2}\,(M^{58} + M^{67}, M^{57} - M^{68}, M^{56} + M^{78})\,,
\end{aligned} \tag{5.25}$$

while one finds coefficients ${c^{1i}}_{st}$ and ${c^{2i}}_{st}$ for the corresponding gauge fields

$$\begin{aligned}
\vec{A}^1_a &= \frac{1}{2}\,(\omega_{58a} - \omega_{67a}, \omega_{57a} + \omega_{68a}, \omega_{56a} - \omega_{78a}) \\
\vec{A}^2_a &= \frac{1}{2}\,(\omega_{58a} + \omega_{67a}, \omega_{57a} - \omega_{68a}, \omega_{56a} + \omega_{78a})\,,
\end{aligned} \tag{5.26}$$

from the relation

$$\vec{\tau}^A \vec{A}^A_m = M^{st}\omega_{stm}\,. \tag{5.27}$$

Taking into account Eq. (5.5) (Ref. [5], Eq. (11)) one finds

$$\begin{aligned}
\vec{\tau}^1 \quad &= \vec{\tau}^{1\,\sigma}\, p_\sigma = {\vec{\tau}^{1\,\sigma}}_\tau x^\tau\, p_\sigma\,, \\
\vec{\tau}^2 \quad &= \vec{\tau}^{2\sigma}\, p_\sigma = {\vec{\tau}^{2\sigma}}_\tau x^\tau\, p_\sigma\,, \\
{\vec{\tau}^{1\,\sigma}}_\tau &= \frac{1}{2}\,({e^5}_\tau f^{\sigma 8} - {e^8}_\tau f^{\sigma 5} - {e^6}_\tau f^{\sigma 7} + {e^7}_\tau f^{\sigma 6}, \\
&\quad {e^5}_\tau f^{\sigma 7} - {e^7}_\tau f^{\sigma 5} + {e^6}_\tau f^{\sigma 8} - {e^8}_\tau f^{\sigma 6}, \\
&\quad {e^5}_\tau f^{\sigma 6} - {e^6}_\tau f^{\sigma 5} - {e^7}_\tau f^{\sigma 8} + {e^8}_\tau f^{\sigma 7}), \\
{\vec{\tau}^{2\sigma}}_\tau &= \frac{1}{2}\,({e^5}_\tau f^{\sigma 8} - {e^8}_\tau f^{\sigma 5} + {e^6}_\tau f^{\sigma 7} - {e^7}_\tau f^{\sigma 6}, \\
&\quad {e^5}_\tau f^{\sigma 7} - {e^7}_\tau f^{\sigma 5} - {e^6}_\tau f^{\sigma 8} + {e^8}_\tau f^{\sigma 6}, \\
&\quad {e^5}_\tau f^{\sigma 6} - {e^6}_\tau f^{\sigma 5} + {e^7}_\tau f^{\sigma 8} - {e^8}_\tau f^{\sigma 7})\,.
\end{aligned} \tag{5.28}$$

The expressions for ${f^\sigma}_m$ are correspondingly

$${f^\sigma}_m = \;({\vec{\tau}^{1\,\sigma}}_\tau\, \vec{\mathcal{A}}^1_m + {\vec{\tau}^{2\sigma}}_\tau\, \vec{\mathcal{A}}^2_m)\, x^\tau\,. \tag{5.29}$$

Expressing the two $SU(2)$ gauge fields, $\vec{A}^1_m$ and $\vec{A}^2_m$, with ω_{stm} as it is required in Eqs. (5.26), then using for each ω_{stm} the expression presented in Eq. (5.16), in which ${f^\sigma}_m$ is replaced by the relation in Eq. (5.29), then taking for ${f^\sigma}_s = f\delta^\sigma_s$, where f is a scalar function of the coordinates x^σ, $\sigma = (5, 6, \ldots, 8)$ (in this case ${e^s}_\mu = -\delta^m_\mu {e^s}_\sigma {f^\sigma}_m$, Eq. (5.10)), it follows after a longer but straightforward calculation that

$$\begin{aligned}
\vec{A}^1_m &= \vec{\mathcal{A}}^1_m\,, \\
\vec{A}^2_m &= \vec{\mathcal{A}}^2_m\,.
\end{aligned} \tag{5.30}$$

One obtains this result for any component of A^{1i}_m and A^{2i}_m, $i = 1, 2, 3$, separately.

It is not difficult to see that the gauge fields, which are superposition of ω_{stm}, $s, t = (5, 6, \ldots, d)$, demonstrate in $d = (3+1)$ the isometry of the space of $SO(d-4)$, Eq. (5.9), with

$$e^{s}{}_{\sigma} = f^{-1} \begin{pmatrix} 1 & 0 & 0 & \cdots 0 \\ 0 & 1 & 0 & \cdots 0 \\ 0 & 0 & 1 & \cdots 0 \\ & & & \cdots 0 \\ & & & \cdots 0 \\ 0 & 0 & \cdots & 1 \end{pmatrix}, \quad f^{\sigma}{}_{s} = f \begin{pmatrix} 1 & 0 & 0 & \cdots 0 \\ 0 & 1 & 0 & \cdots 0 \\ 0 & 0 & 1 & \cdots 0 \\ & & & \cdots 0 \\ & & & \cdots 0 \\ 0 & 0 & \cdots & 1 \end{pmatrix}. \tag{5.31}$$

The space breaks into $SO(3+1) \times SO(d-4)$ and f is any scalar field of the coordinates:

$$f = f\left(\frac{\sum_{\sigma} (x^{\sigma})^2}{\rho_0^2}\right), \tag{5.32}$$

while ρ_0 is the radius of the $(d-4)$ sphere and

$$f^{\sigma}{}_{m} = \sum_{A} \vec{A}^{A}_{m}\, \vec{\tau}^{A\sigma}{}_{\tau}\, x^{\tau}, \tag{5.33}$$

where $\vec{A}^{A}_{m}$ are the superposition of $\omega^{st}{}_{m}$, $A^{Ai}_{m} = \sum_{s,t} c^{Ai}{}_{st}\, \omega^{st}{}_{m}$, demonstrating the symmetry of the space with $s \geq 5$. This completes the proof of the statement in the section 5.2.

5.3 Relations between vielbeins and spin connections for scalars

We demonstrate in this section the relation between the vielbeins and the spin connections for the scalar gauge fields, again for the case that the space $(d-4)$ demonstrates the isometry presented in Eqs. (5.5 - 5.9) and we make a choice of $f^{\sigma}{}_{s} = f\delta^{\sigma}_{s}$ (which solves the Killing equation (5.8), if f is the scalar function of the coordinates). We do not include fermion sources and we put $f^{\sigma}{}_{m} = 0$.

To find the relation among the vielbeins and spin connections we need to express the curvature $R^{\sigma}{}_{\tau\sigma\tau'}\, g^{\tau\tau'}$ for the $(d-4)$ space, where the Riemann tensor and $\Gamma^{\sigma}{}_{\tau\sigma'}$ for this space are

$$\begin{aligned} R^{\sigma}{}_{\tau\sigma'\tau'} &= \Gamma^{\sigma}{}_{\tau[\tau',\sigma']} + \Gamma^{\sigma}{}_{\tau''[\sigma'}\, \Gamma^{\tau''}{}_{\tau\tau']}, \\ \Gamma^{\sigma}{}_{\tau\sigma'} &= \frac{1}{2}\, g^{\sigma\tau'}\, (g_{\sigma'\tau',\tau} + g_{\tau\tau',\sigma'} - g_{\tau\sigma',\tau'}), \end{aligned} \tag{5.34}$$

in terms of vielbeins $g^{\sigma\tau} = f^{\sigma}{}_{s}\, f^{\tau s}$, which is in our case $g^{\sigma\tau} = f^2\, \eta^{\sigma\tau}$, while $g_{\sigma\tau} = f^{-2}\, \eta_{\sigma\tau}$ ($,_{\delta}$ again denotes the derivative with respect to $\frac{\partial}{\partial x^{\delta}}$ and $_{[\,]}$ the anti symmetrization with respect to particular two indexes) and compare this expression with the corresponding one for the spin connections

$$R = \frac{1}{2} \{ f^{\alpha[a} f^{\beta b]}\, (\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\, \omega^{c}{}_{b\beta}) \} + h.c.\,, \tag{5.35}$$

One finds that $\Gamma^{\sigma}{}_{\tau\sigma'}$ is for $f^{\sigma}{}_{s} = f\,\delta^{\sigma}_{s}$ equal to

$$\Gamma^{\sigma}{}_{\tau\sigma'} = f^{-1}\,(\delta^{\sigma}_{\sigma'}\,f,_{\tau} + \delta^{\sigma}_{\tau}\,f,_{\sigma'} - \eta^{\sigma'\tau}\,f,^{\sigma}),$$

while one finds for $\omega^{st}{}_{s'} = -(f,^{t}\,\delta^{s}_{s'} - f,^{s}\,\delta^{t}_{s'})$ and for $\omega^{st}{}_{\sigma} = \omega^{st}{}_{s'}\,e^{s'}{}_{\sigma} = -f^{-1}(f,^{t}\,\delta^{s}_{\sigma} - f,^{s}\,\delta^{t}_{\sigma})$.

It then follows for $R^{\sigma}{}_{\tau\sigma\tau'}\,g^{\tau\tau'}$, Eq. (5.34), since $\Gamma^{\sigma}{}_{\tau[\tau',\sigma']}\,g^{\tau\tau'} = 2\,(d-4-1)\,(f,_{\tau}\,f,^{\tau} - f\,f,_{\tau},^{\tau})$ and $\Gamma^{\sigma}{}_{\tau''[\sigma'}\,\Gamma^{\tau''}{}_{\tau\tau']}\,g^{\tau\tau'} = (-1+d-4)\,(2-d-4)\,f,_{\tau}\,f,^{\tau}$, that

$$R^{\sigma}{}_{\tau\sigma\tau'}g^{\tau\tau'} = (d-4-1)\{[2-(d-4-2)]\,f,_{\tau}\,f,^{\tau} - 2\cdot f\,f,_{\tau},^{\tau}\}. \tag{5.36}$$

Taking into account Eq. (5.4) we also find the same expression, [namely (Eq. (5.35)), $f^{\sigma[s}f^{\tau t]}\;\omega_{st\sigma,\tau} = 2\,(d-4-1)\,(f,_{\tau}\,f,^{\tau} - f\,f,_{\tau},^{\tau})$ and $f^{\sigma[s}f^{\tau t]}\;(-)\omega_{t's\sigma}\,\omega^{t'}{}_{t\tau} = (-1+d-4)\,(2-d-4)\,f,_{\tau}\,f,^{\tau}]$ for $\frac{1}{2}\{f^{\sigma[s}f^{\tau t]}\;(\omega_{st\sigma,\tau} - \omega_{t's\sigma}\,\omega^{t'}{}_{t\tau})\} + \text{h.c.} = (d-4-1)\{[2-(d-4-2)]\cdot f,_{\tau}\,f,^{\tau} - 2\cdot\, f\,f,_{\tau}^{\;\tau}\}$.

We therefore conclude: If there are no spinor sources present and $f^{\sigma}{}_{s} = \delta^{\sigma}_{s}\,f$, where $f = f(x^{\tau}x_{\tau})$, then

$$R^{\sigma}{}_{\tau\sigma\tau'}g^{\tau\tau'} = \frac{1}{2}\{f^{\sigma[s}f^{\tau t]}\;(\omega^{st}{}_{\tau,\sigma} + \omega_{st'\sigma}\,\omega^{t'}{}_{t\tau})\} + \text{h.c.}\,, \tag{5.37}$$

where

$$\omega^{st}{}_{\sigma} = \omega^{st}{}_{s'}\,e^{s'}{}_{\sigma} = -f^{-1}(f,^{t}\,\delta^{s}_{\sigma} - f,^{s}\,\delta^{t}_{\sigma})\,. \tag{5.38}$$

5.4 Conclusions

We presented the proof, that the vielbeins $f^{\sigma}{}_{m}$ (Einstein index $\sigma \geq 5$, $m = 0,1,2,3$) lead in $d = (3+1)$ to the vector gauge fields, which are the superposition of the spin connection fields ω_{stm}: $f^{\sigma}{}_{m} = \sum_{A}\,\vec{A}^{A}_{m}\,\vec{\tau}^{A\sigma}{}_{\tau}\,x^{\tau}$, with $A^{Ai}_{m} = \sum_{s,t} c^{Ai}{}_{st}\,\omega^{st}{}_{m}$, when the metric in $(d-4)$, $g_{\sigma\tau}$, is invariant under the coordinate transformations $x^{\sigma'} = x^{\sigma} + \sum_{A,i,s,t}\varepsilon^{Ai}\,(x^{m})\,c^{Ai}{}_{st}\,E^{\sigma st}(x^{\tau})$ and $c^{Ai}{}_{st}\,E^{\sigma st} = \tau^{Ai\sigma}$, while $\tau^{Ai\sigma}$ solves the Killing equation (Eq. (5.8)): $D_{\sigma}\,\tau^{Ai}_{\tau} + D_{\tau}\tau^{Ai}_{\sigma} = 0$ ($D_{\sigma}\,\tau^{Ai}_{\tau} = \partial_{\sigma} - \Gamma^{\tau'}_{\tau\sigma}\tau^{Ai}_{\tau'}$).

We demonstrated for the case when $SO(7,1)$ breaks into $SO(3,1)\times SU(2)\times SU(2)$ that $\sum_{A,i}\tau^{Ai}\,A^{Ai}_{m} = \sum_{s,t} M^{st}\,\omega_{stm}$ and that the effective action in $(3+1)$ for the vector gauge fields is $\int\,d^{4}x\{-\frac{1}{4}\,F^{Ai}{}_{\mu\nu}\,F^{Ai\mu\nu}\}$, where $F^{Ai}{}_{mn} = \partial_{m}A^{st}_{n} - \partial_{n}A^{st}_{m} - if^{Aijk}\,A^{Aj}_{m}\,A^{Ak}_{n}$.

The generalization of the break of $SO(13,1)$ into $SO(3,1)\times SU(2)\times SU(2)\times SU(3)\times U(1)$, used in the spin-charge-family theory, goes equivalently. In a general case one has $\sum_{A,i}\tau^{Ai}\,A^{Ai}_{m} = \sum^{*}_{s,t} M^{st}\,\omega_{stm}$, where * means that the summation concerns only those (s,t), which appear in τ^{Ai}.

The proof is true for any f which is a scalar function of the coordinates $x^{\sigma}, \sigma \geq 5$.

We also demonstrated that spin connections and vielbeins are related also for the scalar fields. However, while for the vector gauge fields the (effective) low energy action is in $d = (3+1)$ equal to $\int E\,d^{4}x\{-\frac{1}{4}F^{Ai}{}_{\mu\nu}\,F^{Ai\mu\nu}\}$, where $F^{Ai}{}_{mn} = \partial_{m}A^{st}_{n} - \partial_{n}A^{st}_{m} - if^{Aijk}\,A^{Aj}_{m}\,A^{Ak}_{n}$, $A^{Ai}_{m} = \sum_{st}\,c^{Ai}{}_{st}\,\omega^{st}{}_{m}$ and $\tau^{Ai} =$

$\sum_{st} c^{Aist} M_{st}$, it follows for the scalar fields that $\Gamma^{\sigma}{}_{\tau[\tau',\sigma]} + \Gamma^{\sigma}{}_{\tau''[\sigma} \Gamma^{\tau''}{}_{\tau\tau']} \, g^{\tau\tau'}$ $= \frac{1}{2}\{f^{\sigma[s} f^{\tau t]} \, (\omega^{st}{}_{\tau,\sigma} + \omega_{st'\sigma} \, \omega^{t'}{}_{t\tau})\} + h.c.$.

Similar relation follows also for the superposition of the spin connection fields.

We indeed assume an almost (Ref. [11]) compactified space, which means that there are sources which force $(d-4)$ space to compactify [2].

5.5 Appendix: Derivation of the equality $\vec{A}^1_m = \vec{\mathcal{A}}^1_m$

We demonstrate for the case A^{11}_m, equal to $(\omega_{58m} - \omega_{67m})$, Eq. (5.26), that this A^{11}_m is equal to $\mathcal{A}^{11}_m$, appearing in Eq. (5.29)

$$f^{\sigma}{}_m = \sum_{A,i} \mathcal{A}^{Ai}_m \tau^{Ai\sigma}{}_{\tau} x^{\tau} \,. \tag{5.39}$$

When using Eq. (5.16) for $A^{11}_m = \omega_{58m} - \omega_{67m}$ we end up with the expression

$$\begin{aligned} A^{11}_m = \frac{1}{2E} \Big\{ & f^{\sigma}{}_m [e^8{}_{\sigma} \partial_{\tau}(E f^{\tau 5}) - e^5{}_{\sigma} \partial_{\tau}(E f^{\tau 8})] \\ & - f^{\sigma}{}_m [e^7{}_{\sigma} \partial_{\tau}(E f^{\tau 6}) - e^6{}_{\sigma} \partial_{\tau}(E f^{\tau 7})] \\ & + e^5{}_{\sigma} \partial_{\tau}[E(f^{\sigma}{}_m f^{\tau 8}) - f^{\tau}{}_m f^{\sigma 8}] - e^6{}_{\sigma} \partial_{\tau}[E(f^{\sigma}{}_m f^{\tau 7}) - f^{\tau}{}_m f^{\sigma 7}] \\ & - e^8{}_{\sigma} \partial_{\tau}[E(f^{\sigma}{}_m f^{\tau 5}) - f^{\tau}{}_m f^{\sigma 5}] + e^7{}_{\sigma} \partial_{\tau}[E(f^{\sigma}{}_m f^{\tau 6}) - f^{\tau}{}_m f^{\sigma 6}] \Big\} . \end{aligned} \tag{5.40}$$

We must insert for $f^{\sigma}{}_m$ the expression from Eq. (5.39). We obtain when taking into account Eq. (5.28)

$$A^{11}_m = \partial_8(f_{5m}) - \partial_5(f_{8m}) - \partial_7(f_{6m}) + \partial_6(f_{7m}) \,. \tag{5.41}$$

Inserting Eq. (5.39), in which we take into account Eq. (5.28) as well as that $e^s{}_{\sigma} = f^{-1}\delta^s_{\sigma}$ and $f^{\sigma}{}_s = f\delta^{\sigma}_s$, into Eq. (5.41), we and up with

$$A^{11}{}_m = \sum_{A,i} \mathcal{A}^{Ai}{}_m \delta^A_1 \delta^i_1 \,. \tag{5.42}$$

Similarly one obtains for the gauge fields of both subgroups $SU(2) \times SU(2)$ of the group $SO(4)$

$$A^{Ai}{}_m = \sum_{B,j} \mathcal{A}^{Bj}{}_m \delta^A_B \delta^i_j \,. \tag{5.43}$$

Similar derivations go for any $SO(n)$.

[2] We have shown in Ref. [12,11] that for $f = (1 + \frac{\rho^2}{(2\rho_0)^2})$ the symmetry of the space with the coordinate x^{σ}, $\sigma = (5), (6)$, is a surface S^2,with one point missing.

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6 Spin-charge-family Theory is Offering Next Step in Understanding Elementary Particles and Fields and Correspondingly Universe

N.S. Mankoč Borštnik *

Department of Physics, FMF, University of Ljubljana,
Jadranska 19, SI-1000 Ljubljana, Slovenia

Abstract. More than 40 years ago the *standard model* made a successful new step in understanding properties of fermion and boson fields. Now the next step is needed, which would explain what the *standard model* and the cosmological models just assume: **a.** The origin of quantum numbers of massless one family members. **b.** The origin of families. **c.** The origin of the vector gauge fields. **d.** The origin of the higgses and Yukawa couplings. **e.** The origin of the dark matter. **f.** The origin of the matter-antimatter asymmetry. **g.** The origin of the dark energy. **h.** And several other open problems. The *spin-charge-family* theory, a kind of the Kaluza-Klein theories in $(d = (2n-1)+1)$-space-time, with $d = (13+1)$ and the two kinds of the spin connection fields, which are the gauge fields of the two kinds of the Clifford algebra objects anti-commuting with one another, offers this very much needed next step. The talk presents: **i.** A short presentation of this theory. **ii.** The review over the achievements of this theory so far, with some not published yet achievements included. **iii.** Predictions for future experiments.

Povzetek. Pred več kot 40 leti je *standardni model* omogočil uspešen nov korak v razumevanju lastnosti fermionskih in bozonskih polj. Zdaj pa je potreben nov korak, ki bo pojasnil predpostavke, na katerih gradijo *standardni model* ter kozmološki modeli. Pojasniti je potrebno: **a.** Izvor kvantnih števil brezmasnih članov posamezne družine. **b.** Izvor družin. **c.** Izvor vektorskih umeritvenih polj. **d.** Izvor higgsovega delca in Yukawinih sklopitev. **e.** Izvor temne snovi. **f.** Izvor asimetrije med snovjo in antisnovjo. **g.** Izvor temne energije. **h.** Odprtih vprašanj je še več. Teorija *spinov-nabojev-družin*, ki sodi deloma med Kaluza-Kleinove teorije v $(d = (2n-1)+1)$-prostor-času, z $d = (13+1)$, vendar gradi na dveh vrstah objektov Cliffordove algebre, ki med saboj antikomutirajo, ponuja ta potreben korak. V predavanju avtorica predstavi: **i.** Kratek pregled te teorije. **ii.** Pregled dosedanjih dosežkov, vključno z nekaterimi še neobjavljenimi rezultati. **iii.** Napovedi za prihodnje poskuse.

6.1 Introduction

The *standard model* made a great step in understanding properties of fermion and boson fields by: **i.** Starting with massless fields. **ii.** Assuming quantum numbers of one family of massless quarks and leptons and relating the handedness with

* norma.mankoc@fmf.uni-lj.si

charges. **iii.** Postulating the existence of several families. **iv.** Postulating the existence of the vector gauge fields of the charges of quarks and leptons. **v.** Postulating a simple action for fermions and vector bosons under the requirement of the gauge invariance. **vi.** Postulating the existence of the scalar field, which breaks the weak and the hyper charges of the vacuum what makes fermions and heavy bosons massive. **viii.** Postulating the Yukawa couplings.

Properties of fermions and bosons in the *standard model* are presented and commented in 6.7.

Although the assumptions from **i.- v.** are elegant, in particular the assumption that all the elementary fields are massless gaining masses through the interactions only, as well as the choice of simple actions for fermion and boson massless fields, yet these assumptions need the explanation, why has nature "decided" to make this particular choice of fermions and vector gauge fields and in what steps in the evolution.

The assumption **vi.** that there is the massive scalar field, carrying the charges in the fundamental representations of the groups, while all the other bosons (vector bosons) carry charges in the adjoint representations of the groups, and the assumption that the Yukawa couplings take care of the fermion properties, without explaining the origin of these couplings, do not seem either elegant or simple.

The experiments have confirmed so far the existence of three families of fermions with by the *standard model* required properties, of the vector fields, which are the gauge fields of the charges $SU(3), SU(2)$ and $U(1)$, and of the higgs, all in accordance with the *standard model* assumptions.

To be able to predict the outcome of future experiments the next step beyond the *standard model* is needed.

Just adding several new fields by repeating ideas of the *standard model*, without explaining the assumptions of this so far so successful model, has to my understanding a little chance to be the right step.

The *spin-charge-family* theory [1–15] does explain all the assumptions of the *standard model*: **i.** The charges of the left and of the right handed quarks and leptons of one family - the right handed neutrinos are in this theory regular members of each family - and of their antiquarks and antileptons. **ii.** The appearance and properties of families. **iii.** The appearance and properties of the vector gauge fields of the family members charges. **iv.** The appearance and properties of scalars fields, explaining the properties of the higgses carrying charges in the fundamental representations of the groups and the Yukawa couplings.

The *spin-charge-family* theory is offering also the explanation for the existence of the phenomenas not explained by the *standard model*: **a.** For the dark matter [13]. **b.** For the (ordinary) matter-antimatter asymmetry [2].

This theory predicts: **a.** At the low energy regime two decoupled groups of four families; **a.i.** The fourth [1,5,4,9,12] to the already observed three families of quarks and leptons will be measured at the LHC [14]; **a.ii.** The lowest of the upper four families constitutes the dark matter [13]. **b.** New scalar fields with the weak and the hyper charges of the higgs [1,4], some of them will be measured at the LHC. **c.** New $SU(2)$ vector gauge fields, explaining the appearance of the hyper charge and its $U(1)$ gauge field. **d.** New scalar fields, which are in the fundamental representations of the colour charge (triplets), explaining the ordinary matter-antimatter asymmetry and causing the proton decay.

Within this theory many consequences of the *standard model*, like it is the "miraculous" cancellation of the triangle anomalies, can straightforwardly be explained (Subsect. 6.4.3).

6.2 Short presentation of the *spin-charge-family* theory

This section follows a lot the similar one from Ref. [2]. It briefly presents the *spin-charge-family* theory ([1,2,4,5], and the references therein). The details of the theory will follow in Sect. (6.3).

Let me start with the *assumptions* on which the theory is built. Comments, following the assumptions, will explain the meaning of each of the assumptions.

A i. Fermions (ψ) carry in $d = (13+1)$ as the *internal degrees of freedom only two kinds of spins*, no charges, determined by the *two kinds of the Clifford objects* [1], (γ^a and $\tilde{\gamma}^a$), and *interact correspondingly with the two kinds of the spin connection fields* - $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, (the *gauge fields* of $S^{ab} = \frac{i}{4}\,(\gamma^a\gamma^b - \gamma^b\gamma^a)$, the generators of $SO(13,1)$ and $\tilde{S}^{ab} = \frac{i}{4}\,(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$, the generators of $\widetilde{SO}(13,1)$), and the *vielbeins* $f^{\alpha}{}_{a}$.

$$
\begin{aligned}
\mathcal{A} &= \int d^dx\; E\; \mathcal{L}_f + \\
&\quad \int d^dx\; E\; (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \\
\mathcal{L}_f &= \frac{1}{2}\,(\bar{\psi}\,\gamma^a p_{0a}\psi) + \text{h.c.}\,, \\
p_{0a} &= f^{\alpha}{}_{a} p_{0\alpha} + \frac{1}{2E}\,\{p_{\alpha}, E f^{\alpha}{}_{a}\}_-\,, \\
p_{0\alpha} &= p_{\alpha} - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}\,, \\
R &= \frac{1}{2}\,\{f^{\alpha[a}f^{\beta b]}\;(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^{c}{}_{b\beta})\} + \text{h.c.}\,, \\
\tilde{R} &= \frac{1}{2}\,\{f^{\alpha[a}f^{\beta b]}\;(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\,\tilde{\omega}^{c}{}_{b\beta})\} + \text{h.c.}\,. \qquad (6.1)
\end{aligned}
$$

Here [2] $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$. R and $\tilde{R}$ are the two scalars (R is a curvature).

---

[1] There exist only two kinds of the Clifford algebra objects, connected with the left and right multiplication ([1], Sect. IV. Eq.(28)).

[2] $f^{\alpha}{}_{a}$ are inverted vielbeins to $e^{a}{}_{\alpha}$ with the properties $e^{a}{}_{\alpha}f^{\alpha}{}_{b} = \delta^{a}{}_{b},\; e^{a}{}_{\alpha}f^{\beta}{}_{a} = \delta^{\beta}_{\alpha}$, $E = \det(e^{a}{}_{\alpha})$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

A ii. The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)} \times M^{(6)}$ (which manifests as $SO(7,1) \times SU(3) \times U(1)$), affecting both internal degrees of freedom - the one represented by γ^a and the one represented by $\tilde{\gamma}^a$. Since the left handed (with respect to $M^{(7+1)}$) spinors couple differently to scalar (with respect to $M^{(7+1)}$) fields than the right handed ones, the break can leave massless and mass protected $2^{((7+1)/2-1)}$ families [34]. The rest of families get heavy masses [3].

A iii. There are additional breaks of symmetry: The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)} \times M^{(4)}$.

A iv. There is a scalar condensate (Table 6.1) of two right handed neutrinos with the family quantum numbers of the upper four families, bringing masses of the scale $\propto 10^{16}$ GeV to all the vector and scalar gauge fields, which interact with the condensate [2].

A v. There are the scalar fields with the space index $(7,8)$ carrying the weak (τ^{1i}) and the hyper charges ($Y = \tau^{23} + \tau^4$, τ^{1i} and τ^{2i} are generators of the subgroups of $SO(4)$, τ^4 and τ^{3i} are the generators of $U(1)_2$ and $SU(3)$, respectively, which are subgroups of $SO(6)$), which with their nonzero vacuum expectation values change the properties of the vacuum and break the weak charge and the hyper charge. Interacting with fermions and with the weak and hyper bosons, they bring masses to heavy bosons and to twice four groups of families. Carrying no electromagnetic ($Q = \tau^{13} + Y$) and colour (τ^{3i}) charges and no $SO(3,1)$ spin, the scalar fields leave the electromagnetic, colour and gravity fields in $d = (3+1)$ massless.

*Comments (**C**) on the assumptions* (**A**):

C i. The simple starting action, Eq.(6.1), of the *spin-charge-family* theory leads in the low energy regime - after the breaks of the starting symmetry - to the effective action, which is the *standard model* action with the right handed neutrinos included, what offers the explanation for all the *standard model* assumptions, as well as for the appearance of the families, of the higgs and of the Yukawa couplings:

C i.a. One Weyl (massless) representation of $SO(13,1)$ contains [5,3,4,9,1], if analyzed with respect to the subgroups $SO(3,1)$, $SU(2)_I$, $SU(2)_{II}$, $SU(3)$ and $U(1)$ (Eqs. (A1) - (A6) of Ref. [1]), all the family members and anti-members assumed by the *standard model*, with the right handed neutrinos as the regular members of each family in addition: It contains the left handed weak ($SU(2)_I$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons and right handed weak chargeless and $SU(2)_{II}$ charged coloured quarks and colourless leptons, as well as the right handed weak charged and $SU(2)_{II}$ chargeless anti-coloured (anti-triplet) antiquarks and (anti)colourless antileptons, and left handed weak chargeless and $SU(2)_{II}$ charged antiquarks and antileptons. (The anti-fermion states are reachable

[3] A toy model [34,35] was studied in $d = (5+1)$ with the same action as in Eq. (6.1). The break from $d = (5+1)$ to $d = (3+1) \times$ an almost S^2 was studied. For a particular choice of vielbeins and for a class of spin connection fields the manifold $M^{(5+1)}$ breaks into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families remain massless and mass protected. Equivalent assumption, although not yet proved how does it really work, is made in the $d = (13+1)$ case. This study is in progress.

from the fermion states by the application of the discrete symmetry operator $\mathcal{C}_{\mathcal{N}}$ $\mathcal{P}_{\mathcal{N}}$, presented in Ref. [36].)

C i.b. Before the electroweak break are all observable gauge fields massless: the gravity, the colour octet vector gauge fields (of the group $SU(3)$ from $SO(6)$), the weak triplet vector gauge fields (of the group $SU(2)_{I}$ from $SO(4)$), and the hyper singlet vector gauge field (a superposition of $U(1)$ from $SO(6)$ and the third component of $SU(2)_{II}$ triplet). All are the superposition of the $f^{\alpha}{}_{c}\ \omega_{ab\alpha}$ spinor gauge fields.

C i.c. There are before the electroweak break two decoupled massless groups of four families of quarks and leptons, in the fundamental representations of $\widetilde{SU}(2)_{R,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{II,\widetilde{SO}(4)}$ and $\widetilde{SU}(2)_{L,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{I,\widetilde{SO}(4)}$ groups, respectively - the subgroups of $\widetilde{SO}(3,1)$ and $\widetilde{SO}(4)$ (Table 6.4). These eight families remain massless up to the electroweak break due to the "mass protection mechanism", that is due to the fact that the right handed members have no left handed partners with the same charges.

C i.d. There are scalar fields [2,1] with the space index $(7,8)$ and with respect to the space index with the weak charge and the hyper charge of the Higgs's scalar (Eq. (6.21)). They have additional quantum numbers, belonging either to one of the two groups of two triplets or to three singlets.
One group of the two triplets belong to the family groups $\widetilde{SU}(2)_{R\,\widetilde{SO}(3,1)}$ and $\widetilde{SU}(2)_{II\,\widetilde{SO}(4)}$ and couple to the upper four families. The second group of the two triplets belong to the family groups $\widetilde{SU}(2)_{L\,\widetilde{SO}(3,1)}$ and $\widetilde{SU}(2)_{I\,\widetilde{SO}(4)}$ and couple to the lower four families. All these scalars are the superposition of $f^{\sigma}{}_{s}\ \tilde{\omega}_{ab\sigma}$ [4]. Scalars, belonging to three singlets, are the gauge fields of the charges (Q, Q', Y') [5] and couple to the family members of both groups of families.They are the superposition of $f^{\sigma}{}_{s}\ \omega_{ab\sigma}$ [6]. Both kinds of scalar fields determine the fermion masses (Eq. (6.31)), offering the explanation for the Yukawa couplings and the heavy bosons masses.

C i.e. The starting action contains also additional $SU(2)_{II}$ (from $SO(4)$, Eq. (6.68)) vector gauge fields (one of the components contributes to the hyper charge gauge fields as explained above), as well as the scalar fields with the space index $s \in (5,6)$ and $t \in (9,10,\ldots,14)$. All these fields gain masses of the scale of the condensate (Table 6.1) with which they interact. They all are expressible as superposition of $f^{\mu}{}_{m}\ \tilde{\omega}_{ab\mu}$. In the case of free fields (if no spinor source, carrying their quantum numbers, is present) both $f^{\mu}{}_{m}\ \omega_{ab\mu}$ and $f^{\mu}{}_{m}\ \tilde{\omega}_{ab\mu}$ are expressible

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[4] $\vec{\tilde{A}}^{\tilde{1}}_{s} = (\tilde{\omega}_{\tilde{5}\tilde{8}s} - \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} + \tilde{\omega}_{\tilde{6}\tilde{8}s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} - \tilde{\omega}_{\tilde{7}\tilde{8}s})$, $\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{s} = (\tilde{\omega}_{\tilde{2}\tilde{3}s} + i\,\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} + i\,\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} + i\,\tilde{\omega}_{\tilde{0}\tilde{3}s})$, $\vec{\tilde{A}}^{\tilde{2}}_{s} = (\tilde{\omega}_{\tilde{5}\tilde{8}s} + \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} - \tilde{\omega}_{\tilde{6}\tilde{8}s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} + \tilde{\omega}_{\tilde{7}\tilde{8}s})$ and $\vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{s} = (\tilde{\omega}_{\tilde{2}\tilde{3}s} - i\,\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} - i\,\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} - i\,\tilde{\omega}_{\tilde{0}\tilde{3}s})$, where $(s \in (7,8))$ (Ref. [1], Eq. (A8)).

[5] $Q := \tau^{13} + Y$, $Q' := -Y \tan^2\vartheta_1 + \tau^{13}$, $Q' := -Y \tan^2\vartheta_1 + \tau^{13}$, $Y := \tau^4 + \tau^{23}$, $Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}$, $Q := \tau^{13} + Y$ (Ref. [1], Eq. (A7).

[6] $A^{4}_{s} = g^{Q}\, Q A^{Q}_{s} + g^{Q'}\, Q' A^{Q'}_{s} + g^{Y'}\, Y' A^{Y'}_{s}$, $A^{4}_{s} = -(\omega_{9\,10\,s} + \omega_{11\,12\,s} + \omega_{13\,14\,s})$, $A^{13}_{s} = (\omega_{56s} - \omega_{78s})$, $A^{23}_{s} = (\omega_{56s} + \omega_{78s})$, $A^{Q}_{s} = \sin\vartheta_1\, A^{13}_{s} + \cos\vartheta_1\, A^{Y}_{s}$, $A^{Q'}_{s} = \cos\vartheta_1\, A^{13}_{s} - \sin\vartheta_1\, A^{Y}_{s}$, $A^{Y'}_{s} = \cos\vartheta_2\, A^{23}_{s} - \sin\vartheta_2\, A^{4}_{s}$, with $(s \in (7,8))$ (Re. [1], Eq. (A9)).

with vielbeins, Eq. (C9) in Ref. [1], correspondingly only one kind of the three gauge fields are the propagating fields.

C ii., C iii.: There are many ways of breaking symmetries from $d = (13 + 1)$ to $d = (3 + 1)$. The assumed breaks explain why the weak and the hyper charges are connected with the handedness of spinors, manifesting correspondingly the properties of the family members - the quarks and the leptons, left and right handed (Table 6.2) - and of the vector gauge fields.
Since the left handed members are weak charged while the right handed ones are weak chargeless, the family members remain massless and mass protected up to the electroweak break, when the nonzero vacuum expectation values of the scalar fields with the space index $(7, 8)$ break the weak and the hyper charge symmetry.
Antiparticles are accessible from particles by the application of the operator $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$, as explained in Refs. [36,37]. This discrete symmetry operator does not contain $\tilde{\gamma}^a$'s degrees of freedom. To each family member there corresponds the anti-member, with the same family quantum number.

C iv.: It is the condensate of two right handed neutrinos with the quantum numbers of the upper four families (Table 6.1), which makes massive all the scalar gauge fields (with the index (5, 6, 7, 8), as well as those with the index $(9, \ldots, 14)$) and the vector gauge fields, manifesting nonzero τ^4, τ^{23}, $\tilde{\tau}^4$, $\tilde{\tau}^{23}$, $\tilde{N}^3_R$ [7]. Only the vector gauge fields of Y, $SU(3)$ and $SU(2)_I$ remain massless, since they do not interact with the condensate, its corresponding quantum numbers are zero ($Y = 0$, $\tau^{3i} = 0$ and $\tau^{1i} = 0$).

C v.: At the electroweak break the scalar fields with the space index $s = (7, 8)$ - originating in $\tilde{\omega}_{abs}$ [8], as well as some superposition of $\omega_{s's''s}$ with the quantum numbers (Q, Q', Y') (footnotes in this paper and Ref. [1], Eq. (22)), conserving the electromagnetic charge - change their mutual interaction, and gaining nonzero vacuum expectation values change correspondingly also their masses. They contribute to mass matrices of twice the four families, as well as to the masses of the heavy vector bosons (to the two members of the weak triplet and the superposition of the third member of the weak triplet with the hyper vector field (Ref. ([1], Eqs. (17-20)).

All the rest scalar fields keep masses of the scale of the condensate and are correspondingly unobservable in the low energy regime.

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[7] $\vec{\tau}^1 := \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78})$, $\vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78})$, $\vec{N}_{\pm}(= \vec{N}_{(L,R)}) : \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03})$, $\vec{\tau}^3 := \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}$, $\tau^4 := -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$, $\vec{\tilde{\tau}}^1 := \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78})$, $\vec{\tilde{\tau}}^2 := \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78})$, $\vec{\tilde{N}}_{L,R} := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03})$, $\tilde{\tau}^4 := -\frac{1}{3}(\tilde{S}^{9\,10} + \tilde{S}^{11\,12} + \tilde{S}^{13\,14})$ Ref. ([1], Eqs. (A1-A6)).

[8] $\tilde{A}^{N_L \boxplus}_{\stackrel{78}{(\pm)}} = \tilde{A}^{N_L 1}_{\stackrel{78}{(\pm)}} \boxplus i\, \tilde{A}^{N_L 2}_{\stackrel{78}{(\pm)}}$, $\tilde{A}^{\tilde{N}_L \boxplus}_{\stackrel{78}{(\pm)}} = \{(\tilde{\omega}_{\tilde{2}\tilde{3}\stackrel{78}{(\pm)}} + i\, \tilde{\omega}_{\tilde{0}\tilde{1}\stackrel{78}{(\pm)}}) \boxplus i\, (\tilde{\omega}_{\tilde{3}\tilde{1}\stackrel{78}{(\pm)}} + i\, \tilde{\omega}_{\tilde{0}\tilde{2}\stackrel{78}{(\pm)}})\}$, $\tilde{A}^{\tilde{N}_L 3}_{\stackrel{78}{(\pm)}} = (\tilde{\omega}_{\tilde{1}\tilde{2}\stackrel{78}{(\pm)}} + i\, \tilde{\omega}_{\tilde{0}\tilde{3}\stackrel{78}{(\pm)}})$, $A^{Q}_{\stackrel{78}{(\pm)}} = \omega_{56\stackrel{78}{(\pm)}} - (\omega_{9\,10\stackrel{78}{(\pm)}} + \omega_{11\,12\stackrel{78}{(\pm)}} + \omega_{13\,14\stackrel{78}{(\pm)}})$, with $N^3_L\, \tilde{A}^{\tilde{N}_L \boxplus}_{\stackrel{78}{(\pm)}} = \boxplus\, \tilde{A}^{\tilde{N}_L \boxplus}_{\stackrel{78}{(\pm)}}$, $N^3_L\, \tilde{A}^{\tilde{N}_L 3}_{\stackrel{78}{(\pm)}} = 0$ and $Q\, A^{Q}_{\stackrel{78}{(\pm)}} = 0$ (Ref. [1], Eq. (22)).

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^{4}	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^{4}$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$(\|\nu^{\mathbf{VIII}}_{\mathbf{1R}} >_1 \|\nu^{\mathbf{VIII}}_{\mathbf{2R}} >_2)$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\|\nu^{VIII}_{1R} >_1 \|e^{VIII}_{2R} >_2)$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\|e^{VIII}_{1R} >_1 \|e^{VIII}_{2R} >_2)$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

Table 6.1. This table is taken from [2]. The condensate of the two right handed neutrinos ν_R, with the VIII^{th} family quantum numbers, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The right handed neutrino has $Q = 0 = Y$. The triplet carries $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{\tau}^4 = -1$, $\tilde{N}^3_R = 1$, $\tilde{N}^3_L = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$. The family quantum numbers are presented in Table 6.4.

The fourth family to the observed three ones is *predicted to be observed* at the LHC. Its properties are under consideration [14,15]. The *baryons of the stable family of the upper four families offer the explanation for the dark matter* [13]. The *triplet and anti-triplet scalar fields* contribute together with the condensate to the *matter/anti-matter asymmetry* [2].

6.3 Quarks, leptons and vector and scalar gauge fields in the *spin-charge-family* theory

I shall formally rewrite the part of the action in Eq.(6.1), which determines the spinor degrees of freedom, in the way that we can clearly see that the action does manifest in the low energy regime by the *standard model* required degrees of freedom of the fermions (Table 6.9), of the vector gauge fields (Table 6.10) and of the scalar gauge fields (Table 6.11) [5,3,4,10,1,9,6–8,11–14].

$$\begin{aligned}
\mathcal{L}_f = & \bar{\psi}\gamma^m(p_m - \sum_{A,i} g^{Ai}\tau^{Ai}A^{Ai}_m)\psi + \\
& \{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\ \psi\} + \\
& \{\sum_{t=5,6,9,\dots,14} \bar{\psi}\gamma^t p_{0t}\ \psi\}, \qquad (6.2)
\end{aligned}$$

where $p_{0s} = p_s - \frac{1}{2}S^{s's"}\omega_{s's"s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$, $p_{0t} = p_t - \frac{1}{2}S^{t't"}\omega_{t't"t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt}$, with $m \in (0,1,2,3)$, $s \in (7,8)$, $(s', s") \in (5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within either $(0,1,2,3)$ or $(5,6,7,8)$, t runs $\in (5,\dots,14)$, $(t', t")$ run either $\in (5,6,7,8)$ or $\in (9,10,\dots,14)$. The spinor function ψ represents all family members of all the $2^{\frac{7+1}{2}-1} = 8$ families.

The first line of Eq. (6.2) determines (in $d = (3+1)$) the kinematics and dynamics of spinor (fermion) fields, coupled to the vector gauge fields. The generators τ^{Ai} of the charge groups are expressible in terms of S^{ab} through the complex coefficients ${c^{Ai}}_{ab}$ [9],

$$\tau^{Ai} = \sum_{a,b} {c^{Ai}}_{ab}\, S^{ab}\,, \tag{6.3}$$

fulfilling the commutation relations

$$\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk} \tau^{Ak}\,. \tag{6.4}$$

They represent the colour (τ^{3i}), the weak (τ^{1i}) and the hyper (Y) charges (as well as the $SU(2)_{II}$ (τ^{2i}) and τ^4 charges, the gauge fields of these last two groups gain masses interacting with the condensate, Table 6.1. The condensate leaves massless, besides the colour and gravity gauge fields, only the weak and the hyper charge vector gauge fields). The corresponding vector gauge fields A^{Ai}_m are expressible with the spin connection fields ω_{stm}.

$$A^{Ai}_m = \sum_{s,t} {c^{Ai}}_{st}\, {\omega^{st}}_m\,, \tag{6.5}$$

with (s, t) either in $(5, 6, 7, 8)$ or in $(9, \ldots, 14)$, in agreement with the assumptions **A ii.** and **A iii.**. I demonstrate [1,18] in Subsect. 6.3.2 the equivalence between the usual Kaluza-Klein procedure leading to the vector gauge fields through the vielbeins and the procedure with the spin connections used by the *spin-charge-family* theory.

All the vector gauge fields, appearing in the first line of Eq. (6.2), except $A^{2\pm}_m$ and $A^{Y'}_m$ ($= \cos\vartheta_2\, A^{23}_m - \sin\vartheta_2\, A^{4}_m$, Y' and τ^4 are defined in the footnote [10]), are massless before the electroweak break. $\vec{A}^3_m$ carries the colour charge $SU(3)$ (originating in $SO(6)$), $\vec{A}^1_m$ carries the weak charge $SU(2)_I$ ($SU(2)_I$ and $SU(2)_{II}$ are the subgroups of $SO(4)$) and A^Y_m ($= \sin\vartheta_2\, A^{23}_m + \cos\vartheta_2\, A^4_m$) carries the corresponding $U(1)$ charge ($Y = \tau^{23} + \tau^4$, τ^4 originates in $SO(6)$ and τ^{23} is the third component of the second $SU(2)_{II}$ group, A^4_m and $\vec{A}^2_m$ are the corresponding vector gauge fields). The fields $A^{2\pm}_m$ and $A^{Y'}_m$ get masses of the order of the condensate scale through the interaction with the condensate of the two right handed neutrinos with the quantum numbers of one of the group of four families (the assumption **A iv.**, Table 6.1). (See Ref. [1].)

6.3.1 Quarks and leptons in the *spin-charge-family* theory

To offer the explanation for the origin of quantum numbers of one (anyone) family of massless quarks and leptons, assumed by the *standard model* Table(6.9), the

[9] $\vec{\tau}^1 := \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78})\,, \vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78})$, $\vec{\tau}^3 := \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}$, $\tau^4 := -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$. After the electroweak break the nonconserved charges manifest $Y := \tau^4 + \tau^{23}$, $Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}$, $Q' := -Y \tan^2\vartheta_1 + \tau^{13}$, θ_1 is the electroweak angle, breaking the weak $SU(2)_I$ and the hyper charge, θ_2 is the angle of the break of $SU(2)_{II}$ from $SU(2)_I \times SU(2)_{II}$. $Q := \tau^{13} + Y$ remains the conserved charge.

[10] $Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}$, $\tau^4 = -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$.

spin-charge-family theory must answer the question, where do the *standard model* charges originate and why are the weak and the hyper charge connecting with the spin of quarks and leptons. The theory must answer also the question: Where do families of quarks and leptons originate?

This section demonstrates that spinors, which carry in $d = (13+1)$ nothing but spins of two kinds, determined by the two kinds of the Clifford algebra objects ([1], Sect. IV., App. B), explain the origin of spins and charges and of the families: One kind of spins manifests in $d-(3+1)$ at low energies the spin and all the charges, connecting the spin (the handedness) and the charges [6–8,11,31–33]. The second kind explains the origin of families.

To explain the appearance of the electroweak break, which causes that all the families become massive, the properties of the scalar fields must be explained. This is done in Subsects. 6.3.3, where also the appearance of the Yukawa couplings is explained, while masses of the two groups of four families, predicted by the *spin-charge-family* theory, will be discussed in Subsect. 6.4.1.

In Table 6.2 one Weyl representation of spinors in $d = (13+1)$ is presented. The *technique* [6,7,31,32], 6.6, is used, which makes that the states themselves demonstrate properties of spinors. Besides the states also the quantum numbers of the members are presented with respect to the groups $SO(3,1)$, $SU(2)_I$, $SU(2)_{II}$, $U(1)_{II}$ and $SU(3)$, which are the subgroups of the group $SO(13,1)$. One easily sees that the states of one Weyl representation include all the quarks and the leptons, and the antiquarks and the antileptons of one family of quarks and leptons, with just the quantum as assumed by the *standard model*, Table 6.9.

Table 6.2 demonstrates that left handed quarks and leptons carry the weak charge ($SU(2)_I$ with τ^{1i} as generators) and the hyper charge ($Y = \tau^{23} + \tau^4$, τ^{2i} are generators of $SU(2)_{II}$, which is a subgroup of $SO(4)$, τ^4 are generators of $U(1)_{II}$, which is a subgroup of $SO(6)$) just as required by the *standard model*, Table 6.9, while the right handed quarks and leptons are weak chargeless, carrying the $SU(2)_{II}$ charge with τ^{2i} as generators, determining the hyper charges of quarks and leptons, again in agreement with the *standard model*.

Quarks carry the colour charge in the fundamental representation and the "fermion charge" $\tau^4 = \frac{1}{6}$, leptons are colourless carrying the "fermion charge" $\tau^4 = -\frac{1}{2}$. Correspondingly is the hyper charge of either left handed or right handed quarks and leptons in agreement with the *standard model* assumptions.

Table 6.2 demonstrates that left handed antiquarks and antileptons are weak chargeless and the right handed antiquarks and antileptons are weak charged. Antiquarks carry antitriplet charges, while leptons are anticolourless. Correspondingly fermions and anti-fermions carry opposite colour, hyper, electromagnetic and "fermion" charges than fermions.

Handedness is in the one Weyl representation of $SO(13,1)$ strongly related to the weak charge and the hyper charge.

Left and right handed neutrinos (carrying nonzero $Y' = -\tau^4 \tan^2 \vartheta_2 + \tau^{23}$ quantum number) are the regular members of each family, and so are the antineutrinos.

i	$\vert {}^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
	(Anti)octet, $\Gamma^{(1,7)} = (-1)\,1$, $\Gamma^{(6)} = (1)\,-1$ of (anti)quarks and (anti)leptons										
1 u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
2 u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
3 d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
4 d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
5 d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
6 d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
7 u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
8 u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
9 u_R^{c2}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
10 u_R^{c2}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...											
17 u_R^{c3}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
18 u_R^{c3}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...											
25 ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
26 ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
27 e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
28 e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
29 e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
30 e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
31 ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
32 ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0

Table 6.2. The left handed ($\Gamma^{(13,1)} = -1$) ($= \Gamma^{(7,1)} \times \Gamma^{(6)}$) multiplet of spinors - the members of the $SO(13,1)$ group representation, manifesting the subgroup $SO(7,1)$ - of the colour charged quarks and anti-quarks and the colourless leptons and antileptons, is presented in the massless basis using the technique presented in 6.6. It contains the left handed ($\Gamma^{(3,1)} = -1$) weak charged ($\tau^{13} = \pm\frac{1}{2}$) and $SU(2)_{II}$ chargeless ($\tau^{23} = 0$) quarks and the right handed weak chargeless and $SU(2)_{II}$ charged ($\tau^{23} = \pm\frac{1}{2}$) quarks of three colours (c^i $= (\tau^{33}, \tau^{38})$) with the "spinor" charge ($\tau^4 = \frac{1}{6}$) and the colourless left handed weak charged and right handed weak chargeless leptons with the "spinor" charge ($\tau^4 = -\frac{1}{2}$). S^{12} defines the ordinary spin $\pm\frac{1}{2}$. It contains also the corresponding anti-states with opposite charges, reachable from the particle states by the application of the discrete symmetry operator $\mathcal{C}_{\mathcal{N}}$ $\mathcal{P}_{\mathcal{N}}$, presented in Refs. [36,37]. The vacuum state, on which the nilpotents and projectors operate, is not shown. The reader can find this Weyl representation also in Refs. [2,10,1]. Table is separated into two parts.

Let me pay attention to the reader, that the term $\gamma^0 \stackrel{78}{(-)} \tau^{Ai} A^{Ai}_{\stackrel{78}{(-)}}$, where τ^{Ai} $A^{Ai}_{\stackrel{78}{(-)}}$ represent the superposition of either the scalar fields ω_{abs} or the scalar fields $\tilde{\omega}_{abs}$, $s \in (7,8)$, as presented in Subsect. 6.3.3, Eq. (6.20), transforms the right handed u_R^{c1} quark from the first line of Tables 6.2–6.3 into the left handed u_L^{c1}

i	$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^{4}	Y	Q
	(Anti)octet, $\Gamma^{(1,7)} = (-1)\,1\,,\ \Gamma^{(6)} = (1)\,-1$ of (anti)quarks and (anti)leptons										
33 $\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
34 $\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
35 $\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
36 $\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
37 $\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
38 $\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
39 $\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
40 $\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
41 $\bar{d}_L^{\bar{c}2}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
…											
49 $\bar{d}_L^{\bar{c}3}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
…											
57 $\bar{e}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
58 $\bar{e}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
59 $\bar{\nu}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
60 $\bar{\nu}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
61 $\bar{\nu}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
62 $\bar{\nu}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
63 $\bar{e}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1
64 $\bar{e}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Table 6.3. Continuation of Table 6.2

quark from the seventh line of the same table [11], which can, due to the properties of the scalar fields (Eq. (6.21)), be interpreted also in the *standard model* way, namely, that $A^{Ai}_{\stackrel{78}{(-)}}$ "dress" u^{c1}_R giving it the weak and the hyper charge of the left handed u^{c1}_L quark, while γ^0 changes handedness. Equivalently happens to ν_R from the 25^{th} line, which transforms under the action of $\gamma^0\ \stackrel{78}{(-)}\ \tau^{Ai}\ A^{Ai}_{\stackrel{78}{(-)}}$, into ν_L from the 31^{th} line.

The operator $\gamma^0\ \stackrel{78}{(+)}\ \tau^{Ai}\ A^{Ai}_{\stackrel{78}{(+)}}$ transforms d^{c1}_R from the third line of Tables 6.2–6.3 into d^{c1}_L from the fifth line of this table, or e_R from the 27^{th} line into e_L from the 29^{th} line, where $A^{Ai}_{\stackrel{78}{(+)}}$ belong to the scalar fields from Eq. (6.19).

The operator τ^{Ai}, if representing the first three operators in Eq. (6.19), (only) multiplies the right handed family member with its eigenvalue.

The term $\gamma^0\ \stackrel{78}{(\mp)}\ \tau^{Ai}\ A^{Ai}_{\stackrel{78}{(\mp)}}$ of the action (Eqs. (6.1, 6.2)) determines the Yukawa couplings (6.3.3, 6.4.1).

Since spinors (fermions) carry besides the family members quantum numbers also the family quantum numbers, determined by $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$, there

[11] This transformation of the right handed family members into the corresponding left handed partners can easily be calculated by using Eqs. (6.59, 6.57, 6.73).

are correspondingly $2^{(7+1)/2-1} = 8$ families [1], which split into two groups of families, each manifesting the $(\widetilde{SU}(2)_{\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)} \times U(1))$ symmetry.

The eight families of the right handed u_{1R} quark (the first member of the eight-plet of quarks from Tables 6.2 – 6.3) and of the right handed ν_{1R} leptons (the first member of the eight-plet of leptons from Tables 6.2 – 6.3) are presented as an example in Table 6.4 [4].

All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight families separate into two groups of four families: One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$.Another group of four families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

If τ^{Ai} represents the last four operators of Eq. (6.19) in Subsect. 6.3.3, the operators $\gamma^0 \stackrel{78}{(\mp)} \tau^{Ai} A^{Ai}_{\stackrel{78}{(\mp)}}$ ($(\mp)$ for (u_R, ν_R) and (d_R, e_R), respectively) transform the right handed family member of one family into the left handed partner of another family within the same group of four families, since these four operators manifest the symmetry twice $(\widetilde{SU}(2)_{\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)})$. One group of four families carries the family quantum numbers ($\vec{\tilde{\tau}}^1$, $\vec{\tilde{N}}_L$), the other group of four families carries the family quantum numbers ($\vec{\tilde{\tau}}^2$, $\vec{\tilde{N}}_R$).

The contribution of the scalar fields to masses of fermions and to the Yukawa couplings will be discussed in Subsects. (6.4.1, 6.3.3), respectively.

At each break of symmetry fermions can gain masses [38] of the order of the scale of the condensate. In the Refs. [34,35] we discuss possible conditions under which fermion remain massless at the break. This discussion concerns in our case the break of $d = (2(2n+1)-1, 1)$, for $n = 9$ or larger, down to $d = (13+1)$ and from $d = (13+1)$ to $d = (7+1)$ before the symmetry between spinors and antispinors is broken. After that the massless fermions are mass protected, since the left handed and right handed members differ in the weak and hyper charges, until the weak and the hyper charges are no longer conserved quantum numbers.

Let me point out here that there are scalar fields, the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, which couple only to the four families which are doublets with respect to these two groups, while the scalar fields, which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, couple only to the four families which are doublets with respect to these last two groups. Each of the two kinds of scalar contribute after the electroweak transition to their own group of four families, while the scalar gauge fields of (Q, Q', Y') couple to family members of all eight families.

6.3.2 Vector gauge fields in the *spin-charge-family* theory

In the starting action of Eq. (6.1) of the *spin-charge-family* theory all the gauge fields are the gravitational ones: the vielbeins and the spin connections of two kinds. If there are no spinor sources present are both kinds of the spin connection

					$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$	$\tilde{\tau}^4$
I	$u^{c1}_{R\,1}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	$\nu_{R\,2}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	$\nu_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	$\nu_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	$\nu_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
II	$u^{c1}_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	$\nu_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	$\nu_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	$\nu_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	$\nu_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\,\|\,\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$

Table 6.4. Eight families of the right handed u^{c1}_R (6.2–6.3) quark with spin $\frac{1}{2}$, the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, and of the colourless right handed neutrino ν_R of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$), the other (II) is a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a doublet with with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$). All the families of each of the two groups follow from the starting one by the application of one of the two operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$), Eq. (6.73), respectively. The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (6.73)) transform u_{1R} of one family to all the members of the same family of the same colour. The same generators transform equivalently the right handed neutrino ν_{1R} to all the colourless members of the same family.

fields uniquely determined by the vielbeins (Ref. [1], Eqs. ((30)-(32), (C9))). Spinors (fermions) interact with the vielbeins and the two kinds of the spin connection fields. After the break of the starting symmetry $SO(13,1)$ the starting action manifests at low energies the effective action. Eq. (6.2) represents the effective action for fermions in $d=(3+1)$ interacting with the vector gauge fields, which are the superposition of the spin connection gauge fields with the vector index m ($m=(0,1,2,3)$) - $A^{Ai}_{m}=\sum_{s,t} c^{Aist}\,\omega_{stm}$ - and the scalar gauge fields, which are the superposition of the spin connection gauge fields of both kinds ω_{abs}'s and $\tilde{\omega}_{abs}$'s with the scalar index s ($s\ge 5$) - $A^{(Q,Q',Y')}_{s}=\sum_{s,t} c^{(Q,Q",Y')st}\,\omega_{stm}$ (for (Q,Q',Y'), respectively), $\tilde{A}^{\tilde{A}i)}_{s}=\sum_{s,t}\tilde{c}^{\tilde{A}ist}\,\tilde{\omega}_{stm}$.

I comment in this section that the vector gauge fields in the *spin-charge-family* theory appear equivalently either from vielbeins $f^{\sigma}{}_{m}$ - like it is usually proceeded in the Kaluza-Klein-like theories [40,39] (with which the *spin-charge-family* theory has many things in common) - or from the spin connection fields $\sum_{s,t} c^{Ai}{}_{st}\,\omega^{st}{}_{m}$. This is indeed known for a long time [39–41].

This section reviews Refs. [1,18,42], where this equivalence is demonstrated when the spaces of $d\ge 5$ have the metric tensor $g_{\sigma\tau}=\eta_{\sigma\tau}\,f^{-1}$, where (x^{σ}, x^{τ}) determine the coordinates of the (almost [34]) compactified space, $\eta_{\sigma\tau}$ is the diagonal matrix in this space and f is any scalar function of these coordinates.

Let the space with $s\ge 5$ have the symmetry allowing the infinitesimal transformations of the kind

$$x'^{\mu}=x^{\mu}\,,\quad x'^{\sigma}=x^{\sigma}-i\sum_{A,i,s,t}\varepsilon^{Ai}(x^{\mu})\,C_{Ai}{}^{st}M_{st}\,x^{\sigma}\,, \tag{6.6}$$

where $M^{st}=S^{st}+L^{st}$, $L^{st}=x^{s}p^{t}-x^{t}p^{s}$, S^{st} concern internal degrees of freedom of boson and fermion fields, $\{M^{st},M^{s't'}\}_{-}=i(\eta^{st'}M^{ts'}+\eta^{ts'}M^{st'}-\eta^{ss'}M^{tt'}-\eta^{tt'}M^{ss'})$. From Eq. (6.6) it follows

$$\begin{aligned}
-i\sum_{s,t} C_{Ai}{}^{st}M_{st}\,x^{\sigma}=E^{\sigma}_{Ai}=\sum_{s,t} C_{Ai}{}^{st}\,(x_{s}\,f^{\sigma}{}_{t}-x_{t}\,f^{\sigma}{}_{s})\,,\\
\sum_{s,t} C_{Ai}{}^{st}\,M_{st}{}^{\sigma}:=iE^{\sigma}_{Ai}\,,
\end{aligned} \tag{6.7}$$

and correspondingly: $\tau_{Ai}=E^{\sigma}_{Ai}p_{\sigma}$, where $\tau^{Ai}=\sum_{s,t} C_{Ai}{}^{st}\,M_{st}$ with the commutation relations $\{\tau_{Ai},\tau_{Bj}\}_{-}=i\delta^{AB}\,f^{Aijk}\,\tau_{Ak}$, f^{Aijk} are the structure constants of the symmetry group A. One derives, when taking into account Eq. (6.7) and the commutation relations among generators of the infinitesimal transformations τ_{Ai} the equation for the Killing vectors E^{σ}_{Ai} [39]

$$E^{\sigma}_{Ai}p_{\sigma}E^{\tau}_{Bj}p_{\tau}-E^{\sigma}_{Bj}p_{\sigma}E^{\tau}_{Ai}p_{\tau}=i\delta^{AB}\,f^{Aijk}\,E^{\tau}_{Ak}p_{\tau}\,, \tag{6.8}$$

and the Killing equation

$$\begin{aligned}
D_{\sigma}E_{\tau Ai}+D_{\tau}E_{\sigma Bj}=0\,,\\
D_{\sigma}E_{\tau Ai}=\partial_{\sigma}E_{\tau Ai}-\Gamma^{\tau'}_{\tau\sigma}E_{\tau' Ai}\,.
\end{aligned} \tag{6.9}$$

Let the corresponding background field ($g_{\alpha\beta}=e^{a}{}_{\alpha}\,e^{a}{}_{\beta}$) be

$$e^{a}{}_{\alpha}=\begin{pmatrix}\delta^{m}{}_{\mu} & e^{m}{}_{\sigma}=0\\ e^{s}{}_{\mu} & e^{s}{}_{\sigma}\end{pmatrix}\,,\quad f^{\alpha}{}_{a}=\begin{pmatrix}\delta^{\mu}{}_{m} & f^{\sigma}{}_{m}\\ 0=f^{\mu}{}_{s} & f^{\sigma}{}_{s}\,,\end{pmatrix}\,, \tag{6.10}$$

so that the background field in $d = (3+1)$ is flat. From $e^{a}{}_{\mu} f^{\sigma}{}_{a} = \delta^{\sigma}_{\mu} = 0$ it follows

$$e^{s}{}_{\mu} = -\delta^{m}_{\mu} e^{s}{}_{\sigma} f^{\sigma}{}_{m} . \tag{6.11}$$

This leads to

$$g_{\alpha\beta} = \begin{pmatrix} \eta_{mn} + f^{\sigma}{}_{m} f^{\tau}{}_{n} e^{s}{}_{\sigma} e_{s\tau} & -f^{\tau}{}_{m} e^{s}{}_{\tau} e_{s\sigma} \\ -f^{\tau}{}_{n} e^{s}{}_{\tau} e_{s\sigma} & e^{s}{}_{\sigma} e_{s\tau} \end{pmatrix} , \tag{6.12}$$

and

$$g^{\alpha\beta} = \begin{pmatrix} \eta^{mn} & f^{\sigma}{}_{m} \\ f^{\sigma}{}_{m} & f^{\sigma}{}_{s} f^{\tau s} + f^{\sigma}{}_{m} f^{\tau m} \end{pmatrix} . \tag{6.13}$$

We have: $\Gamma^{\tau'}_{\tau\sigma} = \frac{1}{2} g^{\tau'\sigma'} (g_{\sigma\sigma'},_{\tau} + g_{\tau\sigma'},_{\sigma} - g_{\sigma\tau},_{\sigma'})$.

Let us make a choice for the vielbein

$$f^{\sigma}{}_{m} = \sum_{A} \vec{\tau}^{A\sigma} \vec{\mathcal{A}}^{A}_{m} , \tag{6.14}$$

where we expect $\vec{\mathcal{A}}^{A}_{m}$ that they manifest in $d = (3+1)$ as the gauge fields of the charges τ^{Ai}. To prove this we must compare the gauge fields $A^{A}_{m} = C_{Ai}{}^{st} \omega_{stm}$, appearing in Eq. (6.2), with the gauge fields $\vec{\mathcal{A}}^{A}_{m}$.

If there are no fermions present then the vector gauge fields of the family members charges and of the family charges - ω_{abm} and $\tilde{\omega}_{abm}$, respectively - are uniquely expressible with the vielbeins [2,1,42]. We are interested in the vector gauge fields in $d = (3+1)$, for which we find

$$\begin{aligned} \omega_{stm} = \frac{1}{2E} \{ & f^{\sigma}{}_{m} \, [e_{t\sigma} \partial_{\tau} (E f^{\tau}{}_{s}) - e_{s\sigma} \, \partial_{\tau} (E f^{\tau}{}_{t})] \\ & + e_{s\sigma} \partial_{\tau} [E (f^{\sigma}{}_{m} f^{\tau}{}_{t} - f^{\tau}{}_{m} f^{\sigma}{}_{t})] - e_{t\sigma} \partial_{\tau} [E (f^{\sigma}{}_{m} f^{\tau}{}_{s} - f^{\tau}{}_{m} f^{\sigma}{}_{s})] \} . \end{aligned} \tag{6.15}$$

We must show that if we calculate $C_{Ai}{}^{st} \omega_{stm}$ by taking for ω_{stm} from Eq. (6.15) and we put on the right hand side of this equation the vielbeins $f^{\sigma}{}_{m} = \sum_{A} \vec{\tau}^{A\sigma} \vec{\mathcal{A}}^{A}_{m}$ from Eq. (6.14), we must end up with the equality relation

$$A^{Ai}_{m} = \mathcal{A}^{Ai}_{m} . \tag{6.16}$$

It is not difficult to check that Eq. (6.16) follows, if we take into account that $e^{s}{}_{\mu} = -\delta^{m}_{\mu}$ and make a choice of the symmetry of space $(d-4)$: $f^{\sigma}{}_{s} = f \, \delta^{\sigma}_{s}$ (Ref. [1], Sect. II.).

Calculating from Eqs. (6.12, 6.13) the Riemann curvature $R^{(d)}$ in d-dimensional space by taking into account that $(d = (3+1))$ space is flat, one obtains

$$\begin{aligned} R^{(d)} &= R^{(d-4)} - \frac{1}{4} g_{\sigma\tau} E^{\sigma}{}_{Ai} E^{\tau}{}_{A'i'} \, F^{Ai}{}_{mn} F_{A'i'}{}^{mn} , \\ F^{Ai}{}_{mn} &= \partial_{m} A^{st}_{n} - \partial_{n} A^{st}_{m} - i f^{Aijk} \, A^{Aj}_{m} \, A^{Ak}_{n} , \\ A^{Ai}_{m} &= \sum_{st} c^{Ai}{}_{st} \, \omega^{st}{}_{m} , \\ \tau^{Ai} &= \sum_{st} c^{Aist} \, M_{st} . \end{aligned} \tag{6.17}$$

The integration of the action $\int E\, d^4x d^{(d-4)}x\, R^{(d)}$ over $(d-4)$ space (in which it turns out that only even functions of coordinates x^σ give nonzero contributions) leads to the well known effective action for the vector gauge fields in $d=(3+1)$ space: $\int d^4x \{-\frac{1}{4} F^{Ai}{}_{\mu\nu}\, F^{Ai\mu\nu}\}$.

The quadratic form for the vector gauge fields, Eq. (6.17), in $d=(3+1)$, obtained from the curvature $R^{(d)}$ can be found in many text book [39]. In Ref. ([21], Sect. 5.3) the Lagrange function for the gauge vector fields is derived by using the Clifford algebra space. The author allows besides the curvature R also its quadratic form R^2 (Eq. (240)).

6.3.3 Scalar fields in the *spin-charge-family* theory explain the origin of the higgs and Yukawa couplings

In the *spin-charge-family* theory the spin connection fields of both kinds, ω_{abs}'s and $\tilde{\omega}_{abs}$'s, carrying the space index $s=(7,8)$, explain the higgs and the Yukawa couplings of the *standard model*. They all belong to the weak charge doublets ([1,2] and references therein), as will be demonstrated in this section.

After gaining nonzero vacuum expectation values these scalar fields break the weak and the hyper charges of the vacuum (the assumption **A v.** and comments **C v.**) making all the fermions, due to the interaction with the vacuum, massive. Also the heavy bosons, interacting with the vacuum, gain masses.

The gauge scalar fields with the space index $s>8$ contribute to the matter-antimatter asymmetry in the universe [2].

This section follows mainly the equivalent sections in Refs. [2,1].

It turns out [2] that all scalars (the gauge fields with the space index $s \geq 5$) of the action (Eq. (6.1)) carry charges due to the space index in fundamental representations: They are either doublets (Table 6.5), $s=(5,6,7,8)$, or triplets (Table 6.8, also Ref. [2], Sect. II, Table I), $s=(9,10,..,13,14)$. Scalars with the space indices $s \in (7,8)$ and $s \in (5,6)$ are the $SU(2)$ *doublets* (Table 6.5).

All scalars carry additional quantum numbers: Besides the quantum numbers determined by the space index s they carry also the quantum numbers Ai, Eq. (6.3), the states of which belong to the adjoint representations. They originate in S^{ab} or $\tilde{S}^{ab}$, Eq. (6.50), $S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$, $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$, the gauge fields of which are ω_{abs} and $\tilde{\omega}_{abs}$, respectively. S^{ab} determine family members spin and charges, $\tilde{S}^{ab}$ determine family charges.

The infinitesimal generators $\mathcal{S}^{ab}$, which apply on the spin connections ω_{bde} $(= f^\alpha{}_e\ \omega_{bd\alpha})$ and $\tilde{\omega}_{\tilde{b}\tilde{d}e}$ $(= f^\alpha{}_e\ \tilde{\omega}_{\tilde{b}\tilde{d}\alpha})$, on either the space index e or any of the indices $(b, d, \tilde{b}, \tilde{d})$, operates as follows

$$\mathcal{S}^{ab}\, A^{d\ldots e\ldots g} = i\, (\eta^{ae}\, A^{d\ldots b\ldots g} - \eta^{be}\, A^{d\ldots a\ldots g})\,, \tag{6.18}$$

in accordance with the Eqs. (6.56, 6.57, 6.58). The expressions for the infinitesimal operators of the subgroups of the starting group (presented in Eq. (6.3) and the footnote before Eq. (6.3) and determined by the coefficients $c^{Ai}{}_{ab}$ in Eq. (6.3)) are the same for all three kinds of degrees of freedom, S^{ab}, $\tilde{S}^{ab}$ or $\mathcal{S}^{ab}$. Correspondingly are the same also the commutation relations

At the electroweak break all the scalar fields with the space index $(7,8)$, those which belong to one of twice two triplets carrying the family quantum numbers ($\tilde{\tau}^{\tilde{A}i}$) and those which belong to one of the three singlets carrying the family members quantum numbers (Q, Q', Y'), Eq. (6.19), start to self interact, gaining nonzero vacuum expectation values and breaking the weak charge, the hyper charge and the family charges.

Let me introduce a common notation A^{Ai}_s for all the scalar fields with $s = (7,8)$, independently of whether they originate in ω_{abs} - in this case $Ai = (Q,Q',Y')$ - or in $\tilde{\omega}_{\tilde{a}\tilde{b}s}$ - in this case all the family quantum numbers of all eight families contribute.

$$
\begin{aligned}
&A^{Ai}_s \text{ represents } (\, A^{Q}_s \,, A^{Q'}_s \,, A^{Y'}_s \,, \vec{\tilde{A}}^{\tilde{1}}_s \,, \vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_s \,, \vec{\tilde{A}}^{\tilde{2}}_s \,, \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_s \,)\,,\\
&\tau^{Ai} \text{ represents } (Q,\, Q',\, Y',\, \vec{\tilde{\tau}}^{1},\, \vec{\tilde{N}}_L,\, \vec{\tilde{\tau}}^{2},\, \vec{\tilde{N}}_R)\,. \qquad (6.19)
\end{aligned}
$$

Here τ^{Ai} represent all the operators, which apply on the spinor states. These scalars, the gauge scalar fields of the generators τ^{Ai} and $\tilde{\tau}^{Ai}$, are expressible in terms of the spin connection fields (Ref. [1], Eqs. (10,22,A8,A9)).

Let me demonstrate [1] that all the scalar fields with the space index $(7,8)$ carry with respect to this space index the weak and the hyper charge $(\mp\frac{1}{2}, \pm\frac{1}{2})$, respectively. This means that all these scalars have properties as required for the higgs in the *standard model*.

Let me make a choice of the superposition of the scalar fields so that they are eigenstates of $\tau^{13} = \frac{1}{2}(\mathcal{S}^{56} - \mathcal{S}^{78})$ (Eq (6.3) and footnotes at the same page). I rewrite for this purpose the second line of Eq. (6.2) as follows (the momentum p_s is left out [12])

$$
\begin{aligned}
&\sum_{s=(7,8),Ai} \bar{\psi}\,\gamma^{s}\,(-\tau^{Ai}\,A^{Ai}_s\,)\,\psi = \\
&-\bar{\psi}\{\stackrel{78}{(+)}\;\tau^{Ai}\,(A^{Ai}_7 - i\,A^{Ai}_8) + \stackrel{78}{(-)}\;(\tau^{Ai}\,(A^{Ai}_7 + i\,A^{Ai}_8)\}\psi\,,\\
&\stackrel{78}{(\pm)} = \frac{1}{2}\,(\gamma^7 \pm\, i\gamma^8\,)\,,\quad A^{Ai}_{\stackrel{78}{(\pm)}} := (A^{Ai}_7 \mp i\,A^{Ai}_8)\,, \qquad (6.20)
\end{aligned}
$$

with the summation over Ai performed, since A^{Ai}_s represent the scalar fields (A^{Q}_s, $A^{Q'}_s$, $A^{Y'}_s$, $\tilde{A}^{\tilde{4}}_s$, $\vec{\tilde{A}}^{\tilde{1}}_s$, $\vec{\tilde{A}}^{\tilde{2}}_s$, $\vec{\tilde{A}}^{\tilde{N}_R}_s$ and $\vec{\tilde{A}}^{\tilde{N}_L}_s$).

The application of the operators Y ($Y = \tau^{23} + \tau^4$, $\tau^{23} = \frac{1}{2}(\mathcal{S}^{56} + \mathcal{S}^{78})$, $\tau^4 = -\frac{1}{3}(\mathcal{S}^{9\,10} + \mathcal{S}^{11\,12} + \mathcal{S}^{13\,14})$), Q ($Q = \tau^{13} + Y$ and τ^{13} ($\tau^{13} = \frac{1}{2}(\mathcal{S}^{56} - \mathcal{S}^{78})$) on the fields $(A^{Ai}_7 \mp i\,A^{Ai}_8)$ gives ($\mathcal{S}^{ab}$ is defined in Eq. (6.18))

[12] It is expected that solutions with nonzero momentum lead to higher masses of the fermion fields in $d = (3+1)$ [34,35]. We correspondingly pay no attention to the momentum p_s, $s \in (5,\ldots,8)$, when having in mind the lowest energy solutions, manifesting at low energies.

$$\begin{aligned}
\tau^{13}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= \pm\,\frac{1}{2}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8})\,,\\
Y\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= \mp\,\frac{1}{2}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8})\,,\\
Q\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= 0\,.
\end{aligned} \tag{6.21}$$

Since τ^{4}, Y, τ^{13} and τ^{1+}, τ^{1-} give zero if applied on (A^{Q}_{s}, $A^{Q'}_{s}$ and $A^{Y'}_{s}$) with respect to the quantum numbers (Q, Q', Y'), and since Y and τ^{13} commute with the family quantum numbers, one sees that the scalar fields A^{Ai}_{s} (=(A^{Q}_{s}, A^{Y}_{s}, $A^{Y'}_{s}$, $\tilde{A}^{\tilde{4}}_{s}$, $\tilde{A}^{\tilde{Q}}_{s}$, $\vec{\tilde{A}}^{\tilde{1}}_{s}$, $\vec{\tilde{A}}^{\tilde{2}}_{s}$, $\vec{\tilde{A}}^{\tilde{N}_R}_{s}$, $\vec{\tilde{A}}^{\tilde{N}_L}_{s}$)), rewritten as $A^{Ai}_{\substack{78\\(\pm)}} = (A^{Ai}_{7} \mp i\,A^{Ai}_{8})$, are eigenstates of τ^{13} and Y, having the quantum numbers of the *standard model* Higgs' scalar.

These superposition of $A^{Ai}_{\substack{78\\(\pm)}}$ are presented in Table 6.5 as two doublets with respect to the weak charge τ^{13}, with the eigenvalue of τ^{23} (the second $SU(2)_{II}$ charge), equal to either $-\frac{1}{2}$ or $+\frac{1}{2}$, respectively.

	state	τ^{13}	τ^{23}	spin	τ^{4}	Q
$A^{Ai}_{\substack{78\\(-)}}$	$A^{Ai}_{7} + iA^{Ai}_{8}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
$A^{Ai}_{\substack{56\\(-)}}$	$A^{Ai}_{5} + iA^{Ai}_{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1
$A^{Ai}_{\substack{78\\(+)}}$	$A^{Ai}_{7} - iA^{Ai}_{8}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0
$A^{Ai}_{\substack{56\\(+)}}$	$A^{Ai}_{5} - iA^{Ai}_{6}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	+1

Table 6.5. The two scalar weak doublets, one with $\tau^{23} = -\frac{1}{2}$ and the other with $\tau^{23} = +\frac{1}{2}$, both with the "spinor" quantum number $\tau^{4} = 0$, are presented. In this table all the scalar fields carry besides the quantum numbers determined by the space index also the quantum numbers $\mathcal{A}i$ from Eq. (6.19).

The operators $\tau^{1\boxed{\pm}} = \tau^{11} \pm i\tau^{12}$

$$\tau^{1\boxed{\pm}} = \frac{1}{2}[(\mathcal{S}^{58} - \mathcal{S}^{67})\;\boxed{\pm}\;i\,(\mathcal{S}^{57} + \mathcal{S}^{68})]\,, \tag{6.22}$$

transform one member of a doublet from Table 6.5 into another member of the same doublet, keeping τ^{23} ($= \frac{1}{2}\,(\mathcal{S}^{56} + \mathcal{S}^{78})$) unchanged, clarifying the above statement.

That the scalar fields $A^{Ai}_{\substack{78\\(\pm)}}$ are either *triplets* as the gauge fields of the *family quantum numbers* ($\vec{\tilde{N}}_R$, $\vec{\tilde{N}}_L$, $\vec{\tilde{\tau}}^{2}$, $\vec{\tilde{\tau}}^{1}$); or they are singlets as the gauge fields of $Q = \tau^{13} + Y$, $Q' = -\tan^2\vartheta_1 Y + \tau^{13}$ and $Y' = -\tan^2\vartheta_2\tau^{4} + \tau^{23}$, is shown in Ref. [1], Eq. (22).

One finds

$$\begin{aligned}
&\tilde{N}^{3}_{L}\,\tilde{A}^{\tilde{N}_L\boxed{\pm}}_{\substack{78\\(\pm)}} = \boxed{\pm}\,\tilde{A}^{\tilde{N}_L\boxed{\pm}}_{\substack{78\\(\pm)}}\,, \quad \tilde{N}^{3}_{L}\,\tilde{A}^{\tilde{N}_L 3}_{\substack{78\\(\pm)}} = 0\,,\\
&Q\,A^{Q}_{\substack{78\\(\pm)}} = 0\,.
\end{aligned} \tag{6.23}$$

with $Q = \mathcal{S}^{56} + \tau^4 = \mathcal{S}^{56} - \frac{1}{3}(\mathcal{S}^{9\,10} + \mathcal{S}^{11\,12} + \mathcal{S}^{13\,14})$, and with τ^4 defined in the footnote on the page of Eq. (6.3), if replacing S^{ab}, Eq. (6.50), by $\mathcal{S}^{ab}$, Eq. (6.18).

Similarly one finds properties with respect to the Ai quantum numbers for all the scalar fields $A^{Ai}_{\substack{78\\(\pm)}}$.

After the appearance of the condensate (Table 6.1), which breaks the $SU(2)_{II}$ symmetry (bringing masses to all the scalar fields), the weak charge $\vec{\tau}^1$ and the hyper charge Y remain the conserved charges [13].

The nonzero vacuum expectation values of the scalar fields of Eq. (6.19) break the mass protection mechanism of quarks and leptons and determine correspondingly the mass matrices (Eq. (6.31)) of the two groups of quarks and leptons.

Obviously have the scalar fields in the *spin-charge-family* theory all the properties of the higgs. I show below and in Subsect. 6.4.1 that these scalar fields explain also the Yukawa couplings of the *standard model*.

All other scalar fields: $A^{Ai}_s, s \in (5,6)$ and $A^{Ai}_t, t \in (9,\ldots,14)$ have masses of the order of the condensate scale and contribute to matter-antimatter asymmetry [2].

Effective action for scalar fields with the space index $(7,8)$

Since all the scalar fields, as well as their Lagrange density, is included in the starting action, Eq.(6.1), it would be possible, at least in principle, to derive the low energy effective action for scalars by guessing the boundary conditions, under which the universe evolved. This is extremely demanding project.

In what follows the effective Lagrange density for the scalar fields is assumed that it changes from the Lagrange density before the electroweak break $\mathcal{L}_s = E\{(p_m A^{Ai}_s)^\dagger (p^m A^{Ai}_s) - (m'_{Ai})^2 A^{Ai\dagger}_s A^{Ai}_s\}$ to

$$\begin{aligned}\mathcal{L}_{sg} = E \sum_{A,i} \{&(p_m A^{Ai}_s)^\dagger (p^m A^{Ai}_s) - (-\lambda^{Ai} + (m'_{Ai})^2))\, A^{Ai\dagger}_s A^{Ai}_s \\ &+ \sum_{B,j} \Lambda^{AiBj}\, A^{Ai\dagger}_s A^{Ai}_s\, A^{Bj\dagger}_s A^{Bj}_s\},\end{aligned} \tag{6.24}$$

where $-\lambda^{Ai} + m'^2_{Ai} = m^2_{Ai}$ and m_{Ai} manifests as the mass of the A^{Ai}_s scalar.

The Lagrange density leads to the coupled equations of motion for many scalar fields with, in this assumption, harmonic interactions. It requires a lot of effort to extract the dependence of the eigen modes on the parameters of the Lagrange density to see the influence of the parameters on properties of fermions. This work has not yet been done. First attempts are in progress.

Yukawa couplings in the spin-charge-family theory

Let $\psi^\alpha_{(L,R)}$ denote massless and $\Psi^\alpha_{(L,R)}$ massive four vectors for each family member $\alpha = (u_{L,R}, d_{L,R}, \nu_{L,R}, e_{L,R})$, let say for the group of four families among which there are the observed three families, after taking into account loop correc-

[13] It is τ^{23} which determines the hyper charge Y ($Y = \mathcal{S}^{23} + \tau^4$) of these scalar fields, since τ^4, if applied on the scalar index of these scalar fields, gives zero, according to equations in the footnote above Eq. (6.3).

tions in all orders.

$$\psi^{\alpha}_{(L,R)} = V^{\alpha}_{(L,R)}\, \Psi^{\alpha}_{(L,R)}\,, \tag{6.25}$$

and let $(\psi^{\alpha\, k}_{(L,R)}\,,\ \Psi^{\alpha\, k}_{(L,R)})$ be any component of the four vectors, massless and massive, respectively. On the tree level we have $\psi^{\alpha}_{(L,R)} = V^{\alpha}_{(o)}\, \Psi^{\alpha\,(o)}_{(L,R)}$ and

$$< \psi^{\alpha}_{L}|\gamma^0\, \mathcal{M}^{\alpha}_{(o)}\, |\psi^{\alpha}_{R} >=< \Psi^{\alpha\,(o)}_{L}|\gamma^0\, V^{\alpha\,\dagger}_{(o)}\, \mathcal{M}^{\alpha}_{(o)}\, V^{\alpha}_{(o)}\, |\Psi^{\alpha}_{R\,(o)} >, \tag{6.26}$$

with $\mathcal{M}^{\alpha}_{(o)k\,k'} = \sum_{A,i} (-g^{Ai}\, \nu_{Ai\,\mp})\; C^{\alpha}_{k\,k'}\,.\ g^{Ai}\, \nu_{Ai\,\mp}$ represents the nonzero vacuum expactation values of the scalar fields. The coefficients $C^{\alpha}_{k\,k'}$ are determined by the mass matrices, Eq.(6.31), on the tree level in this case. It then follows

$$\begin{aligned} \overline{\Psi}^{\alpha}\, V^{\alpha\dagger}_{(o)}\, \mathcal{M}^{\alpha}_{(o)}\, V^{\alpha}_{(o)}\, \Psi^{\alpha} &= \overline{\Psi}^{\alpha}\, \mathrm{diag}(m^{\alpha}_{(o)1}\,, \cdots, m^{\alpha}_{(o)4})\, \Psi^{\alpha}\,,\\ V^{\alpha\dagger}_{(o)}\, \mathcal{M}^{\alpha}_{(o)}\, V^{\alpha}_{(o)} &= \Phi^{\alpha}_{\Psi(o)}\,. \end{aligned} \tag{6.27}$$

The coupling constants $m^{\alpha}_{(o)k}$ (in some units) of the dynamical scalar fields $\Phi^{\alpha}_{\Psi(o)\,k}$, which are superposition of A^{Ai}_{s}, to the family member $\Psi^{\alpha\, k}$ belonging to the k^{th} family are on the tree level correspondingly equal to

$$(\Phi^{\alpha}_{\Psi(o)})_{k\,k'}\, \Psi^{\alpha\, k'} = \delta_{k\,k'}\, m^{\alpha}_{(o)k}\, \Psi^{\alpha\, k}\,. \tag{6.28}$$

The superposition of scalar fields $(\Phi^{\alpha}_{\Psi(o)})$, which couple to fermions and depend on the quantum numbers α and k, are in general different from the superposition, which are their mass eigenstates. Each family member α of each massive family k couples in general to different superposition of scalar fields.

It turns out that mass matrices of both - quarks and leptons - behave in a very similar way. No additional neutrinos, offering a "sea-saw" mechanism, are needed. All this is already included in the starting action.

6.3.4 The condensate in the *spin-charge-family* theory

The appearance of the condensate of two right handed neutrinos with properties presented in Table 6.1 is in this paper assumed, so that in the low energy regime the *spin-charge-family* theory lead to the effective action, explaining the assumptions of the *standard model* and consequently the observed phenomenas. The condensate should appear during the expansion of the universe due to particular boundary conditions and the conditions in the universe in the time of the appearance of the condensate. This study has not yet been done.

The condensate, presented on Table 6.1, does not influence the colour, the weak and the hyper charges ($\vec{\tau}^3$, $\vec{\tau}^1$, Y, respectively) of the corresponding gauge fields. Since the corresponding vector gauge fields don't interact with the condensate, the colour, the weak and the hyper charges remain the conserved quantities up to the electroweak phase transition.

The condensate changes the properties of the scalar fields, which are before the appearance of the condensate massless scalar gauge fields. Interaction with the condensate makes all the scalar fields massive.

After the electroweak break, when the scalar fields with the space index $s = (7, 8)$ - those with the family quantum numbers $(\tilde{N}^{i}_{(L,R)}, \tilde{\tau}^{(1,2)i})$ and those with the family members quantum numbers (Q, Q', Y') - start to strongly self interact (Eq. (6.24)), gaining nonzero vacuum expectation values (Eq. (6.24)) and correspondingly changing the vacuum and their own masses, so that the weak charge and the hyper charge are no longer conserved quantities. The only conserved charges are then the colour and the electromagnetic charges.

6.4 Summary of the *spin-charge-family* theory achievements so far

To understand better the history of our universe the explanation of the *standard model* assumptions is certainly needed. It is also needed to know the number of families in the low energy regime and to understand the appearance of phenomenas like the existence of the dark matter, the matter-antimatter asymmetry and the dark energy.

I have demonstrated so far, that the *spin-charge-family* theory, starting with the simple action in $d = (13 + 1)$ for fermions - carrying only two kinds of spins (no charges) - and for gauge fields - to which fermions are coupled, vielbeins and two kinds of spin connection fields - offers the explanation for all the assumptions of the *standard model*:

a. The theory explains all the properties of the family members - quarks and leptons, left and right handed, relating handedness and charges, and their right and left handed antiquarks and antileptons.

b. It explains the appearance and properties of the families of family members.

c. It explains the existence of the gauge vector fields of the family members charges.

d. It explains the appearance and properties of the scalar field (the higgs) and the Yukawa couplings.

The *spin-charge-family* theory predicts that there are at the low energy regime two decoupled groups of four families of quarks and leptons, what means that besides the observed three there is the fourth not yet observed family of quarks and leptons.

The existence of two decoupled groups of four families also means that the stable of the upper four families must also be observed. In Subsect. 6.4.4 [13] it is presented that the stable of the upper four families constitute the dark matter.

In this section I present the *spin-charge-family* theory achievements, explaining:

i. The properties of the lower four families, the three of which have already been observed, as they follow from the properties of the scalar fields of this theory, Subsect. 6.4.1,

i. a. presenting the results of the calculations,

i. b. discussing whether or not present experiments speak or not against the existence of the fourth family, in particular I shall comment the contribution of the fourth family to the production of the higgs in the quark-fusion process,

Subsect. 6.4.2, the topics which seems to speak the most against the existence of the fourth family.
ii. The fact that this theory easily explains the "miraculous" cancellation of the triangle anomalies in the *standard model*, Subsect. 6.4.3.
iii. The existence of the dark matter, Subsect. 6.4.4.
iv. The explanation for the matter-antimatter asymmetry, Subsect. 6.4.5.

6.4.1 Masses of the lower four families of quarks and leptons in the *spin-charge-family* theory [12,14,15]

There are two groups of four families. The mass matrix of each family member of each of the group of four families demonstrates in the massless basis the $U(1) \times \widetilde{SU}(2) \times \widetilde{SU}(2)$ symmetry (each of the two $\widetilde{SU}(2)$ is a subgroup, one of $\widetilde{SO}(3,1)$ and the other of $\widetilde{SO}(4)$).

The scalars with the family quantum numbers split the eight families into twice four families. To the masses of the lower four families the scalar fields, which are the gauge fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ contribute. To the masses of the upper four families the gauge fields of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ contribute. The scalars with the family members quantum numbers (Q, Q', Y') contribute to the masses of the lower and upper four families.

I discuss here properties of quarks and leptons of the lower four families, Eq. (6.31).

Let ψ_i, $i = (1,2,3,4)$, denote the massless basis for a particular family member α. And let us denote the two kinds of the operators, which transform the basis vectors into one another as

$$\tilde{N}^i_L\,,\; i = (1,2,3)\,, \qquad \tilde{\tau}^i_L\,,\; i = (1,2,3)\,. \tag{6.29}$$

One finds

$$\begin{aligned}
\tilde{N}^3_L\,(\psi_1,\psi_2,\psi_3,\psi_4) &= \frac{1}{2}(-\psi_1,\ \psi_2,-\psi_3,\ \psi_4)\,,\\
\tilde{N}^+_L\,(\psi_1,\psi_2,\psi_3,\psi_4) &= \ (\psi_2,\ 0,\ \psi_4,\ 0)\,,\\
\tilde{N}^-_L\,(\psi_1,\psi_2,\psi_3,\psi_4) &= \ (0\ ,\ \psi_1,\ 0,\ \psi_3)\,,\\
\tilde{\tau}^{13}\ (\psi_1,\psi_2,\psi_3,\psi_4) &= \frac{1}{2}(-\psi_1,-\psi_2,\ \psi_3,\ \psi_4)\,,\\
\tilde{\tau}^{1+}\ (\psi_1,\psi_2,\psi_3,\psi_4) &= \ (\psi_3,\ \psi_4,\ 0,\ 0)\,,\\
\tilde{\tau}^{1-}\ (\psi_1,\psi_2,\psi_3,\psi_4) &= \ (\ 0,\ 0,\ \psi_1,\ \psi_2)\,.
\end{aligned} \tag{6.30}$$

This is indeed what the two $SU(2)$ operators in the *spin-charge-family* theory do on the lower four families. The gauge scalar fields $A^{Ai}_{\substack{78\\(\pm)}}$, $Ai = [\tilde{N}^i_L, i = (\pm,3), \tilde{\tau}^{1i}, i = (\pm,3)]$, Eqs. (6.20, 6.19), of these operators determine (together with the operators and the corresponding coupling constants hidden in the scalar fields) the off diagonal and diagonal matrix elements after the electroweak phase transition in which scalar fields gain nonzero vacuum expectation values.

In addition to these two kinds of $SU(2)$ scalars there are three $U(1)$ scalars, which distinguish among the family members, contributing on the tree level the same diagonal matrix elements for all the families.

In loop corrections in all orders the symmetry of mass matrices remains unchanged, while the three $U(1)$ scalars manifest in off diagonal elements as well.

All the scalars, the two three-plets and the three singlets, are doublets with respect to the weak charge, contributing to the weak and the hyper charge of the fermions so that they transform the right handed members into the left handed onces with the phases presented in Table 6.2.

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha} . \tag{6.31}$$

Although any accurate 3×3 submatrix of the 4×4 unitary matrix determines the 4×4 matrix uniquely, neither the quark nor (in particular) the lepton 3×3 mixing matrix are measured accurately enough that it would be possible to determine three complex phases of the 4×4 mixing matrix as well as the mixing matrix elements of the fourth family members to the lower three.

We therefore assumed in our calculations [12,14,15] that the mass matrices are symmetric and real. Correspondingly the mixing matrices are orthogonal. We fitted the 6 free parameters of each quark mass matrix, Eq. (6.31), to twice three (6) measured quark masses, and to the 6 (from the experimental data extracted) parameters of the corresponding 4×4 mixing matrix.

While the experimental accuracy of the quark masses of the lower three families does not influence the calculated mass matrices considerably, it turned out that the experimental accuracy of the 3×3 quark mixing matrix is not good enough to trustworthy determine the mass intervals for the fourth family quarks.

Taking into account our calculations, in which we fit parameters of Eq. (6.31) to the experimental data for masses and mixing matrices for quarks and the meson decays evaluations in the literature, as well as our own evaluations, we estimated that the fourth family quarks masses might be above 1 TeV. Choosing the masses of the fourth family quarks we were able not only to calculate the fourth family matrix elements to the lower three families, but also predict towards which values will the matrix elements of the 3×3 submatrix move in more accurate experiments [15].

The two fitted mass matrices, Ref. ([15], Eqs. (23, 27)) lead to masses of Eq. (6.32) for $M_{u_4}/\text{MeV}/c^2 = 700\,000 = M_{d_4}/\text{MeV}/c^2$

$$\begin{aligned} \mathbf{M}^{u}/\text{MeV}/c^2 &= (1.3, 620.0, 172\,000., 700\,000.)\,, \\ \mathbf{M}^{d}/\text{MeV}/c^2 &= (2.88508, 55.024, 2\,899.99, 700\,000.)\,, \end{aligned} \tag{6.32}$$

and to masses of Eq. (6.33) for $M_{u_4}/\text{MeV}/c^2 = 1\,200\,000 = M_{d_4}/\text{MeV}/c^2$

$$\begin{aligned} \mathbf{M}^{u}/\text{MeV}/c^2 &= (1.3, 620.0, 172\,000., 1\,200\,000.)\,, \\ \mathbf{M}^{d}/\text{MeV}/c^2 &= (2.88508, 55.024, 2\,899.99, 1\,200\,000.)\,. \end{aligned} \tag{6.33}$$

They lead to the 4×4 mixing matrix in which we fit two kinds of the experimental - the old data ($\exp_o$) and the new data ($\exp_n$) - each used in calculations for the choice $m_{u_4} = m_{d_4} = 700$ GeV (old_1, new_1) and $m_{u_4} = m_{d_4} = 1\,200$ GeV (old_2, new_2), Eq. (6.34)

$$|V_{(ud)}| = \left(\begin{array}{cccc}
exp_o & 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.00049 \\
exp_n & 0.97425 \pm 0.00022 & 0.2253 \pm 0.0008 & 0.00413 \pm 0.00049 \\ \hline
old_1 & 0.97423 & 0.22531 & 0.003 \\
old_2 & 0.97425 & 0.22536 & 0.00301 \\
new_1 & 0.97423 & 0.22531 & 0.00299 \\
new_2 & 0.97423 & 0.22538 & 0.00299 \\ \hline
exp_o & 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 \\
exp_n & 0.225 \pm 0.008 & 0.986 \pm 0.016 & 0.0411 \pm 0.0013 \\ \hline
old_1 & 0.22526 & 0.97338 & 0.042 \\
old_2 & 0.22534 & 0.97336 & 0.04239 \\
new_1 & 0.22534 & 0.97335 & 0.04245 \\
new_2 & 0.22531 & 0.97336 & 0.04248 \\ \hline
exp_o & 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 \\
exp_n & 0.0084 \pm 0.0006 & 0.0400 \pm 0.0027 & 1.021 \pm 0.032 \\ \hline
old_1 & 0.00663 & 0.04197 & 0.9991 \\
old_2 & 0.00663 & 0.04198 & 0.9991 \\
new_1 & 0.00667 & 0.04203 & 0.99909 \\
new_2 & 0.00667 & 0.04206 & 0.99909
\end{array}\right) . \tag{6.34}$$

It was noticed [15] that the mass matrices of u and d quarks change by a factor of ≈ 1.5 when the masses of the fourth family members grow from 700 GeV to 1 200 GeV. The mixing matrix elements of the 3×3 sub matrix of the 4×4 matrix do not change a lot with the masses of the fourth family quarks, but they do change so that they agree better with the newer [19] than with the older [20] experimental values.

From the above results it follows:

i. The predictionof the calculated mixing matrix elements, obtained by fitting the symmetry of the mass matrices (Eq. (6.31)) to the experimental data [20], was confirmed by more accurate experimental data [19]. In all cases are the calculated 3×3 matrix elements closer to the new experimental values than to the old experimental values.

ii. The fourth family masses change the mass matrices considerably, while their influence on the 3×3 submatrix of the 4×4 mixing matrix is much weaker.

i iii. We expect that more accurate experiments will bring a slightly smaller values for $(V_{u_1 d_1}, V_{u_1 d_3}, V_{u_3 d_3})$, smaller $(V_{u_2 d_2}, V_{u_3 d_1})$, $(V_{u_1 d_2}, V_{u_2 d_1})$ will slightly grow and $(V_{u_2 d_3})$ $V_{u_3 d_2}$ will grow.

iv. The matrix elements $V_{u_i d_4}$ and $V_{u_4 d_i}$ change considerably with the mass of the ourth family members, and they differ quite a lot also when using new instead of the old experimental data for the mixing matrix.

v. Fitting (twice 6) free parameters of the mass matrices to the new experimental data [19] gives smaller uncertainty in fitting procedure than when fitting to the old experimental data [20], while the masses of the fourth family members do not influence the uncertainty of the calculations considerably. Only very accurate mixing matrix elements would allow to determine fourth family quarks masses more accurately.

vi. Since the choice of the fourth family quark masses does not appreciably influence either the fitting procedure or the obtained 3×3 mixing matrix, and also not the accuracy of the masses of the three lower families, it is difficult to predict the interval for the masses of the fourth family members. For the masses of the fourth family quarks to be close or above 1 TeV speak more other experimental data, like decays of mesons.

vii. If the masses of the fourth family members are above 1 TeV, the mass matrices are close to the democratic matrix: The matrix elements are closer to one another the higher is the mass of the fourth family member. In such a case are the fourth family masses mostly determined by the scalars carrying the family quantum numbers. Correspondingly are the masses of the u_4-quarks closer to the masses of the d_4-quarks.

The complex mass matrices would lead to unitary and not to orthogonal mixing matrices. The more accurate experimental data for quarks mixing matrix would allow us to extract also the phases of the unitary mixing matrix, allowing us to predict the fourth family masses.

6.4.2 Is the existence of the fourth family in agreement with the present experiments?

This part is following equivalent part in my contribution to the Proceedings to the Conference on New Physics at the Large Hadron Collider, 29 February – 4 March, **2016**, Nanyang Executive Centre, NTU, Singapore.

The *spin-charge-family* theory predicts the existence of the fourth family to the observed three, while there has been no direct observation of the fourth family quarks with the masses below 1 TeV. The fourth family quarks with masses above 1 TeV contribute according to the *standard model* (the *standard model* Yukawa couplings of quarks to the scalar higgs are proportional to $\frac{m_4^\alpha}{v}$, where m_4^α is the fourth family member ($\alpha = u$, d) mass and v the vacuum expectation value of the scalar) to either the quark-gluon fusion production of the scalar field (the higgs) or to the scalar field decay into two photons ≈ 10 times too much in comparison with the observations. Correspondingly the high energy physicists do not expect the existence of the fourth family members at all [26].

I am stressing [30] in this subsection that the u_i-quarks and d_i-quarks of a i^{th} family, if they couple with the opposite sign (with respect to the "$\pm$" degree of freedom) to the scalar fields, carrying the family $\tilde{A}i$ quantum numbers - $\tilde{A}^{\tilde{A}i}_{\pm}$ ($\tilde{A}i = (\tilde{\tau}^{1i}, \tilde{N}^i_L)$, (Eq. 6.19)) (they are the same for all the family members) - do not contribute to either the quark-gluon fusion production of the scalar fields with the family quantum numbers or to the decay of these scalars into two photons, if the u_i-quarks and d_i-quarks have the same mass. Since the u_4-quarks and d_4-quarks might have similar masses (Subsect. 6.4.1) and since their masses are for $m_{u_4} > 1$ TeV and $m_{d_4} > 1$ TeV mostly determined by the scalars with the family quantum numbers, the observations so far are consequently not in contradiction with the *spin-charge-family* theory prediction that there exists the fourth family coupled to the observed three.

The couplings of u_i and d_i to the scalars carrying the family members quantum numbers are determined besides by the corresponding couplings also by the eigenvalues of the operators (Q, Q', Y') on the quarks states (which do distinguish between u_i and d_i).

The strong influence of the scalar fields carrying the family members quantum numbers on the masses of the lower (observed) three families manifests in the huge differences in the masses of u_i and d_i, $i = (1, 2, 3)$, among families (i) and family members (u, d). For the fourth family quarks, which are more and more decoupled from the observed three families the higher are their masses [15,14], the influence of the scalar fields carrying the family members quantum numbers on their masses is expected to be much weaker. Correspondingly the u_4 and d_4 masses become closer to each other the higher are their masses and the weaker is their couplings (the mixing matrix elements) to the lower three families.

If the masses of the fourth family quarks are close to each other, then u_4 and d_4 contribute in the quark-gluon fusion very little to the production of the scalar field - the higgs - which is mostly superposition of the scalar fields with the family members quantum numbers, what is in agreement with the observation: In the quark-gluon fusion production of the higgs mostly the top (u_3) contributes.

In Tables 6.2–6.3 the phases of all the states are chosen to be 1. Here I use different phases, those which enable the usual presentation of fermions under the change of spin and under $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$.

In Table 6.6, the properties of u and d quarks, needed in Fig. 6.1, are presented. In Fig. 6.1 the properties of the u and d quarks, contributing to the production of

state	τ^{13}	Y	Q
u_{Ri}	0	$\frac{2}{3}$	$\frac{2}{3}$
u_{Li}	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
d_{Ri}	0	$-\frac{1}{3}$	$-\frac{1}{3}$
d_{Li}	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$

Table 6.6. The weak, hyper and electromagnetic charges for quarks in their massless basis are presented, the colour charge is not shown. These and other properties of quarks and leptons can be read from Tables 6.2–6.3.

the dynamical part of the scalar fields - $\Phi^{Ai}_{\substack{78\\(\pm)}}$, Eq.(6.19), ($A^{Ai}_{\substack{78\\(\pm)}} = \Phi^{Ai}_{\substack{78\\(\pm)}} + \nu^{Ai}_{\substack{78\\(\pm)}}$) - in the quark-gluon fusion, are presented. One notices the opposite signs of the couplings of u_i with respect to d_i for ether Φ_- or for Φ_+: The fourth family quarks contribute to the production of the higgs little enough not to be in contradiction with the observation. Correspondingly also the decay of the higgs, to the production of which contribute mostly u_3- quarks while the fourth family quarks are much weaker coupled to this one, into two photons is in agreement with the observations.

The fourth family quarks can still contribute to the production of the scalars of the masses of a few TeV, to which they couple stronger than to the higgs.

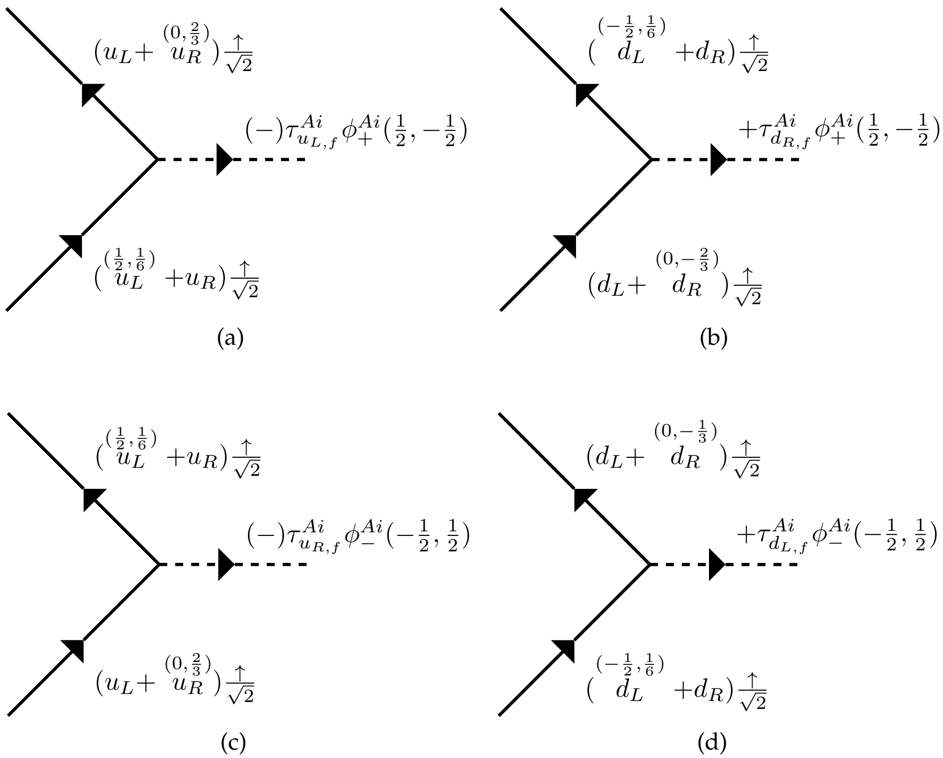

Fig. 6.1. The contributions of u and d quarks to the production of the scalar fields Φ^{Ai}_{-} and Φ^{Ai}_{+}, when τ^{Ai} represent the family quantum numbers (which are for the lower four families $\tilde{\tau}^{1i}$ and $\tilde{N}^{i}_{L}$) or the family members quantum numbers (Q, Q', Y'), are presented: (a) the u-quark contribution to the scalar fields Φ^{Ai}_{+}, (b) the d-quark contribution to Φ^{Ai}_{+}, (c) the u-quark contribution to Φ^{Ai}_{-}, (d) the d-quark contribution to Φ^{Ai}_{-}. $\tau^{Ai}_{u_{(L,R)},f}$ and $\tau^{Ai}_{d_{(L,R)},f}$ denote the application values of the operators $\tilde{\tau}^{1i}$ and $\tilde{N}^{i}_{L}$ and Q, Q', Y' on the states. While $\tilde{\tau}^{1i}$ and $\tilde{N}^{i}_{L}$ do not distinguish among family members u and d so that in this case the contribution of u and d have opposite signs, Q, Q', Y' do, influencing the signs in addition.

The figures are valid for any Ai and correspondingly also for any superposition of $\Phi^{Ai}_{\pm}$.

Let me conclude the above observations that the effective behavior of the scalar fields at the electroweak break - which should be developed from the starting action - can hardly behave in the way (perturbatively, with one or a few higgses repeating the idea of the observed higgs and with several additional assumptions) what the estimations of the experimental data assume.

6.4.3 Anomaly cancellation in the *spin-charge-family* theory

In the *standard model* triangle anomalies "miraculously" disappear due to the fact that the sum of all possible traces $\mathrm{Tr}[\tau^{Ai}\tau^{Bj}\tau^{Ck}]$ (here $\tau^{Ai}, \tau^{Bi}, \tau^{Ck}$ are the generators of one, of two or of three of the groups of $SU(3), SU(2)$ and $U(1)$) over the representations of one family of the left handed fermions and their antifermions (and separately of the right handed fermions and their antifermions), contributing to the triangle currents, are equal to zero [22–25].

Let me demonstrate that this cancellation of the *standard model* triangle anomaly follows straightforwardly, if the $SO(3,1), SU(2), U(1)$ and $SU(3)$ are the subgroups of the orthogonal group $SO(13,1)$.

To the triangle anomaly the right-handed spinors (fermions) and antispinors contribute with the opposite sign than the left handed spinors and their antispinors. Their common contribution to anomalies is proportional to [24]

$$(\sum_{(A,i,B,j,C,k)_{L\,\bar{L}}} \mathrm{Tr}[\tau^{Ai}\,\tau^{Bj}\,\tau^{Ck}] - \sum_{(A,i,B,j,C,k)_{R\,\bar{R}}} \mathrm{Tr}[\tau^{Ai}\,\tau^{Bj}\,\tau^{Ck}]\,)\,, \tag{6.35}$$

where τ^{Ai} are in the *standard model* the generators of the infinitesimal transformation of the groups $SU(3) \times SU(2) \times U(1)$, while in the *spin-charge-family* theory τ^{Ai} are irreducible subgroups of the starting orthogonal group $SO(2(2n+1)-1,1)$, $n = 3$. $_{L\,\bar{L}}$ ($_{R\,\bar{R}}$) denote the left (right) handed spinors and their antispinors (right (left)), respectively.

In the first seven columns (up to to ||) of Table 6.7 the by the *standard model* assumed properties of the one family members, running in the triangle, are presented. The last two columns - taken from Table 6.2 - describe additional properties which quarks and leptons (and antiquqrks and antileptons) would have, if the *standard model* groups $SO(3,1), SU(2), SU(3)$ and $U(1)$ are embedded into the $SO(13,1)$ group. All the properties are needed to demonstrate that the "miraculous" cancellation of the triangle anomalies is a "trivial" one if one takes into account that the *standard model* groups can easily be interpreted (unified) by making the next step beyond the *standard model*. The triangle anomaly of the *standard model* occurs if the traces in Eq.(6.35) are not zero for either the left handed quarks and leptons and their antiparticles or the right handed quarks and leptons and their antiparticles for the Feynman triangle diagrams in which the gauge vector fields of the charges

$$\begin{aligned}
&U(1) \times U(1) \times U(1)\,,\\
&SU(2) \times SU(2) \times U(1)\,,\\
&SU(3) \times SU(3) \times SU(3)\,,\\
&SU(3) \times SU(3) \times U(1)\,,\\
&U(1) \times \text{gravitational}
\end{aligned} \tag{6.36}$$

contribute to the triangle anomaly.

To see that embedding the *standard model* groups into the orthogonal group $SO(13,1)$ makes the cancelltion of the triangle anomalies self evident, let us recognize: The subgroups of the $SO(13,1)$ group are $SO(7,1) \times SO(6)$. The subgroups of

i_L	name	handedness $\Gamma^{(3,1)}$	weak charge τ^{13}	hyper charge Y	colour charge τ^{33}	τ^{38}	elm charge Q	$SU(2)_{II}$ charge τ^{23}	$U(1)_{II}$ charge τ^{4}
1_L	u_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	0	$\frac{1}{6}$
2_L	d_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
3_L	u_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	0	$\frac{1}{6}$
4_L	d_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
5_L	u_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{2}{3}$	0	$\frac{1}{6}$
6_L	d_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
7_L	ν_L	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$-\frac{1}{2}$
8_L	e^L	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	0	$-\frac{1}{2}$
9_L	$\bar{u}L$	-1	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$
10_L	$\bar{d}L$	-1	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$
11_L	$\bar{u}L$	-1	0	$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$
12_L	$\bar{d}L$	-1	0	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$
13_L	$\bar{u}L$	-1	0	$-\frac{2}{3}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$
14_L	$\bar{d}L$	-1	0	$\frac{1}{3}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$
15_L	$\bar{\nu}_L$	-1	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
16_L	$\bar{e}_L$	-1	0	1	0	0	1	$\frac{1}{2}$	$\frac{1}{2}$
1_R	u_R	1	0	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
2_R	d_R	1	0	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$
3_R	u_R	1	0	$\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
4_R	d_R	1	0	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$
5_R	u_R	1	0	$\frac{2}{3}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
6_R	d_R	1	0	$-\frac{1}{3}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$
7_R	ν_R	1	0	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
8_R	e_R	1	0	-1	0	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
9_R	$\bar{u}_R$	1	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	0	$-\frac{1}{6}$
10_R	$\bar{d}_R$	1	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
11_R	$\bar{u}_R$	1	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	0	$-\frac{1}{6}$
12_R	$\bar{d}_R$	1	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
13_R	$\bar{u}_R$	1	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{2}{3}$	0	$-\frac{1}{6}$
14_R	$\bar{d}_R$	1	$\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
15_R	$\bar{\nu}_R$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$
16_R	$\bar{e}_R$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	0	$\frac{1}{2}$

Table 6.7. Properties of the left handed quarks and leptons and their antiparticles and of the right handed quarks and leptons and their antiparticles, as assumed by the *standard model* are presented in the first seven columns. In the last two columns the two quantum numbers are added, which the fermions and antifermions would have if the *standard model* groups $SO(3,1), SU(2), SU(3)$ and $U(1)$ are embedded into the $SO(13,1)$ group. The whole quark part appears, due to the colour charges, three times. One can check that the hyper charge is the sum of $\tau^{4}_{i_{L,R}} + \tau^{23}_{i_{L,R}}$ Table 6.2. The quantum numbers are the same for all the families.

$SO(6)$ are the colour group $SU(3)$ with the generators denoted by τ^{3i}, $i = 1, \ldots, 8$ and the $U(1)$ (we shall call it $U(1)_{II}$) group with the generator τ^4. One sees that all the quarks have $\tau^4 = \frac{1}{6}$, all the antiquarks have $\tau^4 = -\frac{1}{6}$, while the leptons have $\tau^4 = -\frac{1}{2}$ and antileptons have $\tau^4 = \frac{1}{2}$. Correspondingly the trace of τ^4 over all the family members is equal to zero.

The subgroups of the $SO(7,1)$, as seen in Table 6.2, have as subgroups $SO(3,1)\times SU(2)_I \times SU2_{II}$, with the generators τ^{1i} (representing the weak group operators) and τ^{2i} (representing the generators of the additional $SU(2)$ group), respectively. The left handed spinors are $SU(2)_I$ (weak) doublets and $SU(2)_{II}$ singlets, while the right handed spinors are the $SU(2)_I$ (weak) singlets and $SU(2)_{II}$ doublets. Correspondingly are the left handed antispinors the $SU(2)_I$ (weak) singlets and $SU(2)_{II}$ doublets, while the right handed antispinors are the $SU(2)_I$ (weak) doublets and the $SU(2)_{II}$ singlets.

The hypercharge of the *standard model* corresponds to the sum of τ^4 and τ^{23}

$$Y = \tau^4 + \tau^{23} \,. \tag{6.37}$$

For the triangle Feynman diagram, to which three hyper $U(1)$ boson fields contribute, we must evaluate $\sum_i \mathrm{Tr}(Y_i)^3$, in which the sum runs over all the members (i) of the left handed spinors and antispinors, and of the right handed spinors and antispinors separately. In the case of embedding the *standard model* groups into $SO(13,1)$ we have

$$\begin{aligned}
\sum_{i_{L,R}} (Y_{i_{L,R}})^3 &= \sum_{i_{L,R}} (\tau^4_{i_{L,R}} + \tau^{23}_{i_{L,R}})^3 \\
&= \sum_{i_{L,R}} (\tau^4_{i_{L,R}})^3 + \sum_{i_{L,R}} (\tau^{23}_{i_{L,R}})^3 \\
&+ \sum_{i_{L,R}} 3.(\tau^4_{i_{L,R}})^2.\tau^{23}_{i_{L,R}} + \sum_{i_{L,R}} 3.\tau^4_{i_{L,R}}.(\tau^{23}_{i_{L,R}})^2 \,,
\end{aligned} \tag{6.38}$$

for either the left, i_L, or the right, i_R, handed members. Table 6.7 demonstrates clearly (last column) that $\sum_{i_L} (\tau^4_{i_L})^3 = 0$, when the contribution of the left (right) handed spinors and antispinors are taken into account.

Table 6.7 also demonstrates (the last but one column) that $\sum_{i_L} 3.(\tau^4_{i_L})^2.\tau^{23}_{i_L} = 0$ even more trivially since the contribution of either spinors or antispinors - left or right handed - separately are equal to zero.

The easiest is to evaluate $\sum_{i_{L,R}} (\tau^{23}_{i_L})^3 = 0$ and $\sum_{i_L} 3.(\tau^4_{i_L})^2.\tau^{23}_{i_L} = 0$ since, as seen from Table 6.7, the summation separately within the quarks and lepton representations give zero.

Since all the members belong to one spinor representation, it is straightforwardly that all the triangle traces are zero, if the *standard model* groups are the subgroups of the orthogonal group $SO(13,1)$.

From only the *standard model* assumptions point of view the cancellation of the triangle anomalies does look miraculously. For our $\sum_{i_{L,R}} (Y_{i_{L,R}})^3$ one obtains for the left handed members: $[3.2.(\frac{1}{6})^3 + 2.(-\frac{1}{2})^3 + 3.((-\frac{2}{3})^3 + (\frac{1}{3})^3) + 1^3)\ ]$, and for the right handed members: $[3.((\frac{2}{3})^3 + (-\frac{1}{3})^3) + (-1)^3) + 3.2.(-\frac{1}{6})^3 + 2.(\frac{1}{2})^3\ ]$.

6.4.4 Dark matter in the *spin-charge-family* theory

As discussed in Sect. 6.2 the *spin-charge-family* theory [5,3,6–10,4,11–17,2,1] predicts in the low energy region two decoupled groups of four families. In Ref. [13] the possibility that the dark matter consists of clusters of the fifth family - the stable heavy family of quarks and leptons (with (amost) zero Yukawa couplings to the lower group of four families) - is discussed.

I presented here a very short overview through the estimation done in Ref. [13]. In this reference we made a rough, but to our knowledge a reasonable, estimation of the properties of baryons of this fifth family members, following the behaviour of quarks during the evolution of the universe up to 1 GeV and through the colour phase transition. We also estimate the behaviour of the neutral clusters when scattering among themselves and with the ordinary matter. We studied possible limitations on the family properties due to the cosmological evidences, the direct experimental evidences ([13], Sect. IV) and all others known properties of the dark matter.

We used the simple hydrogen-like model to evaluate the properties of these heavy baryons and their interaction among themselves and with the ordinary nuclei, taking into account that for masses of the order of 1 TeV or larger the one gluon exchange determines the force among the constituents of the fifth family baryons ([13], Sect. II). Due to their very large masses "the nuclear interaction" among these baryons has very interesting properties. We concluded that it is the fifth family neutron, which is very probably the most stable nucleon.

We followed the behaviour of the fifth family quarks and antiquarks in the plasma of the expanding universe, through the freezing out procedure solving the Boltzmann equations, through the colour phase transition, while forming neutrons, up to the present dark matter ([13], Sect. III).

The cosmological evolution suggested the limits for the masses of the fifth family quarks

$$10\ \mathrm{TeV} < m_{q_5}\, c^2 < \text{a few} \cdot 10^2\ \mathrm{TeV} \tag{6.39}$$

and for the scattering cross sections

$$10^{-8}\,\mathrm{fm}^2 \ < \sigma_{c_5} \ < 10^{-6}\,\mathrm{fm}^2\,, \tag{6.40}$$

while the measured density of the dark matter does not put much limitation on the properties of heavy enough clusters.

The direct measurements limited the fifth family quark mass to ([13], Sect. IV.)

$$\text{several}\ 10\,\mathrm{TeV} < m_{q_5} c^2 < 10^5\ \mathrm{TeV}\,. \tag{6.41}$$

We also find that our fifth family baryons of the mass of a few $10\ \mathrm{TeV}/c^2$ have for a factor more than 100 times too small scattering amplitude with the ordinary matter to cause a measurable heat flux on the Earth's surface.

6.4.5 Matter-antimatter in the *spin-charge-family* theory

I shortly overview in this section the properties, quantum numbers, and discrete symmetries of those scalar and vector gauge fields appearing in the starting action (Eqs. (6.1, 6.2) which cause transitions of antileptons into quarks and back, and antiquarks into quarks and back. The appearance of the condensate breaks this symmetry making possible under non thermal conditions the ordinary (mostly made of the first family members) matter-antimatter asymmetry. The reader can find details in Ref. [2].

Scalar gauge fields, contributing to matter-antimatter asymmetry and causing also the proton decay carry the triplet or antitriplet colour charges (see Table 6.8) and the fractional hyper and electromagnetic charge.

The Lagrange densities from Eqs. (6.1, 6.2) manifest $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ invariance. All the vector and the spinor gauge fields are before the appearance of the condensate (Subsect. 6.3.4) massless and reactions creating particles from antiparticles and back go in both directions equivalently. Correspondingly there is no matter-antimatter asymmetry. It is the condensate, which breaks this symmetry

Let me analyse the Lagrange density of Eq. (6.2) before the appearance of the condensate. The term $\gamma^t \, \frac{1}{2}\, S^{s's"}\, \omega_{s's"t}$ can be rewritten as follows

$$
\begin{aligned}
\gamma^t \, \frac{1}{2}\, S^{s's"}\, \omega_{s's"t} &= \sum_{+,-} \sum_{(t\,t')} \overset{tt'}{(\oplus)} \, \frac{1}{2}\, S^{s's"}\, \omega_{s"s"\overset{tt'}{(\oplus)}}\,,\\
\omega_{s"s"\overset{tt'}{(\oplus)}} : &= \omega_{s"s"\overset{tt'}{(\pm)}} = (\omega_{s's"t} \mp i\, \omega_{s's"t'})\,,\\
\overset{tt'}{(\oplus)}: &= \overset{tt'}{(\pm)} = \frac{1}{2}\,(\gamma^t \pm \gamma^{t'})\,,\\
(t\,t') &\in ((9\,10), (11\,12), (13\,14))\,.
\end{aligned}
\tag{6.42}
$$

I introduced the notations $\overset{tt'}{(\oplus)}$ and $\omega_{s"s"\overset{tt'}{(\oplus)}}$ to distinguish among different superposition of states in equations below.

The expression $\overset{tt'}{(\oplus)}\; \frac{1}{2}\, S^{s's"}\; \omega_{s"s"\overset{tt'}{(\oplus)}}$ can be further rewritten as follows

$$
\begin{aligned}
&\overset{tt'}{(\oplus)} \quad \frac{1}{2}\, S^{s's"}\, \omega_{s"s"\overset{tt'}{(\oplus)}} = \\
&\overset{tt'}{(\oplus)} \quad \{\tau^{2+}\, A^{2+}_{\overset{tt'}{(\oplus)}} + \tau^{2-}\, A^{2-}_{\overset{tt'}{(\oplus)}} + \tau^{23}\, A^{23}_{\overset{tt'}{(\oplus)}} + \tau^{1+}\, A^{1+}_{\overset{tt'}{(\oplus)}} + \tau^{1-}\, A^{1-}_{\overset{tt'}{(\oplus)}} + \tau^{13}\, A^{13}_{\overset{tt'}{(\oplus)}}\,\}\,,\\
&A^{2\boxplus}_{\overset{tt'}{(\oplus)}} = (\omega_{58\overset{tt'}{(\oplus)}} + \omega_{67\overset{tt'}{(\oplus)}}) \boxplus i(\omega_{57\overset{tt'}{(\oplus)}} - \omega_{68\overset{tt'}{(\oplus)}})\,,\\
&A^{23}_{\overset{tt'}{(\oplus)}} = (\omega_{56\overset{tt'}{(\oplus)}} + \omega_{78\overset{tt'}{(\oplus)}})\,,\\
&A^{1\boxplus}_{\overset{tt'}{(\oplus)}} = (\omega_{58\overset{tt'}{(\oplus)}} - \omega_{67\overset{tt'}{(\oplus)}}) \boxplus i(\omega_{57\overset{tt'}{(\oplus)}} + \omega_{68\overset{tt'}{(\oplus)}})\,,\\
&A^{13}_{\overset{tt'}{(\oplus)}} = (\omega_{56\overset{tt'}{(\oplus)}} - \omega_{78\overset{tt'}{(\oplus)}})\,.
\end{aligned}
\tag{6.43}
$$

Equivalently one expresses the term $\gamma^t \, \frac{1}{2} \, \tilde{S}^{ab} \, \tilde{\omega}_{abt}$ in Eq. (6.2) with $\tilde{S}^{ab}$ as the infinitesimal generators of either $\widetilde{SO}(3,1)$ or $\widetilde{SO}(4)$ and $\tilde{\omega}_{abt}$ belonging to the corresponding gauge fields with $t = (9, \ldots, 14)$, by using Eqs. (6.70 - 6.73), as

$$
\begin{aligned}
\gamma^t \frac{1}{2} \, \tilde{S}^{ab} \, \tilde{\omega}_{abt} =& \overset{tt'}{(\bigoplus)} \; \frac{1}{2} \, \tilde{S}^{ab} \, \tilde{\omega}_{ab \substack{tt' \\ (\oplus)}} = \\
& \overset{tt'}{(\bigoplus)} \; \{ \tilde{\tau}^{2+} \, \tilde{A}^{2+}_{\substack{tt' \\ (\oplus)}} + \tilde{\tau}^{2-} \, \tilde{A}^{2-}_{\substack{tt' \\ (\oplus)}} + \tilde{\tau}^{23} \, \tilde{A}^{23}_{\substack{tt' \\ (\oplus)}} + \\
& \tilde{\tau}^{1+} \, \tilde{A}^{1+}_{\substack{tt' \\ (\oplus)}} + \tilde{\tau}^{1-} \, \tilde{A}^{1-}_{\substack{tt' \\ (\oplus)}} + \tilde{\tau}^{13} \, \tilde{A}^{13}_{\substack{tt' \\ (\oplus)}} + \\
& \tilde{N}^{+}_{R} \, \tilde{A}^{N_R +}_{\substack{tt' \\ (\oplus)}} + \tilde{N}^{-}_{R} \, \tilde{A}^{N_R -}_{\substack{tt' \\ (\oplus)}} + \tilde{N}^{3}_{R} \, \tilde{A}^{N_R 3}_{\substack{tt' \\ (\oplus)}} + \\
& \tilde{N}^{+}_{L} \, \tilde{A}^{N_L +}_{\substack{tt' \\ (\oplus)}} + \tilde{N}^{-}_{L} \, \tilde{A}^{N_L -}_{\substack{tt' \\ (\oplus)}} + \tilde{N}^{3}_{L} \, \tilde{A}^{N_L 3}_{\substack{tt' \\ (\oplus)}} \}, \\
\tilde{A}^{N_R \pm}_{\substack{tt' \\ (\oplus)}} =& (\tilde{\omega}_{23 \substack{tt' \\ (\oplus)}} - i \, \tilde{\omega}_{01 \substack{tt' \\ (\oplus)}}) \mp i(\tilde{\omega}_{31 \substack{tt' \\ (\oplus)}} - i \, \tilde{\omega}_{02 \substack{tt' \\ (\oplus)}}), \\
\tilde{A}^{N_R 3}_{\substack{tt' \\ (\oplus)}} =& (\tilde{\omega}_{12 \substack{tt' \\ (\oplus)}} - i \, \tilde{\omega}_{03 \substack{tt' \\ (\oplus)}}), \\
\tilde{A}^{N_L \pm}_{\substack{tt' \\ (\oplus)}} =& (\tilde{\omega}_{23 \substack{tt' \\ (\oplus)}} + i \, \tilde{\omega}_{01 \substack{tt' \\ (\oplus)}}) \mp i(\tilde{\omega}_{31 \substack{tt' \\ (\oplus)}} + i \, \tilde{\omega}_{02 \substack{tt' \\ (\oplus)}}), \\
\tilde{A}^{N_R 3}_{\substack{tt' \\ (\oplus)}} =& (\tilde{\omega}_{12 \substack{tt' \\ (\oplus)}} + i \, \tilde{\omega}_{03 \substack{tt' \\ (\oplus)}}).
\end{aligned}
\tag{6.44}
$$

The expressions for

$$
\tilde{A}^{2\pm}_{\substack{tt' \\ (\oplus)}}, \tilde{A}^{23}_{\substack{tt' \\ (\oplus)}}, \tilde{A}^{1\pm}_{\substack{tt' \\ (\oplus)}} \text{ and } \tilde{A}^{13}_{\substack{tt' \\ (\oplus)}}
$$

can easily be obtained from Eq.(6.43) by replacing in expressions for

$$
A^{2\pm}_{\substack{tt' \\ (\oplus)}}, A^{23}_{\substack{tt' \\ (\oplus)}}, A^{1\pm}_{\substack{tt' \\ (\oplus)}} \text{ and } A^{13}_{\substack{tt' \\ (\oplus)}},
$$

respectively, $\omega_{s's" \substack{tt' \\ (\oplus)}}$ by $\tilde{\omega}_{s's" \substack{tt' \\ (\oplus)}}$.

The term $\gamma^t \, \frac{1}{2} \, S^{t't"} \, \omega_{t't"t}$ in Eq. (6.2) can be rewritten with respect to the generators $S^{t't"}$ and the corresponding gauge fields $\omega_{s's"t}$ as one colour octet scalar field and one $U(1)_{II}$ singlet scalar field (Eq. 6.69)

$$
\begin{aligned}
\gamma^t \, \frac{1}{2} \, S^{t"t'"} \, \omega_{t"t'"t} = \sum_{+,-} \sum_{(t\,t')} & \overset{tt'}{(\bigoplus)} \; \{ \vec{\tau}^{3} \cdot \vec{A}^{3}_{\substack{tt' \\ (\oplus)}} + \tau^{4} \cdot A^{4}_{\substack{tt' \\ (\oplus)}} \}, \\
& (t\,t') \in ((9\,10), 11\,12), 13\,14)) \, .
\end{aligned}
\tag{6.45}
$$

Considering all the above equations (6.42 - 6.45), and leaving out $p_{\overset{tt'}{(\oplus)}}$ since in the low energy limit the momentum does not play any role, it follows

$$
\begin{aligned}
\mathcal{L}_{f"} = \psi^\dagger \gamma^0 (-) \{ \sum_{+,-} \sum_{(t\,t')} \overset{tt'}{(\oplus)} \cdot \\
&[\tau^{2+} A^{2+}_{\overset{tt'}{(\oplus)}} + \tau^{2-} A^{2-}_{\overset{tt'}{(\oplus)}} + \tau^{23} A^{23}_{\overset{tt'}{(\oplus)}} \\
&+ \tau^{1+} A^{1+}_{\overset{tt'}{(\oplus)}} + \tau^{1-} A^{1-}_{\overset{tt'}{(\oplus)}} + \tau^{13} A^{13}_{\overset{tt'}{(\oplus)}} \\
&+ \tilde{\tau}^{2+} \tilde{A}^{2+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{2-} \tilde{A}^{2-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{23} \tilde{A}^{23}_{\overset{tt'}{(\oplus)}} \\
&+ \tilde{\tau}^{1+} \tilde{A}^{1+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{1-} \tilde{A}^{1-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{13} \tilde{A}^{13}_{\overset{tt'}{(\oplus)}} \\
&+ \tilde{N}^{+}_{R} \tilde{A}^{N_R+}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{R} \tilde{A}^{N_R-}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{R} \tilde{A}^{N_R3}_{\overset{tt'}{(\oplus)}} \\
&+ \tilde{N}^{+}_{L} \tilde{A}^{N_L+}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{L} \tilde{A}^{N_L-}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{L} \tilde{A}^{N_L3}_{\overset{tt'}{(\oplus)}} \\
&+ \sum_{i} \tau^{3i} A^{3i}_{\overset{tt'}{(\oplus)}} + \tau^{4} A^{4}_{\overset{tt'}{(\oplus)}} \\
&+ \sum_{i} \tilde{\tau}^{3i} \tilde{A}^{3i}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{4} \tilde{A}^{4}_{\overset{tt'}{(\oplus)}}]\}\psi \,,
\end{aligned}
\tag{6.46}
$$

where (t, t') run in pairs over $[(9, 10), \ldots (13, 14)]$ and the summation must go over $+$ and $-$ of $\overset{tt'}{(\oplus)}$.

On Table 6.8, taken from Ref. [2], the quantum numbers of the scalar and vector gauge fields, appearing in Eq. (6.2), are presented, where it is taken into account that the spin of gauge fields is determined according to Eq. (6.47),

$$
(S^{ab})^{c}{}_{e}\, A^{d\ldots e \ldots g} = i\,(\eta^{ac}\delta^{b}_{e} - \eta^{bc}\delta^{a}_{e})\, A^{d\ldots e\ldots g}\,, \tag{6.47}
$$

for each index ($\in (d \ldots g)$) of a bosonic field $A^{d\cdots g}$ separately. We must take into account also the relation among S^{ab} and the charges (the relations are, of course, the same for bosons and fermions), presented in Eqs. (6.67, 6.68, 6.69)). The scalar fields with the scalar index $s = (9, 10, \cdots, 14)$, presented in Table 6.8, carry one of the triplet colour charges and the "spinor" charge equal to twice the quark "spinor" charge, or the antitriplet colour charges and the anti "spinor" charge. They carry in addition the quantum numbers of the adjoint representations originating in S^{ab} or in $\tilde{S}^{ab}$ [14].

Let us choose the 57^{th} line of Table 6.2, which represents in the spinor technique the left handed positron, $\bar{e}^{+}_{L}$, to see what do the scalar fields, appearing in Eq. (6.46) and in Table 6.8, do when applying on the left handed members of the

[14] Although carrying the colour charge in one of the triplet or antitriplet states, these fields can not be interpreted as superpartners of the quarks since they do not have quantum numbers as required by, let say, the $N = 1$ supersymmetry. The hyper charges and the electromagnetic charges are namely not those required by the supersymmetric partners to the family members.

field	prop.	τ^4	τ^{13}	τ^{23}	(τ^{33}, τ^{38})	Y	Q	$\tilde\tau^4$	$\tilde\tau^{13}$	$\tilde\tau^{23}$	$\tilde N^3_L$	$\tilde N^3_R$
$A^{1\boxed{\pm}}_{9\,10}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	$\boxed{\pm}\,1$	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}+\boxed{\pm}\,1$	0	0	0	0	0
$A^{13}_{9\,10}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$A^{1\boxed{\pm}}_{11\,12}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	$\boxed{\pm}\,1$	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}+\boxed{\pm}\,1$	0	0	0	0	0
$A^{13}_{11\,12}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$A^{1\boxed{\pm}}_{13\,14}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	$\boxed{\pm}\,1$	0	$(0, \oplus\,\frac{1}{\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}+\boxed{\pm}\,1$	0	0	0	0	0
$A^{13}_{13\,14}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(0, \oplus\,\frac{1}{\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$A^{2\boxed{\pm}}_{9\,10}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	0	$\boxed{\pm}\,1$	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}+\boxed{\pm}\,1$	$\oplus\,\frac{1}{3}+\boxed{\pm}\,1$	0	0	0	0	0
$A^{23}_{9\,10}\,(\oplus)$ $\cdots$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$\tilde A^{1\boxed{\pm}}_{9\,10}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	$\boxed{\pm}\,1$	0	0	0
$\tilde A^{13}_{9\,10}\,(\oplus)$ $\cdots$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$\tilde A^{2\boxed{\pm}}_{9\,10}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	$\boxed{\pm}\,1$	0	0
$\tilde A^{23}_{9\,10}\,(\oplus)$ $\cdots$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$\tilde A^{N_L\boxed{\pm}}_{9\,10}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	$\boxed{\pm}\,1$	0
$\tilde A^{N_L 3}_{9\,10}\,(\oplus)$ $\cdots$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$\tilde A^{N_R\boxed{\pm}}_{9\,10}\,(\oplus)$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	$\boxed{\pm}\,1$
$\tilde A^{N_R 3}_{9\,10}\,(\oplus)$ $\cdots$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$A^{3i}_{9\,10}\,(\oplus)$ $\cdots$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\boxed{\pm}\,1+\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$A^{4}_{9\,10}\,(\oplus)$ $\cdots$	scalar	$\oplus\,\frac{1}{3}$	0	0	$(\oplus\,\frac{1}{2}, \oplus\,\frac{1}{2\sqrt{3}})$	$\oplus\,\frac{1}{3}$	$\oplus\,\frac{1}{3}$	0	0	0	0	0
$\vec{A}^{3}_{m}$	vector	0	0	0	octet	0	0	0	0	0	0	0
A^{4}_{m}	vector	0	0	0	0	0	0	0	0	0	0	0

Table 6.8. Quantum numbers of the scalar gauge fields carrying the space index $t = (9, 10, \cdots, 14)$, appearing in Eq. (6.2), are presented. To the colour charge of all these scalar fields the space degrees of freedom contribute one of the triplets values. These scalars are with respect to the two $SU(2)$ charges, (τ^{13} and $\vec{\tau}^2$), and the two $\widetilde{SU}(2)$ charges, ($\vec{\tilde\tau}^1$ and $\vec{\tilde\tau}^2$), triplets (that is in the adjoint representations of the corresponding groups), and they all carry twice the "spinor" number (τ^4) of the quarks. The quantum numbers of the two vector gauge fields, the colour and the $U(1)_{II}$ ones, are added.

Weyl representation presented on Table 6.2, containing quarks and leptons and antiquarks and antileptons [9,10,36].

If we make, let say, the choice of the term ($\gamma^0 \stackrel{910}{(+)} \tau^{2\boxminus}$) $A^{2\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ (the scalar field $A^{2\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ is presented in the 7^{th} line in Table 6.8 and in the second line of Eq. (6.46)), the family quantum numbers will not be affected and they can be any. The state carries the "spinor" (lepton) number $\tau^4 = \frac{1}{2}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = \frac{1}{2}$ and the colour charge $(\tau^{33}, \tau^{38}) = (0, 0)$. Correspondingly, its hyper charge $(Y (= \tau^4 + \tau^{23}))$ is 1 and the electromagnetic charge $Q(= Y + \tau^{13})$ is 1.

So, what does the term $\gamma^0 \stackrel{910}{(+)} \tau^{2\boxminus}$ $A^{2\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ make on this spinor $\bar{e}^{+}_{L}$? Making use of Eqs. (6.57, 6.59, 6.73) of appendix 6.6 one easily finds that operator $\gamma^0 \stackrel{910}{(+)} \tau^{2-}$ transforms the left handed positron into $\stackrel{03}{(+i)}\stackrel{12}{(+)} | \stackrel{56}{[-]}\stackrel{78}{[-]} || \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$, which is d^{c1}_{R}, presented on line 3 of Table 6.2. Namely, γ^0 transforms $\stackrel{03}{[-i]}$ into $\stackrel{03}{(+i)}$, $\stackrel{910}{(+)}$ transforms $\stackrel{9\,10}{[-]}$ into $\stackrel{9\,10}{(+)}$, while τ^{2-} $(= -\stackrel{56}{(-)}\stackrel{78}{(-)})$ transforms $\stackrel{56}{(+)}\stackrel{78}{(+)}$ into $\stackrel{56}{[-]}\stackrel{78}{[-]}$. The state d^{c1}_{R} carries the "spinor" (quark) number $\tau^4 = \frac{1}{6}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = -\frac{1}{2}$ and the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$. Correspondingly its hyper charge is $(Y = \tau^4 + \tau^{23} =) -\frac{1}{3}$ and the electromagnetic charge $(Q = Y + \tau^{13} =) -\frac{1}{3}$. The scalar field $A^{2\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ carries just the needed quantum numbers as we can see in the 7^{th} line of Table 6.8.

If the antiquark $\bar{u}^{\bar{c}2}_{L}$, from the line 43 (it is not presented, but one can very easily construct it) in Table 6.2, with the "spinor" charge $\tau^4 = -\frac{1}{6}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = -\frac{1}{2}$, the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, -\frac{1}{2\sqrt{3}})$, the hyper charge $Y(= \tau^4 + \tau^{23} =) -\frac{2}{3}$ and the electromagnetic charge $Q(= Y + \tau^{13} =) -\frac{2}{3}$ submits the $A^{2\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ scalar field, it transforms into u^{c3}_{R} from the line 17 of Table 6.2, carrying the quantum numbers $\tau^4 = \frac{1}{6}$, $\tau^{13} = 0$, $\tau^{23} = \frac{1}{2}$, $(\tau^{33}, \tau^{38}) = (0, -\frac{1}{\sqrt{3}})$, $Y = \frac{2}{3}$ and $Q = \frac{2}{3}$. These two quarks, d^{c1}_{R} and u^{c3}_{R} can bind together with u^{c2}_{R} from the 9^{th} line of the same table (at low enough energy, after the electroweak transition, and if they belong to a superposition with the left handed partners to the first family) into the colour chargeless baryon - a proton. This transition is presented in Figure 6.2.

The opposite transition at low energies would make the proton decay.

Similar transitions go also with other scalars from Eq. (6.46) and Table 6.8. The $\vec{\tilde{A}}^{1}_{\stackrel{t',t''}{(+)}}$, $\vec{\tilde{A}}^{2}_{\stackrel{t',t''}{(+)}}$, $\vec{\tilde{A}}^{N_L}_{\stackrel{t',t''}{(+)}}$ and $\vec{\tilde{A}}^{N_L}_{\stackrel{t',t''}{(+)}}$ fields cause transitions among the family members, changing a particular member into the antimember of another colour and of another family. The term $\gamma^0 \stackrel{910}{(+)}$ $\tilde{N}^{-}_{R}$ $A^{\tilde{N}_R -}_{\stackrel{9\,10}{(\oplus)}}$ transforms $\bar{e}^{+}_{R}$ into u^{c1}_{L}, changing the family quantum numbers.

The action from Eqs. (6.1, 6.2) manifests $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ invariance. All the vector and the spinor gauge fields are massless.

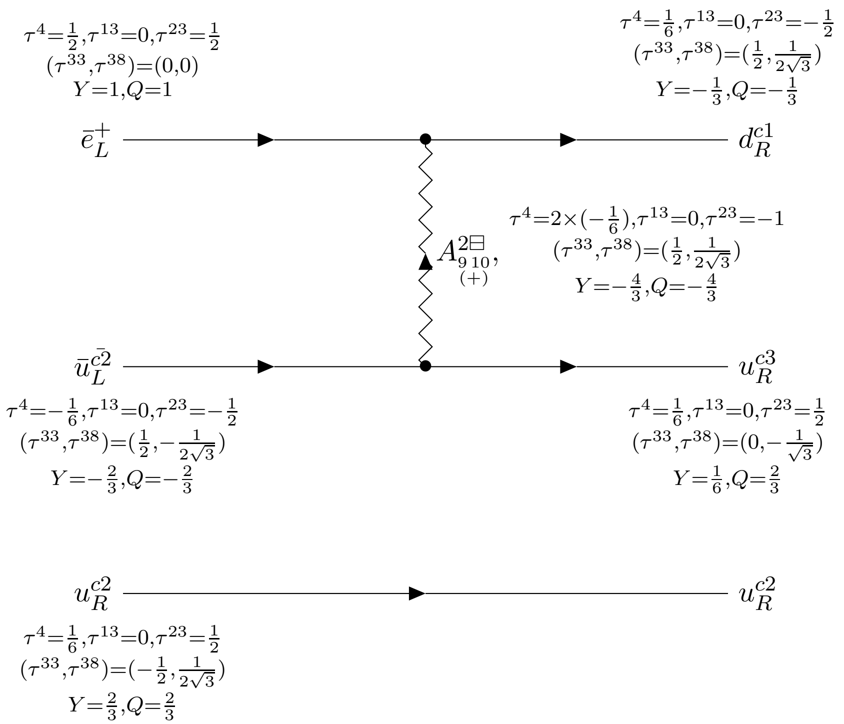

Fig. 6.2. The birth of a "right handed proton" out of an positron $\bar{e}_L^{+}$, antiquark $\bar{u}_L^{\bar{c}2}$ and quark (spectator) u_R^{c2}. The family quantum number can be any.

Since none of the scalar fields from Table 6.8 have been observed and also no vector gauge fields like $\vec{A}^2_m$, A^4_m and other scalar and vector fields, it must exist a mechanism, which makes the non observed scalar and vector gauge fields massive enough.

Scalar fields from Table 6.8 carry the colour and the electromagnetic charge. Therefore their nonzero vacuum expectation values would not be in agreement with the observed phenomena. One, however, notices that all the scalar gauge fields from Table 6.8 and several other scalar and vector gauge fieldscouple to the condensate with the nonzero quantum number τ^4 and τ^{23} and nonzero family quantum numbers.

It is not difficult to recognize that the desired condensate must have spin zero, $Y = \tau^4 + \tau^{23} = 0$, $Q = Y + \tau^{13} = 0$ and $\vec{\tau}^1 = 0$ in order that in the low energy limit the *spin-charge-family* theory would manifest effectively as the *standard model*.

I make a choice of the two right handed neutrinos of the $\mathrm{VIII}^{\mathrm{th}}$ family coupled into a scalar, with $\tau^4 = -1$, $\tau^{23} = 1$, correspondingly $Y = 0$, $Q = 0$ and $\vec{\tau}^1 = 0$, and with family quantum numbers (Eqs. (6.71, 6.70)) $\tilde{\tau}^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{N}^3_R = 1$, and correspondingly with $\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23} = 0$, $\tilde{Q} = \tilde{Y} + \tilde{\tau}^{13} = 0$, and $\vec{\tilde{\tau}}^1 = 0$. The condensate carries the family quantum numbers of the upper four families, see Subsect. 6.3.4.

The condensate made out of spinors couples to spinors differently than to antispinors - "anticondensate" would namely carry $\tau^4 = 1$, and $\tau^{23} = -1$ - breaking correspondingly the $\mathbb{C}_{\mathcal{N}}\cdot \mathcal{P}_{\mathcal{N}}$ symmetry: The reactions creating particles from antiparticles are not any longer symmetric to those creating antiparticle from particles.

Such a condensate leaves the hyper field A^{Y}_{m} $(=\sin\vartheta_2\, A^{23}_{m} + \cos\vartheta_2\, A^{4}_{m})$ (for the choice that $\sin\vartheta_2 = \cos\vartheta_2$ and $g^4 = g^2$, there is no justification for such a choice, $A^{Y}_{m} = \frac{1}{\sqrt{2}}\,(A^{23}_{m} + A^{4}_{m})$ follows) massless, while it gives masses to $A^{2\pm}_{m}$ and $A^{Y'}_{m}$ $(= \frac{1}{\sqrt{2}}\,(A^{4}_{m} - A^{23}_{m})$ for $\sin\vartheta_2 = \cos\vartheta_2)$ and it gives masses also to all the scalar gauge fields from Table 6.8, since they all couple to the condensate through τ^4.

The weak vector gauge fields, $\vec{A}^{1}_{m}$, the hyper charge vector gauge fields, A^{Y}_{m}, and the colour vector gauge fields, $\vec{A}^{1}_{m}$, remain massless.

The scalar fields with the scalar space index $s = (7, 8)$ (there are three singlets which couple to all eight families, two triplets which couple only to the upper four families and another two triplets which couple only to the lower four families) - carrying the weak and the hyper charges of the Higgs's scalar - wait for gaining nonzero vacuum expectation values to change their masses while causing the electroweak break.

The condensate does what is needed so that in the low energy regime the *spin-charge-family* manifests as an effective theory which agrees with the *standard model* to such an extent that it is in agreement with the observed phenomena, explaining the *standard model* assumptions and predicting new fermion and boson fields.

It also may hopefully explain the observed matter-antimatter asymmetry if the conditions in the expanding universe would be appropriate, Ref. ([2], Sect. VI.). The work needed to check these conditions in the expanding universe within the *spin-charge-family* theory is very demanding. Although we do have some experience with following the history of the expanding universe [13], this study needs much more efforts, not only in calculations, but also in understanding the mechanism of the condensate appearance, relations among the velocity of the expansion, the temperature and the dimension of space-time in the period of the appearance of the condensate. This study has not yet been really started.

6.5 Conclusions

To better understand the history of the universe and also to make next step in understanding the dynamics of the elementary fermion fields and boson (vector and scalar) gauge fields it is needed to explain the assumptions of the *standard model*, as well as the phenomena like the existence of the dark matter, matter-antimatter asymmetry and dark energy.

There must be understood the origin of: **A.** the family members quantum numbers, **B.** the family quantum numbers, **C.** the origin of vector gauge fields, **D.** the origin of the higgs and Yukawa couplings.

One of the most urgent questions in the elementary particle physics is: *Where do the families originate?* Explaining the origin of families would answer the question about the number of families which are possibly observable at the low energy regime, about the origin of the scalar field(s) and the Yukawa couplings (the couplings of fermions to the scalar field(s)), about the differences in the fermions properties - the differences in the masses and mixing matrices among family members – quarks and leptons, as well as about the hierarchy in quark and lepton masses.

I demonstrated in this talk, that the *spin-charge-family* theory - starting with the simple action in $d = (13 + 1)$ for fermions and bosons - offers the explanation for all the assumptions of the *standard model*:

a. The theory explains all the properties of the family members - quarks and leptons, left and right handed, and their right and left handed antiquarks and antileptons [15], explaining why the left handed spinors carry the weak charge while the right handed do not (the right handed neutrino is the regular member of each family).

b. It explains the appearance and the properties of the families of family members.

c. It explains the existence of the gauge vector fields of the family members charges.

d. It explains the appearance and the properties of the scalar field (the higgs) and the Yukawa couplings.

All the gauge fields, vector and scalar in $d = (3 + 1)$, origin in vielbeins and the two kinds of spin connection fields in $d = (13 + 1)$ - the gravity. If there are no spinors present, are the two spin connections expressible uniquely with the vielbeins [42].

It also offers the explanation for the phenomena, which are not part of the *standard model*, like:

e. It explains the existence of the dark matter.

f. It explain the origin of the (ordinary) matter-antimatter asymmetry.

6.5.1 Predictions for the future experiments

The theory predicts:

g. There are twice two groups of four families of quarks and leptons at low energies.

g.i. The fourth family with masses above 1 TeV, weakly coupled to the observed three families, will be measured at the LHC.

g.ii. The quarks and leptons of the fifth family - that is of the stable one of the upper four families - form the dark matter. The family members, which form the chargeless clusters, manifest, due to their very heavy masses, a ”new nuclear force”.

[15] One Weyl representation of $SO(13 + 1)$ contains, if analyzed with respect to the *standard model* groups, all the members of one family, the coloured quarks and colourless leptons, and the anticoloured antiquarks and (anti)colourless antileptons, with the left handed leptons carrying the weak charge and the right handed ones weak chargeless, while the left handed antispinors are weak chargeless and the right handed ones carry the weak charge.

h. The predicted scalar fields with the space index $(7, 8)$ manifest in $(d = (3 + 1))$ as the weak and hyper charges doublets (as required by the *standard model* higgs) with respect to the space index. These scalars carry in addition:
h.i. Either they carry one of the three family members quantum numbers, (Q, Q', Y') - belonging correspondingly to one of three singlets.
h.ii. Or they carry family quantum numbers - belonging correspondingly to one of the twice two triplets.
h.iii. The three singlets and the two triplets determine mass matrices of the lower four families, contributing to masses of the heavy vector bosons.
h.iv. These scalars determine the observed higgs and the Yukawa couplings.
i. The predicted scalar fields with the space index $(9, 10, .., 14)$ are triplets with respect to the space index. They cause the transitions from antileptons into quarks and antiquarks into quarks and back.
i.i. The condensate breaks the matter-antimatter symmetry, causing the asymmetry in the (ordinary) matter with respect to antimatter.
i.ii. These condensate is responsible also for the proton decay.
j. The condensate is a scalar of the two right handed neutrinos with the family quantum numbers of the upper four families.
k. The condensate gives masses to all the gauge fields with which it interacts.
k.i. It gives masses to all scalar fields and to vector fields, leaving massless only the colour, the weak, the hyper vector gauge fields and the gravity in $((3 + 1))$.
l. There is the $SU(2)$ (belonging together with the weak $SU(2)$ to $SO(4)$ gauge fields included in $SO(7, 1)$) vector gauge field, which gain masses of the order of the appearance of the condensate.
m. At the electroweak break the scalar fields with the space index $(7, 8)$ change their mutual interaction, and gaining nonzero vacuum expectation values, break the weak and the hyper charges and correspondingly the mass protection of fermions, making them massive.
n. The symmetry of mass matrices allow, in the case that the experimental data for the mixing submatrix 3×3 of the 4×4 mixing matrix would be accurate, to determine the mixing matrix and the masses of the fourth family quarks. The accuracy, with which the masses of the six lower families are measured so far, does not influence the results appreciably. Due to uncertainty of the experimental data for the 3×3 mixing submatrix we are only able to determine the 4×4 quark mixing matrix for a chosen masses of the the fourth family quarks. However, we also predict how will the 3×3 submatrix of the mixing matrix change with more accurate measurements.
n.i. The fourth family quarks mass matrices are for masses above 1 TeV closer and closer to the democratic matrices. The less the scalars with the family members quantum numbers contribute to masses of the fourth family quarks, the closer is m_{u_4} to m_{d_4}.
n.ii. The large contribution of the scalars with the family members quantum numbers (Q, Q', Y') to the masses of the lower four families manifests in the large differences of quarks masses of the lower four families.
n.iii. Although we have done calculations also for leptons, must further analyses of their properties wait for more accurate experimental data.

o. In the case that the u_4 and d_4 quarks have similar masses - determined mostly by the scalar fields carrying the family quantum numbers - they contribute mostly to the production of these scalars, while their contribution to the production of those scalars which carry the family members quantum numbers - to the higgs in particular - is much weaker, which is in agreement with the experiment [16].
p. All the degrees of freedom discussed in this talk are already a part of the simple starting action Eq.(6.1).
p.i. The way of breaking symmetries (ordered by the conditions determining the history of our universe) is assumed so that it leads in $d = (3+1)$ to the observable symmetries, although we could in principle derive it from the starting action and boundary conditions.
p.ii. Also the effective interaction among scalar fields is assumed, although we could derive it in principle from the starting action and the boundary conditions.
r. The *spin-charge-family* theory easily explains what in the *standard model* seems like a miracle: no triangle anomalies.
s. A lot of efforts has been put in this theory to show that it could work as a next step below the *standard model* proving like:
s.i. There is possibility in the Kaluza-Klein-like theory that breaking symmetries can leave fermions massless.
s.ii. That vielbeins in the Kaluza-Klein theories and spin-connections in the *spin-charge-family* theory represent the same vector gauge fields in $d = (3+1)$.

6.5.2 Open questions in the *spin-charge-family* theory

There are several open problem in the *spin-charge-family* theory:
t. Since this theory is, except that fermions carry two kinds of spins - one kind taking care of spin and charges, the second one taking care of families - a kind of the Kaluza-Klein theories, it shares at very high energy with these theories the quantization problem.
u. The dimension of space-time, $d = (13+1)$, is in the *spin-charge-family* theory chosen, since $SO(13,1)$ contains all the members, assumed in the *standard model*. It contains also the right handed neutrino (which carries the Y' quantum number.)
u.i. It should be shown, however, how has nature "made the decision" in evolution to go through this dimension and what is indeed the dimension of space-time (infinite?).
v. There are many other open question, like:
v.i. What is the reason for the (so small) dark energy?
v.ii. At what energy the electroweak phase transition occurs? (Let me add that there is the answer within the *spin-charge-family* theory to this open problem.)
v.iii. Why do we have fermions and bosons? [43]

It is encouraging that the more work is done on this theory the more answers to the open questions is found.

[16] The coupling constants of the singlet scalar fields differ among themselves and also from the coupling constants of the two triplet scalar fields

6.6 Appendix: Short presentation of spinor technique [1,7,31,32]

This appendix is a short review (taken from [4]) of the technique [7,33,31,32], initiated and developed in Ref. [7], while proposing the *spin-charge-family* theory [5,3,6–10,4,11–17,2,1]. All the internal degrees of freedom of spinors, with family quantum numbers included, are describable in the space of d-anticommuting (Grassmann) coordinates [7], if the dimension of ordinary space is also d. There are two kinds of operators in the Grassmann space fulfilling the Clifford algebra and anticommuting with one another Eq.(6.48). The technique was further developed in the present shape together with H.B. Nielsen [33,31,32].

In this last stage we rewrite a spinor basis, written in Ref. [7] as products of polynomials of Grassmann coordinates of odd and even Grassmann character, chosen to be eigenstates of the Cartan subalgebra defined by the two kinds of the Clifford algebra objects, as products of nilpotents and projections, formed as odd and even objects of γ^a's, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s.

The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

Ref. [1], App. B, briefly represents the starting point [7] of this technique. There are two kinds of the Clifford algebra objects, γ^a's and $\tilde{\gamma}^a$'s.

These objects have properties,

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \quad , \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,. \tag{6.48}$$

If B is a Clifford algebra object, let say a polynomial of γ^a, $B = a_0 + a_a\,\gamma^a + a_{ab}\,\gamma^a\gamma^b + \cdots + a_{a_1 a_2 \ldots a_d}\,\gamma^{a_1}\gamma^{a_2}\ldots\gamma^{a_d}$, one finds

$$\begin{aligned}
&(\tilde{\gamma}^a B : = i(-)^{n_B}\, B\gamma^a\,)\, |\psi_0\, >, \\
&\qquad B = a_0 + a_{a_0}\gamma^{a_0} + a_{a_1 a_2}\gamma^{a_1}\gamma^{a_2} + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\cdots\gamma^{a_d}\,,
\end{aligned} \tag{6.49}$$

where $|\psi_0>$ is a vacuum state, defined in Eq. (6.63) and $(-)^{n_B}$ is equal to 1 for the term in the polynomial which has an even number of γ^b's, and to -1 for the term with an odd number of γ^b's, for any d, even or odd, and I is the unit element in the Clifford algebra.

It follows from Eq. (6.49) that the two kinds of the Clifford algebra objects are connected with the left and the right multiplication of any Clifford algebra objects B (Eq. (6.49)).

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$\begin{aligned}
S^{ab} &: = (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,, \\
\tilde{S}^{ab} &: = (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,,
\end{aligned} \tag{6.50}$$

$\{S^{ab}, \tilde{S}^{cd}\}_- = 0, \{S^{ab}, S^{cd}\}_- = i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac}), \{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- = i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})$.

We assume the "Hermiticity" property for γ^a's

$$\gamma^{a\dagger} = \eta^{aa}\gamma^a \,, \tag{6.51}$$

in order that γ^a are compatible with (6.48) and formally unitary, i.e. $\gamma^{a\,\dagger}\,\gamma^a = I$.

One finds from Eq. (6.51) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq.(6.50) that the two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$\begin{aligned}
S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4,\\
S^{03}, S^{12}, \cdots, S^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,,\\
\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4\,,\\
\tilde{S}^{03}, \tilde{S}^{12}, \cdots, \tilde{S}^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,.
\end{aligned} \tag{6.52}$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d

$$\begin{aligned}
\Gamma^{(d)} :&= (i)^{d/2} \qquad \prod_a \quad (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n,\\
\Gamma^{(d)} :&= (i)^{(d-1)/2} \prod_a \quad (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n+1\,.
\end{aligned} \tag{6.53}$$

One proceeds equivalently for $\tilde{\Gamma}^{(d)}$, substituting γ^a's by $\tilde{\gamma}^a$'s. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1\cdots\gamma^d$. It follows from Eq.(6.51) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma,\ \Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$) , while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows

$$\stackrel{ab}{(k)}:= \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \stackrel{ab}{[k]}:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,, \tag{6.54}$$

where $k^2 = \eta^{aa}\eta^{bb}$. It follows then

$$\begin{aligned}
\gamma^a &= \stackrel{ab}{(k)} + \stackrel{ab}{(-k)}\,, \qquad \gamma^b = ik\eta^{aa}\,(\stackrel{ab}{(k)} - \stackrel{ab}{(-k)})\,,\\
S^{ab} &= \frac{k}{2}(\stackrel{ab}{[k]} - \stackrel{ab}{[-k]})\,.
\end{aligned} \tag{6.55}$$

One can easily check by taking into account the Clifford algebra relation (Eq. (6.48)) and the definition of S^{ab} and $\tilde{S}^{ab}$ (Eq. (6.50)) that the nilpotent $\stackrel{ab}{(k)}$ and the projector $\stackrel{ab}{[k]}$ are "eigenstates" of S^{ab} and $\tilde{S}^{ab}$

$$\begin{aligned}
S^{ab}\ \stackrel{ab}{(k)} &= \frac{1}{2}\,k\ \stackrel{ab}{(k)}\,, \qquad S^{ab}\ \stackrel{ab}{[k]} = \frac{1}{2}\,k\ \stackrel{ab}{[k]}\,,\\
\tilde{S}^{ab}\ \stackrel{ab}{(k)} &= \frac{1}{2}\,k\ \stackrel{ab}{(k)}\,, \qquad \tilde{S}^{ab}\ \stackrel{ab}{[k]} = -\frac{1}{2}\,k\ \stackrel{ab}{[k]}\,,
\end{aligned} \tag{6.56}$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by k and $\stackrel{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}. We further recognize that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$\begin{aligned}
&\gamma^a \stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]},\; \gamma^b \stackrel{ab}{(k)} = -ik \stackrel{ab}{[-k]},\; \gamma^a \stackrel{ab}{[k]} = \stackrel{ab}{(-k)},\; \gamma^b \stackrel{ab}{[k]} = -ik\eta^{aa} \stackrel{ab}{(-k)}\,,\\
&\tilde{\gamma^a} \stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]},\; \tilde{\gamma^b} \stackrel{ab}{(k)} = -k \stackrel{ab}{[k]},\; \tilde{\gamma^a} \stackrel{ab}{[k]} = \; i \stackrel{ab}{(k)},\; \tilde{\gamma^b} \stackrel{ab}{[k]} = -k\eta^{aa} \stackrel{ab}{(k)}
\end{aligned} \tag{6.57}$$

From Eq.(6.57) it follows

$$\begin{aligned}
&S^{ac} \stackrel{ab}{(k)}\stackrel{cd}{(k)} = -\frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[-k]}\stackrel{cd}{[-k]}\,, \qquad \tilde{S}^{ac} \stackrel{ab}{(k)}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[k]}\stackrel{cd}{[k]}\,,\\
&S^{ac} \stackrel{ab}{[k]}\stackrel{cd}{[k]} = \frac{i}{2} \stackrel{ab}{(-k)}\stackrel{cd}{(-k)}\,, \qquad \tilde{S}^{ac} \stackrel{ab}{[k]}\stackrel{cd}{[k]} = -\frac{i}{2} \stackrel{ab}{(k)}\stackrel{cd}{(k)}\,,\\
&S^{ac} \stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa} \stackrel{ab}{[-k]}\stackrel{cd}{(-k)}\,, \qquad \tilde{S}^{ac} \stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa} \stackrel{ab}{[k]}\stackrel{cd}{(k)}\,,\\
&S^{ac} \stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc} \stackrel{ab}{(-k)}\stackrel{cd}{[-k]}\,, \qquad \tilde{S}^{ac} \stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc} \stackrel{ab}{(k)}\stackrel{cd}{[k]}\;.
\end{aligned} \tag{6.58}$$

From Eq. (6.58) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

Let us deduce some useful relations

$$\begin{aligned}
&\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0\,, && \stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa} \stackrel{ab}{[k]}\,, && \stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]}\,, && \stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0\,,\\
&\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}\,, && \stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0\,, && \stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0\,, && \stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]}\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0\,, && \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}\,, && \stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}\,, && \stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}\,, && \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, && \stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0\,, && \stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)}\;.
\end{aligned} \tag{6.59}$$

We recognize in Eq. (6.59) the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, respectively. Defining

$$\stackrel{ab}{(\tilde{\pm i})} = \frac{1}{2}\,(\tilde{\gamma}^a \mp \tilde{\gamma}^b)\,, \quad \stackrel{ab}{(\tilde{\pm 1})} = \frac{1}{2}\,(\tilde{\gamma}^a \pm i\tilde{\gamma}^b)\,, \tag{6.60}$$

one recognizes that

$$\stackrel{ab}{(\tilde{k})}\stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{(\tilde{-k})}\stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]}\,, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[k]} = i \stackrel{ab}{(k)}\,, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[-k]} = 0\,. \tag{6.61}$$

Recognizing that

$$\stackrel{ab}{(k)}^{\dagger} = \eta^{aa} \stackrel{ab}{(-k)}\,, \quad \stackrel{ab}{[k]}^{\dagger} = \stackrel{ab}{[k]}\,, \tag{6.62}$$

we define a vacuum state $|\psi_0>$ so that one finds

$$
\begin{aligned}
&< \stackrel{ab}{(k)}^{\dagger} \stackrel{ab}{(k)} >= 1\,,\\
&< \stackrel{ab}{[k]}^{\dagger} \stackrel{ab}{[k]} >= 1\,.
\end{aligned}
\tag{6.63}
$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of $d/2$, let us say, only nilpotents $\stackrel{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eq.(6.52)), applying it on an (unimportant) vacuum state. For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

$$
\begin{array}{c}
\stackrel{0d}{(k_{0d})}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{[-k_{12}]}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
\vdots\\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{[-k_{d-1\;d-2}]}\;|\psi_0>\\
\stackrel{od}{(k_{0d})}\stackrel{12}{[-k_{12}]}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0>\\
\vdots
\end{array}
\tag{6.64}
$$

All the states have the same handedness Γ, since $\{\Gamma, S^{ab}\}_- = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrates that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra (Eq. (6.52)) generate all the states of one representation, $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra (Eq. (6.52)) generate the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set (Eq. (6.52)) of the algebra S^{ab} and $\tilde{S}^{ab}$ $S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}$, $\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14}$, a left handed ($\Gamma^{(13,1)} = -1$) eigenstate of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hyper charge (2/3) and colour $(1/2\,, 1/(2\sqrt{3}))$, for example, can be written as

$$
\begin{aligned}
&\overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{(+)} \parallel \overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}\; |\psi_0\rangle = \\
&\frac{1}{2^7}(\gamma^0 - \gamma^3)(\gamma^1 + i\gamma^2)|(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8)|| \\
&(\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14})|\psi_0\rangle\,. \qquad (6.65)
\end{aligned}
$$

This state is an eigenstate of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (6.52)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (6.52)), generate families from the starting u_R quark, transforming the u_R quark from Eq. (6.65) to the u_R of another family, keeping all of the properties with respect to S^{ab} unchanged. In particular, $\tilde{S}^{01}$ applied on a right handed u_R-quark from Eq. (6.65) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,, 1/(2\sqrt{3}))$

$$
\tilde{S}^{01}\; \overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{(+)} \parallel \overset{9\,10}{(+)}\,\overset{11\,12}{(-)}\,\overset{13\,14}{(-)} = -\frac{i}{2}\; \overset{03}{[+i]}\overset{12}{[+]} \mid \overset{56}{(+)}\overset{78}{(+)} \parallel \overset{9\,10}{(+)}\,\overset{11\,12}{(-)}\,\overset{13\,14}{(-)}\;. \qquad (6.66)
$$

Below some useful relations [9] are presented

$$
\vec{N}_{\pm}(= \vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03})\,, \qquad (6.67)
$$

$$
\begin{aligned}
\vec{\tau}^1 &:= \frac{1}{2}(S^{58} - S^{67},\, S^{57} + S^{68},\, S^{56} - S^{78})\,, \\
\vec{\tau}^2 &:= \frac{1}{2}(S^{58} + S^{67},\, S^{57} - S^{68},\, S^{56} + S^{78})\,, \qquad (6.68)
\end{aligned}
$$

$$
\begin{aligned}
\vec{\tau}^3 :=\; & \frac{1}{2}\{S^{9\,12} - S^{10\,11}\,, S^{9\,11} + S^{10\,12},\, S^{9\,10} - S^{11\,12}, \\
& S^{9\,14} - S^{10\,13},\, S^{9\,13} + S^{10\,14}\,, S^{11\,14} - S^{12\,13}\,, \\
& S^{11\,13} + S^{12\,14},\, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}, \\
\tau^4 :=\; & -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})\,, \qquad (6.69)
\end{aligned}
$$

$$
\vec{\tilde{N}}_{L,R} := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03})\,, \qquad (6.70)
$$

$$
\begin{aligned}
\vec{\tilde{\tau}}^1 &:= \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67},\, \tilde{S}^{57} + \tilde{S}^{68},\, \tilde{S}^{56} - \tilde{S}^{78})\,, \\
\vec{\tilde{\tau}}^2 &:= \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67},\, \tilde{S}^{57} - \tilde{S}^{68},\, \tilde{S}^{56} + \tilde{S}^{78})\,, \qquad (6.71)
\end{aligned}
$$

$$\tilde{\tau}^4 := -\frac{1}{3}(\tilde{S}^{9\,10} + \tilde{S}^{11\,12} + \tilde{S}^{13\,14})\,. \tag{6.72}$$

$$\begin{aligned}
&N^{\pm}_{+} = N^1_{+} \pm i\, N^2_{+} = -\stackrel{03}{(\mp i)}\stackrel{12}{(\pm)}\,, \quad N^{\pm}_{-} = N^1_{-} \pm i\, N^2_{-} = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}\,,\\
&\tilde{N}^{\pm}_{+} = -\stackrel{03}{\tilde{(\mp i)}}\stackrel{12}{\tilde{(\pm)}}\,, \quad \tilde{N}^{\pm}_{-} = \stackrel{03}{\tilde{(\pm i)}}\stackrel{12}{\tilde{(\pm)}}\,,\\
&\tau^{1\pm} = (\mp)\,\stackrel{56}{(\pm)}\stackrel{78}{(\mp)}\,, \quad \tau^{2\mp} = (\mp)\,\stackrel{56}{(\mp)}\stackrel{78}{(\mp)}\,,\\
&\tilde{\tau}^{1\pm} = (\mp)\,\stackrel{56}{\tilde{(\pm)}}\stackrel{78}{\tilde{(\mp)}}\,, \quad \tilde{\tau}^{2\mp} = (\mp)\,\stackrel{56}{\tilde{(\mp)}}\stackrel{78}{\tilde{(\mp)}}\,.
\end{aligned} \tag{6.73}$$

6.7 Appendix: *Standard model* assumptions

α name	**hand-edness** $-4\mathbf{i}\mathbf{S}^{03}\mathbf{S}^{12}$	**weak charge** τ^{13}	**hyper charge** $\mathbf{Y}$	**colour charge**	elm charge Q
$\mathbf{u}^{\mathbf{i}}_{\mathbf{L}}$	$-\mathbf{1}$	$\frac{1}{2}$	$\frac{1}{6}$	**colour triplet**	$\frac{2}{3}$
$\mathbf{d}^{\mathbf{i}}_{\mathbf{L}}$	$-\mathbf{1}$	$-\frac{1}{2}$	$\frac{1}{6}$	**colour triplet**	$-\frac{1}{3}$
$\nu^{\mathbf{i}}_{\mathbf{L}}$	$-\mathbf{1}$	$\frac{1}{2}$	$-\frac{1}{2}$	**colourless**	0
$\mathbf{e}^{\mathbf{i}}_{\mathbf{L}}$	$-\mathbf{1}$	$-\frac{1}{2}$	$-\frac{1}{2}$	**colourless**	-1
$\mathbf{u}^{\mathbf{i}}_{\mathbf{R}}$	**1**	**weakless**	$\frac{2}{3}$	**colour triplet**	$\frac{2}{3}$
$\mathbf{d}^{\mathbf{i}}_{\mathbf{R}}$	**1**	**weakless**	$-\frac{1}{3}$	**colour triplet**	$-\frac{1}{3}$
$\nu^{\mathbf{i}}_{\mathbf{R}}$	**1**	**weakless**	**0**	**colourless**	0
$\mathbf{e}^{\mathbf{i}}_{\mathbf{R}}$	**1**	**weakless**	$-\mathbf{1}$	**colourless**	-1

Table 6.9. Table represents the *standard model* assumptions for each of the three so far observed ($i = 1, 2, 3$), before the electroweak break massless, families of quarks and leptons. Each family contains the left handed weak charged quarks and right handed weak chargeless quarks, each quark belonging to the colour triplet $(1/2, 1/(2\sqrt{3}))$, $(-1/2, 1/(2\sqrt{3}))$, $(0, -1/(\sqrt{3}))$, and the left handed weak charged and right handed weak chargeless colourless leptons. τ^{13} defines the third component of the weak charge, Y is the hyper charge determining the electromagnetic charge $Q = Y + \tau^{13}$. The *standard model* assumes to each family member of each family the corresponding anti-fermions.

More than 40 years ago the *standard model* offered an *elegant new step* in understanding the origin of fermions and bosons by postulating:

name	**hand-edness**	**weak charge**	**hyper charge**	**colour charge**	elm charge
hyper photon	0	0	0	colourless	0
weak bosons	0	**triplet**	0	colourless	**triplet**
gluons	0	0	0	**colour octet**	0

Table 6.10. Vector fields, the gauge fields of the hyper, weak and colour charges, all massless before the electroweak break. They all are vectors in the adjoint representations with respect to the weak, colour and hyper charges.

name	**hand-edness**	**weak charge**	**hyper charge**	**colour charge**	**elm charge**
$0 \cdot$ higgs$_{\rm u}$	0	$\frac{1}{2}$	$\frac{1}{2}$	colourless	1
<higgs$_{\rm d}$ >	0	$-\frac{1}{2}$	$\frac{1}{2}$	colourless	0
< higgs$_{\rm u}$ >	0	$\frac{1}{2}$	$-\frac{1}{2}$	colourless	0
$0 \cdot$ higgs$_{\rm d}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	colourless	-1

Table 6.11. Higgs is the scalar field with the weak charge and the hyper charge $\pm\frac{1}{2}$ and $\mp\frac{1}{2}$, respectively. The $0.$ higgs$_{\rm d}$ and $0.$ higgs$_{\rm u}$ are not assumed. In Table these two are added to manifest the fundamental representation of the charge groups. The two components, <higgs$_{\rm u}$ > and<higgs$_{\rm d}$ >, "dress" in the *standard model* the right handed u-quarks and d-quarks, respectively, giving them the charges of the left partners.

- The existence of *massless family members*: coloured quarks and colourless leptons, both left and right handed, the left handed members distinguishing from the right handed ones in the weak and hyper charges and correspondingly mass protected, Table 6.9.
- The existence of - before the electroweak break - *massless vector gauge fields* to the observed charges of the family members, Table 6.10.
- The existence of a massive *scalar field* carrying the weak charge ($\pm\frac{1}{2}$) and the hyper charge ($\mp\frac{1}{2}$), which by its "nonzero vacuum expectation values" breaks the weak and the hyper charge and correspondingly breaks the mass protection of fermions and those vector bosons which interact with this vacuum, Table 6.11.
- The existence of the *Yukawa couplings* of fermions, which together with (the gluons and) the scalar take care of the properties of fermions after the electroweak break.

The *standard model* assumptions have been confirmed without offering surprises. The last unobserved field, the higgs, detected in June 2012, was confirmed in March 2013.

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7 Do Present Experiments Exclude the Fourth Family Quarks as Well as the Existence of More Than One Scalar?

N.S. Mankoč Borštnik[1] and H.B.F. Nielsen[2]

[1]University of Ljubljana,
Jadranska 19, SI-1000 Ljubljana, Slovenia
[2]The Niels Bohr Institute,
Blegdamsvej 17 - 21, Copenhagen Ø; Denmark

Abstract. The *spin-charge-family* theory [1–17] predicts the existence of the fourth family to the lower three. It also predicts several scalar fields (the mass eigenstates of the three singlets with the family members quantum numbers and the two triplets with the family quantum numbers) with the weak and the hyper charge of the *standard model* higgs field ($\pm\frac{1}{2}, \mp\frac{1}{2}$, respectively). There is so far no experimental evidence for either the existence of the fourth family quarks with masses below 1 TeV or for the existence of more than one scalar field (however, the Yukawa couplings themselves are the signal that several scalars must exist). If the fourth family quarks have masses above 1 TeV then the experimental evidences [18,19] require that they contribute negligible to either the quark-gluon fusion production of the observed scalar higgs or to the decay of this scalar. It is discussed in this contribution why it is too early to say that the present experiments exclude the fourth family quarks predicted by the *spin-charge-family* theory.

Povzetek. Teorija *spinov-nabojev-družin* [1–17] napove obstoj četrte družine k izmerjenim trem. Napove tudi več skalarnih polj (masnih lastnih stanj treh singletov s kvantnimi števili članov družin in dveh tripletov z družinskimi kvantnimi števili), ki imajo šibki in hiper naboj enak ustreznim nabojem higgsovega polja *standardnega modela* ($\pm\frac{1}{2}, \mp\frac{1}{2}$). Zdi se, da dosedanji poskusi izključujejo obstoj kvarkov četrte družine, z maso pod 1 TeV, pa tudi novih skalarnih polj s tako maso. Če pa naj imajo kvarki četrte družine maso nad 1 TeV, mora biti njihov prispevek k nastanku izmerjenega (higsovega) skalarja, kakor tudi k razpadu tega skalarja, zanemarljiv [18,19]. Prispevek pojasnjuje, zakaj je prezgodaj reči, da četrte družine, ki jo napove teorija *spinov-nabojev-družin* ter novih skalarnih polj (njihov obstoj zagotavljajo že Yukawine sklopitve), ni.

7.1 Introduction

The *spin-charge-family* theory [1–17] predicts before the electroweak break four - rather than the observed three - coupled massless families of quarks and leptons. The 4×4 mass matrices of all the family members demonstrate in this theory the same symmetry [14,15], determined by the scalar fields: the two $\widetilde{SU}(2)$ triplets - the gauge fields of the two family groups operating among families - and the three singlets - the gauge fields of the three charges, (Q, Q' and Y'), distinguishing among family members [2,1]. All these scalar fields carry the weak and the hyper charge as does the scalar of the *standard model*: $\pm\frac{1}{2}$ and $\mp\frac{1}{2}$, respectively [17].

Since there is no direct observations of the fourth family quarks masses below 1 TeV, while the fourth family quarks with masses above 1 TeV would contribute, according to the *standard model* (the *standard model* Yukawa couplings of the quarks to the scalar higgs is proportional to $\frac{m_4^\alpha}{v}$, where m_4^α is the fourth family member ($\alpha = u, d$) mass and v the vacuum expectation value of the scalar), to either the quark-gluon fusion production of the scalar field (the higgs) or to the scalar field decay, too much in comparison with the observations, the high energy physicists do not expect the existence of the fourth family members at all [18,19].

Does this mean that there does not exist the fourth family coupled to the observed three?

Before discussing the question to which extent can be the theoretical interpretations of the experimental data, grounded on the *standard model* assumptions, acceptable for four families, while they are obviously working well for three families, let be pointed out what supports the *spin-charge-family* theory to be the right next step beyond the *standard model*. This theory is only able not to explain - while starting from the very simple action in $d \geq (13+1)$, Eqs. (7.8, 7.9) of Sect. 7.4, with massless fermions with the spin of the two kinds (one kind taking care of the spin and of the charges of the family members the second kind taking care of the families (Eq. (6.50))), which couple only to the gravity (through the vielbeins and the two kinds of the corresponding spin connections (Eqs. (7.8, 7.9))) - all the assumptions of the *standard model*, but also to answer several open questions beyond the *standard model*. It offers the explanation for [1–17]:
a. the appearance of all the charges of the left and right handed family members and for their families and their properties,
b. the appearance of all the corresponding vector and scalar gauge fields and their properties (explaining the appearance of the higgs and the Yukawa couplings),
c. the appearance and properties of the dark matter,
d. the appearance of the matter/antimatter asymmetry in the universe.

The theory predicts for the low energy regime:
i. The existence of the fourth family to the observed three.
ii. The existence of twice two triplets and three singlets of scalars, all with the properties of the higgs with respect to the weak and hyper charges, explaining the existence of the Yukawa couplings. Besides the higgs also a few of the others will be observed at the LHC.
iii. There are several other predictions.

Since the experimental accuracy of the (3×3 submatrix of the 4×4) mixing matrices is not high enough, it is not yet possible to estimate masses of the fourth family members by fitting the experimental data to the parameters of mass matrices, determined by the symmetry as predicted by the *spin-charge-family* [15,14]. While the fitting procedure is not influenced considerably by the accuracy of the measured masses of the lower three families, the accuracy of the measured values of mixing matrices do influence, as expected the fitting results very much. The fact that the fourth family quarks have not yet been observed - directly or indirectly - pushes the fourth family quarks masses to ≈ 2 TeV or higher.

The more effort and work is put into the *spin-charge-family* theory, the more explanations of the observed phenomena and the more predictions for the future observations follow out of it. Offering the explanation for so many observed phenomena - keeping in mind that all the explanations for the observed phenomena originate in a simple starting action - qualifies the *spin-charge-family* theory as the candidate for the next step beyond the *standard model*.

Since in the *spin-charge-family* theory all the low energy degrees of freedom of elementary fields - fermions and vector and scalar bosons - follow from a simple starting action, and since also the dynamics is determined in the starting action, it would in principle be possible to calculate all the properties of the fields in the low energy regime, if we would know the boundary conditions. This is, of course, too ambitious program, not only because the boundary conditions are not known, but also because of too many degrees of freedom of fermions and bosons, in particular at phase transitions (as teaches us physics of fluids and condensed matter).

This paper argues for the existence of the fourth family and of several scalar fields predicted by the *spin-charge-family* theory, discussing the arguments why the contribution of the fourth family quarks to the quark-gluon fusion at LHC, as well as the contribution of this family to the decay of the Higgs's scalar might not disagree with the observations, as long as the interpretations of the events rely on the *standard model* assumptions Ref. [20], which are not in agreement with the *spin-charge-family* theory.

Sect. 7.2 discusses the arguments why the fourth family might exist although has not yet been observed - directly or indirectly.

The *spin-charge-family* theory is presented in the main talk of the author of this contribution.

7.2 The fourth family in the *spin-charge-family* theory and the experimental constraints against it

The *spin-charge-family* theory predicts the fourth family to the observed three. The calculation of the fourth family properties to the observed three, when taking into account the symmetry of mass matrices predicted by this theory and fitting the consequently allowed parameters of mass matrices to the experimental data, shows [14,15], that the measured matrix elements of the 3×3 - submatrices of the 4×4 - mixing matrices are far from being accurate enough even for quarks to determine masses of the fourth family members. In Subsect. 7.2.3 a short report on this calculation is presented. More can be found on Refs. [27,1,15] and the references cited there.

Since there has been no direct observation of the fourth family quarks with the masses below 1 TeV, while the *standard model* without the fourth family is in much better agreement with the experiments than with the fourth family included, the high energy physicists do not expect the existence of the fourth family members at all [18,19].

Should the explanation of the so far obtained experimental data by using the *standard model* assumptions be accepted as the definite experimental evidence that there are only three families and that the fourth family of quarks and leptons does not exist?

Let us try to understand how far are the interpretations of the experimental data trustworthy, if following more or less the *standard model* assumptions:
i. The *standard model* assumes one scalar doublet, the higgs (there are also models using two scalar doublets or more, all more or less following the idea of the *standard model* higgs [21,22]).
ii. In calculations the validity of the Yukawa couplings in the perturbation calculations are assumed.
iii. Calculations have been done in the leading order, next to the leading order and even some in one order more.
iv. The 4×4 mixing matrices elements of the fourth family members to the observed three were just assumed (or neglected).

The review article [20] discusses, reporting on many papers (≈ 200), a possibility of the existence of the fourth family due to the experimental data and theoretical analyses of the data, presenting also assumptions on which the theoretical analyses were done. The author discusses the (non)existence of the fourth family quarks and leptons due to direct searches for the fourth family members, due to changes in mixing matrices if there exists the fourth family, due to measurements testing the electroweak precision data with and without the existence of the fourth family, and in particular due to the analyzes of the higgs boson production and decay. The author concludes pointing out that if taking seriously that there exists only one scalar doublet and if assuming perturbativity of the Yukawa couplings and the Dirac mass of the heavy neutrino, "then the fourth family of fermions can not accommodate the data for the higgs searches" and its decay.

This review article [20] appeared in 2013. The new data [24], reported also in the review talk [25], are not in contradiction with the conclusions of Ref. [20], while the measured mixing matrix elements for quarks - averaged over data of several experimental groups - are still far from being accurate enough to allow the *spin-charge-family* theory to predict the fourth family quarks masses.

Although the assumptions, used to analyze the experimental data, might seem to most of high energy physicists acceptable, the assumptions do not appear so trustworthy when looking at them from the point of view of the *spin-charge-family* theory, what it will be done in this section.

Let us point out the differences between the generally used assumptions in the analyses of the experimental data searching for new scalars and new family and the properties of the fermions and scalar fields in the *spin-charge-family* theory.

Most commonly accepted assumptions [20,24]:
A. There is only one Higgs doublet. If there are more, their properties (their Lagrange function) resemble the properties of the *standard model* higgs.
B. The Yukawa couplings are used in the higher order corrections.
C. The Dirac neutrino masses is used.
D. The perturbativity of the theory is assumed.

Arguments against these common assumptions:
A′. Already the higgs enters into the *standard model* by "hand", with assumed Lagrange function which couples the higgs to the weak bosons ($W^{\pm}_m$ and Z^0_m) and "dresses" fermions with the (appropriate) weak and hyper charges. The higgs does not carry the family quantum numbers. To our understanding, just repeating the "game of the higgs" for several higges without a deeper understanding of the origin of scalars can hardly be the right way beyond the *standard model*.
B′. The Yukawa couplings are also put by "hand" into the *standard model* to compensate the higgs independence of the family quantum numbers. Using the Yukawa couplings in perturbative way might have no theoretical support in calculations with corrections next to leading and next to next to leading orders.
C′. The Dirac neutrino masses seems natural, since all the other family members do have the Dirac masses, but either the Dirac or the Majorana mass of neutrinos must be grounded in a deeper understanding of the origin of fermion masses.
D′. The perturbativity of the theory, originating in effective Lagrange function, at and around the phase transition of the electroweak break, is also questionable, since the acceptance of the effective theory might easily break down.

Calculations done under the assumptions presented from A.-D. lead, due to Ref. [20], to the conclusion:
i. The ratio of the gluon-gluon fusion generating the higgs and decaying into two ZZ if taking into account the four families or only three is $\approx 5-8$.
ii. The ratio of the gluon-gluon fusion generating higgs and decaying into $b\bar{b}$ if taking into account the four families or only three is ≈ 5.
iii.The most stringent is, due to Ref. Lenz, the predicted underproduction (for a factor of 5) in the two γ's channel, if the fourth family is included into calculations with respect to the calculations with only three families.
The author of Ref. [20] reports also some additional drawbacks (calling them minor) of the theoretical interpretation of experiments, like: not taking into account the change of the mixing matrix if there are four families instead of three, not allowing a large enough interval the masses of the fourth family, not taking into account the decays of higgs through the fourth family neutrinos if they appear to be light enough (smaller than $\frac{m_H}{2}$).

The spin-charge-family theory is disagrees with the assumptions A.- D.:
A″. In the *spin-charge-family* theory there are three singlet and two triplet scalar fields originating like all the gauge fields (vectors, tenzors and scalars) in the starting action (this can be read in Eqs. (7.8, 7.9) and in the talk of N.S.M.B. in this Proc., Eqs. (19,20)) as the gauge spin connection fields and manifesting in $(3+1)$ as scalars with the space index $s = (7, 8)$, all carrying the weak and the hyper charges (determined by the space index $s = (7, 8)$) of the *standard model* higgs (in the talk of N.S.M.B. in this Proc., Eq. (21)). The three singlets carry besides the higgs quantum numbers also the family members quantum numbers (Q, Q', Y'), the two triplets carry besides the higgs quantum numbers also the family quantum numbers, Eq. (5) in the talk of N.S.M.B. in this Proc..
B″. Scalars start as massless gauge fields, gaining masses when interacting with the condensate (Table 1. in the talk of N.S.M.B. in this Proc.) and change their properties when obtaining at the electroweak break the nonzero vacuum expectation

values. Each family member of each family couples to a different superposition of the scalar mass eigenstates, what correspondingly determines the Yukawa couplings (Eqs. (25-28) in the talk of N.S.M.B. in this Proc.). Since it is not known at which scale does the electroweak break occur, the perturbativity of the Yukawa couplings might not be an acceptable assumption, in particular since at phase transitions all the systems manifest the long range properties.
C″. In the *spin-charge-family* theory all the family members have the Dirac masses. However, to the mass matrices of each family member besides the scalar fields carrying the family quantum numbers - the two triplets - also the scalars carrying the family members quantum numbers - the three singlets (Q, Q', Y') - contribute. To neutrino masses besides the two triplets only the singlet - the gauge field of Y' - contribute. All these contributions are highly nonperturbative [12–15]. The three singlets contribute to the off diagonal mass matrix elements only in the higher order corrections, what makes masses of the family members so different.
D″. Scalars with the space index $s = (7, 8)$ (the two triplets and the three singlets) gain in a highly nonperturbative way nonzero vacuum expectation values, obeying after the electroweak break the approximate effective Lagrange function (Eq. (24)), for which one can't expect that close to the phase transition the perturbativity would work.

Let us look at the properties of the scalar fields contributing to the masses of the lower four families and of the $W^{\pm}_{m}, Z^{0}_{m}$ in the *spin-charge-family* theory in more details.

7.2.1 Scalar fields contributing to the mass matrices of the lower four families in the *spin-charge-family* theory

The *spin-charge-family* theory predicts twice (almost) decoupled four families in the low energy regime (Refs. [1,3,2,26]. We discuss here only properties of the lower four families.

To understand better how do in the *spin-charge-family* theory the scalar fields determine the properties of families of each family member - after the loop corrections are taken into account in all orders - the Lagrange density of the fermion mass term, Eq. (7.9) and Eq. (20) in the talk of N.S.M.B. in this Proc., the quarks part in particular, is rewritten so that (massless) quark states, u^k and d^k, k is the family index, enter explicitly into expressions.

$$
\begin{aligned}
&\mathcal{L}_{f=qm} = \\
&\frac{1}{2}\{[\psi^{\dagger}_{L}\gamma^{0}(\sum_{A,i,+,-} \stackrel{78}{(\pm)}\, \tau^{Ai} g^{Ai} A^{Ai}_{\pm})\, \psi_{R}] + [\psi^{\dagger}_{L}\gamma^{0}(\sum_{A,i,+,-} \stackrel{78}{(\pm)}\, \tau^{Ai} g^{Ai} A^{Ai}_{\pm})\, \psi_{R}]^{\dagger}\} = \\
&= \frac{1}{2}\sum_{k,l}\{[u^{k\dagger}_{L}\gamma^{0}(\sum_{A,i} \stackrel{78}{(-)}\, \tau^{Ai} g^{Ai} A^{Ai}_{-})\, u^{l}_{R}] + [u^{k\dagger}_{L}\gamma^{0}(\sum_{A,i} \stackrel{78}{(-)}\, \tau^{Ai} g^{Ai} A^{Ai}_{-})\, u^{l}_{R}]^{\dagger} + \\
&\quad [d^{k\dagger}_{L}\gamma^{0}(\sum_{A,i} \stackrel{78}{(+)}\, \tau^{Ai} g^{Ai} A^{Ai}_{+})\, d^{l}_{R}] + [d^{k\dagger}_{L}\gamma^{0}(\sum_{A,i} \stackrel{78}{(+)}\, \tau^{Ai} g^{Ai} A^{Ai}_{+})\, d^{l}_{R}]^{\dagger}\}. \quad (7.1)
\end{aligned}
$$

Operators $\tau^{Ai} = \sum_{s,t} c^{Ai}{}_{st}\, S^{st}$ are defined in Eqs. (3,4) in the talk of N.S.M.B. in this Proc. and in Eqs. (7.10, 7.11), scalar fields $A^{Ai}_{\pm} = \sum_{s,t} c^{Ai}{}_{st}\, \omega^{st}{}_{\pm}$ and

$\gamma^0 \overset{78}{(\pm)}$ are defined in Eqs. (19-21) in the talk of N.S.M.B. in this Proc.. The coupling constants of the two triplets ($\tilde{A}^{\tilde{1}i}_{\pm}$, $\tilde{A}^{i}_{\tilde{N}_L \pm}$) and three singlets ($A^{Q}_{\pm}$, $A^{Q'}_{\pm}$, $A^{Y'}_{\pm}$) are in Eq. (7.1) written explicitly.

Operators τ^{Ai} and $\gamma^0 \overset{78}{(\pm)}$ are Hermitian. In what follows it is assumed that the scalar fields A^{Ai}_s are Hermitian as well and consequently it follows $(A^{Ai}_{\pm})^{\dagger} = A^{Ai}_{\mp})$. While the family operators $\tilde{\tau}^{1i}$ and $\tilde{N}^i_L$ commute with $\gamma^0 \overset{78}{(\pm)}$, the three family members operators (Q, Q', Y') do not, but one sees that

$$
\begin{aligned}
(u^{k\dagger}_L \gamma^0 \sum_{A,i,+,-} (Q, Q', Y')\, g^{(Q,Q',Y')} \overset{78}{(-)}\, A^{(Q,Q',Y')}_{-} u^l_R)^{\dagger} = \\
u^{l\dagger}_R \gamma^0\, (Q, Q', Y')\, g^{(Q,Q'Y')} \overset{78}{(+)}\, A^{(Q,Q',Y')\dagger}_{-} u^k_L \\
= u^{l\dagger}_R\, (Q^k_R, Q'^k_R, Y'^k_R)\, g^{Q,Q',Y'}\, A^{(Q,Q',Y')}_{+}\, u^k_R\,, \quad (7.2)
\end{aligned}
$$

where (Q^k_R, Q'^k_R, Y'^k_R) denote the eigenvalues of the spinor state u^k_R.

The off diagonal matrix elements of mass matrices, Eq. (7.7), start to be dependent on the family members quantum numbers only in loop corrections, consequently the contributions of the loop corrections to all orders are indeed important.

Couplings of u_k and d_k to the scalars carrying the family members quantum numbers are determined also by the eigenvalues of the operators (Q, Q', Y') on the family members states. Strong influences of the scalar fields carrying the family members quantum numbers on the masses of the lower (observed) three families of quarks manifest in huge differences in masses of u_k and d_k, $k = (1, 2, 3)$, among family members (u, d). For the fourth family quarks, which are more and more decoupled from the observed three families the higher are their masses [15,14], the influence of the scalar fields carrying the family members quantum numbers on their masses is expected to be much weaker. *Correspondingly might become the* u_4 *and* d_4 *masses closer to each other the higher are their masses and the weaker is their couplings* (the mixing matrix elements) *to the lower three families.*

The superposition of the scalar eigenstates which couple to the fourth family quarks might therefore differ a lot from those which couple to the lower three families.

Although the gluons couple in the gluon-gluon fusion to all the quarks in an equivalent way, yet *the family members with different family quantum number contribute to the production of different scalar mass eigenstates differently*, which might not be in agreement with the simple *standard model* prediction, that the fourth family couples to the observed higgs proportionally to their masses $(\frac{m_i}{v})$.

If the masses of the fourth family quarks are close to each other, then u_4 *and* d_4 *contribute in the quark-gluon fusion very little to the production of the observed scalar field - the higgs - if the higgs is a superposition of different scalar fields mass eigenstates then the scalar, to which the fourth family quarks mostly couple, as it is expected.*

The scalar fields from the starting action to the effective action Let us discuss the scalar fields, which contribute to the electroweak break with nonzero vacuum expectation values, from the point of view of the starting action (Eq. (7.8)) in order to try to understand better their properties at the electroweak break.

The action (Eq. (7.8)) manifests that there are only $\tilde{\omega}_{mn\sigma}$, which are coupled to the vector gauge fields $\mathcal{A}^{Ai}_m$ ([27], Eqs. (19,20) in [28], as well as Subsect. 6.3.2, page 88, in the talk of N.S.M.B. in this Proc.) on the tree level.

The vector gauge fields $\mathcal{A}^{Ai}_m$, namely, appear in the action, Eq. (7.8), as (Subsect. 3.2. in my talk in this Proc.)

$$f^{\sigma}{}_m = \sum_A \vec{\tau}^{A\sigma}\, \vec{\mathcal{A}}^A_m\,, \tag{7.3}$$

where $\tau^{Ai} = \sum_{st} c^{Ai}{}_{st}\, M^{st}$, $M^{st} = S^{st} + L^{st}$, $\{\tau^{Ai}, \tau^{Bj}\}_- = i f^{Aijk} \tau^{Ak}\, \delta^{AB}$, $\vec{\tau}^A = \vec{\tau}^{A\sigma}\, p_\sigma = \vec{\tau}^{A\sigma}{}_\tau\, x^\tau\, p_\sigma$, $\tau^{Ai\sigma} = \sum_{st} c^{Ai}{}_{st}\, M^{st\sigma} = \sum_{st} c^{Ai}{}_{st}\, (e_{s\tau}\, f^\sigma{}_t - e_{t\tau}\, f^\sigma{}_s) x^\tau$, $\mathcal{A}^{Ai}_m = \sum_{st} c^{Ai}{}_{st}\, \omega^{st}{}_m$. The relation between $\omega^{st}{}_m$ and vielbeins is determined by Eq. (7.4), if there are no spinor sources present.

Correspondingly the vector gauge fields gain masses on the tree level through $f^\sigma{}_m f^\tau{}_n\, (\tilde{\omega}^{mn}{}_{[\sigma,\tau]} - \tilde{\omega}^m{}_{m'[\tau} \tilde{\omega}^{m'n}{}_{\sigma]})$ ([ ] means that the two indices must be exchanged), when at the electroweak break the two triplets and the three singlets gain the nonzero vacuum expectation values. Indeed only one superposition - one triplet - is involved. Since only $f^\sigma{}_m$ represents the vector gauge fields ($f^\mu{}_s = 0$ [1,28]) in the low energy regime, all the rest of scalar fields (the second triplet and the three singlets) contribute to masses of $W^\pm_m$, Z^0_m only in loop corrections.

The action (Eq. (7.8)) leads to the equations of motion (Ref. [1], Eqs. (31,32))

$$\begin{aligned}
0 &= 2\alpha \left[f^\beta{}_b\, R^{ba}{}_{[\beta\alpha]} - \frac{1}{2}\, e^a{}_\alpha\, R \right] \\
&+ 2\tilde{\alpha} \left[f^\beta{}_b\, \tilde{R}^{ba}{}_{[\beta\alpha]} - \frac{1}{2}\, e^a{}_\alpha\, \tilde{R} \right] \\
&+ \bar{\Psi}\gamma^a p_{0\alpha}\Psi - f^\beta{}_b e^a{}_\alpha \left(p_\beta(\bar{\Psi}\gamma^b\Psi) - p_\alpha(\bar{\Psi}\gamma^a\Psi) \right)\,, \\
p_{0\alpha} &= p_\alpha - \frac{1}{2}\, S^{cd}\, \omega_{cd\alpha} - \frac{1}{2}\, \tilde{S}^{cd}\, \tilde{\omega}_{cd\alpha}\,, \\
R^{ab}{}_{[\alpha\beta]} &= \partial_{[\alpha}\, \omega^{ab}{}_{\beta]} + \omega^a{}_{c[\alpha} \omega^{cb}{}_{\beta]}\,, \\
\tilde{R}^{ab}{}_{[\alpha\beta]} &= \partial_{[\alpha}\, \tilde{\omega}^{ab}{}_{\beta]} + \tilde{\omega}^a{}_{c[\alpha} \tilde{\omega}^{cb}{}_{\beta]}\,,
\end{aligned} \tag{7.4}$$

$$\begin{aligned}
f^\alpha{}_c\, \omega_{[a}{}^c{}_{b]} + f^\alpha{}_{[a}\, \omega_{b]}{}^c{}_c &= \frac{1}{E}\, \partial_\beta (E f^\alpha{}_{[a} f^\beta{}_{b]}) + \frac{1}{2}\, \bar{\Psi}\, f^\alpha{}_c \gamma^c\, S_{ab}\psi\,, \\
f^\alpha{}_c\, \tilde{\omega}_{[a}{}^c{}_{b]} + f^\alpha{}_{[a}\, \tilde{\omega}_{b]}{}^c{}_c &= \frac{1}{E}\, \partial_\beta (E f^\alpha{}_{[a} f^\beta{}_{b]}) + \frac{1}{2}\, \bar{\Psi}\, f^\alpha{}_c \gamma^c\, \tilde{S}_{ab}\psi\,.
\end{aligned} \tag{7.5}$$

One can read in Eqs. (7.4, 7.5) the interactions among the gauge fields and the interactions of the gauge fields with the fermion fields (in particular we point out the condensate [1] and Table 6.1 (on page 83) in the N.S.M.B. talk in this Proc.).

The appearing of the condensate, its interaction with the scalars and the behavior of scalars at the electroweak phase transition are expected to be highly nonperturbative effects. It is assumed so far (estimating very roughly the degrees of freedom and the interactions among scalars and among scalars and fermions) that the effective Lagrange density of scalars, contributing to the electroweak break, might after the phase transition manifest the *standard model* assumptions, changing from the starting action Lagrange density Eq. (7.8) $\mathcal{L}_s = E\{(p_m A^{Ai}_s)^\dagger (p^m A^{Ai}_s) - (m'_{Ai})^2 A^{Ai\dagger}_s A^{Ai}_s\}$ to

$$\begin{aligned}\mathcal{L}_{sg} = E \sum_{A,i} \{ &(p_m A^{Ai}_s)^\dagger (p^m A^{Ai}_s) - (-\lambda^{Ai} + (m'_{Ai})^2))\, A^{Ai\dagger}_s A^{Ai}_s \\ &+ \sum_{B,j} \Lambda^{AiBj} A^{Ai\dagger}_s A^{Ai}_s A^{Bj\dagger}_s A^{Bj}_s\}, \end{aligned} \tag{7.6}$$

where $-\lambda^{Ai} + m'^2_{Ai} = m^2_{Ai}$ and m_{Ai} manifests as the mass of the A^{Ai}_s scalar.

Whether or not this is an acceptable effective Lagrange function or not remains to be proved.

7.2.2 The contribution of the fourth family quarks of equal masses to the production of the scalar fields

In the N.S.M.B. talk in this proceedings, Subsect. 6.4.2 (page 101), a possibility is discussed that if the fourth family quarks have approximately equal masses (the u_4-quarks and d_4-quarks might have similar masses, if their masses are mostly determined by the scalars with the family quantum numbers, as discussed in Subsect. 7.2.1), while u_4 and d_4 couple to the scalar fields determining their masses with the opposite phases, the contribution of the fourth family quarks to the production of scalar fields in the gluon-gluon fusion can be negligible.

Since the family quantum numbers ($\tilde{\tau}^1$, $\tilde{N}^L$) commute with the weak and the hyper charges, the scalar fields carrying the family quantum numbers, $\vec{\tilde{A}}^{1,N_L}_{\substack{78\\(\pm)}}$, distinguish among u_4 and d_4 only due to the operator $\overset{78}{(\pm)}$. Couplings of u_4 and d_4 to those scalar fields, which carry in addition to the weak and the hyper charge the family members quantum numbers - to the three singlets $(A^{Q}_{\substack{78\\(\pm)}}, A^{Q'}_{\substack{78\\(\pm)}}, A^{Y'}_{\substack{78\\(\pm)}})$ - depend on the eigenvalues of (Q, Q', Y') on the quark states, which are different for u_i and d_i quarks.

Since the masses of u_4 and d_4 are only approximately equal, the fourth family quarks can still weakly contribute to the production of the scalar fields, in particular to those which mostly determine masses of the fourth family members.

7.2.3 Mass matrices of family members and the masses of the fourth family quarks

The *spin-charge-family* theory [2,1,14,15] predicts the mass matrices of the family members α for each groups of four families, Eq. (7.7).

$$\mathcal{M}^\alpha = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^\alpha . \tag{7.7}$$

The mass matrices are determined at the electroweak break, when the scalar fields with the space index $s = (7,8)$ (the three singlets carrying the family members quantum numbers and the two triplets carrying the family quantum numbers, the two triplets and the three singlets interacting among themselves, Eq. (7.6)) get nonzero vacuum expectation values. In loop corrections the singlets influence all the matrix elements of each mass matrix while keeping the $\widetilde{SU}_{\widetilde{SO}(3,1)}(2) \times \widetilde{SU}_{\widetilde{SO}(4)}(2) \times U(1)$ symmetry unchanged.

In Refs. [14,15] the twice 6 parameters of the two mass matrices of the lower group of four families of u and d quarks, in general non Hermitean, presented in Eq. (7.7), were fitted to:

i. twice three masses of the u_i, $i = 1, 2, 3$ (u, c, t) and d_i, $i = 1, 2, 3$ (d, s, b) quarks and

ii. 3×3 submatrix of the 4×4 quark mixing matrix. Although the $(n-1) \times (n-1)$ submatrix of the $n \times n$ unitary matrix, if accurately known, determines uniquely the $n \times n$ matrix for $n \geq 4$, we were not able to determine the masses of the fourth families, even not after assuming that the mass matrices are real, since the 3×3 submatrices are not known accurately enough. We only could tell the fourth family matrix elements of the mixing matrix after assuming the masses of the fourth family quarks.

It turned out that for the masses of the fourth family quarks above 1 TeV the mass matrices are more and more democratic and the fourth family quarks are more and more decoupled from the lower three families the larger are the fourth family masses. It correspondingly appears that the masses of u_4 and d_4 are closer to each other the smaller is contribution of the scalar fields with the family members quantum numbers to their masses. The results are presented in Ref. [15].

7.3 Concluding remarks

In this contribution the arguments against the conclusions of most high energy physicists that present experiments can hardly leave any room for the existence of the fourth family members is discussed.

The analysis of experiments, which are based on the assumptions of the *standard model* - i. on the existence of one scalar doublet, if there are several they follow properties of the higgs, ii. on the perturbativity of the theory, iii. on guessing the mixing matrices elements of the fourth family members to the observed three - might from the point of view of the *spin-charge-family* theory not be acceptable.

The main arguments against the *standard model* assumptions through the "eyes" of the *spin-charge-family* are:

i. Assuming the existence of one scalar fields, or even several scalars repeating the idea of the *standard model* higgs, is too restrictive. In the *spin-charge-family* theory there are three singlet and two triplet scalar fields, which all originate (like all the gauge fields and the gravity in $(3+1)$ do) in the starting action as the gauge spin connection fields and manifesting in $(3+1)$ as scalars with the space index $s = (7, 8)$, all carrying the weak and the hyper charges (determined by the space index $s = (7, 8)$) of the *standard model* higgs. The three singlets carry besides the higgs quantum numbers also the family members quantum numbers (Q, Q', Y'), the two triplets carry besides the higgs quantum numbers also the family quantum

numbers.
ii. Scalars start as massless gauge fields, gaining masses when interacting with the condensate and changing their properties when at the electroweak break when gaining the nonzero vacuum expectation values. Each family member of each family couples to a different superposition of the scalar mass eigenstates, what correspondingly determines the Yukawa couplings. All these contributions are highly nonperturbative.
In addition, the three singlets contribute to the off diagonal mass matrix elements only in the higher order corrections, what makes masses of the family members so different. Since it is also not known at which scale does the electroweak break occur, the perturbativity might not be an acceptable assumption even if at the low enough energies the effective Lagrange density behaves perturbatively.
iii. Also the mixing matrix elements from the fourth family members to the the rest three might influence considerably the interpretation of the experimental data. Also since each family member couples to different superposition of the scalar mass eigenstates.
Let us add that each family member can couple to the scalar fields with its own phase. In the case that the fourth family quark u_4 couples to the scalar fields with the opposite phase than the d_4 quark, and that their masses are closed to each other, what seems to be the case in the *spin-charge-family* theory, then the fourth family quarks contribution to the production of higgs and its decay might be very small, also since the superposition of the scalar mass eigenstates which couple to the fourth family quarks differ a lot from those which couple to the lower three families.

Although the gluons couple in the gluon-gluon fusion to all the quarks in an equivalent way, yet the family members with different family quantum number contribute to the production of different scalar mass eigenstates differently, what means that the simple *standard model* prediction, that the fourth family couples to the observed higgs proportionally to their masses ($\frac{m_i}{v}$), is not acceptable.

Let us point out at the end that the ability of the *spin-charge-family* theory, which starts with a simple action with fermions carrying two kinds of spins and no charges in $d > (3+1)$ and interacting with only gravitational field, to offer the explanation **a.i.** for all the assumptions of the standard model, **a.ii.** for the appearance of the family members and the families, **a.iii.** for the appearance of the gauge vector fields and their properties, **a.iv.** for the appearance of the scalar fields explaining the higgs and the Yukawa couplings, **a.v.** for the appearance of the dark matter, **a.vi.** for the appearance of the matter/antimatter asymmetry in the universe, suggests that this theory must be taken as a candidate showing next step beyons the *standard model*. Correspondingly must the prediction of this theory that there exists the fourth family coupled to the observed three and that there exist several scalar fields, which explain besides the origin of the higgs also the Yukawa couplings, be taken seriously.

7.4 Appendix: Spin-charge-family theory, action and assumptions

I present in this appendix the *assumptions* of the *spin-charge-family* theory, on which the theory is built - following a lot the equivalent sections in Refs. [2,1] - starting with the simple action for fermions and the gravity fields.

A i. In the action [2–4,1], Eq. (7.8), fermions ψ carry in $d=(13+1)$ as the *internal degrees of freedom only two kinds of spins* (no charges), which are determined by the two kinds of the Clifford algebra objects (there exist no additional Clifford algebra objects (6.48)) - γ^a and $\tilde{\gamma}^a$ - and *interact correspondingly with the two kinds of the spin connection fields* - $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, the *gauge fields* of $S^{ab}=\frac{i}{4}\,(\gamma^a\gamma^b-\gamma^b\gamma^a)$ (the generators of $SO(13,1)$) and $\tilde{S}^{ab}=\frac{i}{4}\,(\tilde{\gamma}^a\tilde{\gamma}^b-\tilde{\gamma}^b\tilde{\gamma}^a)$ (the generators of $\widetilde{SO}(13,1)$) - and the *vielbeins* $f^{\alpha}{}_a$.

$$\begin{aligned}
\mathcal{A} &= \int d^dx\, E\, \mathcal{L}_f + \int d^dx\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,,\\
\mathcal{L}_f &= \frac{1}{2}\,(\bar{\psi}\,\gamma^a p_{0a}\psi) + \text{h.c.},\\
p_{0a} &= f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\,\{p_\alpha, E f^{\alpha}{}_a\}_-, \quad p_{0\alpha} = p_\alpha - \frac{1}{2} S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha},\\
R &= \frac{1}{2}\{f^{\alpha[a}f^{\beta b]}\,(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^{c}{}_{b\beta})\} + \text{h.c.}\,,\\
\tilde{R} &= \frac{1}{2}\{f^{\alpha[a}f^{\beta b]}\,(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\,\tilde{\omega}^{c}{}_{b\beta})\} + \text{h.c.}\,.
\end{aligned}\tag{7.8}$$

Here [1] $f^{\alpha[a}f^{\beta b]}=f^{\alpha a}f^{\beta b}-f^{\alpha b}f^{\beta a}$. R and $\tilde{R}$ are the two scalars (the two curvatures) [2].

A ii. The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)}$ times $M^{(6)}$ (manifesting as $SO(7,1)\times SU(3)\times U(1)$), affecting both internal degrees of freedom - the one represented by (the superposition of) S^{ab} and the one represented by (the superposition of) $\tilde{S}^{ab}$. Since the left handed (with respect to $M^{(7+1)}$) spinors couple differently to scalar (with respect to $M^{(7+1)}$) fields than the right handed ones, the break can leave massless and mass protected $2^{((7+1)/2-1)}$ massless families (which decouple into twice four families). The rest of families get heavy masses [3].

[1] $f^{\alpha}{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^{\alpha}{}_b=\delta^a{}_b,\ e^a{}_\alpha f^{\beta}{}_a=\delta^\beta_\alpha$, $E=\det(e^a{}_\alpha)$. Latin indices $a,b,..,m,n,..,s,t,..$ denote a tangent space (a flat index), while Greek indices $\alpha,\beta,..,\mu,\nu,..\sigma,\tau,..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a,b,c,..$ and $\alpha,\beta,\gamma,..$), from the middle of both the alphabets the observed dimensions $0,1,2,3$ ($m,n,..$ and $\mu,\nu,..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s,t,..$ and $\sigma,\tau,..$). We assume the signature $\eta^{ab}=diag\{1,-1,-1,\cdots,-1\}$.

[2] R and $\tilde{R}$ are expressible with vielbeins and their derivatives, when there are no fermions present [1,34].

[3] A toy model [29,30] was studied in $d=(5+1)$ with the same action as in Eq. (7.8). The break from $d=(5+1)$ to $d=(3+1)\times$ an almost S^2 was studied. For a particular choice of vielbeins and for a class of spin connection fields the manifold $M^{(5+1)}$ breaks

A iii. The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)} \times M^{(4)}$.

A iv. The scalar condensate (Table 7.1) of two right handed neutrinos with the family quantum numbers of one of the two groups of four families, brings masses of the scale of the unification ($\approx 10^{16}$ GeV or higher) to all the vector and scalar gauge fields, which interact with the condensate [2].

A v. There are nonzero vacuum expectation values of the scalar fields with the space index $s = (7, 8)$, conserving the electromagnetic and colour charge, which cause the electroweak break and bring masses to all the fermions and to the heavy bosons.

Comments on the assumptions:

C i. The starting action contains all degrees of freedom, either for fermions or for bosons, needed to manifest at low energy regime in $d = (3 + 1)$ all the vector and scalar gauge fields and the one family members as well as families of quarks and leptons as assumed by the *standard model*: **a.** One representation of $SO(13, 1)$ contains, if analyzed with respect to the *standard model* groups ($SO(3, 1) \times SU(2) \times U(1) \times SU(3)$) all the members of one family (Table 6.4, page 89), left and right handed, quarks and leptons (the right handed neutrino is one of the family members), anti-quarks and anti-leptons, with the quantum numbers required by the *standard model* [4]. **b.** The action explains the appearance of families due to the two kinds of the infinitesimal generators of groups: S^{ab} and $\tilde{S}^{ab}$ [5]. **c.** The action explains the appearance of the gauge fields of the *standard model* [2,1] [6]. **d.** It explains the appearance of the scalar higgs and Yukawa couplings [7].

into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families remain massless and mass protected. Equivalent assumption, its proof is in progress, is made in the $d = (13 + 1)$ case.

[4] It contains the left handed weak ($SU(2)_I$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons (neutrinos and electrons), and the right handed weak chargeless and $SU(2)_{II}$ charged coloured quarks and colourless leptons, as well as the right handed weak charged and $SU(2)_{II}$ chargeless colour anti-triplet anti-quarks and (anti)colourless anti-leptons, and the left handed weak chargeless and $SU(2)_{II}$ charged anti-quarks and anti-leptons. The anti-fermion states are reachable from the fermion states by the application of the discrete symmetry operator $\mathcal{C_N}\, \mathcal{P_N}$, presented in Ref. [38,39].

[5] There are before the electroweak break two decoupled groups of four massless families of quarks and leptons, in the fundamental representations of $\widetilde{SU}(2)_{R,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{II,\widetilde{SO}(4)}$ and $\widetilde{SU}(2)_{L,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{I,\widetilde{SO}(4)}$ groups, respectively - the subgroups of $\widetilde{SO}(3, 1)$ and $\widetilde{SO}(4)$ (Table 6.4, page 89). These eight families remain massless up to the electroweak break due to the "mass protection mechanism", that is due to the fact that the right handed members have no left handed partners with the same charges.

[6] Before the electroweak break are all observable gauge fields massless: the gravity, the colour octet vector gauge fields (of the group $SU(3)$ from $SO(6)$), the weak triplet vector gauge field (of the group $SU(2)_I$ from $SO(4)$), and the hyper singlet vector gauge field (a superposition of $U(1)$ from $SO(6)$ and the third component of $SU(2)_{II}$ triplet). All are the superposition of the $f^{\alpha}{}_{c}\ \omega_{ab\alpha}$ spinor gauge fields.

[7] There are scalar fields with the space index $(7, 8)$ and with respect to the space index with the weak and the hyper charge of the Higgs's scalar. They belong with respect to

e. The starting action contains also the additional $SU(2)_{II}$ (from $SO(4)$) vector gauge triplet (one of the components contributes to the hyper charge gauge fiels as explained above in the footnote of **d**. of **C.i.**), as well as the scalar fields with the space index $s \in (5,6)$ and $t \in (9,10,\ldots,14)$. All these fields gain masses of the scale of the condensate (Table 7.1), which they interact with. They all are expressible with the superposition of $f^{\mu}{}_{m}\, \omega_{ab\mu}$ or of $f^{\mu}{}_{m}\, \tilde{\omega}_{ab\mu}$ [8].

C ii., C iii. There are many ways of breaking symmetries from $d = (13+1)$ to $d = (3+1)$. The assumed breaks explain the connection between the weak and the hyper charge and the handedness of spinors, manifesting correspondingly the observed properties of the family members - the quarks and the leptons, left and right handed (Table 6.2, page 86) - and of the observed vector gauge fields. After the break from $SO(13,1)$ to $SO(3,1)\times SU(2)\times U(1)\times SU(3)$ the anti-particles are accessible from particles by the application of the operator $\mathbb{C}_{\mathcal{N}}\cdot\mathcal{P}_{\mathcal{N}}$, as explained [9] in Refs. [38].

C iv. It is the condensate (Table 7.1) of two right handed neutrinos with the quantum numbers of one group of four families, which makes massive all the scalar gauge fields (those with the space index s equal to $(5,6,7,8)$, as well as those with the space indexs equal to $(9,\ldots,14)$) and those vector gauge fields, manifesting nonzero τ^4, τ^{23}, $\tilde{\tau}^4$, $\tilde{\tau}^{23}$, $\tilde{N}^3_R$ [2,1]. Only the vector gauge fields of Y, $SU(3)$ and $SU(2)$ remain massless, since they do not interact with the condensate.

C v. At the electroweak break the scalar fields with the space index $s = (7,8)$ - originating in $\tilde{\omega}_{abs}$, as well as some superposition of $\omega_{s's"s}$ with the quantum numbers (Q, Q', Y'), conserving the colour and the electromagnetic charge - change their mutual interaction, and gaining nonzero vacuum expectation values change correspondingly also their masses. They contribute to mass matrices of twice the four families, as well as to the masses of the heavy vector bosons.

All the rest scalar fields keep masses of the scale of the condensate and are correspondingly unobservable in the low energy regime.

The fourth family to the observed three ones is *predicted to be observed* at the LHC. Its properties are under consideration [14,15], the *baryons of the stable family of the upper four families is offering the explanation for the dark matter* [13]. The

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additional quantum numbers either to one of the two groups of two triplets, (either to one of the two triplets of the groups $\widetilde{SU}(2)_{R\,\widetilde{SO}(3,1)}$ and $\widetilde{SU}(2)_{II\,\widetilde{SO}(4)}$, or to one of the two triplets of the groups $\widetilde{SU}(2)_{L\,\widetilde{SO}(3,1)}$ and $\widetilde{SU}(2)_{I\,\widetilde{SO}(4)}$, respectively), which couple through the family quantum numbers to one (the first two triplets) or to another (the second two triplets) group of four families - all are the superposition of $f^{\sigma}{}_{s}\ \tilde{\omega}_{ab\sigma}$, or they belong to three singlets, the scalar gauge fields of (Q, Q', Y'), which couple to the family members of both groups of families - they are the superposition of $f^{\sigma}{}_{s}\ \omega_{ab\sigma}$. Both kinds of scalar fields determine the fermion masses (Eq. (7.7)), offering the explanation for the higgs, the Yukawa couplings and the heavy bosons masses.

[8] In the case of free fields (if no spinor source, carrying their quantum numbers, is present) both $f^{\mu}{}_{m}\, \omega_{ab\mu}$ and $f^{\mu}{}_{m}\, \tilde{\omega}_{ab\mu}$ are expressible with vielbeins, correspondingly only one kind of the three gauge fields are the propagating fields.

[9] The discrete symmetry operator $\mathbb{C}_{\mathcal{N}}\cdot\mathcal{P}_{\mathcal{N}}$, Refs. [38,39], does not contain $\tilde{\gamma}^a$'s degrees of freedom. To each family member there corresponds the anti-member, with the same family quantum number.

triplet and anti-triplet scalar fields contribute together with the condensate to the *matter/antimatter assymetry*.

Let us (formally) rewrite that part of the action of Eq.(7.8), which determines the spinor degrees of freedom, in the way that we can clearly see that the action does in the low energy regime manifest by the *standard model* required degrees of freedom of the fermions, vector and scalar gauge fields [4,5,3,1,9,6–8,11–14].

$$\begin{aligned}\mathcal{L}_f = &\bar{\psi}\gamma^m(p_m - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_m)\psi + \\ &\{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\, \psi\} + \\ &\{\sum_{t=5,6,9,\dots,14} \bar{\psi}\gamma^t p_{0t}\, \psi\},\end{aligned} \tag{7.9}$$

where

$$\begin{aligned}p_{0s} &= p_s - \frac{1}{2}S^{s's"}\omega_{s's"s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs},\\ p_{0t} &= p_t - \frac{1}{2}S^{t't"}\omega_{t't"t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt},\end{aligned}$$

with $m \in (0,1,2,3)$, $s \in (7,8)$, $(s', s") \in (5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within either $(0,1,2,3)$ or $(5,6,7,8)$, t runs $\in (5,\dots,14)$, $(t', t")$ run either $\in (5,6,7,8)$ or $\in (9,10,\dots,14)$. The spinor function ψ represents all family members of all the $2^{\frac{7+1}{2}-1} = 8$ families.

The first line of Eq. (7.9) determines (in $d = (3+1)$) the kinematics and dynamics of spinor (fermion) fields, coupled to the vector gauge fields. The generators τ^{Ai} of the charge groups are expressible in terms of S^{ab} through the complex coefficients ${c^{Ai}}_{ab}$ [10],

$$\tau^{Ai} = \sum_{a,b} {c^{Ai}}_{ab}\, S^{ab}\,, \tag{7.10}$$

fulfilling the commutation relations

$$\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk} \tau^{Ak}\,. \tag{7.11}$$

They represent the colour, the weak and the hyper charges (as well as the $SU(2)_{II}$ and τ^4 charges, the gauge fields of which gain masses interacting with the condensate, Table 7.1, leaving massless only the hyper charge vector gauge field). The corresponding vector gauge fields A^{Ai}_m are expressible with the spin connection fields ω_{stm}, with (s,t) either $\in (5,6,7,8)$ or $\in (9,\dots,14)$, in agreement with the

[10] $\vec{\tau}^1 := \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78})\,, \vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78})$, $\vec{\tau}^3 := \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}$, $\tau^4 := -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$. After the electroweak break the charges $Y := \tau^4 + \tau^{23}\,, Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}\,, Q := \tau^{13} + Y\,, Q' := -Y\tan^2\vartheta_1 + \tau^{13}$ manifest. θ_1 is the electroweak angle, breaking $SU(2)_I$, θ_2 is the angle of the break of the $SU(2)_{II}$ from $SU(2)_I \times SU(2)_{II}$.

assumptions **A ii.** and **A iii.**. I demonstrate in Ref. [1] the equivalence between the usual Kaluza-Klein procedure leading to the vector gauge fields through the vielbeins and the procedure with the spin connections proposed by the *spin-charge-family* theory.

All vector gauge fields, appearing in the first line of Eq. (7.9), except $A^{2\pm}_{m}$ and $A^{Y'}_{m}$ ($= \cos\vartheta_2\, A^{23}_{m} - \sin\vartheta_2\, A^{4}_{m}$, Y' and τ^4 are defined in [11]), are massless before the electroweak break. $\vec{A}^{3}_{m}$ carries the colour charge $SU(3)$ (originating in $SO(6)$), $\vec{A}^{1}_{m}$ carries the weak charge $SU(2)_{I}$ ($SU(2)_{I}$ and $SU(2)_{II}$ are the subgroups of $SO(4)$) and A^{Y}_{m} ($= \sin\vartheta_2\, A^{23}_{m} + \cos\vartheta_2\, A^{4}_{m}$) carries the corresponding $U(1)$ charge ($Y = \tau^{23} + \tau^4$, τ^4 originates in $SO(6)$ and τ^{23} is the third component of the second $SU(2)_{II}$ group, A^{4}_{m} and $\vec{A}^{2}_{m}$ are the corresponding vector gauge fields). The fields $A^{2\pm}_{m}$ and $A^{Y'}_{m}$ get masses of the order of the condensate scale through the interaction with the condensate of the two right handed neutrinos with the quantum numbers of one - the upper one - of the two groups of four families (the assumption **A iv.**, Table 7.1). (See Ref. [1].)

Since spinors (fermions) carry besides the family members quantum numbers also the family quantum numbers, determined by $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$, there are correspondingly $2^{(7+1)/2-1} = 8$ families [1], which split into two groups of families, each manifesting the $(\widetilde{SU}(2)_{\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)} \times U(1))$ symmetry.

If there are no fermions present then the vector gauge fields of the family members and of the family charges - ω_{abm} and $\tilde{\omega}_{abm}$, respectively - are uniquely expressible with the vielbeins [2,1].

The scalar fields, the gauge fields with the space index $s = (7, 8)$, which are either superposition of $\tilde{\omega}_{abs}$ or of $\omega_{s'ts}$, determine - after gaining nonzero vacuum expectation values (the assumption **A v.** and comments **C v.**) - masses of fermions (belonging to two groups of four families of family members of spinors) and weak bosons.

The condensate (the assumption **A iv.**), Table 7.1, gives masses of the order of the scale of its appearance to all the scalar gauge fields, presented in the second and the third line of Eq. (7.9).

The vector gauge fields of the (before the electroweak break) conserved the colour, the weak and the hyper charges ($\vec{\tau}^3$, $\vec{\tau}^1$, Y) do not interact with the condensate and stay correspondingly massless. After the electroweak break - when the scalar fields (those with the family quantum numbers and those with the family members quantum numbers (Q, Q', Y')) with the space index $s = (7, 8)$ start to strongly self interact, gaining nonzero vacuum expectation values - only the charges $\vec{\tau}^3$ and $Q = Y + \tau^{13}$ are the conserved charges. No family quantum numbers are conserved, since all the scalar fields with the family quantum numbers and the space index $s = (7, 8)$ gain nonzero vacuum expectation values.

Quarks and leptons have the "spinor" quantum number (τ^4, originating in $SO(6)$), presented in Table 6.2, page 86) equal to $\frac{1}{6}$ and $-\frac{1}{2}$, respectively.

[11] $Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}, \tau^4 = -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$.

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^{4}	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^{4}$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$(\vert\nu^{\mathbf{VIII}}_{\mathbf{1R}}>_{\mathbf{1}}\ \vert\nu^{\mathbf{VIII}}_{\mathbf{2R}}>_{\mathbf{2}})$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\vert\nu^{VIII}_{1R}>_1\ \vert e^{VIII}_{2R}>_2)$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\vert e^{VIII}_{1R}>_1\ \vert e^{VIII}_{2R}>_2)$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

Table 7.1. This table is taken from [2]. The condensate of the two right handed neutrinos ν_R, with the VIIIth family quantum numbers, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The right handed neutrino has $Q = 0 = Y$. The triplet carries $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{\tau}^4 = -1$, $\tilde{N}^3_R = 1$, $\tilde{N}^3_L = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$. The family quantum numbers are presented in Table 6.4, page 89.

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8 The *Spin-charge-family* Theory Offers Understanding of the Triangle Anomalies Cancellation in the *Standard Model*

N.S. Mankoč Borštnik[1] and H.B.F. Nielsen[2]

[1]Department of Physics, FMF, University of Ljubljana,
Jadranska 19, SI-1000 Ljubljana, Slovenia
[2]The Niels Bohr Institute,
Blegdamsvej 17 - 21, Copenhagen Ø; Denmark

Abstract. The *standard model* has for massless quarks and leptons "miraculously" no triangle anomalies due to the fact that the sum of all possible traces $\mathrm{Tr}[\tau^{Ai}\tau^{Bj}\tau^{Ck}]$ — where τ^{Ai}, τ^{Bi} and τ^{Ck} are the generators of one, of two or of three of the groups of $SU(3), SU(2)$ and $U(1)$ — over the representations of one family of the left handed fermions and their antifermions (and separately of the right handed fermions and their antifermions), contributing to the triangle currents, are equal to zero [1–4]. It is demonstrated in this paper that this cancellation of the *standard model* triangle anomaly follows straightforwardly if the $SO(3,1), SU(2), U(1)$ and $SU(3)$ are the subgroups of the orthogonal group $SO(13,1)$, as it is assumed in the *spin-charge-family* [5,6,9–12,15–21].

Povzetek. V *standardnem modelu* za brezmasne kvarke in leptone anomalij kot po "čudežu" ni, ker je vsota vseh možnih sledi $\mathrm{Tr}[\tau^{Ai}\tau^{Bj}\tau^{Ck}]$ — kjer so τ^{Ai}, τ^{Bi} in τ^{Ck} generatorji ene, dveh ali treh izmed grup $SU(3), SU(2)$ in $U(1)$ — po upodobitvah ene družine levoročnih kvarkov in leptonov ter njihovih antidelcev (in ločeno po upodobitvah ustreznih desnoročnih delcev in antidelcev), ki prispevajo k tokovom v trikotniku, enaka nič [1–4]. Prispevek pokaže, da to "čudežno" izničenje trikotniške anomalije *standardnega modela* preprosto sledi, če obravnavamo (vložimo) grupe $SO(3,1), SU(2), U(1)$ in $SU(3)$ kot podgrupe ortogonalne grupe $SO(13,1)$, tako kot to velja v teoriji *spinov-nabojev-družin* [5,6,9–12,15–21].

8.1 Introduction

In $d = (2n)$-dimensional space-time massless fermions contribute through the one-loop (n + 1)-angle diagram an anomalous (infinite) function, which causes the current non-conservation and contributes to the gauge non-invariance of the action [1–4].

We discuss in this contribution anomalies in $[d = (3+1)]$-dimensional space-time, that is the triangle anomalies, from the point of view of the *spin-charge-family* theory (which starts with a simple action in $d = (13+1)-$dimensional space-time) to demonstrate that embedding the *standard model* groups into the orthogonal group $SO(13+1)$ explains elegantly the "miraculous" cancellation of the triangle anomalies in the *standard model*. We add as an illustration the case when $SO(13,1)$ breaks into $SO(1,7) \times SU(3) \times U(1)$.

To the triangle anomaly the right-handed spinors (fermions) and anti-spinors (anti-fermions) contribute with the opposite sign than the left handed spinors and their anti-spinors. Their common contribution to anomalies is proportional to [3]

$$\Big(\sum_{(A,i,B,j,C,k)_{L\,\bar{L}}} Tr[\tau^{Ai}\,\tau^{Bj}\,\tau^{Ck}] - \sum_{(A,i,B,j,C,k)_{R\,\bar{R}}} Tr[\tau^{Ai}\,\tau^{Bj}\,\tau^{Ck}] \,\Big)\,, \tag{8.1}$$

where τ^{Ai} are in the *standard model* the generators of the infinitesimal transformation of the groups $SU(3) \times SU(2) \times U(1)$, while in the *spin-charge-family* theory τ^{Ai} are irreducible subgroups of the starting orthogonal group $SO(2(2n+1)-1,1)$, $n = 3$. $_{L\,\bar{L}}$ ($_{R\,\bar{R}}$) denote the left (right) handed spinors and their antispinors (right (left)), respectively.

In $d = (2n)$-dimensional space-time one has summation over products of $(n+1)$ products of generators.

We demonstrate that the *triangle anomaly cancellation in* $d = (3+1)$ *appears naturally, if the standard model groups* - $SO(3,1) \times SU(2) \times SU(3) \times U(1)$ - *are embedded in the orthogonal group* $SO(13,1)$, as it is assumed by the *spin-charge-family* theory [5,6,9–12,15–21], while it appears in the *standard model* almost like a miracle.

One Weyl representation of spinors of $SO(13,1)$ (in the fourteen dimensional space-time) contains, if analyzed from the point of view of the *standard model* groups, all the left and the right handed quarks and leptons, and the right handed anti-partners of the left handed quarks and leptons and the left handed anti-partners of the right handed quarks and leptons of one family. This can be seen in Table 6.2 (see page 86), where the spinor part of quarks and leptons states (wave functions) and their quantum numbers are presented in the "technique" with nilpotents and projectors [22]. Table 6.2 (see page 86) presents spinor handedness ($\Gamma^{(3,1)}$), their spin (S^{12}), weak charge (τ^{13}), the second $SU(2)_{II}$ (τ^{23}) (arising together with $SU(2)_{I}$ from $SO(4)$), their colour charge (τ^{33}, τ^{38}) (arising together with $U(1)_{II}$ from $SO(6)$), and the "fermion" charge (τ^{4}, the generator of $U(1)_{II}$). The hyper charge is $Y = (\tau^{23} + \tau^{4})$, the electromagnetic charge is $Q = (\tau^{13} + Y)$.

Let be pointed out that the *spin-charge-family* theory is able - while starting from a very simple action in $d = (13+1)$ with massless fermions with the spin of the two kinds (one kind taking care of the spin and the charges of the family members in $d = (3+1)$, the second kind taking care of the families and family charges), coupling correspondingly only to the gravity (through the vielbeins and the two kinds of the corresponding spin connections) - to explain not only all the assumptions of the *standard model*, but answers also several open questions beyond the *standard model* [6,5,25].

In Sect. 8.2 the contribution to the triangle anomalies of the *standard model* massless fermions and anti-fermions are discussed from the point of view that the $SO(3,1), SU(2), SU(3)$ and $U(1)$ are the subgroups of the orthogonal group $SO(13,1)$.

We illustrate in Subsect. 8.2.2 the five-angle anomaly cancellation when $SO(13,1)$ breaks into $SO(7,1) \times SU(3) \times U(1)$.

In App. 6.6 (page 118) the technique to represent spinors, used in this contribution, is explained.

8.2 Standard model triangle anomaly cancellation looks natural if $SO(3,1)$, $SU(2)$, $SU(3)$ and $U(1)$ are embedded in $SO(13,1)$

The cancellation of the triangle anomalies of the massless quarks and leptons and their anti-particles looks from the point of view of the *standard model* "miraculous".

The triangle anomaly of the *standard model* occurs if the traces in Eq.(8.1) are not zero for either the left handed quarks and leptons and their anti-particles or the right handed quarks and leptons and their anti-particles for the Feynman triangle diagrams in which the gauge vector fields of the type

$$\begin{aligned}
&U(1) \times U(1) \times U(1)\,,\\
&SU(2) \times SU(2) \times U(1)\,,\\
&SU(3) \times SU(3) \times SU(3)\,,\\
&SU(3) \times SU(3) \times U(1)\,,\\
&U(1) \times \text{gravitational}
\end{aligned} \tag{8.2}$$

contribute to the triangle anomaly.

Let us first in Table 8.1 present by the *standard model* assumed properties of a one family members, running in the triangle. The corresponding data are represented in the first seven columns (up to to ||). The last two columns describe additional properties which quarks and leptons (and anti-quarks and anti-leptons) would have, if the *standard model* groups $SO(3,1)$, $SU(2), SU(3)$ and $U(1)$ are embedded into the $SO(13,1)$ group. The last two columns are taken from Table 6.2 (see page 86). We shall comment these last two columns later. In the *spin-charge-family* theory are the family quantum numbers determined with the second kind of the Clifford algebra objects ($\tilde{S}^{ab}$). Correspondingly are the spins and charges the same for all the families.

To calculate the traces required in Eq. (8.1) for the triangle anomalies of Eq. (8.2) the quantum numbers of the left handed spinors and antispinors, as well as of the right handed spinors and antispinors, presented in Table 8.1 are needed.

Before demonstrating the well known results that the required traces are equal to zero, assuring the "miraculous" anomaly free triangle diagrams, let us comment the quantum numbers of the same spinors, if the *standard model* subgroups - $SO(3,1) \times SU(2) \times SU(3) \times U(1)$ - are embedded into $SO(13,1)$. In this case each of the *standard model* family members (of any family) have additional quantum numbers, as one can see in Table 6.2 (page 86), where the quarks and leptons, left and right handed and the corresponding anti-quarks and anti-leptons are presented.

The subgroups of the $SO(13,1)$ group are $SO(7,1) \times SO(6)$. The subgroups of $SO(6)$ are the colour group $SU(3)$ with the generators denoted by τ^{3i}, $i = 1,\ldots,8$ and the $U(1)$ (we shall call it $U(1)_{II}$) group with the generator τ^4. One sees that

i_L	name	hand-edness $\Gamma^{(3,1)}$	weak charge τ^{13}	hyper charge Y	colour τ^{33}	charge τ^{38}	elm charge Q	$SU(2)_{II}$ charge τ^{23}	$U(1)_{II}$ charge τ^4
1_L	u_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	0	$\frac{1}{6}$
2_L	d_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
3_L	u_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	0	$\frac{1}{6}$
4_L	d_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
5_L	u_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{2}{3}$	0	$\frac{1}{6}$
6_L	d_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
7_L	ν_L	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$-\frac{1}{2}$
8_L	e^L	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	0	$-\frac{1}{2}$
9_L	$\bar{u}L$	-1	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$
10_L	$\bar{d}L$	-1	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$
11_L	$\bar{u}L$	-1	0	$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$
12_L	$\bar{d}L$	-1	0	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$
13_L	$\bar{u}L$	-1	0	$-\frac{2}{3}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$
14_L	$\bar{d}L$	-1	0	$\frac{1}{3}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$
15_L	$\bar{\nu}_L$	-1	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
16_L	$\bar{e}_L$	-1	0	1	0	0	1	$\frac{1}{2}$	$\frac{1}{2}$
1_R	u_R	1	0	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
2_R	d_R	1	0	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$
3_R	u_R	1	0	$\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
4_R	d_R	1	0	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$
5_R	u_R	1	0	$\frac{2}{3}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
6_R	d_R	1	0	$-\frac{1}{3}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$
7_R	ν_R	1	0	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
8_R	e_R	1	0	-1	0	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
9_R	$\bar{u}_R$	1	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	0	$-\frac{1}{6}$
10_R	$\bar{d}_R$	1	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
11_R	$\bar{u}_R$	1	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	0	$-\frac{1}{6}$
12_R	$\bar{d}_R$	1	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
13_R	$\bar{u}_R$	1	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{2}{3}$	0	$-\frac{1}{6}$
14_R	$\bar{d}_R$	1	$\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
15_R	$\bar{\nu}_R$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$
16_R	$\bar{e}_R$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	0	$\frac{1}{2}$

Table 8.1. Properties of the left handed quarks and leptons and their anti-particles and of the right handed quarks and leptons and their anti-particles, as assumed by the *standard model*, are presented in the first seven columns. In the last two columns the two quantum numbers are added, which the fermions and anti-fermions would have if the *standard model* groups $SO(3,1), SU(2), SU(3)$ and $U(1)$ are embedded into the $SO(13,1)$ group. The whole quark part appears, due to the colour charges, three times. One can check that the hyper charge is the sum of $\tau^4_{i_{L,R}} + \tau^{23}_{i_{L,R}}$ Table 6.2 (see page 86). The quantum numbers are the same for all the families also in the *spin-charge-family* theory.

all the quarks have $\tau^4 = \frac{1}{6}$, the anti-quarks have $\tau^4 = -\frac{1}{6}$, while the leptons have $\tau^4 = -\frac{1}{2}$ and anti-leptons have $\tau^4 = \frac{1}{2}$. Correspondingly the trace of τ^4 over all the family members is equal to zero. In one Weyl (spinor) representation of $SO(13,1)$ is the right handed neutrino (and correspondingly its anti-particle) the regular member of the representation.

The subgroups of the $SO(7,1)$, as seen in Table 6.2 (on page 86), have as subgroups $SO(3,1) \times SU(2)_I \times SU2_{II}$, with the generators τ^{1i} (representing the weak group operators) and τ^{2i} (representing the generators of the additional

$SU(2)$ group), respectively. The left handed spinors are $SU(2)_I$ (weak) doublets and $SU(2)_{II}$ singlets, while the right handed spinors are the $SU(2)_I$ (weak) singlets and the $SU(2)_{II}$ doublets. Correspondingly are the left handed anti-spinors the $SU(2)_I$ (weak) singlets and the $SU(2)_{II}$ doublets, while the right handed anti-spinors are the $SU(2)_I$ (weak) doublets and the $SU(2)_{II}$ singlets.

The hyper charge of the *standard model* corresponds to the sum of τ^4 and τ^{23}

$$Y = \tau^4 + \tau^{23}\,, \tag{8.3}$$

which manifests after the $SU(2)_{II}$ symmetry is broken [5,6,9–12,15–21].

A short presentation of the properties of the one family members, appearing in a Weyl representation of $SO(13,1)$ (Table 6.2, see page 86), when the analyses of the members with respect to the subgroups $SO(1,3), SU(2)_I, SU(2)_{II}$ and $U(1)_{II}$ is done, can be found in Table 8.1 when also the last two columns are taken into account. Looking at this table one easily recognizes where does the "miraculous" cancellation of the triangle anomalies emerges. Making a "miraculous" cancellation of triangle anomalies a "trivial" one, if one takes into account that the *standard model* groups have the origin in $SO(13,1)$, supports that the *spin-charge-family* theory might be the right next step beyond the *standard model*.

It is worthwhile to point out that within the one Weyl representationof $SO(13+1)$, Table 6.2 (see page 86), the handedness of the particles as well as of the anti-particles is uniquely connected with their charges, while in $SO(10)$, for example, such a connection must be put by hand.

8.2.1 Traces of the left haded spinors and their antispinors and of the right handed spinors and their antispinors of one family of quarks and leptons

Let us calculate the traces for possible anomalous triangle diagrams, presented in Eq. (8.2). We must evaluate the trace of the product of three generators and sum the trace over all the representations of either the left handed members - the first part of the Table 8.1 - or the right handed members - the second part of the Table 8.1. Let us recognize again that in the case of embedding the *standard model* groups into $SO(13,1)$ we have $Y = (\tau^4 + \tau^{23})$.

For the triangle Feynman diagram, to which three hyper $U(1)$ boson fields contribute, we must evaluate $\sum_i \mathrm{Tr}(Y_i)^3$, in which the sum runs over all the members (i) of the left handed spinors and antispinors, and of the right handed spinors and anti-spinors separately. In the case of embedding the *standard model* groups into $SO(13,1)$ we have

$$\begin{aligned}
\sum_{i_{L,R}} (Y_{i_{L,R}})^3 &= \sum_{i_{L,R}} (\tau^4_{i_{L,R}} + \tau^{23}_{i_{L,R}})^3 \\
&= \sum_{i_{L,R}} (\tau^4_{i_{L,R}})^3 + \sum_{i_{L,R}} (\tau^{23}_{i_{L,R}})^3 \\
&+ \sum_{i_{L,R}} 3 \cdot (\tau^4_{i_{L,R}})^2 \cdot \tau^{23}_{i_{L,R}} + \sum_{i_{L,R}} 3 \cdot \tau^4_{i_{L,R}} \cdot (\tau^{23}_{i_{L,R}})^2\,,
\end{aligned} \tag{8.4}$$

for either the left, i_L, or the right, i_R, handed members. Table 8.1 demonstrates clearly that $(Y_{i_{L,R}})^3 = 0$ without really make any algebraic evaluation. Namely, the last column manifests that $\sum_{i_L} (\tau^4_{i_L})^3 = 0$ [in details $\sum_{i_L} (\tau^4_{i_L})^3 = 0 = 2\cdot 3\cdot (\frac{1}{6})^3 + 2\cdot 3\cdot (-\frac{1}{6})^3 + 2\cdot (-\frac{1}{2})^3 + 2\cdot (\frac{1}{2})^3] = \sum_{i_R} (\tau^4_{i_R})^3 = 0$. Table 8.1 also demonstrates (the last but one column) that $\sum_{i_L} (\tau^{23}_{i_L})^3 = 0 = [(3+1)\cdot ((-\frac{1}{2})^3 + (\frac{1}{2})^3))]$, and that also $\sum_{i_R} (\tau^{23}_{i_R})^3 = 0 = [(3+1)\cdot ((\frac{1}{2})^3 + (-\frac{1}{2})^3)]$.

From Table 8.1 one sees also that $\sum_{i_L} 3\cdot (\tau^4_{i_L})^2 \cdot \tau^{23}_{i_L} = 0 = [3.\{((\frac{1}{2})^2 \cdot (-\frac{1}{2} + \frac{1}{2}) + 3\cdot (-\frac{1}{6})^2 \cdot (-\frac{1}{2} + \frac{1}{2})\}]$, as well as $\sum_{i_R} 3\cdot (\tau^4_{i_R})^2 . \tau^{23}_{i_R} = 0 = [3.\{((-\frac{1}{2})^2 . (\frac{1}{2} + (-\frac{1}{2})) + 3\cdot (\frac{1}{6})^2 \cdot (\frac{1}{2} + (-\frac{1}{2}))\}]$.

That the last term in Eq. (8.4) is zero for either the left or the right handed spinors can also easily be seen: $\sum_{i_L} 3\cdot \tau^4_{i_L} \cdot (\tau^{23}_{i_L})^2 = 0 = [3\cdot \{(\frac{1}{2}((\frac{1}{2})^2 + (-\frac{1}{2})^2) + 3\cdot (-\frac{1}{6})((\frac{1}{2})^2 + (-\frac{1}{2})^2))\}]$, as well as $\sum_{i_R} 3\cdot \tau^4_{i_L} \cdot (\tau^{23}_{i_R})^2 = 0 = [3\cdot \{(-\frac{1}{2}((\frac{1}{2})^2 + (-\frac{1}{2})^2) + 3\cdot (\frac{1}{6})((\frac{1}{2})^2 + (-\frac{1}{2})^2))\}]$.

Since all the members belong to one spinor representation, it is straightforwardly that all the triangle tracesare zero, if the *standard model* groups are the subgroups of the orthogonal group $SO(13,1)$.

There is no need for a detailed calculations, since having a look on Table 8.1 gives immediately the answer.

From only the *standard model* assumptions point of view the cancellation of the triangle anomalies does look miraculously. For our $\sum_{i_{L,R}} (Y_{i_{L,R}})^3$ one obtains for the left handed members: $[3\cdot 2\cdot (\frac{1}{6})^3 + 2\cdot (-\frac{1}{2})^3 + 3\cdot ((-\frac{2}{3})^3 + (\frac{1}{3})^3) + 1^3)$], and for the right handed members: $[3\cdot ((\frac{2}{3})^3 + (-\frac{1}{3})^3) + (-1)^3) + 3\cdot 2\cdot (-\frac{1}{6})^3 + 2\cdot (\frac{1}{2})^3]$.

8.2.2 Anomaly cancellation in case of $SO(7,1), SU(3), U(1)\ U(1)$ are embedded in $SO(13,1)$

Let us add for an illustration the anomaly cancellation for the case when $SO(13,1)$ breaks into $SO(7,1)\times SU(3)\times U(1)$.

We should evaluate

$$
\Big(\sum_{(A,i,B,j,C,k,D,l,E,k)_{L\,\bar{L}}} Tr[\tau^{Ai}\,\tau^{Bj}\,\tau^{Ck}\,\tau^{Dl}\,\tau^{EK}] - \Big(\sum_{(A,i,B,j,C,k,D,l,E,k)_{R\,\bar{R}}} Tr[\tau^{Ai}\,\tau^{Bj}\,\tau^{Ck}\,\tau^{Dl}\,\tau^{EK}]\Big)\,. \quad (8.5)
$$

Let us treat the five-angle anomalies for the less trivial case

$$
U(1)\times U(1)\times U(1)\times U(1)\times U(1)\,. \quad (8.6)
$$

Now only the last column of Table 8.1 is neeeded. Taking into account all the left handed partners and anti-partners one immediately sees that the corresponding trace in Eq. (8.5) is zero [in details: $3\cdot 8\cdot (\,(\frac{1}{6}^5 + (-\frac{1}{6})^5) + 8\cdot (\,(-\frac{1}{2})^5 + (\frac{1}{2})^5) = 0.$]. For all the rest possibilities is even easier to see that they are trivially zero as long as the coupling constants for particles and antiparticles are the same, as well as of doublets.

8.3 Discussions and conclusions

Not only the assumptions of the *standard model* also the "miraculous" cancellation of the triangle anomaly in the *standard model* calls for the explanation. Each successful model, offering the next step beyond the *standard model*, should offer all these explanations. Since the *spin-charge-family* theory does offer all these and many others explanations of the phenomenas in the field of elementary fermions and bosons, seems really promising to be the next step beyond the *standard model*.

In this work we demonstrate that the *standard model* triangle "miraculous" anomaly cancellation can trivially be explained within the *spin-charge-family* theory since one Weyl representation of $SO(13, 1)$ (the model starts with the simple action in $d = (13 + 1)$, leading in the low energy regime to the *standard model* action with the right handed neutrino and his anti-neutrino included, explains also the appearance of the families, of the scalar field and the Yukawa couplings) contains, if analyzed from the point of view of the *standard model* groups $SO(3, 1), SU(2)$, $SU(3)$ and $U(1)$, all the quarks and leptons and their anti-particles with the properties assumed by the *standard model* (with the right handed neutrinos in addition).

In this theory the hyper charge Y appears as a sum of the two quantum numbers: τ^4 (a "fermion" quantum number) and τ^{23} (the second $SU(2)$ charge). Taking this into account and the fact that one Weyl representation has the traces of all subgroups equal to zero, makes a simple explanation for the traceless products of all contributions to the triangle anomalies of the *standard model*, with the $U(1) \times U(1) \times U(1)$ included.

It should also be pointed out that embedding the *standard model* groups into $SO(13, 1)$ makes the weak and hyper charges of particles and anti-particles connected with their handedness, which does not happen in the models which rely on $SO(10)$.

One Weyl representation of $SO(13 + 1)$ contains left handed weak charged and the second $SU(2)$ chargeless coloured quarks and colourless leptons and right handed weak chargeless and the second $SU(2)$ charged quarks and leptons (electrons and neutrinos). It carries also the family quantum numbers, not mentioned in this table. The table is taken from Ref. [13].

The eight families of the first member of the eight-plet of quarks from Table 6.2 (see page 86), for example, that is of the right handed u_{1R} quark, are presented in the left column of Table 8.2 [5]. In the right column of the same table the equivalent eight-plet of the right handed neutrinos ν_{1R} are presented. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight-plets separate into two group of four families: One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to these two groups. The scalar fields

					$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$	$\tilde{\tau}^4$
I	$u^{c1}_{R\,1}$	$\overset{03}{(+i)}\overset{12}{[+]}\vert\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,2}$	$\overset{03}{(+i)}\overset{12}{[+]}\vert\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,2}$	$\overset{03}{[+i]}\overset{12}{(+)}\vert\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,2}$	$\overset{03}{[+i]}\overset{12}{(+)}\vert\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,3}$	$\overset{03}{(+i)}\overset{12}{[+]}\vert\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,3}$	$\overset{03}{(+i)}\overset{12}{[+]}\vert\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,4}$	$\overset{03}{[+i]}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,4}$	$\overset{03}{[+i]}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
II	$u^{c1}_{R\,5}$	$\overset{03}{[+i]}\overset{12}{[+]}\vert\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,5}$	$\overset{03}{[+i]}\overset{12}{[+]}\vert\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,6}$	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,6}$	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,7}$	$\overset{03}{[+i]}\overset{12}{[+]}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,7}$	$\overset{03}{[+i]}\overset{12}{[+]}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,8}$	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$	$\nu_{R\,8}$	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$

Table 8.2. Eight families of the right handed u^{c1}_R (Table 6.2 (see page 86)) quark with spin $\frac{1}{2}$, the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, and of the colourless right handed neutrino ν_R of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. They belong to two groups of four families, one (II) is a doublet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$), the other (I) is a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a doublet with with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$). All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$), Eq. (6.73). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (6.73)) transform u_{1R} to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino ν_{1R} to all the colourless members of the same family.

which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ couple only to the four families which are doublets with respect to these last two groups.

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9 The New LHC-Peak is a Bound State of 6 Top + 6 Anti top

H.B. Nielsen *

Niels Bohr Institute, Blegdamsvej 15 -21
DK 2100 Copenhagen Ø

Abstract. The point of the present talk was that the at the time of the talk still statistically significant digamma resonance F(750) observed in ATLAS and CMS should be identified with the bound state of 6 top and 6 anti top quarks, which we have long speculated to exist. Since then my calculations have suggested that the mass of the bound state is indeed in the range about 750 GeV[1,2]. If the story would be supported by there exisiting a resonance into one of our suggested channels $\gamma\gamma$, pair of weak vector bosons, Higgs+ Higgs, or t + $\bar{t}$, ...with a mass in the neighbourhood of 750 GeV, then it would be an indication of the truth of our suggested new law of nature called "Multiple Point Principle". As a proposal it is not really new since we used it even in the 90's to PREdict the mass of the Higgs long before it were found to 135 ± 10 GeV. But it is "new" in the sense that yet nobody believes in it. It says that there are several vacua - in Standard Model case 3 - all having same energy density.

Povzetek. Avtor pojasnjuje domnevno izmerjen dogodek resonance pri 750 GeV z razpadom vezanega stanja dvanajstih kvarkov — 6 t in 6 $\bar{t}$ v dva fotona — ki ga je skupaj s sodelavci v modelu "Multiple Point Principle" napovedal prav v tem energijskem območju že dolgo pred meritvami [1,2]. Poleg razpada v dva fotona so napovedali razpade v pare dveh težkih in dveh skalarnih bozonov, v t in $\bar{t}$,... Z modelom "Multiple Point Principle" so že dolgo pred meritvijo mase higgsa napovedali njegovo maso pri (135 ± 10) GeV. Model "Multiple Point Principle" predpostavi, da eksistirajo različna vakuumska stanja, vsa z isto energijsko gostoto, *standardni model* pa velja do Planckove energije. (V času predavanja so meritve veljale kot statistično sprejemljive, nove meritve pa obstoja tega dogodka niso potrdile.)

9.1 Introduction

The main point of the talk were, when I gave it in July 2016 that the - at that time still statistically promising - *New Diphoton Resonance F(750) of Mass 750 GeV should be interpreted as a 6 Top + 6 Anti top Bound state* is by now so much worth in as far as the resonance F(750) now seem to have been washed out so that there is no more statistical evidence for it.

We long worked on a picture based on the *Standard Model alone*, but involving a bound state of 6 top + 6 antitop quarks bound by Higgs exchange and helped by

* hbech@nbi.dk

gluon exchange. Thus at first it would seem that the whole content of the article broke down when the statistics of the digamma-spectrum improved and turned out no longer to support significantly any resonance any more.

However, let us immediately review that calculations[1,2] done partly after the finishing of the Bled-workshop estimated in remarkable well coinciding methods using our multiple point principle to be discussed plus a kind of bag model, that a consistent mass just close to 770 GeV is called for. Let me also give the hope of pointing to a newer statistical fluctuation[4] in the maas spectrum for two vector boson ZV via two two hadron jets and a lepton pair, though only seen in CMS with a mass 650 GeV. (With the accuracy, with which we may so far estimate the mass of our bound state of 6 top + 6 anti top, there is no difference between 750 GeV and 650 GeV, but the experimental accuracy is good enough that the experimentally you can NOT identify 650 GeV with 750 GeV)

9.2 Plan for article

The Main Content of Talk on the Diphoton being the 6 top + 6 antitop Bound State is:

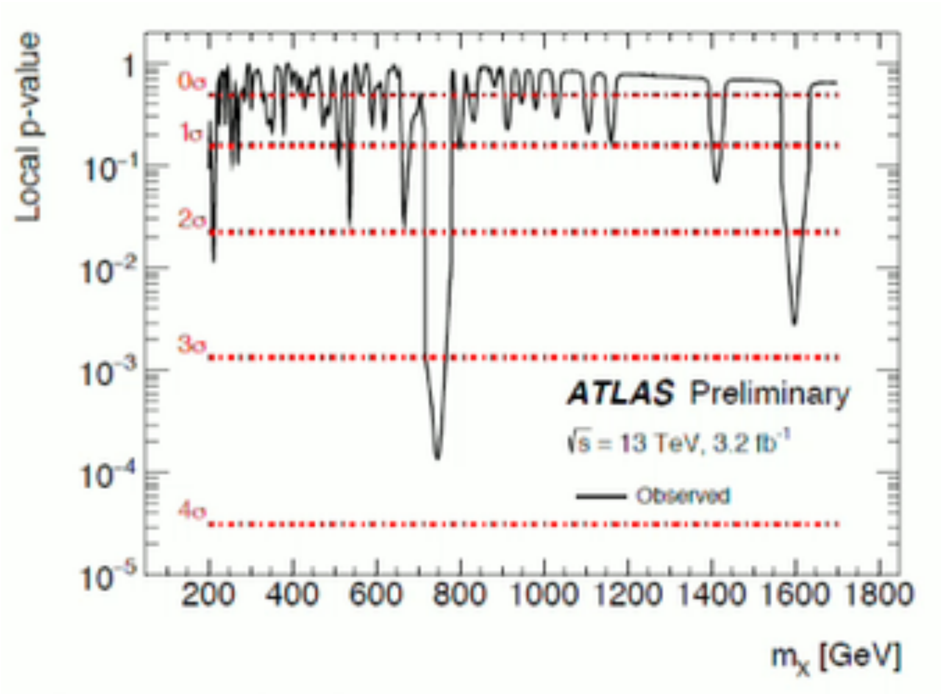
- In this scenario it is possible/natural, that the diphoton resonance of mass 750 GeV has not yet been seen in other channels; but it is very close, and at the $\sqrt{s} = 13$TeV soon to be investigated it can no longer be hidden, if we are right!
- Laperashvili, Das and I calculated a little correction to the observed Higgs mass relative to the one connected to the effective Higgs potential $V_{eff}(\phi_H)$. By an appropriate mass (and radius) of this bound state "diphoton"- particle the observed Higgs mass of 125 GeV could be just compatible with the high Higgs-field vacuum having just the same energy density as the present/physical vacuum, in which we live. Fitting the mass of the bound state to this only barely instability of vacuum leads to a mass compatible with 750 GeV!
- We (thus) suggest that there is new law of nature the Multiple Point Principle saying that there are several vacua with essentially zero energy density (to the accuracy meant here the three quarters of the energy density of the universe today is considered "zero").

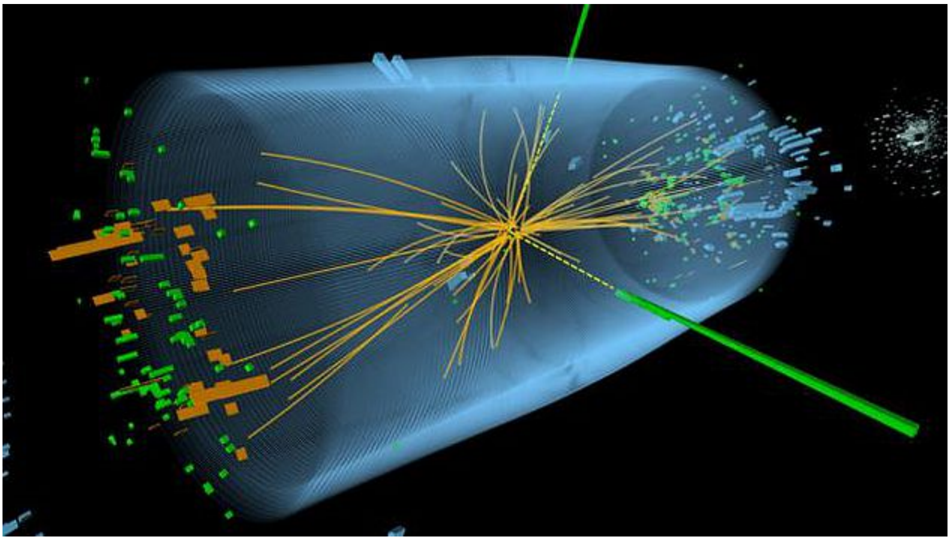
Plan of Talk on "New Resonance ?":

- Intro: Introduction about main thesis: New Particle is Bound State of 6 top + 6 antitop.
- New: Reviewing a bit doubtful peaks from recent LHC experiments.
- 12 tops: Froggatt's and mine crude estimates of the decay and production of our speculated bound state.
- MPP: Our long proposed new law of nature of several degenerate vacua.
- MPP mass: Our calculations using MPP to get mass predictions for the new peak, and for Higgs itself.
- Conclusion: Conclude, that you ought to believe in our long proposed but otherwise new law of nature, MPP (="Multiple Point Principle").

9.3 New Particle or Statistical Fluctuations?

The quite new particle - December 2015 - was a seemingly *new particle* , which decays into two photons and has a mass 750 GeV/c^2 just found at ATLAS and also seen by CMS. But which unfortunately got washed out in august 2016. We shall interprete it as a bound state of 6 top + 6 anti top quark, but nobody knows at present, what kind of particle it would be even, if it were not a statistical fluctuation.

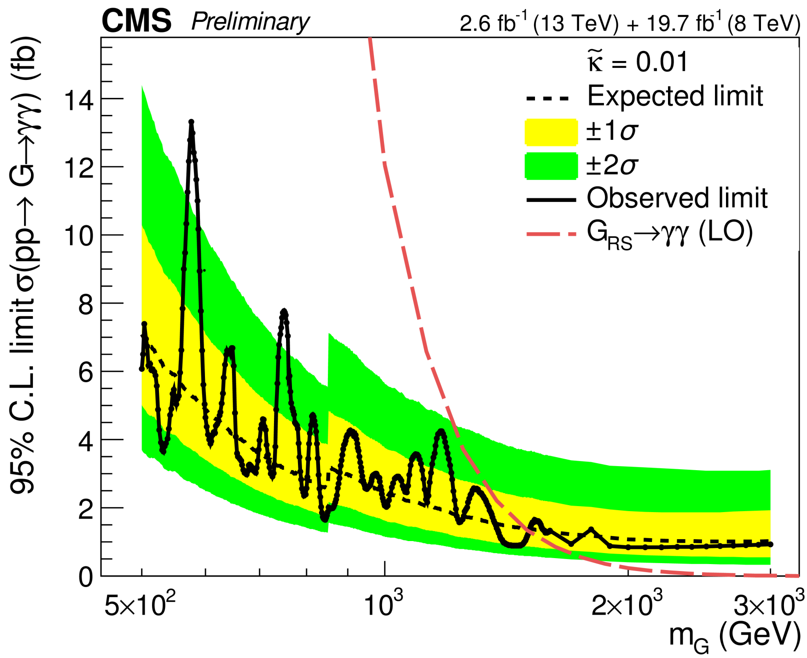




9.3.1 There are a couple of further presumably fluctuations or resonances?

The newest and most trustworthy deviation from the Standard Model - but nevertheless probably just a statistical fluctuation - is a little top/excess in the number of pairs of photons that comes out of the LHC collisions, when this number is plotted versus the collected mass of the two photons.

The mass of the peak is 750 GeV/c^2.



9.3.2 Has LHC shown anything in excess of the Standard Model ?

Not convincing, But there are Statistical Fluctuations, or is it New Physics ???

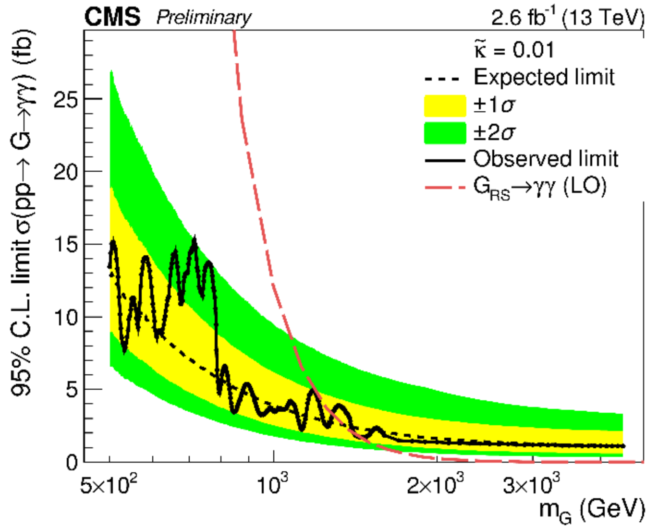
A couple of may be new physics observations

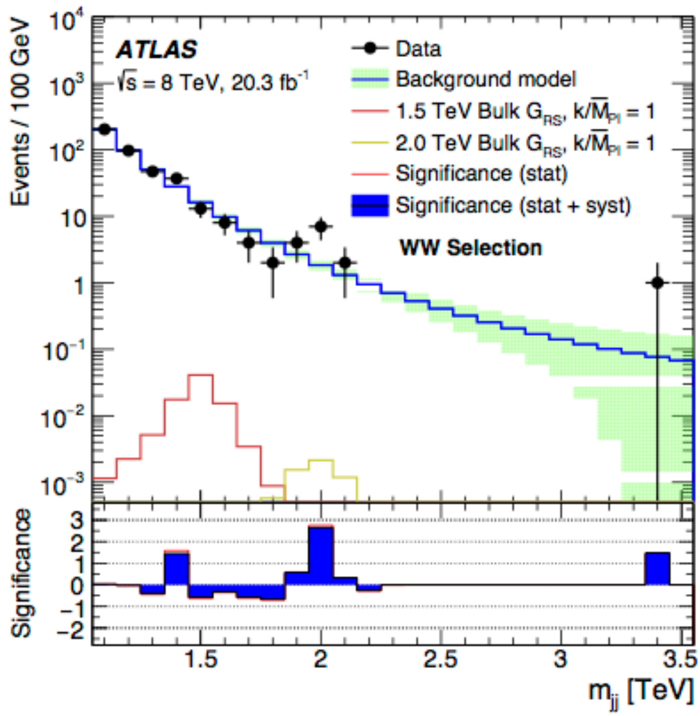
- **A Resonance with mass 1.8 TeV to 2 TeV** ca. 3 σ
- **A Resonance (or something else) Decaying into e.g. two Higgs bosons of W's ...** It is a single bin with an exceptional high number of events at a bit under 0.3 TeV in mass. It is for decay to two particles that could be Higgs's or W's or Z's This particle could easily be the particle which Colin Froggatt and I imagined as a bound state consisting of 6 top + 6 antitop quarks. (but now we shifted our hope to the 750 GeV excess)

9.4 An early Deviation from Standard Model

- **A excess of Higgs decay to $\gamma\gamma$ at ATLAS** The first deviation found from Standard Model was that ATLAS found a bit higher number of Higgs decays to two photons than predicted from Standard Model But CMS did not confirm that.

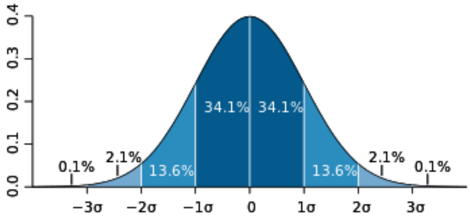
- The very newest is a resonance $\rightarrow \gamma\gamma$ with mass 750 to 760 GeV.



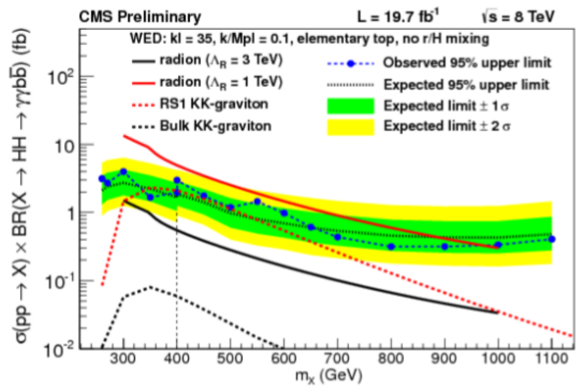


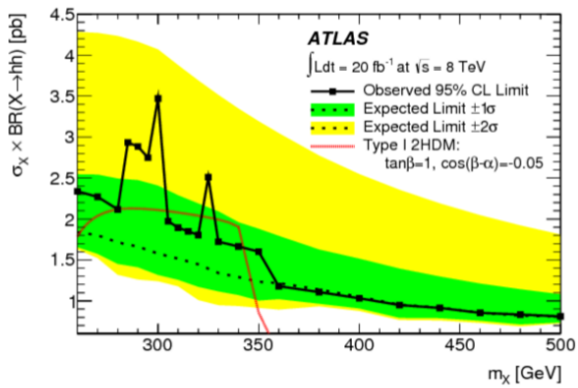
This Atlas plot shows the mass spectrum for pairs of particles WW, WZ or ZZ.

If the little relative increase in the number of events - i.e. of numbers of WW, WZ, or ZZ pairs - at 1.8 TeV were statistically significant. But it is only 3 standard deviations. Our/my ? hope is that we can identify this 1.8 TeV heavy peak as a resonance in two of the 750 GeV ones, but this may be too early to talk about now. We have calculated more on the 750 GeV peaks so far.



In the following two one look for the collective mass but seek to look for decays into two Higgses $b\bar{b}$ and to $\gamma\gamma$.





If you want to illustrate the main result of LHC that the Standard Model works perfectly almost one can show the two following not so easy to overview tabels just expressing that there are now good bounds for many theoretical hopes for new physics, and nothing seen so far. Typically the new physics scale of energy would have to be at least about 1 TeV, if it shall not be excluded already (see Figs. 9.1, 9.2).

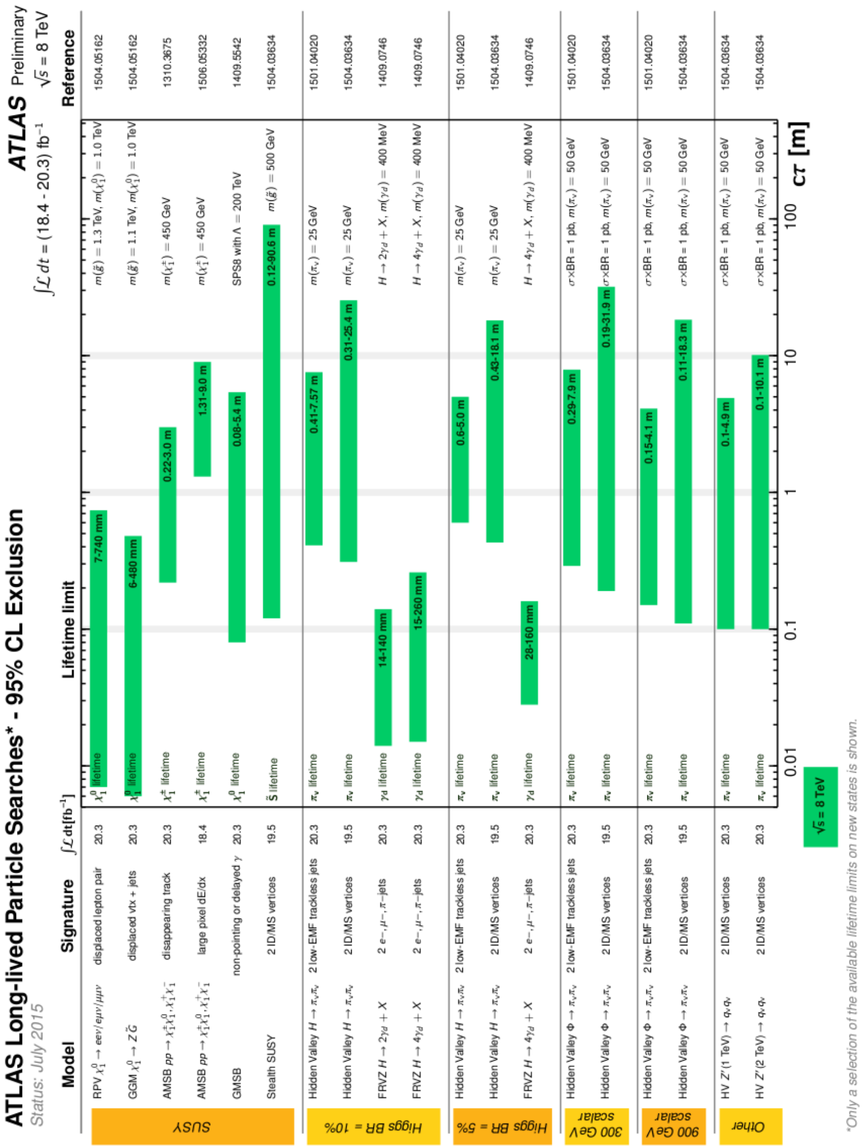


Fig. 9.1. ATLAS long-lived particle searches.

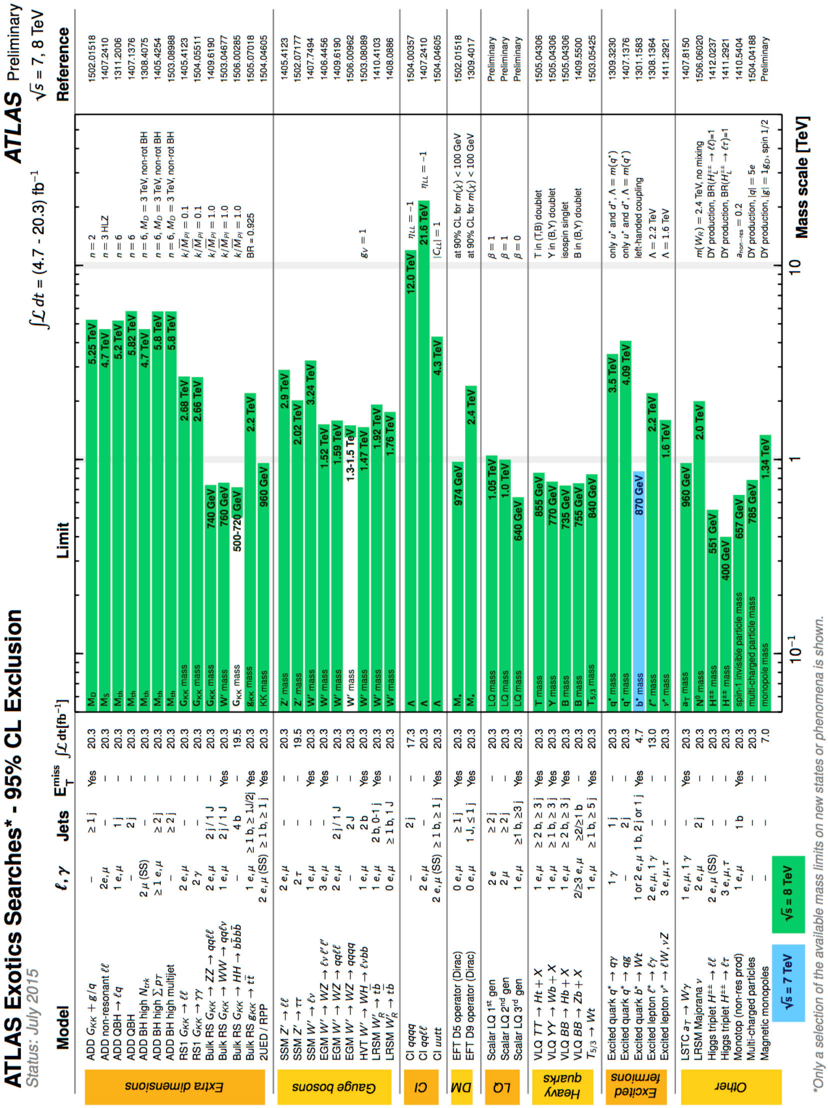
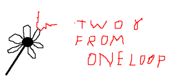


Fig. 9.2. ATLAS exotics searches.

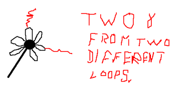
9.5 12 tops

Colin Froggatt and I did an attempt to estimate the relative rates of decay of our hypothesised bound state of the 6 top and 6 anti tops. We assume as reasonable that the dominant decays are the two-particle decays because they have the best phase space. [1] Nevertheless of course all the tops and anti tops have to annihilate away before the bound state disappears. So with a low number, 2, of decay product particles compared to the number 12 of constituents, most of the top anti top pairs have to annihilate into nothing so to speak. a major new point of Froggatts and mine work is the division of the decay amplitudes into two slightly different types of decay: The two final state particles can come from the same top anti top annihilation, which we call "From same top-loop", or they can be emitted from two different annihilating top anti top pairs, and the latter we call " From TWO different loops of tops". A major point to have in mind is, that, if the decay particles have some quantum numbers, then that quantum number has to be transfered from one annihilation loop to another one. The difference between these two cases is illustrated by the figures with the "flowers" on. One effect that could have been important is that e.g. weak gauge bosons W and the $SU(2)$ coupling superposition of the photon and the Z^0 have a weak gauge charge, which is conserved as long as the Higgs vacuum expectation value is not included in the considered diagram. That means that the Higgs expectation value or some exchange of the quantum number from one annihilation loop to another one is required in order for say WW decay occurring by use of the "From TWO different loops of tops" type of diagram. The major part of the photon which couples via the U(1) part, however, has no such "charge". The main part of the $\gamma\gamma$ decay should therefore without any problem be possible with each photon coming from a different annihilation loop. Since there are $12/2 = 6$ annihilation loops this a priori gives the main part of the (di)photon decay amplitude an extra factor 6; but it is even so that the number of loops that must annihilate into quite nothing is 5 for both particles going from one loop while only 4 in the case of the two decay particles coming from different loops.

From same top-loop:



From TWO different loops of tops:



Photon and transverse Z. The electric charge of the top quark is $q = 2e/3$ and the effective coupling for of the photon to the $t\bar{t}$ loop is $4\alpha/9$.
The corresponding effective coupling of Z to the $t\bar{t}$ loop is

$$\frac{\alpha}{2\sin^2\theta_W\cos^2\theta_W}\left[\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)^2+\left(-\frac{2}{3}\sin^2\theta_W\right)^2\right]=\frac{4\alpha}{9}*0.92. \tag{9.1}$$

We take $\alpha = 1/129$ and the Weinberg angle to be given by $\sin^2\theta_W = 0.23$.

[1] I thank Li for discussions at CERN long time ago where we especially discussed that the two-particle decays at the end tended to dominate.

Final st. f	Bound	Relative prediction	$\frac{\Gamma(S\to f)}{\Gamma(S\to\gamma\gamma)}$	Comt.
$\gamma\gamma$	$< 0.8(r/5)$	$(4\alpha/9)^2 = 1.2 * 10^{-5}$	1	
gl + gl	$< 1300 \cdot \frac{r}{5}$	$8\alpha_s/6)^2 = 2.3 * 10^{-3}$	190	
H + H	$< 20(r/5)$	$\alpha_h^2/4 = 3 * 10^{-4}$	25	Higgs
ZZ	$< 6(r/5)$	$\alpha_h^2/4 = 3 * 10^{-4}$	25	longtl.
WW	$< 20(r/5)$	$\alpha_h^2/2 = 6 * 10^{-4}$	50	longtl.
$Z\gamma$	$< 2(r/5)$	$2(4\alpha/9)^2 * 0.92$	1.8	
ZZ	$< 6(r/5)$	$(4\alpha/9)^2 * (0.92)^2$	0.8	tran.
WW	$< 20(r/5)$	$2(0.54\alpha)^2 = 3.5 * 10^{-5}$	3	tran.
$t + \bar{t}$	$< 300(r/5)$	$3\alpha_{t\bar{t}}^2 T_2 = 6.5 * 10^{-5}$	5	
		$\Gamma_{total}(S)/\Gamma(S \to \gamma\gamma)$:	302	

Table 9.1. Assuming dominance of *one* top anti top pair giving the final state, relative predictions are given for the partial decay widths of S and for the branching ratios relative to the diphoton decay width compared to the experimental upper bounds.

Gluon. The vertex for a gluon of color i coupling to a top quark is $g_s\lambda^i/2$. Averaging over the colors of the top quark, the effective coupling of the gluon to the $t\bar{t}$ loop becomes

$$\frac{\alpha_s}{3}\mathrm{Tr}\left(\frac{\lambda^i}{2}\right)^2 = \frac{\alpha_s}{6}. \tag{9.2}$$

We take $\alpha_s = 0.1$ and then sum over the 8 color states of the gluon.

Higgs and longitudinal $W^\pm$ and Z^0. According to the Goldstone Boson Equivalence Theorem in the high energy limit the couplings of the longitudinal $W^\pm$ and Z^0 become equal to those of the corresponding eaten Higgs fields. The Higgs field coupling to the $t\bar{t}$ loop is

$$\alpha_h = \frac{g_t^2/2}{4\pi} = 0.035 \tag{9.3}$$

where g_t is the top quark Yukawa coupling constant.

Transverse $W^\pm$. The $W^\pm$ gauge fields are formed from two real fields, W_1 and W_2, lying in the adjoint representation of SU(2). So their effective coupling to the $t\bar{t}$ loop is

$$\frac{1}{2} * \frac{\alpha}{\sin^2\theta_W}\left(\left(\frac{\sigma^i}{2}\right)^2\right)_{t_L t_L} = \frac{\alpha}{8\sin^2\theta_W} = 0.54\alpha, \tag{9.4}$$

where the extra factor of $1/2$ is due to $W^\pm$ only interacting with left-handed top quarks. The final sum over $i = 1, 2$ gives a factor of 2 in the decay rate.

Photon and transverse Z. The hypercharge coupled superposition of the photon and Z^0 is described by the field $B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$. It couples with an average squared charge $[(2/3)^2 + (1/6)^2]/2 = 0.236$ to a top quark. The two loop diphoton decay is dominated by the production of this B_μ component and so the effective coupling for the photon is 0.236α.
The corresponding effective coupling of Z is $0.236\alpha\tan\theta_W$.

Fin. f	Bound	Relative pred.	$\frac{\Gamma(S\to f)}{\Gamma(S\to\gamma\gamma)}$	Com.
$\gamma\gamma$	$< 0.8(r/5)$	$(0.236\alpha)^2 = 3.35 * 10^{-6}$	1	
gl + gl	$< 1300 \cdot \frac{r}{5}$	$8(\alpha_s/18)^2 = 2.5 * 10^{-4}$	74	
H + H	$< 20(r/5)$	$\alpha_h^4/(4T_2) = 3.4 * 10^{-5}$	10	Higgs
ZZ	$< 6(r/5)$	$\alpha_h^4/(4T_2) = 3.4 * 10^{-5}$	10	longtn.
WW	$< 20(r/5)$	$\alpha_h^4/(2T_2) = 6.8 * 10^{-5}$	20	longtn.
$Z\gamma$	$< 2(r/5)$	$2(0.236\alpha)^2 \tan^2\theta_W$	0.6	
ZZ	$< 6(r/5)$	$(0.236\alpha)^2 \tan^2\theta_W$	0.09	transv.
WW	$< 20(r/5)$	$2(0.54\alpha)^4/T_2 = 6 * 10^{-8}$	0.02	transv.
$t + \bar{t}$	$< 300(r/5)$	$3\alpha_{t\bar{t}}^4/T_2 = 1.06 * 10^{-3}$	316	
		$\Gamma_{total}(S)/\Gamma(S \to \gamma\gamma)$:	432	

Table 9.2. Assuming dominance of *two* top anti top pairs giving the final state, relative predictions are given for the partial decay widths of S and for the branching ratios relative to the diphoton decay width compared to the experimental upper bounds(Franceschini).

Gluon. Averaging over the colors of the two (anti)top pairs, the effective coupling of a gluon of color i for the "crossed" diagram is

$$\frac{\alpha_s}{9}\mathrm{Tr}\left(\frac{\lambda^i}{2}\right)^2 = \frac{\alpha_s}{18}. \tag{9.5}$$

Higgs, longitudinal Z^0, $W^{\pm}$, top antitop. We use the same effective couplings as in the one loop case.

Final state f	Bound	$\frac{\Gamma(S\to f)}{\Gamma(S\to\gamma\gamma)}$	Comment
$\gamma\gamma$	$< 0.8(r/5)$	1	
gluon + gluon	$< 1300(r/5)$	117	
Higgs + Higgs	$< 20(r/5)$	15	Higgs-particles
ZZ	$< 6(r/5)$	15	longitudinal
WW	$< 20(r/5)$	30	longitudinal
$Z\gamma$	$< 2(r/5)$	1.0	
ZZ	$< 6(r/5)$	0.3	transverse
WW	$< 20(r/5)$	1.1	transverse
top + anti top	$< 300(r/5)$	208	
	$\Gamma_{total}(S)/\Gamma(S \to \gamma\gamma)$:	387	

Table 9.3. Benchmark model with $\epsilon^2 = 0.15$. Predictions are given for the decay branching ratios of S relative to the diphoton decay width and compared to the experimental upper bounds from ref. [5].

9.5.1 Production

We assume the production rate to be of an order calculated analogous to the production of a fourth family just taking into account that our bound state consist of 12 quarks. Using our decay ratio estimates the rate for $S \to \gamma\gamma$ is order of magnitude o.k.

9.6 MPP

Multiple Point Principle In general, a quantum field theory allows an existence of several minima of the effective potential, which is a function of a scalar field.

If all vacua, corresponding to these minima, are degenerate, having zero cosmological constants, then we can speak about the existence of *a multiple critical point (MCP) in the phase diagram*. See:[7].

We postulated a Multiple Point Principle (MPP) for many degenerate vacua. This principle should solve the finetuning problem by actually making a rule for finetuning.

9.6.1 Multiple Point Principle

The Multiple Point Model (MPM) of the Universe evokes simply the Standard Model up to the scale $\sim 10^{18}$ GeV.

If the MPP is very accurate, we may have a new law of Nature, that can help us to restrict coupling constants from theoretical principles.

Assuming the existence of two degenerate vacua in the SM:

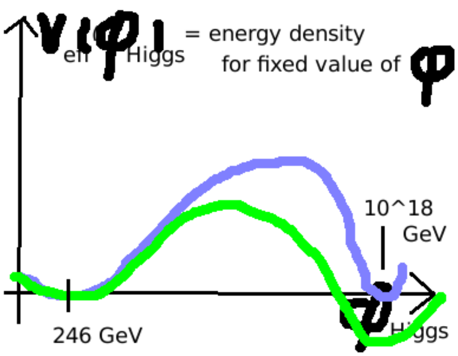
1. the Electroweak vacuum at $\nu = 246$ GeV, and
2. the Planck scale vacuum at $\nu_2 \simeq 10^{18}$ GeV,

See [8].

We predicted the top-quark and Higgs boson masses:

$$M_t = 173 \pm 5 \text{ GeV}, \;\; M_H = 135 \pm 9 \text{ GeV}.$$

Multiple Point Principle



Here it is shown the existence of the second (non-standard) minimum of the effective potential of the pure SM at the Planck scale. Multiple Point Principle: The tree-level Higgs potential with the standard "Electroweak minimum" at $\phi_{min\ 1} = \nu$ is given by:

$$V_1 = \frac{\lambda}{4}\left(\phi^2 - \nu\right)^2 + C_1.$$

The new minimum at the Planck scale:

$$V_2 = V_{eff}(\text{at Planck scale}) = \frac{\lambda_{run}}{4}\left(\phi^2 - \nu_2\right)^2 + C_2,$$

can be higher or lower than the Electroweak one, showing a stable Electroweak vacuum (in the first case), or metastable one (in the second case).

In accordance with cosmological measurements, Froggatt and Nielsen assumed that cosmological constants C_1 and C_2 for both vacua are equal to zero (or approximately zero): $C_{1,\,2} = 0$, or $C_{1,\,2} \approx 0$.

This means that vacua $\nu = \nu_1$ & ν_2, and they are degenerate. Multiple Point Principle The following requirements must be satisfied in order to, the effective potential have two degenerate minima:

$$V_{eff}(\phi^2_{min\ 1}) = V_{eff}(\phi^2_{min\ 2}) = 0,$$

and

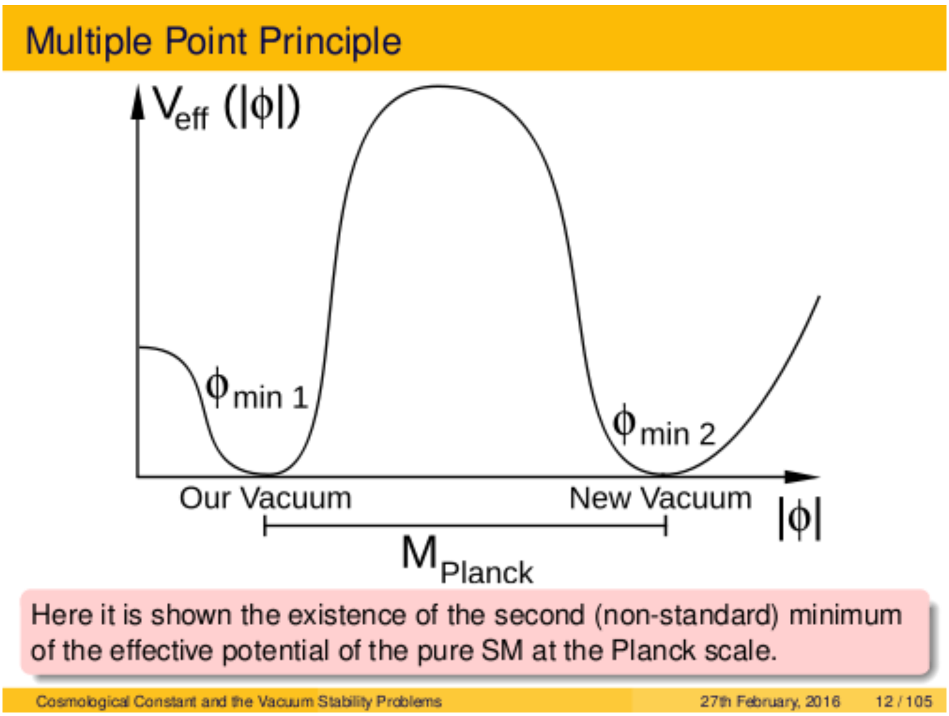
$$V'_{eff}(\phi^2_{min\ 1}) = V'_{eff}(\phi^2_{min\ 2}) = 0,$$

where,

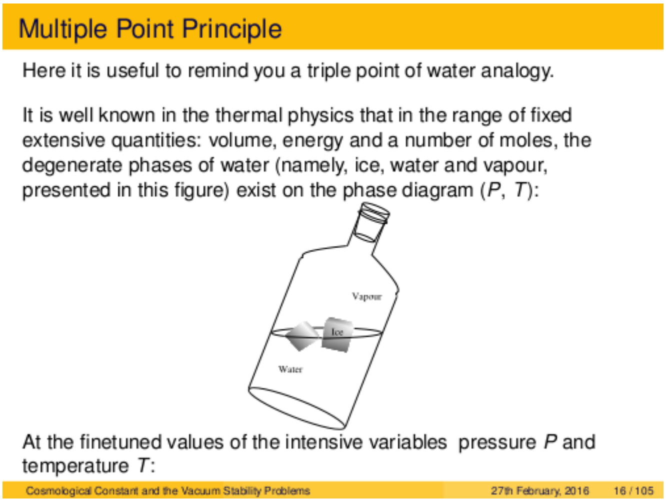
$$V'(\phi^2) = \frac{\partial V}{\partial \phi^2}$$

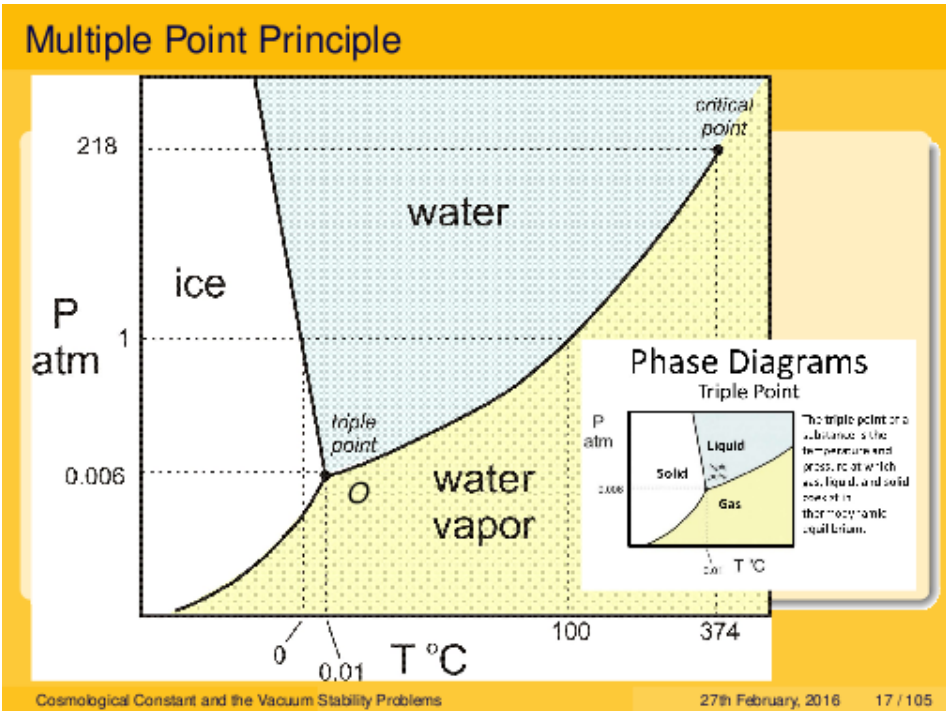
[6]

Multiple Point Principle postulates: There are many vacua with the same energy density, or cosmological constant, and all cosmological constants are zero, or approximately zero.



Multiple Point Principle If several vacua are degenerate, then the phase diagram contains a special point – the Multiple Critical Point (MCP), at which the corresponding phases assembly together.





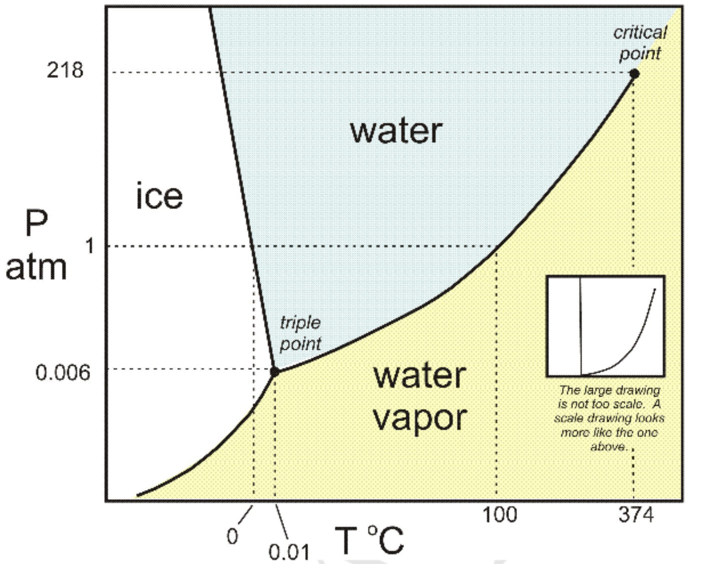


Fig. 5: The phase diagram $(P,\ T)$ of water analogy. The triple point O with $T_c = 0.01^C$ and $P_c = 4.58\ mm\ Hg$ is shown in Fig. 4.

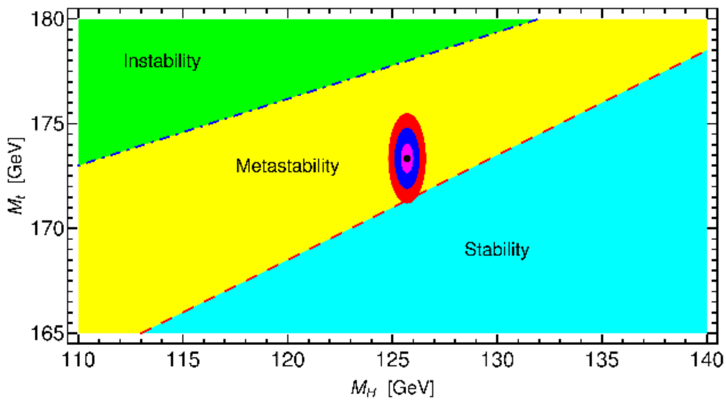


Fig. 6: Stability phase diagram $(M_H,\ M_t)$ is divided into three different sectors: 1) an absolute stability region – blue region of figure; 2) a metastability (yellow) region, and 3) an instability (green) region. The black dot indicates current experimental values $M_H \simeq 125.7$ GeV and $M_t \simeq 173.34$ GeV. The ellipses take into account 1σ, 2σ and 3σ, according to the current experimental errors.

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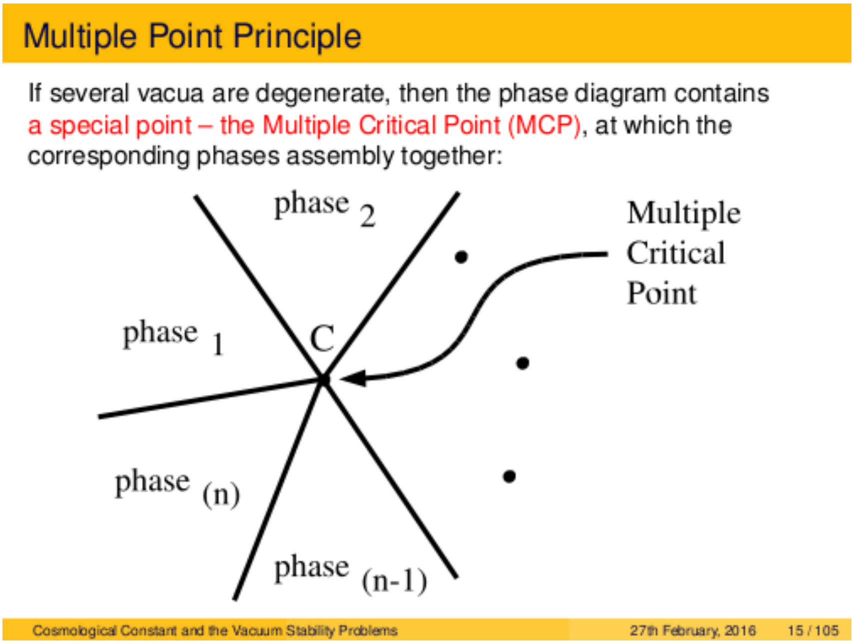
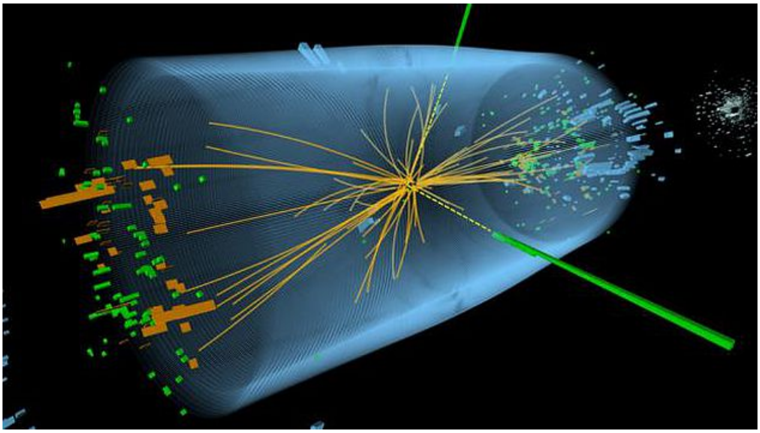
At the finetuned values of the variables pressure P and temperature T – we have:

$$T_c \approx 0.01^\circ C,\ \ P_c \approx 4.58 \text{ mm Hg},$$

giving the critical (triple) point O shown in the previous figure.

This is a triple point of water analogy. The idea of the Multiple Point Principle has its origin from the lattice investigations of gauge theories. In particular, Monte Carlo simulations of U(1), SU(2) and SU(3) gauge theories on lattice, indicate the existence of the triple critical point.





9.7 MPPmass — Post/Pre- dicting Masses from Multiple Point Principle

9.7.1 Claim 3 Post or Pre-dictions of Masses from MPP

We use/assume three different “vacua” which we may name: “physical”, “Higg Higgs” and “S condensate”:

- 1.Mass of Higgs from degeneracy of "physical" and "High Higgs" **Prediction !**
- 2. Mass of the new resonance S of 750 GeV from degeneracy of "physical" and "Higgs Higgs" with improved accuracy. (postdiction only, but...)
- 3. Mass of the new 750 GeV resonance from degeneracy of "physical" and "S condensate".(now postdiction) (Actually Colin Froggatt and I made this calculation as PREdiction to 285 GeV for the mass, but without attaching much belief to it.)

kunstmaler lars andersen http://www.23.dk/skak.htm

Lars Andersen

Historiemaler Portrætmaler Provokunstner Om Lars Andersen CV/omtale Kontakt

"Skak" (Mogens Lykketofte og Holger Bech Nielsen)

Tilhører Frederiksborg Museet

Hænger i moderne samling i det Nationalhistoriske Museum på Frederiksborg Slot.

Bragt i årbogen "Dansk Kunst 1998", Jyllandsposten, Politiken, BT, Ud og Se,

Berlingske Tidende, Ekstra Bladet, Kristeligt Dagblad, Alt for Damerne.,

Frederiksborg Amts Avis, DR1, DR2, TV Lorry , Kunstavisen Billed Bladet....

Trykt som plakat 50x70 cm.

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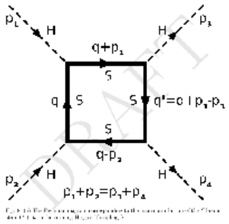
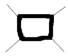
Higgs-Mass Correction:



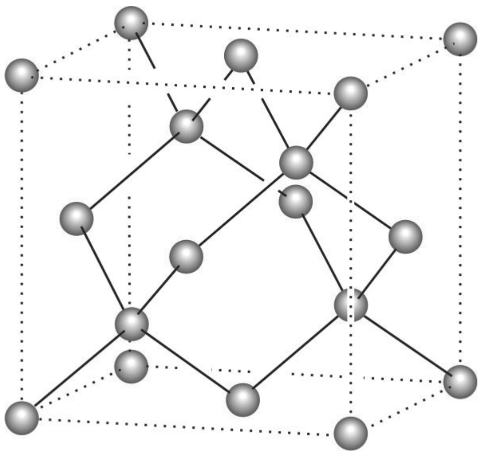
$$m_H^2 = 2(\delta\lambda + \lambda)v^2$$

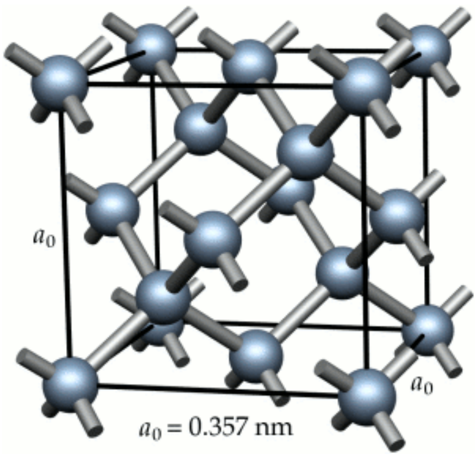
$\delta\lambda =$  $+ \dots$

9.8 Getting the Bound state S Mass from Requiring Degeneracy of Vacua "Physical" and "S Condensate" to 4 m_t

With the right not so obvious approximations one gets in a very simple way that the bound state S shall have the mass $m_S = 12m_t/3 = 4m_t$. Thes assumptions:

- The "S condensate" vacuum is a lattice of same structure as diamond crystal.
- We can count the binding energy as if the neighbouring S-states in the crystal have there constituent top and anti tops in the n=2 level of the surrounded S.
- We can ignore the effective Higgs mass for the exchange up to the n=2 level, but from n=3 and on the Higgs mass may be taken infinite.
- The MPP - degeneracy of "physical" and "S condensate" vacua - requires the binding in the crystal to just cancel the Einstein masses of the S particles.
- We can take the S's in the "S condensate " vacuum as at rest.





9.8.1 Mass from MPP of S-condensate

We want to estimate the condition in a non-relativistic ansatz for the vacuum with the S-condensate as a diamond-structure pattern of S-bound states. The binding energy between the neighbours in this pattern is estimated by assuming that the four nearest neighbours S's to a certain S have their top and anti tops effectively filling up the 4 $n = 2$ states surrounding the S considered. Since $n = 2$ states in the Bohr atom have a binding $1/n^2 = 1/4$ times that in the $n = 1$ states, we take it that the binding per quark of a neighbouring S to a given one is in the Higgs mass zero approximation just 1/4 times that binding of one of the constituent quarks inside its S. Thus the binding of an S to its neighbour must be with a potential 1/4 of the binding energy of an S from its constituents.

In the assumed diamond lattice each carbon atom has 4 nearest neighbours, but each "binding-link" is attached to two carbon atoms. So the number of "binding-links" is twice as large as the number of carbon atoms. If we therefore as argued have one quarter of the binding energy in these "binding links" as in the S's or the carbon atoms in the analogue, there will be 2/4 = 1/2 as much binding in the "binding -links" as in the S's themselves.

If the Einstein energy $E = mc^2$ of a sample of bound states S consisting of top and anti tops shall be just compensated by the binding energy between these quarks, then the total binding energy per quark must add up to this Einstein energy numerically. Such compensation is required by our new law of nature "Multiple point principle".

If the bindings in the "binding-links" make up 1/2 of the binding of the constituents inside their respective S bound states, the latter must make up 2/3 of the Einstein energy. The a priori Einstein energy of the twelve top or anti top quarks in an S bound state is of course 12 m_t. The binding energy the S-bound state should thus from MPP be 12 m_t * 2/3. Thus the left over mass of the S bound state shall be 12 m_t - 2/3 *12m_t = 12/3 m_t = 4 m_t, which is indeed very close to the observed mass $m_S = 750\text{GeV}$. In fact $4m_t = 4 * 173\text{GeV} = 692\text{GeV}$.

9.9 Conclusion

Conclusion

- We have argued for that in our interpretation of the diphoton peak as a bound state of $6t + 6\bar{t}$ bound state there are two arguments using the "multiple point principle" that independently lead to the bound state having a mass near the 750 GeV:
  - A. The correction to the to be observed Higgs mass needed to make degeneracy (MPP) of the physical and the high Higgs vacua is close to requiring the bound state mass 760 GeV.
  - B. To have the degeneracy (MPP) of the S-condensate vacuum with the physical one an S mass of $\sim 4m_t$ is needed.
- Even ignoring the little correction from the bound state S to the relation between the Higgs mass and the energy density of the high Higgs vacuum is so well approximately in correspondance with the MPP-required degeneracy of the physical and the high higgs vacua that we - Colin Froggatt and I - PREdited the Higgs mass correctly within 10 GeV!
- That the 750 GeV peak is so far only seen in the diphoton channel is so far barely consistent with the bounds from LHC, because one has not yet analysed the other relevant channels at 13 TeV.
- The production rate is in crude agreement with our estimate.

Acknowledgements

I would like to thank the Niels Bohr Institute for being allowed to stay as emeritus and even coming to Bled. Also I thank my collaborators Colin Froggatt, Larisa Laperashvili and Chitta Das for the work without which the information and

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10 Progressing Beyond the Standard Models

B.A. Robson *

The Australian National University, Canberra, ACT 2601, Australia

Abstract. The Standard Model of particle physics (SMPP) has enjoyed considerable success in describing a whole range of phenomena in particle physics. However, the model is considered incomplete because it provides little understanding of other empirical observations such as the existence of three generations of leptons and quarks, which apart from mass have similar properties. This paper examines the basic assumptions upon which the SMPP is built and compares these with the assumptions of an alternative model, the Generation Model (GM). The GM provides agreement with the SMPP for those phenomena which the SMPP is able to describe, but it is shown that the assumptions inherent in the GM allow progress beyond the SMPP. In particular the GM leads to new paradigms for both mass and gravity. The new theory for gravity provides an understanding of both dark matter and dark energy, representing progress beyond the Standard Model of Cosmology (SMC).

Povzetek. Standardni Model elektrošibke in barvne interakcije zelo uspešno opiše veliko pojavov v fiziki osnovnih delcev. Model imajo kljub temu za nepopoln, ker ne pojasni vrste empiričnih dejstev, kot je obstoj treh generacij leptonov in kvarkov, ki imajo, razen različnih mas, zelo podobne lastnosti. V prispevku obravnavamo osnovne predpostavke, na katerih so zgradili ta model in jih primerjamo s predpostavkami alternativnega modela, generacijskega modela. Generacijski model se v napovedih ujema z napovedmi standardnega modela za tiste pojave, ki jih slednji dobro opiše. Drugačne predpostavke omogočijo generacijskemu modelu napovedi, ki niso v dosegu standardnega modela: generacijski model ponudi drugačno paradigmo za maso in energijo. To vodi k novi teoriji gravitacije, ki ponuja novo razumevanje problemov temne snovi in temne energije, ter s tem k razširitvi standardnega modela kozmologije.

10.1 Introduction

The two models in the title are the Standard Model of Particle Physics (SMPP) and the Standard Model of Cosmology (SMC).

In this paper the SMPP [1] will be briefly described, indicating its *incompleteness* and the need for an improved model such as the Generation Model (GM) [2] in which the elementary particles of the SMPP have a substructure. During the last decade an alternative model, the GM, has been developed, although the current version has not changed since 2011. This model allows the elementary particles of the SMPP to have a substructure, suggested by indirect evidence. This version of the GM leads to new paradigms for both mass [3] and gravity [4]. In particular

* brian.robson@anu.edu.au

the new theory of gravity provides an understanding of both dark matter [5] and dark energy [6], and also solves the cosmological matter-antimatter asymmetry problem [7]. These represent progress beyond both the SMPP and the SMC.

10.2 Standard Model of Particle Physics

The SMPP [1,2] was developed throughout the 20th century, although the current formulation was essentially finalized in the mid-1970s following the experimental confirmation of the existence of quarks. The SMPP has enjoyed considerable success in describing the interactions of leptons and the multitude of hadrons (baryons and mesons) with each other as well as the decay modes of the unstable leptons and hadrons. However, the model is considered to be *incomplete* in the sense that it provides little understanding of several empirical observations: it does not explain the occurrence of three generations of the elementary particles: the first generation comprising the up and down quarks, the electron and its neutrino; the second generation comprising the charmed and strange quarks, the muon and its neutrino and the third generation comprising the top and bottom quarks, the tauon and its neutrino. Each generation behaves similarly except for mass. Second, it does not provide a unified description of the origin of mass nor describe the mass hierarchy of leptons and quarks. It also fails to describe the nature of gravity, dark matter, dark energy or the cosmological matter-antimatter asymmetry problem.

Because of the incompleteness of the SMPP, I have closely examined the basic assumptions upon which the SMPP has been erected [8]. There are three basic assumptions, which I consider to be dubious and also present major stumbling blocks preventing progress beyond the SMPP. These are (i) the assumption of a diverse complicated scheme of additive quantum numbers to classify its elementary particles; (ii) the assumption of weak isospin doublets in the quark sector to accommodate the universality of the charge-changing weak interactions and (iii) the assumption that the weak interactions are fundamental interactions described by a local gauge theory.

The additive quantum numbers allotted in the SMPP to classify the six leptons and the six quarks, which constitute the elementary matter particles of the SMPP, are charge Q, lepton number L, muon lepton number L_μ and tau lepton number L_τ for the leptons, and charge Q, baryon number A, strangeness S, charm C, bottomness B and topness T for the quarks. Antiparticles have opposite quantum numbers to the corresponding particle.

It should be noted that except for charge, leptons and quarks have different kinds of quantum numbers so that this classification is *non-unified*. Each of the additive quantum numbers is conserved in any interaction, except for S, C, B and T, which may undergo a change of one unit in weak interactions.

I consider that the basic problem with the SMPP is this classification of its elementary particles employing a diverse complicated scheme of additive quantum numbers, some of which are not conserved in weak interaction processes; and at the same time failing to provide any physical basis for this scheme.

Another problem with the SMPP concerns the method it employs to accommodate the universality of the charge-changing (CC) weak interactions. The CC weak interactions are mediated by the *W* bosons which have zero additive quantum numbers apart from charge.

In the SMPP, the observed universality of the CC weak interactions in the lepton sector is described by assuming that the mass eigenstate leptons form weak isospin doublets. The leptons have weak isospin 1/2, whose third component is related to both charge and lepton number. Restricting the discussion in this paper to only the first two generations for simplicity, means that the two neutrinos interact with their corresponding charged leptons with the full strength of the CC weak interaction and do not interact at all with the other charged lepton. This is guaranteed by the conservation of lepton numbers.

On the other hand the universality of the CC weak interactions in the quark sector is treated differently. It is assumed that the u and c quarks form weak isospin doublets with so-called weak eigenstate quarks d' and s', respectively, where

$$d' = d\cos\theta + s\sin\theta,$$

and

$$s' = -d\sin\theta + s\cos\theta,$$

and θ is a mixing angle introduced by Cabibbo in 1963 into the transition amplitudes prior to the development of the quark model in 1964. In the quark case the third component of weak isospin is related to both charge and baryon number.

The SMPP assumes that the u and c quarks interact with d' and s', respectively, with the full strength of the CC weak interaction and that the u and c quarks do not interact at all with s' and d', respectively. However, this latter assumption is dubious since, unlike the lepton sector, there are no conserved quantum numbers to guarantee this.

A third problem with the SMPP concerns the origin of mass. In the SMPP, the masses of hadrons arise mainly from the energy content of their constituent quarks and gluons, in agreement with Einstein's 1905 conclusion. On the other hand the masses of the elementary particles, the leptons, the quarks and the *W* and *Z* bosons are interpreted differently, arising from the existence of the so-called Higgs field [9,10]. The Higgs field was introduced mathematically to spontaneously break the U(1) x SU(2) local gauge symmetry of the electroweak interaction to generate the masses of the *W* and *Z* bosons. The Higgs field also cured the associated fermion mass problem: by coupling, with appropriate strength, originally massless fermions to the scalar Higgs field, it is possible to produce the observed fermion masses and to maintain local gauge invariance.

I consider that there are several problems with the SMPP's interpretation of the origin of mass. First, there is no clear evidence for the existence of the hypothetical Higgs field. Second, the model provides no *unified* origin of mass. Third, the fermion-Higgs coupling strength is dependent upon the mass of the fermion so that a new parameter is introduced into the SMPP for each fermion mass. In fact fourteen new parameters are required, if one includes two more parameters to describe the masses of the W boson and the Higgs particle. Fourth, the Higgs mechanism does *not provide any physical explanation* for the origin of the masses of the elementary particles.

The assumption that the weak interactions are fundamental interactions arising from a local gauge theory, unlike both the electromagnetic and strong colour interactions, is at variance with the experimental facts: both the W and Z particles, mediating the weak interactions, are massive, and this conflicts with the requirement of a local gauge theory that the mediating particles should be massless in order to guarantee the gauge invariance. I consider this assumption very dubious, especially since it leads to more problems than it solves. It also leaves several questions unanswered: *How does the spontaneous symmetry breaking mechanism occur within the electroweak theory? What is the principle that determines the large range of fermion masses exhibited by the leptons and quarks?*

10.3 Generation Model of Particle Physics

The GM of particle physics [2,8] overcomes many of the problems inherent in the SMPP. In the GM the three dubious assumptions of the SMPP discussed previously are replaced by three different and simpler assumptions. These are (i) the assumption of a *simpler unified* classification of leptons and quarks; (ii) the assumption that the *mass eigenstate quarks* form weak isospin doublets and that hadrons are composed of *weak eigenstate quarks* and (iii) the assumption that the weak interactions are *not fundamental interactions*.

Table 10.1 shows the additive quantum numbers allotted to both leptons and quarks in the GM. This is a much simpler and unified classification scheme involving only three additive quantum numbers: charge Q, particle number p and generation quantum number g. All three quantum numbers are conserved in all interactions. In particular this classification scheme allows the development of a composite model of leptons and quarks, which I consider a necessary condition for a simpler model.

particle	Q	p	g	particle	Q	p	g
ν_e	0	-1	0	u	$+\frac{2}{3}$	$\frac{1}{3}$	0
e^-	-1	-1	0	d	$-\frac{1}{3}$	$\frac{1}{3}$	0
ν_μ	0	-1	± 1	c	$+\frac{2}{3}$	$\frac{1}{3}$	± 1
μ^-	-1	-1	± 1	s	$-\frac{1}{3}$	$\frac{1}{3}$	± 1
ν_τ	0	-1	$0, \pm 2$	t	$+\frac{2}{3}$	$\frac{1}{3}$	$0, \pm 2$
τ^-	-1	-1	$0, \pm 2$	b	$-\frac{1}{3}$	$\frac{1}{3}$	$0, \pm 2$

Table 10.1. GM additive quantum numbers for leptons and quarks.

The conservation of the generation quantum number in weak interactions was only achieved by making two postulates, which means that the GM differs fundamentally from the SMPP in two more ways. First the GM postulates that it is the mass eigenstate quarks of the same generation, which form weak isospin doublets: (u, d) and (c, s). Thus the GM assumes, in the two generation approximation, that the u and c quarks interact with d and s, respectively, with the *full* strength of the CC weak interaction and that the u and c quarks do *not* interact at all with s and d, respectively. This is guaranteed by the conservation of the generation quantum number. Second, the GM postulates that hadrons are composed of weak eigenstate quarks such as d' and s' rather than the corresponding mass eigenstate quarks, d and s, as in the SMPP. Essentially, in the GM the roles of the mass eigenstate quarks and the weak eigenstate quarks are interchanged from that in the SMPP. These two postulates overcome the second dubious assumption of the SMPP.

The GM assumes that the leptons, quarks and the W and Z bosons are *composites*. Consequently, the weak interactions are *not* fundamental interactions arising from an SU(2) local gauge theory. They are residual interactions of the strong colour interaction binding the constituents of the leptons, quarks and the W and Z bosons together. This strong colour interaction is completely analogous to that of QCD in the SMPP. The composite nature of leptons, quarks and the W bosons overcomes the dubious assumption of the SMPP that the weak interactions are fundamental.

10.4 Composite Generation Model

In 2005 I began construction of a GM in which the leptons and quarks are composite particles. This composite GM was based on the unified classification scheme and also on early 1979 composite models of Harari [11] and Shupe [12]. The current composite GM was proposed in 2011 and is described in detail in Chapter 1 of the book *Particle Physics* published by InTech in 2012 [2] and in a review paper published in *Advances in High Energy Physics* in 2013 [8]. Unfortunately, today I have only the time to indicate some of the features of the composite GM (CGM) that are relevant for today's talk.

In the CGM the elementary particles of the SMPP have a *substructure* consisting of massless "rishons" bound together by strong colour interactions, mediated by massless hypergluons. Each rishon carries a colour charge: red, green or blue like a quark in the SMPP. This model is very similar to the SMPP in which hadrons have a *substructure* consisting of quarks bound together by strong colour interactions, mediated by massless gluons. Today I shall only have time to give a very brief outline of the development of the CGM.

There are numerous models in the literature. However, the CGM is based on the 1979 two-particle schematic models of Harari and Shupe, which are very similar and provide arguably the most economical and impressive description of the first generation of leptons and quarks. Both models treat leptons and quarks as composites of two kinds of spin-1/2 particles that Harari named "rishons" from the Hebrew for primary. The two kinds of rishons are labelled T with charge $Q = 1/3$ and V with $Q = 0$.

Table 10.2 shows the structures given to the first generation of leptons and quarks. It should be noted that no composite particle involves mixtures of rishons and antirishons. Also it should be noted that quarks contain mixtures of the two types of rishons, whereas leptons do not. Essentially, the Harari-Shupe model (HSM) describes the charge character of the first generation of particles.

particle	structure	Q
e^+	TTT	$+1$
u	TTV, TVT, VTT	$+\frac{2}{3}$
$\bar{d}$	TVV, VTV, VVT	$+\frac{1}{3}$
ν_e	VVV	0
$\bar{\nu}_e$	$\bar{V}\bar{V}\bar{V}$	0
d	$\bar{T}\bar{V}\bar{V}$, $\bar{V}\bar{T}\bar{V}$, $\bar{V}\bar{V}\bar{T}$	$-\frac{1}{3}$
$\bar{u}$	$\bar{T}\bar{T}\bar{V}$, $\bar{T}\bar{V}\bar{T}$, $\bar{V}\bar{T}\bar{T}$	$-\frac{2}{3}$
e^-	$\bar{T}\bar{T}\bar{T}$	-1

Table 10.2. HSM of first generation of leptons and quarks.

The CGM is a major extension of the HSM: the introduction of a *third* kind of rishon (U) and all three additive quantum numbers are allotted to each kind of rishon (see Table 10.3).

rishon	Q	p	g
T	$+\frac{1}{3}$	$+\frac{1}{3}$	0
V	0	$+\frac{1}{3}$	0
U	0	$+\frac{1}{3}$	-1

Table 10.3. CGM additive quantum numbers for rishons.

Table 10.4 gives the structures of the first generation of leptons and quarks in the CGM. Antiparticles are denoted in the usual manner by a "bar" placed above the particle identifier. The u-quark has $p = 1/3$ since it contains two T-rishons and one $\bar{V}$-rishon. It is essential that the u-quark should contain an $\bar{V}$-rishon rather than a V-rishon as in the HSM, since its particle number is required to agree with its baryon number 1/3. It should be noted that leptons are composed of three rishons, while quarks are composed of one rishon and one rishon-antirishon pair. Each lepton of the first generation is *colourless*, composed of three rishons carrying different colours. Each quark of the first generation is *coloured*, composed of one rishon and one colourless rishon-antirishon pair. The first generation of particles are all built out of T and V rishons and their antiparticles so that each particle has $g = 0$. The second and third generations are identical to the first generation plus one and two *colourless* rishon-antirishon pair(s): $\bar{U}V$ or $\bar{V}U$ with $Q = p = 0$ but $g = \pm 1$ so that the second and third generations have $g = \pm 1$ and $g = 0, \pm 2$, respectively. This gives *three repeating patterns*.

particle	structure	Q	p	g
e^+	TTT	$+1$	$+1$	0
u	$TT\bar{V}$	$+\frac{2}{3}$	$+\frac{1}{3}$	0
$\bar{d}$	$T\bar{V}\bar{V}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0
ν_e	$\bar{V}\bar{V}\bar{V}$	0	-1	0
$\bar{\nu}_e$	VVV	0	$+1$	0
d	$\bar{T}VV$	$-\frac{1}{3}$	$+\frac{1}{3}$	0
$\bar{u}$	$\bar{T}\bar{T}V$	$-\frac{2}{3}$	$-\frac{1}{3}$	0
e^-	$\bar{T}\bar{T}\bar{T}$	-1	-1	0

Table 10.4. CGM of first generation of leptons and quarks.

10.5 Mass

Since the mass of a hadron arises mainly from the energy of its constituents, the CGM suggests that the mass of a lepton, quark or vector boson arises from a characteristic energy E associated with its constituent rishons and hypergluons, according to $m = E/c^2$. Thus the CGM provides a *new paradigm* and a *unified* description for the origin of *all* mass: the mass of a body arises from the energy content E of its constituents. The mass is given by $m = E/c^2$ in agreement with Einstein's 1905 conclusion, so that there is no need for the existence of a Higgs field with its accompanying problems. A corollary of this idea is: *If a particle has mass, then it is composite.*

The CGM suggests that the mass hierarchy of the three generations arises from the substructures of the leptons and quarks. The mass of each composite particle is expected to be *greater* if the constituents are on average more widely spaced: this is a consequence of the nature of the strong colour interactions, which are stronger for larger separations of the colour charges, and higher generation particles are more massive than lower generations. Particles with two or more charged rishons will have larger structures due to electric repulsion.

Qualitatively, for the same generation, one expects that (i) a charged lepton will have a greater mass than the corresponding neutral lepton; (ii) a $Q = +2/3$ quark will have a greater mass than the corresponding $Q = -1/3$ quark. These are both generally true: (i) the electron has a larger mass than its corresponding neutrino, and (ii) the top quark mass (175 GeV) is $>$ the bottom quark mass (4.5 GeV), the charmed quark mass (1.3 GeV) is $>$ the strange quark mass (200 MeV), although the up quark mass (5 MeV) is $<$ the down quark mass (10 MeV). The first generation quarks seem to present an anomaly since the proton consists of two up quarks and one down quark while the neutron consists of two down quarks and one up quark so that the proton is only stable if the down quark ($Q = -1/3$) is more massive than the up quark ($Q = +2/3$). In the CGM, this anomaly is accounted for by the constituents of hadrons being weak-eigenstate quarks rather than mass-eigenstate quarks. The proton is stable since the weak eigenstate quark d' has a larger mass than the up quark, containing about 5% of the strange quark mass.

10.6 Gravity

Let us now consider the nature of gravity within the framework of the GM. It is envisaged that the rishons of each colourless lepton, i.e., a particle with total colour charge zero, are very strongly localized since to date there is no direct evidence for any substructure of these particles. The rishons are expected to be distributed according to quantum-mechanical wave functions, for which the product wave function is significant for only an extremely small volume of space so that the corresponding colour fields are almost cancelled. It should be noted that the colour fields would only cancel completely if each of the rishons occupied the same position, but quantum mechanics prevents this. This raises a question: *What is the residual interaction arising from the incomplete cancellation of the strong interactions?*

Between any two colourless leptons (electrons) there will be a very weak residual interaction, arising from the colour interactions acting between the rishons of one lepton and the rishons of the other lepton. In two papers [3,4] I suggested that this residual interaction gives rise to the usual gravitational interaction. There will be a similar residual interaction between any two colourless hadrons such as neutrons and protons, each containing three differently coloured quarks.

Gravity acts between bodies with mass. The mass of a body of ordinary matter is essentially the total mass of its constituent electrons, neutrons and protons. In the GM, each of these three particles is composite and colourless. Indeed, all three particles are in a three-colour antisymmetric state so that their behaviour with respect to the colour interactions is basically the same. This suggests that the residual colour interactions between electrons, neutrons and protons have several properties associated with the usual gravitational interaction: universality, very weak strength and attraction.

In the GM gravity essentially arises from the residual colour forces between all electrons, neutrons and protons. This leads to a *new law of gravity*: the residual colour interactions between any two bodies of masses m_1 and m_2, separated by a distance r, leads to a universal law of gravitation, which closely resembles Newton's original law given by: $F = H(r)m_1 m_2/r^2$, where Newton's gravitational constant (G) is replaced by a function of r, $H(r)$.

The new gravitational interaction of the GM is based upon the residual colour interactions acting between electrons, neutrons and protons. The GM assumes that the colour interactions acting between rishons have the same characteristics as the colour interactions acting between quarks in the SMPP. These colour interactions have two important properties that differ from the Newtonian interaction: (i) asymptotic freedom and (ii) colour confinement. These determine the nature of the function $H(r)$.

Asymptotic freedom is rather a misnomer. A better term is antiscreening as used by Wilczek in his 2004 Nobel lecture. Antiscreening arises from the self-interactions of the hypergluons mediating the residual colour interactions. These antiscreening effects lead to an increase in the strength of the residual colour interactions acting so that H becomes an increasing function of r. The 'flat' rotation curves observed for galaxies imply that $H(r) = G(1 + kr)$.

10.7 Galaxy Rotation Problem and Dark Matter

For galaxies there is a major gravitational problem [5], which has been around for about forty years. It was found that the rotation curves for galaxies disagreed with Newton's gravitational law for large r: the stars and gas were rotating much faster than expected from Newton's law and their orbital velocities were roughly constant. These observations implied that either Newton's law was incorrect at large distances or some considerable mass was missing.

The rotation curve for a galaxy is the dependence of the orbital velocity of the visible matter in the galaxy on its radial distance from the centre of the galaxy. What the observations showed was that the rotation curves were essentially 'flat' at the extremities of the visible matter, i.e., at large distances. This implies gross disagreement with Newton's universal law of gravitation, which predicts a fall-off as $1/\sqrt{r}$ as in the solar system.

Two solutions, which have been very successful, are first the *dark matter hypothesis* that proposes that a galaxy is embedded within a giant halo of dark matter. This matter is considered to be non-atomic but otherwise its nature is unknown and so far has not been detected. The second solution is the *Modified Newtonian Dynamics* (MOND) hypothesis: in 1983 Milgrom [13] proposed that gravity varies from Newton's law for low accelerations. This is an empirical hypothesis without physical understanding.

It was found that the *new law of gravity* is essentially equivalent to the MOND hypothesis so that the GM gravitational interaction provides a physical basis for the MOND hypothesis. In my opinion, the continuing success [14,15] of the MOND hypothesis is a strong argument against the existence of undetected dark matter haloes, consisting of unknown matter embedding galaxies.

In the GM $H(r) = G(1+kr)$ arises from the self-interactions of the hypergluons mediating the gravitational interaction and explains the dark matter problem of the galaxy rotational curves: for small r, $H(r)$ is approximately G and gravity is approximately Newtonian; for large r, $H(r)$ is approximately Gkr and gravity is approximately $1/r$ rather than $1/r^2$, and the $1/r$ dependence gives the flat rotation curves observed.

10.8 Dark Energy Problem

Colour confinement is the phenomenon that colour charged particles (e.g., quarks in the SMPP, rishons in the GM) cannot be isolated and consequently form colourless composite particles (e.g., mesons and baryons in the SMPP and also leptons in the GM). Colour confinement leads to another phenonmenon analogous to the 'hadronization process', i.e., the formation of hadrons out of quarks and gluons in the SMPP and implies $H(r) = 0$ for sufficiently large r in the GM.

In the GM, $H(r) = 0$ arises if the gravitational field energy is sufficient that it is energetically favourable to produce the mass of a particle-antiparticle colourless pair rather than the colour field to extend further. This implies that gravity ceases to exist for sufficiently large cosmological distances.

The strong colour interaction is known to have a finite range of approximately 10^{-15}m. Gravity is about 10^{-41} times weaker at 10^{-15}m [16] than the strong colour interaction. This suggests that the 'hadronization process' for gravity occurs at about 10^{26}m, i.e., roughly ten billion light years.

The new law of gravity implies that gravity ceases to exist for cosmological distances exceeding several billion light years, resulting in less slowing down of galaxies than expected from Newton's law. This result agrees well with observation [17,18] of distant Type Ia supernovae, which indicate the onset of an accelerating expansion of the universe at about six billion light years.

10.9 Summary and Conclusion

The GM allows progress beyond the Standard Models of both particle physics and cosmology by the development of a composite model of the elementary particles of the SMPP. This led to (i) a unified description of all mass and a qualitative understanding of the mass hierarchy of the three generations of leptons and quarks and (ii) a new law of gravity and an understanding of both dark matter and dark energy.

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Discussion Section

The discussion section is reserved for those open problems discussed during the workshop, that they might start new collaboration among participants or at least stimulate participants to start to think about possible solutions of particular open problems in a different way. Since the time between the workshop and the deadline for contributions for the proceedings is very short and includes for most of participants also their holidays, it is not so easy to prepare besides their presentations at the workshop also the common contributions to the discussion section. However, the discussions, even if not presented as a contribution to this section, influenced participants' contributions, published in the main section.

As it is happening every year also this year quite a lot of started discussions have not succeeded to appear in this proceedings. Organizers hope that they will be developed enough to appear among the next year talks, or will just stimulate the works of the participants.

The author uses the old idea, that an appropriate parametrization of almost democratic mass matrices might help to better understand how fermions gain masses, upgrading this idea by the appropriate rotations of mass matrices and a specific parametrization of the Cabibbo-Kobayashi-Maskawa mixing matrix.

When studying the electrostatic interaction among two point charges in a nonlinear model, taking into account QED, the authors find that the repulsion force between two equal charges which are ∞ close to each other surprisingly disappears, while it remains infinite if the charges are not equal.

Using the model based on the recognition that the binary code can well be used to clasify quarks and leptons of the standard model as well as the corresponding gauge fields, the authors try to evaluate the unification scale between the weak and the electromagnetic field.

A test is proposed which would show whether and how will the break of the $SU(3)$ local gauge symmetry, originating in the family quantum number (the model is described in the talk section), influence the cosmological observations.

All discussion contributions are arranged alphabetically with respect to the authors' names.

Ta razdelek je namenjen odprtim vprašanjem, o katerih smo med delavnico izčrpno razpravljali. Problemi, o katerih smo razpravljali, bodo morda privedli do novih sodelovanj med udeleženci, ali pa so pripravili udeležence, da razmislijo o možnih rešitvah odprtih vprašanj na drugačne načine. Ker je čas med delavnico in rokom za oddajo prispevkov zelo kratek, vmes pa so poletne počitnice, je zelo težavno poleg prispevka, v katerem vsak udeleženec predstavi lastno delo, pripraviti še prispevek k temu razdelku.

Tako se velik del diskusij ne bo pojavil v letošnjem zborniku. So pa gotovo vplivale na prispevek marsikaterega udeleženca. Organizatorji upamo, da bodo te diskusije do prihodnje delavnice dozorele do oblike, da jih bo mogoče na njej predstavit.

Avtorica prispevka uporabi dobro poznano idejo, da bi s primerno parametrizacijo masnih matrik, ki so skoraj podobne edinkam, morda lahko razumeli, kako fermioni z maso postanejo masivni. Idejo nadgradi s primerno izbiro rotacijskih matrik in parametrov mešalnih matrik.

Avtorji naletijo pri študiju electrostatske interacije med dvema točkastima nabojema, ko uporabijo nelinearni model ter upoštevajo popravke kvantne elektrodinamke, na presenetljiv rezultat: Odbojna sila, ki je med različnima nabojema neskončna, ko sta naboja tesno skupaj, postane med enakima nabojema enaka nič.

Avtor uporabi model z binarno kodo, ki ga pojani v predavanju, da oceni energijsko skalo, pri kateri postaneta šibka in elektromagnetna sila enako močni.

Avtorja predlagata kozmoške meritve, ki bodo pokazale, če je model, ki uporabi grupo $SU(3)$ za opis družinskega kvantnega števila (model je opisan v enem od predavanj) smiselen. Zlomitev družinskega kvantnega števila vpliva na zgodovino razvoja vesolja in s predlaganimi meritvami ga je mogoče opaziti.

Prispevki v tej sekciji so, tako kot prispevki v glavnem delu, urejeni po abecednem redu priimkov avtorjev.

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11 Discreteness of Point Charge in Nonlinear Electrodynamics

A.I. Breev[1] ⋆ and A.E. Shabad[2,1] ⋆⋆

[1]Tomsk State University, 36 Lenin Prospekt, 634050, Tomsk, Russia
[2]P. N. Lebedev Physical Institute, 53 Leninskiy prospekt,
119991, Moscow, Russia

Abstract. We consider two point charges in electrostatic interaction between them within the framework of a nonlinear model, associated with QED, that provides finiteness of their field energy. We argue that if the two charges are equal to each other the repulsion force between them disappears when they are infinitely close to each other, but remains as usual infinite if their values are different. This implies that within any system to which such a model may be applicable the point charge is fractional, it may only be 2^n-fold of a certain fundamental charge, $n = 0, 1, 2....$

We find the common field of the two charges in a dipole approximation, where the separation between them is much smaller than the observation distance.

Povzetek. Avtorja obravnavata elekstrostatično interakcijo med točkastima nabojema v okviru nelinearnega modela, pridruženega kvantni elektrodinamiki, ki poskrbi, da je energija polja končna. V limiti, ko sta enaka točkasta neskončno blizu, je odbojna sila med njima končna. Sila med različnima točkasima nabojema v isti limiti je sila neskončna, tako kot običajno. Avtorja povzameta, da so v vsakem sistemu, ki mu tak model ustreza, točkasti naboji 2^n-kratniki ustreznega osnovnega naboja, $n = 0, 1, 2....$

Poiščeta skupno polje dveh nabojev v dipolnem približku z mnogo manjšo medsebojno razdaljo od razdalje opazovanja.

11.1 Introduction

Recently a class of nonlinear electrodynamic models was proposed [1] wherein the electrostatic field of a point charge is, as usual, infinite in the point where the charge is located, but this singularity is weaker than that of the Coulomb field, so that the space integral for the energy stored in the field converges. In contrast to the Born-Infeld model, the models from the class of Ref. [1] refer to nonsingular Lagrangians that follow from the Euler-Heisenberg (E-H) effective Lagrangian [2] of QED truncated at any finite power of its Taylor expansion in the field. This allows us to identify the self-coupling constant of the electromagnetic field with a definite combination of the electron mass and charge and to propose that such models may be used to extend QED to the extreme distances smaller than those for

⋆ breev@mail.tsu.ru
⋆⋆ shabad@lpi.ru

which it may be thought of as a perfectly adequate theory. More general models based on the Euler-Heisenberg Lagrangian, but fit also for considering non-static nonlinear electromagnetic phenomena, where not-too-fast-varying in space and time fields are involved, received attention as well. Among the nonlinear effects studied, there are the linear and quadratic electric and magnetic responses of the vacuum with a strong constant field in it to an applied electric field [3] , with the emphasis on the magneto-electric effect [4–6] and magnetic monopole formation [7]. Also self-interaction of electric and magnetic dipoles was considered with the indication that the electric and magnetic moments of elementary particles are subjected to a certain electromagnetic renormalization [8] after being calculated following a strong interaction theory, say, QCD or lattice simulations. Interaction of two laser beams against the background of a slow electromagnetic wave was studied along these lines, too [9].

In the present paper we are considering the electrostatic problem of two point charges that interact following nonlinear Maxwell equations stemming from the Lagrangian of the above [1] type, their common field not being, of course, just a linear combination of the individual fields. The problem is outlined in the next Section 11.2. Once the field energy is finite we are able to define the attraction or repulsion force between charges as the derivative of the field energy with respect to the distance $\mathbf{R}$ between them. Contrary to the standard linear electrodynamics, this is evidently not the same as the product of one charge by the field strength produced by the other! Based on the permutational symmetry of the problem that takes place in the special case where the values of the two charges are exactly the same we establish that the repulsion force between equal charges disappears when the distance between them is zero. This may shed light to the ever-lasting puzzle of whether a point-like electric charge may exist without flying to pieces due to mutual repulsion of its charged constituents. The optional answer proposed by the present consideration might be that after admitting that these exists a certain fundamental charge q, every other point charge should be fractional, equal to $2^n q$, with n, being zero or positive integer. In Section 11.3 we are developing the procedure of finding the solution to the above static two-body problem in the leading approximation with respect to the ratio of the distance R, to the coordinate of the observation point r, where this ratio is small – this makes the dipole-like approximation of Subsection 11.3.1. The simplifying circumstance that makes this approximation easy to handle is that it so happens that one needs, as a matter of fact, to solve only the second Maxwell equation, the one following from the least action principle, while the first one, $[\nabla \times \mathbf{E}] = 0$, is trivially satisfied. The above general statement concerning the nullification of the repulsion force at $R = 0$ for equal charges is traced at the dynamical level of Subsection 11.3.1 [1]

[1] Throughout the paper, Greek indices span Minkowski space-time, Roman indices span its three-dimensional subspace. Boldfaced letters are three-dimensional vectors, same letters without boldfacing and index designate their lengths, except the coordinate vector $\mathbf{x} = \mathbf{r}$, whose length is denoted as r. The scalar product is $(\mathbf{r} \cdot \mathbf{R}) = x_i R_i$, the vector product is $\mathbf{C} = [\mathbf{r} \times \mathbf{R}]$, $C_i = \epsilon_{ijk} x_i R_k$

11.2 Nonlinear Maxwell equations

11.2.1 Nonlinear Maxwell equations as they originate from QED

It is known that QED is a nonlinear theory due to virtual electron-positron pair creation by a photon. The nonlinear Maxwell equation of QED for the electromagnetic field tensor $F_{\nu\mu}(x) = \partial^{\mu}A^{\nu}(x) - \partial^{\nu}A^{\mu}(x)$ ($\tilde{F}_{\tau\mu}(x)$ designates its dual tensor $\tilde{F}^{\mu\nu} = (1/2)\,\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$) produced by the classical source $J_{\mu}(x)$ may be written as, see *e.g.* [3].

$$\partial^{\nu}F_{\nu\mu}(x) - \partial^{\tau}\left[\frac{\delta\mathcal{L}(\mathfrak{F},\mathfrak{G})}{\delta\mathfrak{F}(x)}F_{\tau\mu}(x) + \frac{\delta\mathcal{L}(\mathfrak{F},\mathfrak{G})}{\delta\mathfrak{G}(x)}\tilde{F}_{\tau\mu}(x)\right] = J_{\mu}(x)\ . \qquad (11.1)$$

Here $\mathcal{L}(\mathfrak{F},\mathfrak{G})$ is the effective Lagrangian (a function of the two field invariants $\mathfrak{F} = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ and $\mathfrak{G} = (1/4)\,\tilde{F}^{\mu\nu}F_{\mu\nu}$), of which the generating functional of one-particle-irreducible vertex functions, called effective action [10], is obtained by the space-time integration as $\Gamma[A] = \int \mathcal{L}(x)\,d^4x$. Eq. (11.1) is the realization of the least action principle

$$\frac{\delta S[A]}{\delta A^{\mu}(x)} = \partial^{\nu}F_{\nu\mu}(x) + \frac{\delta\Gamma[A]}{\delta A^{\mu}(x)} = J_{\mu}(x)\ , \qquad (11.2)$$

where the full action $S[A] = S_{\mathrm{Max}}[A] + \Gamma[A]$ includes the standard classical, Maxwellian, electromagnetic action $S_{\mathrm{Max}}[A] = -\int \mathfrak{F}(x)\,d^4x$ with its Lagrangian known as $L_{\mathrm{Max}} = -\mathfrak{F} = \frac{1}{2}\left(E^2 - B^2\right)$ in terms of the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$.

Eq. (11.1) is reliable only as long as its solutions vary but slowly in the space-time variable x_{μ}, because we do not include the space and time derivatives of $\mathfrak{F}$ and $\mathfrak{G}$ as possible arguments of the functional $\Gamma[A]$ treated approximately as local. This infrared, or local approximation shows itself as a rather productive tool [3]– [9]. The calculation of one electron-positron loop with the electron propagator taken as solution to the Dirac equation in an arbitrary combination of constant electric and magnetic fields of any magnitude supplies us with a useful example of $\Gamma[A]$ known as the E-H effective action [2]. It is valid to the lowest order in the fine-structure constant α, but with no restriction imposed on the the background field, except that it has no nonzero space-time derivatives. Two-loop expression of this local functional is also available [11].

The dynamical Eq. (11.1), which makes the "second pair" of Maxwell equations, may be completed by postulating also their "first pair"

$$\partial_{\nu}\widetilde{F}^{\nu\mu}(x) = 0\,, \qquad (11.3)$$

whose fulfillment allows using the 4-vector potential $A^{\nu}(x)$ for representation of the fields: $F_{\nu\mu}(x) = \partial^{\mu}A^{\nu}(x) - \partial^{\nu}A^{\mu}(x)$. This representation is important for formulating the least action principle and quantization of the electromagnetic field. From it Eq. (11.3) follows identically, unless the potential has singularity like the Dirac string peculiar to magnetic monopole. In the present paper we keep to Eq. (11.3), although its denial is not meaningless, as discussed in Ref. [7], where a magnetic charge is produced in nonlinear electrodynamics.

We want now to separate the electrostatic case. This may be possible if the reference frame exists where all the charges are at rest, $J_0(x) = J_0(\mathbf{r})$. (We denote $\mathbf{r} = \mathbf{x}$).Then in this "rest frame" the spacial component of the current disappears, $\mathbf{J}(x) = 0$, and the purely electric time-independent configuration $F_{ij}(\mathbf{r}) = 0$ would not contradict to equation (11.1). With the magnetic field equal to zero, the invariant $\mathfrak{G} = (\mathbf{E} \cdot \mathbf{B})$ disappears, too. In a theory even under the space reflection, to which class QED belongs, also we have $\left.\frac{\partial \mathcal{L}(\mathfrak{F},\mathfrak{G})}{\partial \mathfrak{G}(x)}\right|_{\mathfrak{G}=0} = 0$, since the Lagrangian should be an even function of the pseudoscalar $\mathfrak{G}$.Then we are left with the equation for a static electric field $E_i = F_{i0}(\mathbf{x})$

$$\partial_i F_{i0}(\mathbf{r}) - \partial_i \frac{\delta \mathcal{L}(\mathfrak{F},0)}{\delta \mathfrak{F}(\mathbf{r})} F_{i0}(\mathbf{r}) = J_0(\mathbf{r}). \tag{11.4}$$

11.2.2 Model approach

Equation (11.4) is seen to be the equation of motion stemming directly from the Lagrangian

$$L = -\mathfrak{F} + \mathcal{L}(\mathfrak{F},0) \tag{11.5}$$

with the constant external charge $J_0(\mathbf{r})$. In the rest of the paper we shall be basing on this Lagrangian in understanding that it may originate from QED as described above or, alternatively, be given *ad hoc* to define a certain model. In the latter case, if treated seriously as applied to short distances near a point charge where the field cannot be considered as slowly varying, in other words, beyond the applicability of the infrared approximation of QED outlined above, the Lagrangian (11.5) may be referred to as defining an extension of QED to short distances once $\mathcal{L}(\mathfrak{F},0)$ is the E-H Lagrangian (or else its multi-loop specification) restricted to $\mathfrak{G} = 0$.

It was shown in [1] that the important property of finiteness of the field energy of the point charge is guarantied if $\mathcal{L}(\mathfrak{F},0)$ in (11.5) is a polynomial of any power, obtained, for instance, by truncating the Taylor expansion of the H-E Lagrangian at any integer power of $\mathfrak{F}$. On the other hand, it was indicated in [12] that a weaker condition is sufficient: if $\mathcal{L}(\mathfrak{F},0)$ grows with -$\mathfrak{F}$ as $(-\mathfrak{F})^w$, the field energy is finite provided that $w > \frac{3}{2}$. The derivation of this condition is given in [14] and in [13]. As a matter of fact a more subtle condition suffices: $\mathcal{L}(\mathfrak{F}) \sim (-\mathfrak{F})^{\frac{3}{2}} \ln^u(-\mathfrak{F})$, $u > 2$. In what follows any of these sufficient conditions is meant to be fulfilled.

In the present paper we confine ourselves to the simplest example of the nonlinearity generated by keeping only quadratic terms in the Taylor expansion of the E-H Lagrangian in powers of the field invariant $\mathfrak{F}$

$$\mathcal{L}(\mathfrak{F}((x),0) = \frac{1}{2} \left.\frac{d^2 \mathcal{L}(\mathfrak{F},0)}{d^2 \mathfrak{F}}\right|_{\mathfrak{F}=0} \mathfrak{F}^2(x),$$

where the constant and linear terms are not kept, because their inclusion would contradict the correspondance principle that does not admit changing the Maxwell Lagrangian $L_{\text{Max}} = -\mathfrak{F}$ for small fields. The correspondance principle is laid into the calculation of the E-H Lagrangian via the renormalization procedure.

Finally, we shall be dealing with the model Lagrangian quartic in the field strength

$$L = -\mathfrak{F}(x) + \frac{1}{2}\gamma\mathfrak{F}^2(x) \tag{11.6}$$

with γ being a certain self-coupling coefficient with the dimensionality of the fourth power of the length, which may be taken as

$$\gamma = \left.\frac{d^2\mathcal{L}(\mathfrak{F},0)}{d^2\mathfrak{F}}\right|_{\mathfrak{F}=0} = \frac{e^4}{45\pi^2 m^4},$$

where e and m are the charge and mass of the electron, if $\mathcal{L}$ is chosen to be the E-H one-loop Lagrangian. We do not refer to this choice henceforward. Generalization to general Lagrangians can be also done in a straightforward way.

The second (11.4) and the first (11.3) Maxwell equations for the electric field $\mathbf{E}$ with Lagrangian (11.6) are

$$\nabla \cdot \left[\left(1 + \frac{\gamma}{2}E^2(\mathbf{r})\right)\mathbf{E}(\mathbf{r})\right] = j_0(\mathbf{r}), \tag{11.7}$$

$$\nabla \times \mathbf{E}(\mathbf{r}) = 0. \tag{11.8}$$

Denoting the solution of the linear Maxwell equations as $\mathbf{E}^{lin}(\mathbf{r})$

$$\nabla \cdot \mathbf{E}^{lin}(\mathbf{r}) = j_0(\mathbf{r}), \quad \nabla \times \mathbf{E}^{lin}(\mathbf{r}) = 0, \tag{11.9}$$

we write the solution of (11.7), in the following way [3] – [8]

$$\left(1 + \frac{\gamma}{2}E^2(\mathbf{r})\right)\mathbf{E}(\mathbf{r}) = \mathbf{E}^{lin}(\mathbf{r}) + [\nabla \times \Omega(\mathbf{r})], \tag{11.10}$$

where the vector function $\Omega(\mathbf{r})$ may be chosen in such a way that $\nabla \cdot \Omega(\mathbf{r}) = 0$. Imposing equation (11.8) we get

$$\Omega(\mathbf{r}) = \frac{1}{\nabla^2}[\nabla \times \boldsymbol{\mathcal{E}}(\mathbf{r})] = -\frac{1}{4\pi}\int \frac{[\nabla' \times \boldsymbol{\mathcal{E}}(\mathbf{r}')]d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}, \tag{11.11}$$

where we have introduced the auxiliary electric field as the cubic combination

$$\boldsymbol{\mathcal{E}}(\mathbf{r}) = \frac{\gamma}{2}E^2(\mathbf{r})\mathbf{E}(\mathbf{r}).$$

(In the case of a general Lagrangian that would be a more complicated function of $\mathbf{E}(\mathbf{r})$, namely $\boldsymbol{\mathcal{E}}(\mathbf{r}) = \frac{\delta\mathcal{L}(\mathfrak{F},0)}{\delta\mathfrak{F}(\mathbf{x})}\mathbf{E}(\mathbf{r})$). From (11.10), (11.11) it follows that

$$\mathbf{E}(\mathbf{r}) + \boldsymbol{\mathcal{E}}(\mathbf{r}) = \mathbf{E}^{lin}(\mathbf{r}) + [\nabla \times \Omega(\mathbf{r})] = \mathbf{E}^{lin}(\mathbf{r}) + \frac{[\nabla \times [\nabla \times \boldsymbol{\mathcal{E}}(\mathbf{r})]]}{\nabla^2}, \tag{11.12}$$

or, in components,

$$E_i(\mathbf{r}) = E_i^{lin}(\mathbf{r}) + \frac{\partial_i \partial_j}{\nabla^2}\frac{\gamma}{2}E^2(\mathbf{r})E_j(\mathbf{r}). \tag{11.13}$$

In the centre-symmetric case of a single point charge considered in [1], [12], the projection operator $\frac{\partial_i \partial_j}{\nabla^2}$ in the latter equation is identity ($\Omega(\mathbf{r}) = 0$), and Eq. (11.13) is no longer an integral equation. The same will be the case in the cylindric-symmetric problem of two point charges within the approximations to be considered in the next Section. This simplification makes solution possible. In this case it is sufficient present the solution of the differential part of Eq. (11.7) in the form (11.10) setting $\Omega(\mathbf{r}) = 0$ in it, then the first Maxwell equation (11.8) is fulfilled automatically.

11.3 Two-body problem

By the two point charge problem we mean the one, where the current $j_0(\mathbf{r})$ in (11.7) is the sum of delta-functions centered in the positions $\mathbf{r} = \pm\mathbf{R}$ of two charges q_1 and q_2 separated by the distance $2R$ (with the origin of coordinates x_i placed in the middle between the charges)

$$\nabla \cdot \left[\left(1 + \frac{\gamma}{2}E^2(\mathbf{r})\right)\mathbf{E}(\mathbf{r})\right] = q_1\delta^3\left(\mathbf{r} - \mathbf{R}\right) + q_2\delta^3\left(\mathbf{r} + \mathbf{R}\right). \tag{11.14}$$

In what follows we shall be addressing this equation accompanied by (11.8) for the combined field of two charges.

We shall be separately interested in the force acting between them. The force $F_i = \frac{dP^0}{dR_i}$ should be defined as the derivative of the field energy $P^0 = \int \Theta^{00} d^3x$ stored in the solution of Eqs. (11.14), (11.8) over the distance between them.

The Noether energy-momentum tensor for the Lagrange density (11.5) is

$$T^{\rho\nu} = (1 - \gamma\mathfrak{F}(x))F^{\mu\nu}\partial^\rho A_\mu - \eta^{\rho\nu}L(x). \tag{11.15}$$

By subtracting the full derivative $\partial_\mu\left[(1 - \gamma\mathfrak{F}(x)F^{\mu\nu})A^\rho\right]$, equal to

$$\left[(1 - \gamma\mathfrak{F}(x)F^{\mu\nu})\partial_\mu A^\rho\right]$$

due to the field equations (11.1) (without the source and with no dependence on $\mathfrak{G}$), the gauge-invariant and symmetric under the transposition $\rho \leftrightarrows \nu$ energy-momentum tensor

$$\Theta^{\rho\nu} = (1 - \gamma\mathfrak{F}(x))F^{\mu\nu}F_\mu{}^\rho - \eta^{\rho\nu}L(x) \tag{11.16}$$

is obtained. This is the expression for the electromagnetic energy proper, without the interaction energy with the source, the same as in the reference book [15]. When there is electric field alone, the energy density is

$$\Theta^{00} = (1 + \frac{\gamma E^2}{2})E^2 - \frac{E^2}{2}\left(1 + \frac{\gamma E^2}{4}\right) = \frac{E^2}{2} + \frac{3\gamma E^4}{8}. \tag{11.17}$$

The integral for the full energy of two charges $P^0 = \int \Theta^{00} d^3x$ converges since it might diverge only when integrating over close vicinities of the charges. But in each vicinity the field of the nearest charge dominates, and we know from the previous publication [1] (also to be explained below) that the energy of a separate charge converges in the present model. When the charges are in the same point, $R = 0$, they make one charge $q_1 + q_2$, whose energy coverges, too.

The energy

$$P^0 = \int \Theta^{00} d^3x \tag{11.18}$$

is rotation-invariant. Hence it may only depend on the length R, in other words, be an even function of $\mathbf{R}$. Then, in the point of coincidence $\mathbf{R}$ =0, the force $F_i = \frac{dP^0}{dR_i}$ must either disappear – if P^0 is a differentiable function of R that point– or be infinite – if not. Crucial to distinguish these cases is the value of the charge difference $\delta q = q_2 - q_1$. If the two charges are equal, $\delta q = 0$, the solution of equation (11.14) for the field is an even function of $\mathbf{R}$, since this equation is invariant under the reflection $\mathbf{R} \to -\mathbf{R}$. We shall see in the next subsection that the linear term in the expansion of the solution in powers of the small ratio $\frac{\mathbf{R}}{r}$ is identical zero in this special case, and so is the linear term of P^0.

11.3.1 Large distance case $r \gg R$ (dipole approximation)

We shall look for the solution in the form

$$\mathbf{E} = \mathbf{E}^{(0)} + \mathbf{E}^{(1)} + ...$$

where $\mathbf{E}^{(0)}$ and $\mathbf{E}^{(1)}$ are contributions of the zeroth and first order with respect to the ratio $\frac{\mathbf{R}}{r}$, respectively.

The zero-order term is spherical-symmetric, because it corresponds to two charges in the same point that make one charge,

$$\mathbf{E}^{(0)} = \frac{\mathbf{r}}{r}\mathbf{E}^{(0)}(r). \tag{11.19}$$

Eq. (11.8) is automatically fulfilled for this form.

Let us write the first-order term $E_i^{(1)}$ in the following general cylindric-symmetric form, linear in the ratio $\frac{\mathbf{R}}{r}$

$$\mathbf{E}^{(1)} = \mathbf{r}\left(\mathbf{R} \cdot \mathbf{r}\right) a(r) + \mathbf{R}g(r), \tag{11.20}$$

where a and g are functions of the only scalar r, and the cylindric axis is fixed as the line passing through the two charges. Let us subject (11.20) to the equation (11.8) $\nabla \times \mathbf{E}^{(1)} = 0$. This results in the relation

$$a(r) = \frac{1}{r}\frac{d}{dr}g(r), \tag{11.21}$$

provided that the vectors $\mathbf{r}, \mathbf{R}$ are not parallel. We shall see that with the ansatzes (11.20) and (11.19) equation (11.10) can be satisfied with the choice $\Omega(\mathbf{r}) = 0$:

$$\left(1 + \frac{\gamma}{2}E^2(\mathbf{r})\right)\mathbf{E}(\mathbf{r}) = \mathbf{E}^{lin}(\mathbf{r}), \tag{11.22}$$

namely, we shall find the coefficient functions a, g from Eq. (11.22) and then ascertain that the relation (11.21) is obeyed by the solution.

The inhomogeneity in (11.22)

$$\mathbf{E}^{lin}(\mathbf{r}) = \frac{q_1}{4\pi}\frac{\mathbf{r}-\mathbf{R}}{|\mathbf{r}-\mathbf{R}|^3} + \frac{q_2}{4\pi}\frac{\mathbf{r}+\mathbf{R}}{|\mathbf{r}+\mathbf{R}|^3}$$

satisfies the linear ($\gamma = 0$) limit of equation (11.14)

$$\nabla \cdot \mathbf{E}^{lin}(\mathbf{r}) = q_1\delta^3(\mathbf{r}-\mathbf{R}) + q_2\delta^3(\mathbf{r}+\mathbf{R}) \tag{11.23}$$

and also (11.8). The inhomogeneity is expanded in $\frac{\mathbf{R}}{r}$as

$$\begin{aligned}\mathbf{E}^{lin}(\mathbf{r}) &= \frac{(q_1+q_2)}{4\pi r^2}\frac{\mathbf{r}}{r} + \frac{(q_2-q_1)}{4\pi r^2}\left(\frac{\mathbf{R}}{r} - 3\frac{\mathbf{r}}{r}\frac{(\mathbf{R}\cdot\mathbf{r})}{r^2}\right) + ... = \\ &= \frac{(q_1+q_2)}{4\pi r^2}\frac{\mathbf{r}}{r} + \frac{1}{4\pi}\left(\frac{\mathbf{d}}{r^3} - \frac{3(\mathbf{d}\cdot\mathbf{r})}{r^5}\mathbf{r}\right) + ...,\end{aligned} \tag{11.24}$$

where $\mathbf{d} = (q_2 - q_1)\mathbf{R}$ is the dipole moment, while the dots stand for the disregarded quadrupole and higher multipole contributions.

The zero-order term satisfies the equation

$$\left(1 + \frac{\gamma}{2}E^{(0)2}(r)\right)E^{(0)}(r) = \frac{(q_1+q_2)}{4\pi r^2}, \tag{11.25}$$

with the first term of expansion (11.24) taken for inhomogeneity. This is an algebraic (not differential) equation, cubic in the present model (11.6), solved explicitly for the field $E^{(0)}$ as a function of r in this case, but readily solved for the inverse function $r(E^{(0)})$ in any model, which is sufficient for many purposes. Even without solving it we see that for small $r \ll \gamma^{\frac{1}{4}}$ the second term in the bracket dominates over the unity, therefore the asymptotic behavior in this region follows from (11.25) to be

$$E^{(0)}(r) \sim \left(\frac{q_1+q_2}{2\pi\gamma}\right)^{\frac{1}{3}} r^{-\frac{2}{3}}.$$

This weakened – as compared to the Coulomb field $\frac{q_1+q_2}{4\pi}r^{-2}$ – singularity is not an obstacle for convergence of the both integrals in (11.18), (11.17) for the proper field energy of the equivalent point charge $q_1 + q_2$.

With the zero-order equation (11.25) fulfilled, we write a linear equation for the first-order correction $\mathbf{E}^{(1)}$ from (11.22), to which the second, dipole part in (11.24) serves as an inhomogeneity

$$\mathbf{E}^{(1)} = \frac{(q_2-q_1)}{r^2}\left(\frac{\mathbf{R}}{r} - 3\frac{\mathbf{r}}{r}\frac{(\mathbf{R}\cdot\mathbf{r})}{r^2}\right) - \frac{\gamma}{2}\left[2\left(\mathbf{E}^{(1)}\cdot\mathbf{E}^{(0)}\right)\mathbf{E}^{(0)} + E^{(0)2}\mathbf{E}^{(1)}\right].$$

This equation is linear and it does not contain derivatives. We use (11.20) as the ansatz. After calculating

$$2\left(\mathbf{E}^{(1)}\cdot\mathbf{E}^{(0)}\right)\mathbf{E}^{(0)} + E^{(0)2}\mathbf{E}^{(1)} = \mathbf{r}E^{(0)2}\frac{(\mathbf{R}\cdot\mathbf{r})}{r^2}\left(2g + 3r^2 a\right) + \mathbf{R}gE^{(0)2},$$

we obtain two equations, along $\mathbf{R}$ and $\mathbf{r}$, with the solutions ($\delta q = q_2 - q_1$, $Q = q_2 + q_1$) :

$$g = \frac{\delta q}{r^3}\frac{1}{1 + \frac{\gamma}{2}E^{(0)2}} = \frac{\delta q}{Qr}E^{(0)}, \tag{11.26}$$

$$a = -\frac{\delta q}{r^5} \frac{3 + \frac{5\gamma}{2} E^{(0)2}}{\left(1 + \frac{\gamma}{2} E^{(0)2}\right)\left(1 + \frac{3\gamma}{2} E^{(0)2}\right)} \tag{11.27}$$

From (11.25) we obtain

$$\frac{d}{dr} E^{(0)} = -\frac{2Q}{r^3 \left(1 + \frac{\gamma}{2} E^{(0)2}\right)} - \frac{\gamma E^{(0)2}}{1 + \frac{\gamma}{2} E^{(0)2}} \frac{d}{dr} E^{(0)}.$$

Hence

$$\frac{d}{dr} E^{(0)} = -\frac{2Q}{r^3 \left(1 + \frac{3\gamma}{2} E^{(0)2}\right)}. \tag{11.28}$$

With the help of this relation the derivative of (11.26) can be calculated to coincide with (11.27) times r. This proves Eq. (11.21) necessary to satisfy the first Maxwell equation (11.8).

By comparing this with (11.27) we see that Eq. (11.21) necessary to satisfy the first Maxwell equation (11.8) has been proved.

Finally, relations (11.26) and (11.27) as substituted in the general cylindric covariant decomposition (11.20) give the linear in $\frac{R}{r}$(11.27) correction $\mathbf{E}^{(1)}$ to the zero-order field $\mathbf{E}^{(0)}$, subject to the equation (11.25), in terms of $E^{(0)}$,which is explicitly known in our special model. These results may be considered as giving nonlinear correction to the electric dipole field (the second term in (11.24)) due to nonlinearity.

Coming back to the discussion on the repulsion force we have to analyze the contribution of the found linear term $\mathbf{E}^{(1)}$ into the energy. The contribution of $\mathbf{E}^{(1)}$ into the energy density (11.17) linear in $\mathbf{R}$ contains the factor $\left(\mathbf{E}^{(1)} \cdot \mathbf{E}^{(0)}\right) = \left(\mathbf{E}^{(1)} \cdot \frac{\mathbf{r}}{r}\right) E^{(0)}$. According to the result (11.20) this factor is linear with respect to the scalar product $(\mathbf{R} \cdot \mathbf{r}) = Rr\cos\theta$. It would give zero contribution into the energy (11.18) due to the angle integration. This does not imply, however, that the force at the point of coincidence $\mathbf{R} = 0$ is zero, because the linear contribution into the integrand in (11.18) would create divergence of the integral (11.18) near $r = 0$. The interchange of the integration over r and of the limiting transition $\frac{R}{r} \to 0$ is not permitted. In the region $r < R$ of integration the linear approximation in the ratio $\frac{R}{r}$ is not relevant. This region gives the infinite contribution into the repulsion force between two charges when the approach each other infinitely close. The case where these charges are equal, $q_1 - q_2 = 0$, is different. Then the solution for $\mathbf{E}^{(1)}$ is just zero, and we confirm the conclusion made above following general argumentation that equal point charges do not repulse when their positions coincide.

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12 The Hypothesis of Unity of the Higgs Field With the Coulomb Field

E.G. Dmitrieff *

29 88 Postyshev bvd., Irkutsk, Russia

Abstract. In this discussion we propose a hypothesis about the physical unity of the Coulomb field and the Higgs field, which leads to the concept of physical vacuum with the periodic domain structure. Estimating the domain radius and the thickness of cross-domain wall in the spheric approximation, we find them both about 10^{-21} m.

Povzetek. V diskusiji avtor obravnava hipotezo fizikalne povezanosti Coulombskega in Higgsovega polja, kar vodi k fizikalnemu vakuumu s periodično strukturo domen. Za radij domene in debelino sten med domenami dobi oceno reda 10^{-21} m.

12.1 Introduction

Developing the model of vacuum and fundamental particles, that is based either on boolean algebra objects or spatial polyhedral tessellations, we were looking for a physical structure in the Nature, which would behave like a discrete field of binary digits, or as a foam with electrically charged near-to-polyhedral cells. Combining the ideas of vacuum domains [1], Kelvin problem of minimal-surfaced foam [2] and its best solution [3], and specific alteration of electric charges in our model [4], we supposed the following model of the physical vacuum.

12.2 United Higgs - Coulomb field

We suppose that scalar Higgs field and field of Coulomb scalar potential are the same physical entity. Also we suppose that the complex phase of Higgs field is limited to 0 and π (like angle in sphere S_0), forming just real values, positive and negative. In this case, the only possibility to get from positive to negative value and back, is to go through zero value.

The space, being in average electrically neutral, in this case would be either structureless 'empty space' with $\varphi = 0$ everywhere (may be, with fluctuations), or the mixture of pieces, or domains [1], having different and alternating values of this field.

Following the effective energy density of Higgs field as

$$V(\varphi) = \lambda^2(\varphi^2 - \eta^2)^2, \tag{12.1}$$

* eliadmitrieff@gmail.com

in empty space phase it would be positive: $V = \lambda^2\eta^4$.

Inside domains, where the potential keeps near $\pm\varphi$, the electrical field and energy density is about zero.

Near the walls between domains, the potential changes from positive to negative value – or back – through zero. So the walls will have extra energy, since zero potential corresponds to local maximum of effective energy density.

The electric charge in this case will be distributed mostly along the walls, causing pieces to be charged.

12.3 Evaluation of characteristic structure sizes

Equating the expression of potential for the charged sphere with radius r to the vacuum expectation value [5]:

$$\frac{q}{4\pi\varepsilon_0 r} = \eta \approx 246\text{GeV}, \tag{12.2}$$

we estimate the domain size:

$$r = \frac{q}{4\pi\varepsilon_0 \eta} = \frac{qc^2}{10^7\eta} = \frac{ec^2}{6\cdot 10^7\eta}; \tag{12.3}$$

$$r = \frac{1.602\cdot 10^{-19}\cdot 2.998^2\cdot 10^{16}}{6\cdot 10^7\cdot 246\cdot 10^9} = 9.755\cdot 10^{-22} \approx 10^{-21}\text{m}. \tag{12.4}$$

As the value of charge q we use $\frac{1}{6}e$ since it is the charge carried by one b-type bit in our 8-bit model.

The wall between pieces can be treated as a charged capacitor. Equating the capacity of the spherical capacitor with the charge to potential ratio:

$$C = \frac{q}{\eta} = \frac{4\pi\varepsilon_0 r^2}{d} = \frac{10^7 r^2}{dc^2} = \frac{q^2c^2}{10^7 d\eta^2}, \tag{12.5}$$

we express d to estimate the wall width, which occurs equal to r:

$$d = \frac{qc^2}{10^7\eta} = r \tag{12.6}$$

12.4 Spontaneous origination of the structure

Probably, the Coulomb character of field causes the instability of domains with radius larger than r: being charged, the different parts of any domain would experience the Coulomb repulsion from each other, so the domain would trend to split into smaller parts. Since this process increases the specific square of inter-domain walls surface, it also leads to increased Higgs energy. So the fragmentation must stop when the Higgs energy equals to Coulomb one.

In this case, the model is supposed to be isomorphic with the solution of the Kelvin problem [2], that is about of the minimal-surfaced foam of bubbles with equal volume. The 'air' phase corresponds to volumes, and 'liquid' phase corresponds to walls.

Because of equality of the estimated cell radius to the wall width, the potential is probably changes smoothly in a function close to a sine wave, forming no sharp walls.

The best known solution of Kelvin problem is the foam with Weaire-Phelan structure [3]. We found that this structure has two variants of optimal charge alterations, with different handedness. This providing possibility for the asymmetric baryogenesis.

12.5 Beyond the Standard Model

According to our models [4] and the assumptions noted above, vacuum probably has a structure of periodic Weaire-Phelan foam with alternating positive and negative charged cells, and the single or multiple anti-structure defects in it are experimentally observed as fundamental particles. Their properties are determined by the defect count and structure.

We also suppose that the vacuum structure may experience distortion and wave processes, that are relevant to the phenomena of gravity.

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13 What Cosmology Can Come From the Broken SU(3) Symmetry of the Three Known Families?

A. Hernández Galeana[1] ⋆ and M.Yu. Khlopov[2,3,4] ⋆⋆

[1]Departamento de Física, ESFM - Instituto Politécnico Nacional
U. P. "Adolfo López Mateos". C. P. 07738, Ciudad de México, México.
[2]National Research Nuclear University "MEPHI" (Moscow Engineering Physics Institute), 115409 Moscow, Russia
[3]Centre for Cosmoparticle Physics "Cosmion" 115409 Moscow, Russia
[4]APC laboratory 10, rue Alice Domon et Léonie Duquet 75205 Paris Cedex 13, France

Abstract. The question, why there are three quark-lepton families, cannot find answer in the Standard model. Extension of the symmetry of the Standard model by local gauge SU(3) symmetry of families involves new physics at high energy scales. Even elusive from direct experimental probes such construction can be tested in the combination of physical, astrophysical and cosmological signatures, in which cosmological probes play important role. In the case of chiral local gauge $SU(3)_H$ family symmetry such probes not only provided complete test of the model, but also demonstrated a possibility to describe the physical basis of modern cosmology, involving inflationary models with baryosynthesis and nonbaryonic dark matter. We formulate the programme of studies of cosmological impact of broken local gauge vector-like $SU(3)_F$ family symmetry.

Povzetek. Standardni model ne more odgovoriti na vprašanje, zakaj obstajo tri družine kvarkov in leptonov. Razširitev simetrij standardnega modela z lokalno umeritveno družinsko simetrijo SU(3) ponudi nove možnosti za opis dogodkov pri visokih energijah. Lokalna umeritvena družinska simetrija $SU(3)_H$ ponudi drugačen opis vesolja, ko so v model vključeni tudi inflacija, sinteza barionov in nebarionska temna snov. Delo predstavi program študija kozmoloških posledic zlomljene družinske simetrije $SU(3)_F$.

13.1 Introduction

The structure of the ordinary matter does not need more than one (first) family. We need quarks to build nucleons, electrons to bind with nuclei in neutral atoms and we have to add neutrino as a necessary element of beta processes, in which all the chemical elements of Mendeleev periodic table can be formed. However, the Nature gives us three families - symmetric by their interactions and asymmetric in their mass pattern. Extending the symmetry of the "Standard Model" (SM) by family symmetry and putting this additional symmetry on the same local gauge

⋆ albino@esfm.ipn.mx
⋆⋆ khlopov@apc.univ-paris7.fr

symmetry footing as in the Standard model we immediately find that though the family symmetry breaking scales in quark and lepton masses are within the range of electroweak symmetry breaking the absence of Flavor Changing Neutral Currents (FCNC) makes us to consider much larger scale of symmetry breaking for horizontal gauge bosons, which should be much heavier, than W and Z. To solve this problem one should assume the existence of heavy partners for ordinary quarks and leptons. These heavy particles should get their masses from the same Higgs mechanism, which makes very massive horizontal bosons, while mixing of these heavy partners with ordinary quarks and leptons reflects the pattern of family symmetry breaking in the mass hierarchy of quarks and leptons of the three known generations. Cosmological impact of this construction provides additional source of information on possible parameters and signatures of a family symmetry model. Here after brief review of the cosmology of horizontal unification, developed by [1,2] (see [3–5] for review and references) we turn to the approach by [Albino] and formulate open questions for successive study of cosmological impact of his vector-like approach to a gauge model of broken family symmetry.

13.2 Cosmology of chiral $SU(3)_H$ symmetry of known families

The existence and observed properties of the three known quark-lepton families appeal to the broken $SU(3)_H$ family symmetry [1,8,9], which should be involved in the extension of the Standard model. It provides the possibility of the *Horizontal unification* in the "bottom-up" approach to the unified theory [2]. Even in its minimal implementation the model of *Horizontal unification* has demonstrated its principal possibility to reproduce the main necessary elements of the modern cosmology. It provided the physical mechanisms for inflation and baryosynthesis as well as it offered unified framework to describe Cold, Warm, Hot and Unstable Dark Matter candidates. Methods of cosmoparticle physics [3,4] have provided the complete test of this model, proving such possibility for any physical model, hiding its basis in super high energy scales.

13.2.1 Horizontal hierarchy

The approach of Refs. [1,2,8,9] (and its revival in Refs. [10–12]) followed from the concept of local chiral gauge symmetry $SU(3)_H$, first proposed by Chkareuli[13]. Under the action of this symmetry the left-handed quarks and leptons transform as $SU(3)_H$ triplets and the right-handed as antitriplets. Their mass term transforms as $3 \bigotimes 3 = 6 \bigotimes \bar{3}$ and, therefore, can only form in the result of horizontal symmetry breaking.

This approach can be trivially extended to the case of n generations, assuming the proper $SU(n)$ symmetry. For three generations, the choice of horizontal symmetry $SU(3)_H$ seemed to be the only possible choice because the orthogonal and vector-like gauge groups could not provide different representations for the left- and right-handed fermion states. However, it turns out [Albino] that vector-like implementation for family $SU(3)_F$ is also possible, what we discuss in the successive sections.

The mass hierarchy between generations was related to the hypothesis of a hierarchy of such symmetry breaking. This hypothesis was called - the hypothesis of horizontal hierarchy (HHH)[14–16].

The model was based on the gauge $SU(3)_H$ flavor symmetry, which was additional to the symmetry of the Standard model. It means that there exist 8 heavy horizontal gauge bosons and there are three multiplets of heavy Higgs fields $\xi_{ij}^{(n)}$ (i,j - family indexes,$n = 1, 2, 3$) in nontrivial (sextet or triplet) representations of $SU(3)_H$. These heavy Higgs bosons were singlets relative to electroweak symmetry and don't have Yukawa couplings with ordinary light fermions. They had direct coupling to heavy fermions. The latter were singlets relative to electroweak symmetry. Ordinary Higgs ϕ of the Standard model was singlet relative to $SU(3)_H$. It coupled left-handed light fermions f_L^i to their heavy right-handed partners F_R^i, which were coupled by heavy Higgses ξ_{ij} with heavy left handed states F_L^j. Heavy left-handed states F_L^j were coupled to right handed light states f_R^j by a singlet scalar Higgs field η, which was singlet both relative to $SU(3)_H$ and electroweak group of symmetry. The described succession of transitions realized Dirac see-saw mechanism, which reproduced the mass matrix m_{ij} of ordinary light quarks and charged leptons f due to mixing with their heavy partners F. It fixed the ratio of vacuum expectation values of heavy Higgs fields, leaving their absolute value as the only main free parameter, which was determined from analysis of physical, astrophysical and cosmological consequences.

The $SU(3)_H$ flavor symmetry was assumed to be chiral to eliminate the flavor symmetric mass term. The condition of absence of anomalies implied heavy partners of light neutrinos, and the latter acquired mass by Majorana see-saw mechanism. The natural absence in the heavy Higgs potentials of triple couplings, which did not appear as radiative effects of any other (gauge or Yukawa) interaction, supported additional global U(1) symmetry, which could be associated with Peccei-Quinn symmetry and whose breaking resulted in the Nambu-Goldstone scalar filed, which shared the properties of axion, Majoron and singlet familon.

Horizontal unification The model provided complete test (in which its simplest implementation was already ruled out) in a combination of laboratory tests and analysis of cosmological and astrophysical effects. The latter included the study of the effect of radiation of axions on the processes of stellar evolution, the study of the impact of the effects of primordial axion fields and massive unstable neutrino on the dynamics of formation of the large-scale structure of the Universe, as well as analysis of the mechanisms of inflation and baryosynthesis based on the physics of the hidden sector of the model.

The model resulted in physically self-consistent inflationary scenarios with dark matter in the baryon-asymmetric Universe. In these scenarios, all steps of the cosmological evolution corresponded quantitatively to the parameters of particle theory. The physics of the inflaton corresponded to the Dirac see-saw mechanism of generation of the mass of the quarks and charged leptons, leptogenesis of baryon asymmetry was based on the physics of Majorana neutrino masses. The parameters of axion CDM, as well as the masses and lifetimes of neutrinos corresponded to the hierarchy of breaking of the $SU(3)_H$ symmetry of families.

The experience gained in the development of this model encourages us to elaborate similar programme for the case of vector-like $SU(3)_F$ family symmetry.

13.3 Horizontal hierarchy in vector-like model

In the approach of vector-like and universal $SU(3)_F$ gauge family symmetry [6,7], all the Standard Model left handed and right handed quarks and leptons transform as the fundamental triplet representation of the $SU(3)_F$ family symmetry. This is the basic starting difference in comparison to the chiral one of Refs. [1,2,8,9], which also defines the different hidden sectors, scalar fields and heavy fermions in order to break symmetries and generate masses for quarks and leptons, as well as the different possible cosmology consequences.

Right handed neutrinos in this approach of vector-like gauge $SU(3)_F$ family symmetry are introduced to cancel anomalies. The fermion content assume a new set of vector - like quarks and leptons U, D, E and N singlets under the $SU(3)_F \times SU(2)_L$ symmetry.

$SU(3)_F$ family symmetry is broken spontaneously in two stages by heavy SM singlet scalars $\eta_i^{(n)}$(i - family index, $n = 2, 3$) in the fundamental representation of $SU(3)_F$, generating 5 extremely heavy boson masses ($\gtrsim$ 100 TeV's) and 3 almost degenerate boson masses of few TeV's. The "Electroweak Symmetry Breaking" (EWSB) is achieved by the Higgs fields Φ_i^u and Φ_i^d, which transform simultaneously as triplets under $SU(3)_F$ and as the ϕ and $\tilde{\phi} = i\,\sigma_2\,\phi^*$ Higgs doublets under the SM, respectively.

The gauge symmetry $G \equiv SU(3)_F \times G_{SM}$, the fermion content, and the transformation of the scalar fields, all together, avoid Yukawa couplings between SM fermions. The tree level allowed Dirac Yukawa couplings involve terms between the SM fermions $\psi_{SM,(L,R)}$ and the corresponding vector-like fermion F (F=U, D, E and N) : $h_F\,\bar{\psi}_{SM,L}\,\Phi^{u,d}\,F_R \;+\; h^{(n)}\,\bar{\psi}_{SM,R}\,\eta^{(n)}\,F_L \;+\; M_F\,\bar{F}_L\,F_R \;+ h.c$, which yield tree level Dirac See-saw mass matrices for quarks and leptons, including neutrinos, with two massless eigenvalues. Therefore, in this scenario ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level, while light fermions, including light neutrinos obtain masses from radiative corrections mediated by the massive gauge bosons of the $SU(3)_F$ family symmetry.

Neutrinos may also obtain left-handed and right-handed Majorana masses both from tree level and radiative corrections.

13.4 Towards cosmology of vector-like $SU(3)_F$ family symmetry

Here we formulate the open questions for cosmological impact of the model of vector-like $SU(3)_F$ family symmetry. We can stipulate the following cosmologically interesting aspects:

- Is there a candidate for inflaton?
- Which new particles are stable or metastable in vector-like $SU(3)_F$ model
- Are there $SU(3)_F$ instantons or sphalerons?

- What kind of phase transitions can take place in the early Universe with Family symmetry breaking
- Can there be a new mechanism of baryosynthesis? Due to Majorana mass term - there can be leptogenesis with successive redistribution of lepton excess to baryon and lepton excess due to electroweak B and L nonconservation
- Is there additional global symmetry and if, yes, what kind of Nambu-Goldstone (pseudo Nambu Goldstone) solutions can exist?

This list can be modified and extended in the course of our studies.

13.5 Conclusion

Here we have formulated the programme of our joint studies during Albino's sabbatical year in the APC, Paris, France. We hope to present their first results at the coming XX Bled Workshop.

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14 Phenomenological Mass Matrices With a Democratic Warp

A. Kleppe *

SACT, Oslo

Abstract. Taking into account all available data on the mass sector, we obtain unitary rotation matrices that diagonalize the quark matrices by using a specific parametrization of the Cabibbo-Kobayashi-Maskawa mixing matrix. In this way, we find mass matrices for the up- and down-quark sectors of a specific, symmetric form, with traces of a democratic texture.

Povzetek. Avtorica uporabi razpoložljive podatke o kvarkih, da določi unitarne matrike rotacij, ki diagonalizirajo masne matrike. Izbere parametrizacijo mešalne matrike Cabibba-Kobayashija-Maskawe, ki poskrbi, da imata masni matriki kvarkov u in d, ki sta zelo blizu demokratičnima matrikama, posebno simetrično obliko.

14.1 Introduction

The Standard Model of particle physics is flawed by the large number of free parameters, for which there is at present no explanation. Most of these free parameters reside in flavour space, the structure of which is determined by the fermion mass matrices, i.e. by the form that the mass matrices take in the "weak basis" where mixed fermion states interact weakly. This basis differs from the mass bases, where the mass matrices are diagonal, with entries corresponding to the masses of the physical fermions.

The information content of a matrix is contained in its matrix invariants, which in the case of a $N \times N$ matrix M are the N sums and products of the eigenvalues λ_j, such as $traceM$, $detM$,

$$\begin{aligned}
I_1 &= \textstyle\sum_j \lambda_j = \lambda_1 + \lambda_2 + \lambda_3 ... \\
I_2 &= \textstyle\sum_{jk} \lambda_j \lambda_k = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + ... \\
I_3 &= \textstyle\sum_{jkl} \lambda_j \lambda_k \lambda_l = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + ... \\
&\ \vdots \\
I_N &= \lambda_1\lambda_2 \cdots \lambda_N
\end{aligned} \tag{14.1}$$

The search for the "right" mass matrices is based on the assumption that even if the information content of a matrix is contained in its invariants, the form of a

* kleppe@nbi.dk

matrix also carries important information. The hope is that the form of the mass matrices in the "weak basis" can give some hint about the origin of the fermion masses.

The crux is that we don't know which of the flavour space bases is the weak basis, we consequently don't know what form of the mass matrices have in this unknown basis. The different mass matrix ansätze found in the literature correspond to different choices, based on different assumptions, as to which flavour space basis is the weak basis.

14.2 Phenomenology

The Standard Model might not be a fundamental theory, but it certainly is an exceedingly successful model. In our approach, we follow the phenomenlogical track, and scrutinize all available data that are relevant for the mass sector. In addition to numerical mass values, there is also the mixing matrix V that appears in the flavour changing charged current Lagrangian

$$\mathcal{L}_{cc} = -\frac{g}{2\sqrt{2}}\bar{f}_L \gamma^\mu V f'_L W_\mu + h.c. \tag{14.2}$$

where as before f and f' are fermion fields with charges Q and $Q-1$, correspondingly. In the case of the quarks, the mixing matrix is the Cabbibo-Kobayashi-Maskawa (CKM) [1] mixing matrix.

That $V \neq \mathbf{1}$ implies that the up-sector mass basis is different from the down-sector mass basis, the CKM matrix being the bridge between the two mass bases. As we go from the weak basis to the two different mass bases by rotating the matrices by the unitary matrices U and U', respectively,

$$M \to UMU^\dagger = D = diag(m_u, m_c, m_t) \tag{14.3}$$
$$M' \to U'M'U'^\dagger = D' = diag(m_d, m_s, m_b)$$

we have that $V = UU^{'\dagger}$. A given choice of the weak basis - i.e. of the mass matrices, thus corresponds to choosing a factorization of the mixing matrix, and since $U = U(M)$ and $U' = U'(M')$, $V = U(M)U'^\dagger(M') = V(M, M')$.

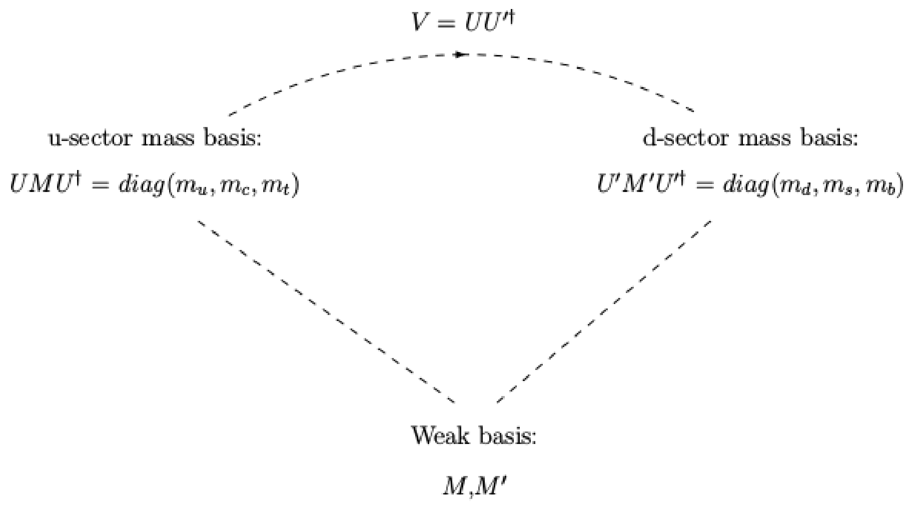
The charged current Lagrangian (14.2) can be interpreted as describing the interaction between the physical up-sector particles $\bar{\psi}_L = (\bar{u}, \bar{c}, \bar{t})_L$ with the mixed down-sector states, or equivalently as the interaction between the up-sector mixed states and the down-sector mass states $\bar{\psi}'_L = (\bar{d}, \bar{s}, \bar{b})_L$,

If we take the definition of the CKM matrix at face value, $V = UU'^\dagger$, it is however more natural to perceive the charged current interactions as taking place between mixed up-sector states and mixed down-sector states,

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\bar{\psi}_L \gamma^\mu V \psi'_L W_\mu + h.c. = -\frac{g}{\sqrt{2}}\bar{\varphi}_L \gamma^\mu \varphi'_L W_\mu + h.c. \tag{14.4}$$

where

$$\varphi = U^\dagger \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad \text{and} \qquad \varphi' = U'^\dagger \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



are the fermion fields in the weak basis in flavour space, and ψ and ψ' are the corresponding mass eigenstates.

Mass eigenstates are defined as "physical", corresponding to particles with definite masses; while the weakly interacting mixings of mass states are referred to as "flavour states". Physical particles are thus identified as mass eigenstates. In the case of neutrinos the situation is however somewhat different, since neutrino mass eigenstates do not appear on stage, they merely propagate in free space. In the realm of neutral leptons it is actually the flavour states ν_e, ν_μ, ν_τ that we perceive as "physical", since they are the only neutrinos that we "see", as they appear together with the charged leptons. As the charged leptons e, μ, τ are assumed to be both weak eigenstates and mass eigenstates, the only mixing matrix that appears in the lepton sector is the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix U [2], which only operates on neutrino states,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where (ν_1, ν_2, ν_3) are mass eigenstates, and $(\nu_e, \nu_\mu, \nu_\tau)$ are the weakly interacting "flavour states". In the lepton sector, the charged currents are thus interpreted as charged lepton flavours (e, μ, τ) interacting with the neutrino "flavour states" $(\nu_e, \nu_\mu, \nu_\tau)$.

For quarks as well as leptons, the relation between the weakly interacting fermion fields φ and the mass eigenstates ψ is determined by the unitary rotation matrix U which diagonalizes the mass matrix M,

$$\mathcal{L}_{mass} = \bar{\varphi} M \varphi = \bar{\varphi} U^{\dagger} (UMU^{\dagger}) U \varphi = \bar{\psi} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} \psi,$$

in the quark sector the (physical) mass eigenstates thus are the fields $\psi = U\varphi$ and $\psi' = U'\varphi'$,

$$\psi = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \quad \text{and} \quad \psi' = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The CKM matrix plays an important role in relating the mass matrices for the up- and down-sectors, since once the form of the mass matrix of one of the charge sectors is established, also the form of the mass matrix of the other charge sector is determined, via the CKM mixing matrix. Once the form of the up-sector mass matrix M is established,the unitary matrix U that diagonalizes M is determined. And since $V = UU'^{\dagger}$, this also determines $U' = V^{\dagger}U$, whereby we have $M' = U'^{\dagger}diag(d, s, b)U'$. In this sense M and M' are determined together.

14.3 Factorizing the mixing matrix

The Cabbibo-Kobayashi-Maskawa mixing matrix can of course be parametrized and factorized in many different ways, and different factorizations correspond to different rotation matrices U and U'. The most obvious and "symmetric" factorization of the CKM mixing matrix, following the "standard parametrization" [3] with three Euler angles α, β, 2θ,

$$V = \begin{pmatrix} c_\beta c_{2\theta} & s_\beta c_{2\theta} & s_{2\theta} e^{-i\delta} \\ -c_\beta s_\alpha s_{2\theta} e^{i\delta} - s_\beta c_\alpha & -s_\beta s_\alpha s_{2\theta} e^{i\delta} + c_\beta c_\alpha & s_\alpha c_{2\theta} \\ -c_\beta c_\alpha s_{2\theta} e^{i\delta} + s_\beta s_\alpha & -s_\beta c_\alpha s_{2\theta} e^{i\delta} - c_\beta s_\alpha & c_\alpha c_{2\theta} \end{pmatrix} = UU'^{\dagger} \tag{14.5}$$

is to take the diagonalizing rotation matrices for the up- and down-sectors as

$$\begin{aligned} U &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} = \\ &= \begin{pmatrix} c_\theta e^{-i\gamma} & 0 & s_\theta e^{-i\gamma} \\ -s_\alpha s_\theta e^{i\gamma} & c_\alpha & s_\alpha c_\theta e^{i\gamma} \\ -c_\alpha s_\theta e^{i\gamma} & -s_\alpha & c_\alpha c_\theta e^{i\gamma} \end{pmatrix} \end{aligned} \tag{14.6}$$

and

$$\begin{aligned} U' &= \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \\ &= \begin{pmatrix} c_\beta c_\theta e^{-i\gamma} & -s_\beta & -c_\beta s_\theta e^{-i\gamma} \\ s_\beta c_\theta e^{-i\gamma} & c_\beta & -s_\beta s_\theta e^{-i\gamma} \\ s_\theta e^{i\gamma} & 0 & c_\theta e^{i\gamma} \end{pmatrix} \end{aligned} \tag{14.7}$$

respectively, where α, β, θ and γ correspond to the parameters in the standard parametrization, with $\gamma = \delta/2$, $\delta = 1.2 \pm 0.08$ rad, and $2\theta = 0.201 \pm 0.011^\circ$, while $\alpha = 2.38 \pm 0.06^\circ$ and $\beta = 13.04 \pm 0.05^\circ$. In this factorization scheme, α and β are rotation angles operating in the up-sector and the down-sector, respectively.

Now, with the rotation matrices U and U', we obtain the the up- and down-sector mass matrices

$$M = U^\dagger \mathrm{diag}(m_u, m_c, m_t)U \;\text{ and }\; M' = U'^\dagger \mathrm{diag}(m_d, m_s, m_b)U',$$

such that

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} = \begin{pmatrix} Xc_\theta^2 + Ys_\theta^2 & -Zs_\theta\, e^{-i\gamma} & (X-Y)c_\theta s_\theta \\ -Zs_\theta\, e^{i\gamma} & Y + 2Z\cot 2\alpha & Zc_\theta\, e^{i\gamma} \\ (X-Y)c_\theta s_\theta & Zc_\theta\, e^{-i\gamma} & Xs_\theta^2 + Yc_\theta^2 \end{pmatrix} \tag{14.8}$$

where $X = m_u$, $Z = (m_c - m_t)\sin\alpha\cos\alpha$ and $Y = m_t + Z\tan\alpha = m_c \sin^2\alpha + m_t\cos^2\alpha$, and

$$M' = \begin{pmatrix} M'_{11} & M'_{12} & M'_{13} \\ M'_{21} & M'_{22} & M'_{23} \\ M'_{31} & M'_{32} & M'_{33} \end{pmatrix} = \begin{pmatrix} X's_\theta^2 + Y'c_\theta^2 & Z'c_\theta\, e^{i\gamma} & (X'-Y')c_\theta s_\theta \\ Z'c_\theta\, e^{-i\gamma} & Y' + 2Z'\cot 2\beta & -Z's_\theta\, e^{-i\gamma} \\ (X'-Y')c_\theta s_\theta & -Z's_\theta\, e^{i\gamma} & X'c_\theta^2 + Y's_\theta^2 \end{pmatrix} \tag{14.9}$$

where $X' = m_b$, $Z' = (m_s - m_d)\sin\beta\cos\beta$ and $Y' = m_d + Z'\tan\beta = m_d\cos^2\beta + m_s\sin^2\beta$.

The two mass matrices thus have similar textures, or forms, and there is even a relational equality,

$$M_{32}/M_{12} = M'_{12}/M'_{32} = -\cot\theta$$

which is independent of the quark masses.

From

$$\begin{aligned} Y &= m_c\sin^2\alpha + m_t\cos^2\alpha, \\ Z &= (m_c - m_t)\sin\alpha\cos\alpha, \\ Y' &= m_d\cos^2\beta + m_s\sin^2\beta \\ \text{and} \\ Z' &= (m_s - m_d)\sin\beta\cos\beta, \end{aligned}$$

we moreover have

$$\begin{aligned} m_u &= X, \quad m_c = Y + Z\cot\alpha, \quad m_t = Y - Z\tan\alpha \\ m_d &= Y' - Z'\tan\beta, \quad m_s = Y' + Z'\cot\beta, \quad m_b = X' \end{aligned} \tag{14.10}$$

14.4 Numerical matrices

Using the numerical values $\beta = 13.04^o$, $\alpha = 2.38^o$, $\delta = 1.2 \pm 0.08$ rad, and $2\theta = 0.201 \pm 0.011^\circ$ for the the angles, and using the mass values (Jamin 2014) [4] for the up- and down-sectors,

$$\begin{aligned} m_u(M_Z) &= 1.24\text{MeV} \;\; m_c(M_Z) = 624\text{MeV} \;\; m_t(M_Z) = 171550\text{MeV} \\ m_d(M_G) &= 2.69\text{MeV} \;\; m_s(M_G) = 53.8\text{MeV} \;\; m_b(M_G) = 2850\text{MeV} \end{aligned} \tag{14.11}$$

we get the numerical values for the mass matrices (14.8) and (14.9)

$$M = \begin{pmatrix} 1.767 & 12.439e^{-i\gamma} & -300.389 \\ 12.439e^{i\gamma} & 918.759 & -7091.892e^{i\gamma} \\ -300.389 & -7091.892e^{-i\gamma} & 171254.714 \end{pmatrix} \mathrm{MeV} \tag{14.12}$$

and

$$M' = \begin{pmatrix} 5.299 & 11.23e^{i\gamma} & 4.99 \\ 11.23e^{-i\gamma} & 51.18 & -0.0197e^{-i\gamma} \\ 4.99 & -0.0197e^{i\gamma} & 2849.99, \end{pmatrix} \mathrm{MeV} \tag{14.13}$$

where in M,

$M_{11} = m_u + \sigma$, $M_{22} = m_c + Q - \sigma$, $M_{33} = m_t - Q$,
$M_{22} + M_{33} = m_c + m_t - \sigma$ and $|M_{33}M_{12}| \approx |M_{13}M_{32}|$,
with $\sigma \simeq 0.53\mathrm{MeV}$, $Q \simeq 295.3\mathrm{MeV}$.

Likewise, in M',

$M'_{11} = m_d + R$, $M'_{22} = m_s + \eta - R$, $M'_{33} = m_b - \eta$,
$M'_{11} + M'_{22} = m_d + m_s + \eta$, and $|M'_{33}M'_{32}| \approx |M'_{13}M'_{12}|$,
with $R \simeq 2.61\mathrm{MeV}, \eta \simeq 0.011$ MeV.

14.5 Traces of a democratic structure

Our factorization of the Cabbibo-Kobayashi-Maskawa mixing matrix is only one of many possible choices, in (14.5) we can moreover sandwich any number of unitary matrices between U and U',

$$V = UU'^{\dagger} = UO_1O_1^{\dagger}U'^{\dagger} = UO_1O_2O_2^{\dagger}O_1^{\dagger}U'^{\dagger} = ...$$

where O_j are unitary matrices such that each set of sandwiched $O_jO_j^{\dagger}$ corresponds to a new set of unitary matrices diagonalizing the mass matrices, and thus to yet another type of mass matrix texture.

In our first approach [9], the sandwich principle was used with the purpose of investigating democratic mass matrix textures. In the democratic scenario, it is assumed that both the up- and down-sector mass matrices have an initial structure of the type $M_0 = k\mathbf{N}$ and $M'_0 = k'\mathbf{N}$ where

$$\mathbf{N} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

with the mass spectra $(0, 0, 3k)$, $(0, 0, 3k')$, and a mixing matrix equal to unity (i.e. no CP-violation). The flavour symmetry displayed by the matrices M_0 and M'_0 is subsequently broken, whereby both mass spectra contain the three observed non-zero values, and the mixing matrix becomes the CKM matrix (with a CP-violating phase).

Our sandwiching procedure started from the factorization $V = UU'^{\dagger}$, with U and U' as in (14.6) and (14.7), and the matrix

$$U_{dem} = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \tag{14.14}$$

which diagonalizes the democratic matrix **N**. When U_{dem} and its Hermitian conjugate were put into the mixing matrix,

$$V = UU'^{\dagger} \Rightarrow V = UU_{dem}U_{dem}^{\dagger}U'^{\dagger}$$

we obtained new rotation matrices UU_{dem} and $U'U_{dem}$ which indeed correspond to mass matrices with democratic textures.

In simplest case (14.5), without any $U_{dem}U_{dem}^{\dagger}$ or other matrices sandwiched between U and U' in $V = UU'^{\dagger}$, there is however already some interesting, democracy-like structure present, which is can be made visible by a slight reformulation of the matrices (14.8) and (14.9). Even though the matrix elements are dominated by the hierarchical family structure, which does not look very "democratic", rewriting the matrices by extracting the dimensional coefficients ρ and μ unveils this structure:

$$M = \rho \begin{pmatrix} A & Be^{-i\gamma} & -C \\ Be^{i\gamma} & H & -BCe^{i\gamma} \\ -C & -BCe^{-i\gamma} & C^2 \end{pmatrix} \tag{14.15}$$

and

$$M' = \mu \begin{pmatrix} A' & B'Ce^{i\gamma} & C \\ B'Ce^{-i\gamma} & H' & -B'e^{-i\gamma} \\ C & -B'e^{i\gamma} & C^2 \end{pmatrix}, \tag{14.16}$$

with

$\rho = (Y - X)s_\theta^2, A = (X\cot_\theta^2 + Y)/(Y - X), B = Z/(Y - X)s_\theta,$
$H = (Y + 2Z\cot 2\alpha)/(Y - X)s_\theta^2, C = \cot\theta$
$\mu = (X' - Y')s_\theta^2, A' = (X' + Y'\cot_\theta^2)/(X' - Y'), B' = Z'/(X' - Y')s_\theta$
$H' = (Y' + 2Z'\cot 2\beta)/(X' - Y')s_\theta^2$

Numerically, with the mass values (14.11), this corresponds to

$\rho = 0.5269$ MeV, $A = 3.3533, B = 23.608, H = 1743.71, C = \cot\theta \simeq 570.1$
$\mu = 0.00875$ MeV, $A' = 605.6, B' = 2.2514, H' = 5849.14,$

where incidentally $H' = AH$. The up-sector mass matrix (14.15) can be rewritten as as

$$M = \rho \left[\begin{pmatrix} 1 & & \\ & Be^{i\gamma} & \\ & & -C \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & Be^{-i\gamma} & \\ & & -C \end{pmatrix} + \Lambda \right] = \rho \left[\hat{M} + \Lambda \right] \tag{14.17}$$

where

$$\Lambda = \begin{pmatrix} A-1 & & \\ & H-B^2 & \\ & & 0 \end{pmatrix} \tag{14.18}$$

Noticing that the matrix

$$\hat{M} = \begin{pmatrix} 1 & & \\ & Be^{i\gamma} & \\ & & -C \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & Be^{-i\gamma} & \\ & & -C \end{pmatrix} = D\mathbf{N}D^* \tag{14.19}$$

has only one non-zero eigenvalue, and that

$$\mathbf{N} = D^* \; \hat{M} \; D,$$

we can relate $\hat{M}$ to the democratic matrix $M_0 = k\mathbf{N}$, by equating the one non-zero eigenvalue of $\hat{M}$, $1 + B^2 + C^2 = \mathrm{trace}(DD^*)$, to the one non-zero eigenvalue $3k$ of the democratic matrix M_0, which gives

$$3k = \rho(1 + B^2 + C^2),$$

i.e. $k = 57181.4 MeV$. Thus identifying the matrix $\hat{M}$ as having a kind of democratic texture, we determine the matrix Λ as the symmetry breaking term which finally gives the mass spectrum with the three observed non-zero masses. Λ has two non-zero eigenvalues $\Lambda_1 = A - 1$ and $\Lambda_2 = H - B^2$; where $\Lambda_1 = m_u/\rho$, and Λ_2 is related to $\hat{M}$ by

$$\frac{k^3}{m_u m_c m_t} = (H - B^2)^2 = \Lambda_2^2$$

If we in this way interpret the mass matrix

$$M = \rho[D\mathbf{N}D^* + \Lambda]$$

as starting out as a democratic matrix $M_0 = k\mathbf{N}$, the first flavour symmetry breaking is identified as

$$M_0 \Rightarrow \hat{M} = D\mathbf{N}D^*$$

where $\hat{M}$ has the same, one non-zero eigenvalue as M_0, $3k = \rho(1 + B^2 + C^2)$, but the flavour symmetry of the fields $(\varphi_1, \varphi_2, \varphi_3)$ in the weak basis is broken. By adding Λ, with the two non-zero eigenvalues Λ_1 and Λ_2, we finally get the full mass spectrum of M.

The down-sector can be treated in a similar fashion, though here the traces of democracy are less transparent.

14.6 Conclusion

Without introducing any new assumptions, by just factorizing the “standard parametrization” of the CKM weak mixing matrix in a specific way, we obtain mass matrices with a specific type of democratic texture. This is a work in progress, and the implications of this democratic structure remain to be analyzed.

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Virtual Institute of Astroparticle Physics Presentation

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15 Virtual Institute of Astroparticle Physics — Scientific-Educational Platform for Physics Beyond the Standard Model

M.Yu. Khlopov[1,2,3]

[1]Centre for Cosmoparticle Physics "Cosmion"
[2]National Research Nuclear University "Moscow Engineering Physics Institute", 115409 Moscow, Russia
[3]APC laboratory 10, rue Alice Domon et Léonie Duquet
75205 Paris Cedex 13, France

Abstract. Being a unique multi-functional complex of science and education online, Virtual Institute of Astroparticle Physics (VIA) operates on website http://viavca.in2p3.fr/site.html. It supports presentation online for the most interesting theoretical and experimental results, participation online in conferences and meetings, various forms of collaborative scientific work as well as programs of education at distance, combining online videoconferences with extensive library of records of previous meetings and Discussions on Forum. Since 2014 VIA online lectures combined with individual work on Forum acquired the form of Open Online Courses. Aimed to individual work with students it is not Massive, but the account for the number of visits to VIA site converts VIA in a specific tool for MOOC activity. VIA sessions are now a traditional part of Bled Workshops' programme. At XIX Bled Workshop it provided a world-wide discussion of the open questions of physics beyond the standard model, supporting world-wide propagation of the main ideas, presented at this meeting.

Povzetek. Virtual Institute of Astroparticle Physics (VIA), ki deluje na spletni strani http://viavca.in2p3.fr/site.html, je vsestranski sistem za podporo znanosti in izobraževanju na spletu. Sistem podpira neposredne spletne predstavitve najbolj zanimivih teoretičnih in eksperimentalnih rezultatov, omogoči virtualno udeležbo na konferencah in srečanjih ter različne oblike znanstvenega sodelovanja, pa tudi programe izobraževanja na daljavo. Zapisi srečanj in diskusij ostanejo na spletnem forumu VIA. Od leta 2014 se je kombinacija neposrednih spletnih predavanj in individualnega dela na spletnih forumih VIA razvila v obliko odprtih spletnih serij predavanj—predmetov (oblika znana kot Open Online Courses). Dostopna so preko uporabniškega računa na VIA in usmerjena bolj v individualno in manj v množično (Massive) orodje za množično spletno izobraževanje na daljavo (MOOC). VIA se redno učinkovito uporablja na blejskih delavnicah. Na letošnji, devetnajsti, delavnici je VIA omogočila diskusijo med udeleženci iz vseh koncev sveta o odprtih vprašanjih fizike onkraj standardnih modelov osnovnih delcev in kozmologije ter podporo predstavitev na daljavo in s tem svetovno dostopnost diskusij, ki so tekle na delavnici.

15.1 Introduction

Studies in astroparticle physics link astrophysics, cosmology, particle and nuclear physics and involve hundreds of scientific groups linked by regional networks (like ASPERA/ApPEC [1,2]) and national centers. The exciting progress in these studies will have impact on the knowledge on the structure of microworld and Universe in their fundamental relationship and on the basic, still unknown, physical laws of Nature (see e.g. [3,4] for review). The progress of precision cosmology and experimental probes of the new physics at the LHC and in nonaccelerator experiments, as well as the extension of various indirect studies of physics beyond the Standard model involve with necessity their nontrivial links. Virtual Institute of Astroparticle Physics (VIA) [5] was organized with the aim to play the role of an unifying and coordinating platform for such studies.

Starting from the January of 2008 the activity of the Institute takes place on its website [6] in a form of regular weekly videoconferences with VIA lectures, covering all the theoretical and experimental activities in astroparticle physics and related topics. The library of records of these lectures, talks and their presentations was accomplished by multi-lingual Forum. In 2008 VIA complex was effectively used for the first time for participation at distance in XI Bled Workshop [7]. Since then VIA videoconferences became a natural part of Bled Workshops' programs, opening the virtual room of discussions to the world-wide audience. Its progress was presented in [8–14]. Here the current state-of-art of VIA complex, integrated since 2009 in the structure of APC Laboratory, is presented in order to clarify the way in which discussion of open questions beyond the standard model at the XIX Bled Workshop were presented with the of VIA facility to the world-wide audience.

15.2 VIA structure and its activity

15.2.1 VIA activity

The structure of VIA complex is illustrated by the Fig. 15.1. The home page, presented on this figure, contains the information on the coming and records of the latest VIA events. The menu links to directories (along the upper line from left to right): with general information on VIA (About VIA), entrance to VIA virtual rooms (Rooms), the library of records and presentations (Previous) of VIA Lectures (Previous → Lectures), records of online transmissions of Conferences(Previous → Conferences), APC Colloquiums (Previous → APC Colloquiums), APC Seminars (Previous → APC Seminars) and Events (Previous → Events), Calender of the past and future VIA events (All events) and VIA Forum (Forum). In the upper right angle there are links to Google search engine (Search in site) and to contact information (Contacts). The announcement of the next VIA lecture and VIA online transmission of APC Colloquium occupy the main part of the homepage with the record of the most recent VIA events below. In the announced time of the event (VIA lecture or transmitted APC Colloquium) it is sufficient to click on "to participate" on the announcement and to Enter as Guest (printing your name) in

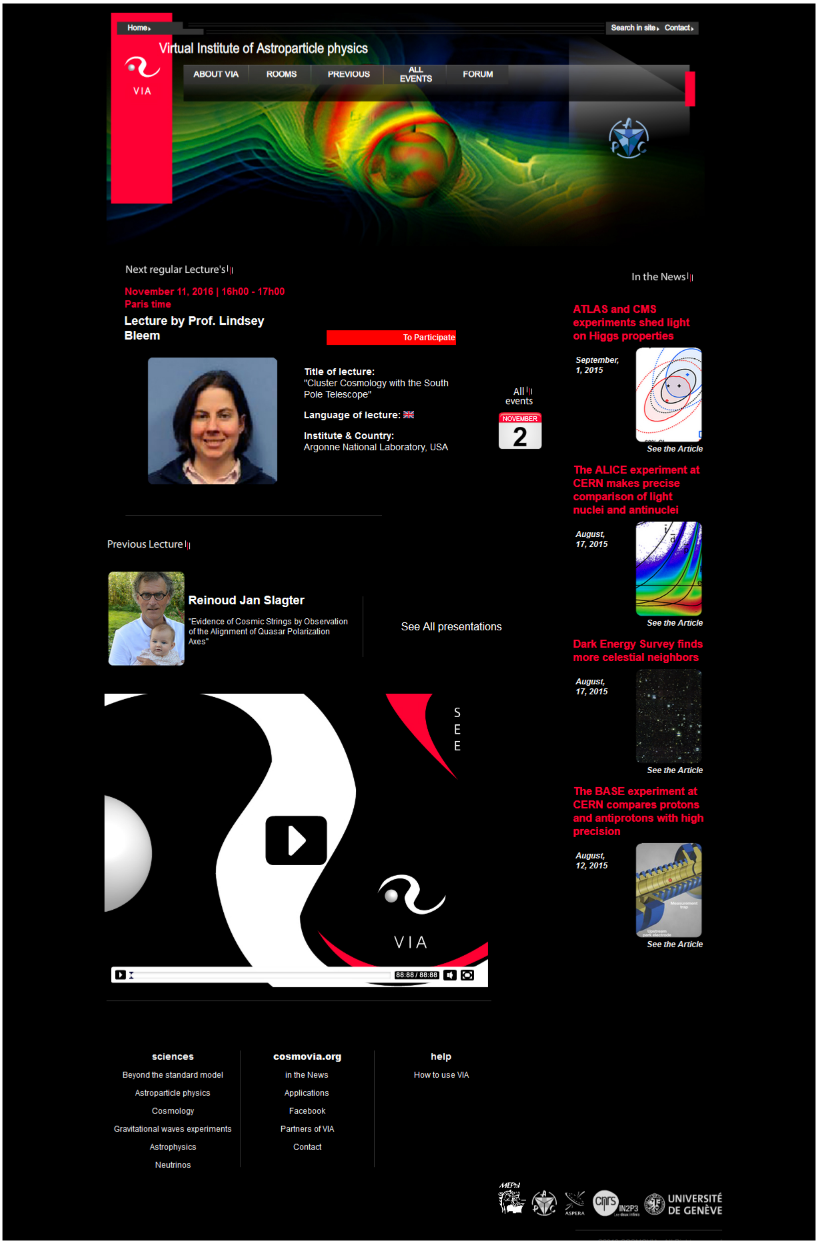

Fig. 15.1. The home page of VIA site

the corresponding Virtual room. The Calender shows the program of future VIA lectures and events. The right column on the VIA homepage lists the announcements of the regularly up-dated hot news of Astroparticle physics and related areas.

In 2010 special COSMOVIA tours were undertaken in Switzerland (Geneva), Belgium (Brussels, Liege) and Italy (Turin, Pisa, Bari, Lecce) in order to test stability of VIA online transmissions from different parts of Europe. Positive results of these tests have proved the stability of VIA system and stimulated this practice at XIII Bled Workshop. The records of the videoconferences at the XIII Bled Workshop are available on VIA site [15].

Since 2011 VIA facility was used for the tasks of the Paris Center of Cosmological Physics (PCCP), chaired by G. Smoot, for the public programme "The two infinities" conveyed by J.L.Robert and for effective support a participation at distance at meetings of the Double Chooz collaboration. In the latter case, the experimentalists, being at shift, took part in the collaboration meeting in such a virtual way.

The simplicity of VIA facility for ordinary users was demonstrated at XIV Bled Workshop in 2011. Videoconferences at this Workshop had no special technical support except for WiFi Internet connection and ordinary laptops with their internal video and audio equipments. This test has proved the ability to use VIA facility at any place with at least decent Internet connection. Of course the quality of records is not as good in this case as with the use of special equipment, but still it is sufficient to support fruitful scientific discussion as can be illustrated by the record of VIA presentation "New physics and its experimental probes" given by John Ellis from his office in CERN (see the records in [16]).

In 2012 VIA facility, regularly used for programs of VIA lectures and transmission of APC Colloquiums, has extended its applications to support M.Khlopov's talk at distance at Astrophysics seminar in Moscow, videoconference in PCCP, participation at distance in APC-Hamburg-Oxford network meeting as well as to provide online transmissions from the lectures at Science Festival 2012 in University Paris7. VIA communication has effectively resolved the problem of referee's attendance at the defence of PhD thesis by Mariana Vargas in APC. The referees made their reports and participated in discussion in the regime of VIA videoconference. In 2012 VIA facility was first used for online transmissions from the Science Festival in the University Paris 7. This tradition was continued in 2013, when the transmissions of meetings at Journes nationales du Dveloppement Logiciel (JDEV2013) at Ecole Politechnique (Paris) were organized [18].

In 2013 VIA lecture by Prof. Martin Pohl was one of the first places at which the first hand information on the first results of AMS02 experiment was presented [17].

In 2014 the 100th anniversary of one of the foundators of Cosmoparticle physics, Ya. B. Zeldovich, was celebrated. With the use of VIA M.Khlopov could contribute the programme of the "Subatomic particles, Nucleons, Atoms, Universe: Processes and Structure International conference in honor of Ya. B. Zeldovich 100th Anniversary" (Minsk, Belarus) by his talk "Cosmoparticle physics: the Universe as a laboratory of elementary particles" [19] and the programme of "Conference

YaB-100, dedicated to 100 Anniversary of Yakov Borisovich Zeldovich" (Moscow, Russia) by his talk "Cosmology and particle physics" [20].

In 2015 VIA facility supported the talk at distance at All Moscow Astrophysical seminar "Cosmoparticle physics of dark matter and structures in the Universe" by Maxim Yu. Khlopov and the work of the Section "Dark matter" of the International Conference on Particle Physics and Astrophysics (Moscow, 5-10 October 2015). Though the conference room was situated in Milan Hotel in Moscow all the presentations at this Section were given at distance (by Rita Bernabei from Rome, Italy; by Juan Jose Gomez-Cadenas, Paterna, University of Valencia, Spain and by Dmitri Semikoz, Martin Bucher and Maxim Khlopov from Paris) and its work was chaired by M.Khlopov from Paris [23]. In the end of 2015 M. Khlopov gave his distant talk "Dark atoms of dark matter" at the Conference "Progress of Russian Astronomy in 2015", held in Sternberg Astronomical Institute of Moscow State University.

In 2016 distant online talks at St. Petersburg Workshop "Dark Ages and White Nights (Spectroscopy of the CMB)" by Khatri Rishi (TIFR, India) "The information hidden in the CMB spectral distortions in Planck data and beyond", E. Kholupenko (Ioffe Institute, Russia) "On recombination dynamics of hydrogen and helium", Jens Chluba (Jodrell Bank Centre for Astrophysics, UK) "Primordial recombination lines of hydrogen and helium", M. Yu. Khlopov (APC and MEPHI, France and Russia)"Nonstandard cosmological scenarios" and P. de Bernardis (La Sapiensa University, Italy) "Balloon techniques for CMB spectrum research" were given with the use of VIA system [24]. At the defence of PhD thesis by F. Gregis VIA facility made possible for his referee in California not only to attend at distance at the presentation of the thesis but also to take part in its successive jury evaluation.

The discussion of questions that were put forward in the interactive VIA events is continued and extended on VIA Forum. Presently activated in English,French and Russian with trivial extension to other languages, the Forum represents a first step on the way to multi-lingual character of VIA complex and its activity. Discussions in English on Forum are arranged along the following directions: beyond the standard model, astroparticle physics, cosmology, gravitational wave experiments, astrophysics, neutrinos. After each VIA lecture its pdf presentation together with link to its record and information on the discussion during it are put in the corresponding post, which offers a platform to continue discussion in replies to this post.

15.2.2 VIA e-learning, OOC and MOOC

One of the interesting forms of VIA activity is the educational work at distance. For the last seven years M.Khlopov's course "Introduction to cosmoparticle physics" is given in the form of VIA videoconferences and the records of these lectures and their ppt presentations are put in the corresponding directory of the Forum [21]. Having attended the VIA course of lectures in order to be admitted to exam students should put on Forum a post with their small thesis. In this thesis students are proposed to chose some BSM model and to analyze its cosmological consequences. The list of possible topics for such thesis is proposed to students, but they are also invited to chose themselves any topic of their own on possible links between cosmology and particle physics. Professor's comments and proposed corrections are put in a Post reply so that students should continuously present on Forum improved versions of work until it is accepted as satisfactory. Then they are admitted to pass their exam. The record of videoconference with their oral exam is also put in the corresponding directory of Forum. Such procedure provides completely transparent way of evaluation of students' knowledge.

Since 2014 the second semester of this course is given in English and converted in an Open Online Course. In 2016 not only students from Moscow, but also from France and Sri Lanka attended this course. It is aimed to develop VIA system as a possible tool for Massive Online Open Courses (MOOC) activity [22]. The students must write their small thesis, present it and being admitted to exam pass it in English. The restricted number of online connections to videoconferences with VIA lectures is compensated by the wide-world access to their records on VIA Forum and in the context of MOOC VIA Forum and videoconferencing system can be used for individual online work with advanced participants. Still the form of individual educational work makes VIA facility most appropriate for PhD courses and it is planned to be involved in the International PhD programme on Fundamental Physics to be in opeeation on the basis of RussianFrench collaborative agreement.

15.2.3 Organisation of VIA events and meetings

First tests of VIA system, described in [5,7–9], involved various systems of videoconferencing. They included skype, VRVS, EVO, WEBEX, marratech and adobe Connect. In the result of these tests the adobe Connect system was chosen and properly acquired. Its advantages are: relatively easy use for participants, a possibility to make presentation in a video contact between presenter and audience, a possibility to make high quality records, to use a whiteboard tools for discussions, the option to open desktop and to work online with texts in any format.

Initially the amount of connections to the virtual room at VIA lectures and discussions usually didn't exceed 20. However, the sensational character of the exciting news on superluminal propagation of neutrinos acquired the number of participants, exceeding this allowed upper limit at the talk "OPERA versus Maxwell and Einstein" given by John Ellis from CERN. The complete record of this talk and is available on VIA website [25]. For the first time the problem of necessity in extension of this limit was put forward and it was resolved by creation of a virtual "infinity room", which can host any reasonable amount of participants. Starting from 2013 this room became the only main virtual VIA room, but for specific events, like Collaboration meetings or transmissions from science festivals, special virtual rooms can be created. This solution strongly reduces the price of the licence for the use of the adobeConnect videoconferencing, retaining a possibility for creation of new rooms with the only limit to one administrating Host for all of them.

The ppt or pdf file of presentation is uploaded in the system in advance and then demonstrated in the central window. Video images of presenter and participants appear in the right window, while in the lower left window the list of all the attendees is given. To protect the quality of sound and record, the participants are required to switch out their microphones during presentation and to use the upper left Chat window for immediate comments and urgent questions. The Chat window can be also used by participants, having no microphone, for questions and comments during Discussion. The interactive form of VIA lectures provides oral discussion, comments and questions during the lecture. Participant should use in this case a "raise hand" option, so that presenter gets signal to switch out his microphone and let the participant to speak. In the end of presentation the central window can be used for a whiteboard utility as well as the whole structure of windows can be changed, e.g. by making full screen the window with the images of participants of discussion.

Regular activity of VIA as a part of APC includes online transmissions of all the APC Colloquiums and of some topical APC Seminars, which may be of interest for a wide audience. Online transmissions are arranged in the manner, most convenient for presenters, prepared to give their talk in the conference room in a normal way, projecting slides from their laptop on the screen. Having uploaded in advance these slides in the VIA system, VIA operator, sitting in the conference room, changes them following presenter, directing simultaneously webcam on the presenter and the audience.

15.3 VIA Sessions at XIX Bled Workshop

VIA sessions of XIX Bled Workshop have developed from the first experience at XI Bled Workshop [7] and their more regular practice at XII, XIII, XIV, XV, XVI, XVII and XVIII Bled Workshops [8–14]. They became a regular part of the Bled Workshop's programme.

In the course of XIX Bled Workshop meeting the list of open questions was stipulated, which was proposed for wide discussion with the use of VIA facility. The list of these questions was put on VIA Forum (see [26]) and all the participants of VIA sessions were invited to address them during VIA discussions. During the XIX Bled Workshop the announcement of VIA sessions was put on VIA home page, giving an open access to the videoconferences at VIA sessions. At the Workshop the test of not only minimal necessary equipment, but either of the use of VIA facility by ordinary non-experienced users was continued following the experience of the previous XVIII Workshop. VIA Sessions were supported by personal laptop with WiFi Internet connection only, as well as in 2016 the members of VIA team were physically absent in Bled and all the videoconferences were directed by M.Khlopov at distance. It principally confirmed a possibility to provide effective interactive online VIA videoconferences even in the absence of any special equipment and qualified personnel at place. Only laptop with microphone and webcam together with WiFi Internet connection was proved to support not only attendance, but also VIA presentations and discussions.

In the framework of the program of XIX Bled Workshop, M. Khlopov, gave his talk "Nonstandard cosmologies from BSM physics" (Fig. 15.2). It provided an additional demonstration of the ability of VIA to support the creative non-formal atmosphere of Bled Workshops (see records in [27]).

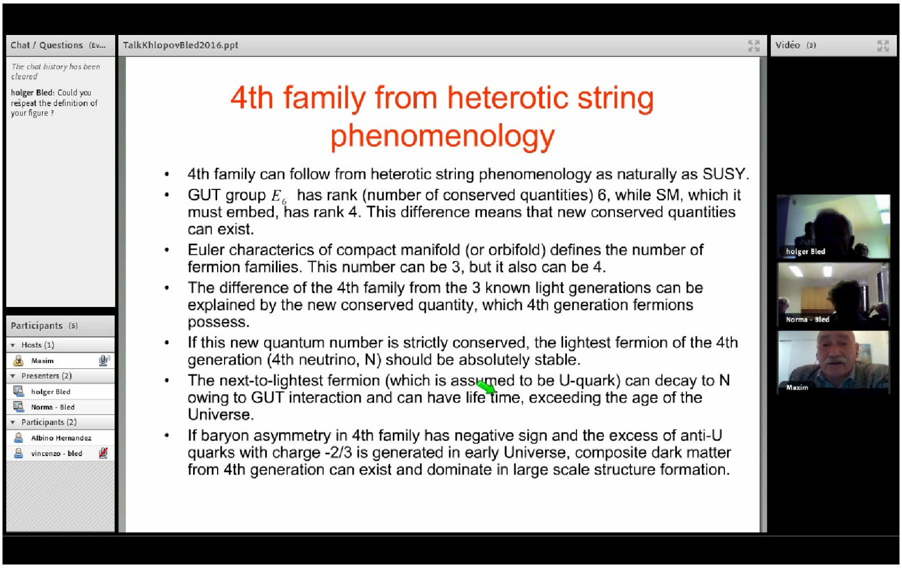

Fig. 15.2. VIA talk "Nonstandard cosmologies from BSM physics" by M.Khlopov given from Paris at XIX Bled Workshop

VIA sessions also included talks of Bled participants of the Workshop "Interpretation of a newly found excess of diphoton decays in the LHC as a bound state of 6 top + 6 anti top quarks." by Holger Bech Nielsen (Fig. 15.3) and "Do no observations so far of the fourth family quarks speak against the spin-charge-family theory?" by Norma Mankoc-Borstnik (Fig. 15.4).

The records of all these lectures and discussions can be found in VIA library [27].

15.4 Conclusions

The Scientific-Educational complex of Virtual Institute of Astroparticle physics provides regular communication between different groups and scientists, working in different scientific fields and parts of the world, the first-hand information on the newest scientific results, as well as support for various educational programs at distance. This activity would easily allow finding mutual interest and organizing task forces for different scientific topics of astroparticle physics and related topics. It can help in the elaboration of strategy of experimental particle, nuclear, astrophysical and cosmological studies as well as in proper analysis of experimental data. It can provide young talented people from all over the world to get the highest level education, come in direct interactive contact with the world known scientists and to find their place in the fundamental research. These educational aspects of VIA activity is now being evolved in a specific tool for International PhD programme for Fundamental physics. VIA applications can go far beyond the particular tasks of astroparticle physics and give rise to an interactive system of mass media communications.

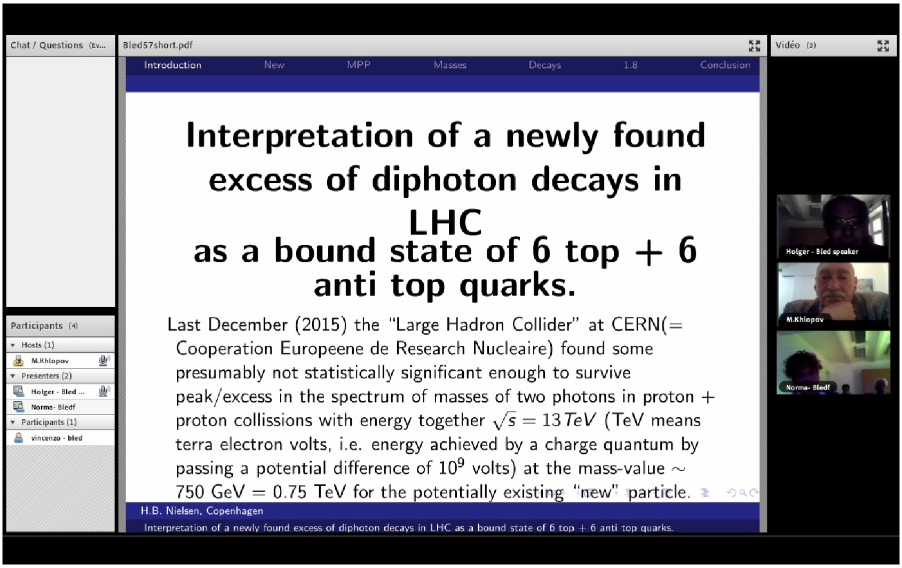

Fig. 15.3. VIA talk by Holger Bech Nielsen at XIX Bled Workshop

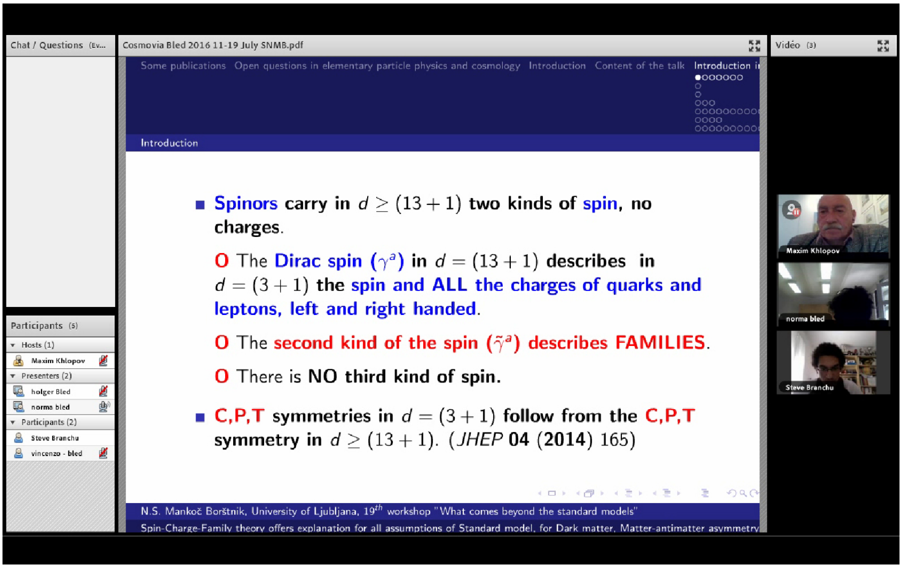

Fig. 15.4. VIA talk "Do no observations so far of the fourth family quarks speak against the spin-charge-family theory?" by N. Mankoc-Borstnik at XIX Bled Workshop

VIA sessions became a natural part of a program of Bled Workshops, maintaining the platform of discussions of physics beyond the Standard Model for distant participants from all the world. This discussion can continue in posts and post replies on VIA Forum. The experience of VIA applications at Bled Workshops plays important role in the development of VIA facility as an effective tool of e-science and e-learning.

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Uredili Norma Susana Mankoč Borštnik, Holger Bech Nielsen in Dragan Lukman

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