BRST QUANTIZATION OF PREGEOMETRY AND TOPOLOGICAL PREGEOMETRY

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Abstract. We investigate the Dirac bracket algebra of the scalar pregeometry including topological pregeometry with BRST formalism, as the first step to its quantization. We derive the precise expression for induced gravity, and discuss how the gravity is induced from the topological theory.

1. Introduction

The quantum theory and the relativity are two of the greatest successes of physics in this century. The problem is, however, that we have no realistic quantum theory of general relativity. Tremendous efforts have been devoted to this subjects, canonical quantization approaches, superstring theories, searches for fundamental clues in lower dimensional systems, etc. Faced with a so difficult problem, we may come to the question "Is the general relativity really fundamental?" "Should it really be quantized as the fundamental object?" In fact, there exist theoretical schemes, called "pregeometry" by Wheeler, where Einstein's general relativity is not fundamental but is induced from more fundamental ingredients [1]. In Sakharov's idea, the Einstein gravity is induced through quantum fluctuations of the matters. The metric is a composite of the fundamental matter fields, and the quantum properties of gravitation are secondary effects due to those of the fundamental matters, just like the quantum properties of the hadrons stem from those of the quarks. Then we have first to establish the quantized theory of pregeometric matters rather than the gravity itself. In this talk, we would like to make a first step towards the quantization by applying the BRST formalism to the scalar pregeometry [2].

2. Scalar pregeometry

The fundamental action for pregeometry should be invariant under diffeomorphisms, and be written without metric, but with the matter fields only. If the matters are the scalar fields, the fundamental action is given by the Nambu-Goto type one [3],

$$\mathcal{L}_\phi = \sqrt{-\det_{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J \eta_{IJ}} F(\phi). \tag{1}$$

where ϕ^I is the fundamental scalar field, $F(\phi)$ is a function of ϕ^I, and η_{IJ}=diag(1,-1,-1,$\cdots$,-1). In (1), μ,ν=0,1,$\cdots$,$D-1$,D, and I,J=0,1,$\cdots N-1$, where D is the number of the spacetime dimensions, and N is the number of the field ϕ. Now we briefly review how the Einstein gravity is induced in this system. The Lagrangian $\mathcal{L}_\phi$ is equivalent to the following Lagrangian with the auxiliary field $g_{\mu\nu}$:

$$\mathcal{L}_{\phi g} = \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J\eta_{IJ} - G(\phi)\right), \tag{2}$$

where $g_{\mu\nu}$ plays the role of the metric, g = det$g_{\mu\nu}$, and $G(\phi) = (D/2 - 1)(F(\phi))^{-2/(D-2)}$. They are equivalent because their equations of motion as well as their commutator algebras of the fields coincide with those of each other. The quantum effects of this $\mathcal{L}_{\phi g}$ give rise to this effective Lagrangian of Einstein gravity,

$$\mathcal{L}_R = \sqrt{-g}\left(\lambda + \frac{1}{16\pi G_{\rm N}}R\right), \tag{3}$$

where R is the scalar curvature, and λ and $G_{\rm N}^{-1}$ are divergent coefficients, which plays the roles of the cosmological and the Newtonian constants, respectively. We introduce a momentum cutoff which we take as realistic one connected with the fundamental scale. Thus the Einstein gravity is induced with a composite metric.

3. BRST Formalism

We should quantize the pregeometric matter Lagrangian $\mathcal{L}_\phi$ or $\mathcal{L}_{\phi g}$ rather than the Lagrangian $\mathcal{L}_R$ for the Einstein gravity.

$$\mathcal{L}_{\phi g} = \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\cdot\partial_\nu\phi - G(\phi)\right). \tag{4}$$

The action $S_{\phi g} = \int \mathcal{L}_{\phi g} {\rm d}^D x$ is invariant under diffeomorphisms

$$\delta\phi^I = \varepsilon^\lambda\partial_\lambda\phi^I, \quad \delta g_{\mu\nu} = g_{\mu\lambda}\partial_\nu\varepsilon^\lambda + g_{\nu\lambda}\partial_\mu\varepsilon^\lambda + \varepsilon^\lambda\partial_\lambda g_{\mu\nu}. \tag{5}$$

where ε^λ is an arbitrary infinitesimal function of x^μ.

Our strategy of quantization is as follows. 1) we fix the gauge by adding the gauge fixing term, 2) add the Faddeev-Popov ghost term to make the action invariant under BRST transformations, 3) exhaust the constraints of the system, 4) work out their Poisson-bracket algebra, 5) define the Dirac bracket, and assign the commutators.

We fix the gauge by the de Donder condition:

$$\mathcal{L}_{\mathrm{GF}} = b_\mu \partial_\nu (\sqrt{-g} g^{\mu\nu}). \tag{6}$$

where b_ν is an auxiliary field. The BRST transformations of ϕ^I and $g_{\mu\nu}$ are given by replacing ε^λ in (5) by the Faddeev-Popov ghost c^λ:

$$\delta_{\mathrm{B}} \phi^I = c^\lambda \partial_\lambda \phi^I, \quad \delta_{\mathrm{B}} g_{\mu\nu} = g_{\mu\lambda} \partial_\nu c^\lambda + g_{\nu\lambda} \partial_\mu c^\lambda + c^\lambda \partial_\lambda g_{\mu\nu}. \tag{7}$$

To make the total action invariant under the BRST transformation, we add to the Lagrangian the Faddeev-Popov term

$$\mathcal{L}_{\mathrm{FP}} = i\sqrt{-g} g^{\mu\nu} \partial_\mu \bar{c}_\lambda \partial_\nu c^\lambda, \tag{8}$$

and define the BRST transformations for c^μ, $\bar{c}_\mu$ and b_μ by

$$\delta_{\mathrm{B}} c^\mu = c^\lambda \partial_\lambda c^\mu, \quad \delta_{\mathrm{B}} \bar{c}_\mu = i b_\mu + c^\lambda \partial_\lambda \bar{c}_\mu, \quad \delta_{\mathrm{B}} b_\mu = c^\lambda \partial_\lambda b_\mu. \tag{9}$$

The BRST transformations δ_{B} are nilpotent.

4. Constraints

Now the system is described by the Lagrangian (with $\tilde{g}^{\mu\nu} \equiv \sqrt{-g} g^{\mu\nu}$)

$$\begin{aligned} \mathcal{L} &\equiv \mathcal{L}_{\phi g} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}} \\ &= -\frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \cdot \partial_\nu \phi - \sqrt{-g} G + \partial_\mu \tilde{g}^{\mu\nu} b_\nu + i \tilde{g}^{\mu\nu} \partial_\mu \bar{c}_\lambda \partial_\nu c^\lambda. \end{aligned} \tag{10}$$

The canonical conjugate variables of ϕ_J, c^ν, $\bar{c}_\nu$, and $\tilde{g}^{\mu\nu}$, are, respectively,

$$\begin{aligned} &\pi^\phi_J = -\tilde{g}^{0\mu} \partial_\mu \phi_J, \quad \pi^c_\nu = i \tilde{g}^{0\mu} \partial_\mu \bar{c}_\nu, \quad \pi^\nu_{\bar{c}} = -i \tilde{g}^{0\mu} \partial_\mu c^\nu, \\ &\pi^\mu_b = 0, \quad \tilde{p}_{mn} = 0, \quad \tilde{p}_{00} = b_0, \quad \tilde{p}_{0m} = b_m/2. \end{aligned} \tag{11}$$

where $\mu, \nu \cdots = 0, 1, \cdots, D-1$, and $m, n, \cdots = 1, \cdots, D-1$. Among them, the first three are solved for the time derivatives, $\partial_0 \phi^I$, $\partial_0 \bar{c}_\mu$, and $\partial_0 c^\mu$, while the last four give the constraints

$$\pi^\mu_b \approx 0, \quad \tilde{p}_{mn} \approx 0, \quad \chi_0 \equiv \tilde{p}_{00} - b_0 \approx 0, \quad \chi_m \equiv \tilde{p}_{0m} - b_m/2 \approx 0. \tag{12}$$

The only non-vanishing Poisson bracket among the constraints is

$$[\chi_\mu(x), \pi^\nu_b(y)]_{\mathrm{P}} = \delta^\nu_\mu \delta(\mathbf{x} - \mathbf{y}). \tag{13}$$

At this stage, π_b^μ, and χ_μ belong to the second class, while $\widetilde{p}_{mn}$ belongs to the first class.

The Hamiltonian density reads

$$\begin{aligned}\mathcal{H} &= -\frac{1}{2\widetilde{g}^{00}}(\pi^\phi + \widetilde{g}^{0m}\partial_m\phi)\cdot(\pi^\phi + \widetilde{g}^{0n}\partial_n\phi) \\ &+\frac{1}{2}\widetilde{g}^{mn}\partial_m\phi\cdot\partial_n\phi + (-\widetilde{g})^{\frac{1}{D-2}}G - \partial_m\widetilde{g}^{m\nu}b_\nu \\ &+i\frac{1}{\widetilde{g}^{00}}(\pi_\lambda^{\bar{c}} - i\widetilde{g}^{0m}\partial_m\bar{c}_\lambda)(\pi_c^\lambda + i\widetilde{g}^{0n}\partial_n c^\lambda) - i\widetilde{g}^{mn}\partial_m\bar{c}_\lambda\partial_n c^\lambda, \qquad (14)\end{aligned}$$

which governs the time evolution of the physical quantities. For consistency, the constraints should remain vanishing during the time evolution. As for the second class constraints, we can make them so by adding appropriate constraints to the Hamiltonian. On the other hand, the condition that the first class constraint $\widetilde{p}_{mn}$ remain vanishing implies the secondary constraint

$$\Phi_{mn} \equiv \frac{D-2}{G}\left(-\frac{1}{2}\partial_m\phi\cdot\partial_n\phi - \partial_{(m}b_{n)} + i\partial_{(m}\bar{c}_\lambda\partial_{n)}c^\lambda\right) - g_{mn} \approx 0. \qquad (15)$$

At this stage, the constraints are χ_μ, π_b^μ, Φ_{mn}, and $\widetilde{p}_{mn}$. Though the Poisson bracket algebra of them are complicated, it is diagonalized by introducing $\widehat{\chi}_\mu$ and $\widehat{\pi}_b^\mu$ defined by

$$\widehat{\chi}_0 = \chi_0 + \widetilde{p}_{mn}\widetilde{g}^{mn}g_{00}, \qquad \widehat{\chi}_k = \chi_k + 2\widetilde{p}_{mn}\widetilde{g}^{mn}g_{0k},$$

$$\widehat{\pi}_b^0 = \pi_b^0, \qquad \widehat{\pi}_b^k = \pi_b^k - (D-2)\partial_l(\widetilde{p}_{mn}\overline{C}^{mnkl}/G), \qquad (16)$$

where

$$\overline{C}^{mnkl} = \sqrt{-g}\left(\overline{g}^{m(k}\overline{g}^{l)n} - \overline{g}^{mn}\overline{g}^{kl}\right), \quad \overline{g}^{mn} = g^{mn} - g^{m0}g^{n0}/g^{00}. \qquad (17)$$

The only non-vanishing Poisson brackets among them are

$$[\widehat{\chi}_\mu(x), \widehat{\pi}_b^\nu(y)]_{\mathrm{P}} \approx \delta_\mu^\nu\delta(\mathbf{x}-\mathbf{y}), \qquad (18)$$

$$[\Phi_{kl}(x), \widetilde{p}_{mn}(y)]_{\mathrm{P}} = C_{klmn}\delta(\mathbf{x}-\mathbf{y}), \qquad (19)$$

where

$$C_{klmn} = \frac{1}{\sqrt{-g}}\left(g_{k(m}g_{n)l} - \frac{1}{D-2}g_{kl}g_{mn}\right). \qquad (20)$$

Now all the constraints belong to the second class, and can be kept vanishing during the time evolution. Then, it is convenient to define the Dirac bracket for arbitrary fields $A(x)$ and $B(x)$ by

$$\begin{aligned}[A(x), B(y)]_{\mathrm{D}} &= [A(x), B(y)]_{\mathrm{P}} \\ &- \int \Bigg([A(x), \widehat{\pi}_b^\mu(z)]_{\mathrm{P}}[\widehat{\chi}_\mu(z), B(y)]_{\mathrm{P}}\end{aligned}$$

$$
\left. \begin{aligned}
&-[A(x),\widehat{\chi}_\mu(z)]_{\rm P}[\widehat{\pi}_b^\mu(z),B(y)]_{\rm P} \\
&+[A(x),\widetilde{p}_{kl}(z)]_{\rm P}\overline{C}^{klmn}(z)[\Phi_{mn}(z),B(y)]_{\rm P} \\
&-[A(x),\Phi_{kl}(z)]_{\rm P}\overline{C}^{klmn}(z)[\widetilde{p}_{mn}(z),B(y)]_{\rm P}
\end{aligned} \right) d\mathbf{z} \tag{21}
$$

which vanishes if $A(x)$ or $B(x)$ is a constraint.

5. Quantization

We assign the equal-time (anti-)commutators in terms of the Dirac bracket as

$$
[A(x),B(y)] \quad \text{or} \quad \{A(x),B(y)\} = i[A(x),B(y)]_{\rm D}. \tag{22}
$$

Then we obtain for ϕ^I, c^μ, $\overline{c}_\mu$, $\widetilde{g}^{0\mu}$, and the conjugates π_I^ϕ, π_μ^c, $\pi_{\overline{c}}^\mu$, $b_\mu(=\widetilde{p}_{00}$ or $=2\widetilde{p}_{0m})$

$$
\begin{aligned}
&[\phi^I(x),\pi_J^\phi(y)] = i\delta_I^J\delta(\mathbf{x}-\mathbf{y}), \quad [\widetilde{g}^{0\mu}(x),b_\nu(y)] = i\delta_\nu^\mu\delta(\mathbf{x}-\mathbf{y}), \\
&\{c^\lambda(x),\pi_\kappa^c(y)\} = i\delta_\kappa^\lambda\delta(\mathbf{x}-\mathbf{y}), \quad \{\overline{c}_\lambda(x),\pi_{\overline{c}}^\kappa(y)\} = i\delta_\lambda^\kappa\delta(\mathbf{x}-\mathbf{y}),
\end{aligned} \tag{23}
$$

and $\widetilde{g}^{mn}$ depend on ϕ^I, c^μ, $\overline{c}_\mu$, b_μ through the constraint $\Phi_{mn}\approx 0$, and obey

$$
\begin{aligned}
&[\widetilde{g}^{kl}(x),\pi_I^\phi(y)] = i\widetilde{C}^{klmn}(x) \\
&\quad\times\left(\partial_m\phi_I(x)\partial_n\delta(\mathbf{x}-\mathbf{y}) + \frac{1}{D-2}g_{mn}\frac{\partial G}{\partial\phi^I}\delta(\mathbf{x}-\mathbf{y})\right), \\
&[\widetilde{g}^{kl}(x),\pi_\mu^c(y)] = \widetilde{C}^{klmn}(x)\partial_m\overline{c}_\mu(x)\partial_n\delta(\mathbf{x}-\mathbf{y}), \\
&[\widetilde{g}^{kl}(x),\pi_{\overline{c}}^\mu(y)] = -\widetilde{C}^{klmn}(x)\partial_m c^\mu(x)\partial_n\delta(\mathbf{x}-\mathbf{y}), \\
&[\widetilde{g}^{kl}(x),\widetilde{p}_{0\mu}(y)] = -\widetilde{g}^{kl}g_{0\mu}\delta(\mathbf{x}-\mathbf{y}), \\
&[\widetilde{g}^{kl}(x),\widetilde{g}^{0m}(y)] = -\widetilde{C}^{klmn}(x)\partial_n\delta(\mathbf{x}-\mathbf{y}),
\end{aligned} \tag{24}
$$

with $\widetilde{C}^{klmn} = (D-2)\overline{C}^{klmn}/G$. The BRST charge is given by $Q_{\rm B} = \int J_{\rm B}^0 d\mathbf{x}$ with

$$
\begin{aligned}
J_{\rm B}^0 = c^0\Bigg(&-\frac{1}{2\widetilde{g}^{00}}(\pi^\phi + \widetilde{g}^{0m}\partial_m\phi)\cdot(\pi^\phi + \widetilde{g}^{0n}\partial_n\phi) \\
&+\frac{i}{\widetilde{g}^{00}}\,(\pi_\lambda^c - i\widetilde{g}^{0m}\partial_m\overline{c}_\lambda)(\pi_{\overline{c}}^\lambda + i\widetilde{g}^{0n}\partial_n c^\lambda) \\
&+\frac{1}{2}\widetilde{g}^{mn}\partial_m\phi\cdot\partial_n\phi - \sqrt{-g}G - i\widetilde{g}^{mn}\partial_m\overline{c}_\rho\partial_n c^\rho\Bigg) \\
+c^n&\left(\pi_\phi\cdot\partial_n\phi + \pi_\mu^c\partial_n c^\mu + \pi_{\overline{c}}^\mu\partial_n\overline{c}_\mu\right) \\
+b_\mu&\left(-i\pi_{\overline{c}}^\mu + \partial_n(c^n\widetilde{g}^{0\mu} - c^0\widetilde{g}^{n\mu})\right).
\end{aligned} \tag{25}
$$

6. Quantum transition amplitude

The transition amplitude from the state Ψ_i to Ψ_f is given by

$$\begin{aligned} T_{fi} &= \int \mathcal{D}\phi^I \mathcal{D}\pi_I^\phi \mathcal{D}\tilde{g}^{\mu\nu} \mathcal{D}\tilde{p}_{\mu\nu} \mathcal{D}b_\mu \mathcal{D}\pi_b^\mu \mathcal{D}c^\mu \mathcal{D}\pi_\mu^c \mathcal{D}\bar{c}_\mu \mathcal{D}\pi_{\bar{c}}^\mu \\ &\quad \Psi_f^* \Psi_i \prod_x \left[\delta(\hat{\pi}_b^\mu)\delta(\hat{\chi}_\mu)\delta(\tilde{p}_{mn})\delta(\Phi_{mn}) \det C_{ijkl}\right] \\ &\quad \exp i \int \mathrm{d}^D x \left(\pi^\phi \cdot \dot{\phi} + \tilde{p}_{\mu\nu}\dot{\tilde{g}}^{\mu\nu} + \pi_b^\mu \dot{b}_\mu + \pi_\mu^c \dot{c}^\mu + \pi_{\bar{c}}^\mu \dot{\bar{c}}_\mu - \mathcal{H}\right), \end{aligned} \tag{26}$$

Using the explicit expression of $\det C_{ijkl}$, we get

$$\det C_{ijkl} = (\det_{mn} g_{mn})^D (\sqrt{-g})^{-D(D-1)/2} \times \text{constant}. \tag{27}$$

The integrations by π_b^μ, $\tilde{p}_{0\mu}$, and $\tilde{p}_{mn}$ are trivial. We perform the integrations by π_I^ϕ, π_μ^c, and $\pi_{\bar{c}}^\mu$

$$\begin{aligned} &\int \mathcal{D}\pi_I^\phi \mathcal{D}\pi_\mu^c \mathcal{D}\pi_{\bar{c}}^\mu \exp i \int \mathrm{d}^D x \left(\pi^\phi \cdot \dot{\phi} + \pi_\mu^c \dot{c}^\mu + \pi_{\bar{c}}^\mu \dot{\bar{c}}_\mu + b_\mu \dot{\tilde{g}}^{0\mu} - \mathcal{H}\right) \\ &\qquad = \left(\tilde{g}^{00}\right)^{-D+N/2} \exp i \int \mathrm{d}^D x \mathcal{L}, \end{aligned} \tag{28}$$

where $\mathcal{L} = \mathcal{L}_{\phi g} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}$. We rewrite the $\delta(\Phi_{mn})$ as

$$\prod_x \delta(\Phi_{mn}) = \int \mathcal{D}(\sqrt{-g}u^{mn}) \exp i \int \mathrm{d}^D x \sqrt{-g} u^{mn} \Phi_{mn}, \tag{29}$$

where u^{mn} is the Lagrange multiplier. Then we obtain

$$\begin{aligned} T_{fi} &= \int \mathcal{D}\phi^I \mathcal{D}\tilde{g}^{\mu\nu} \mathcal{D}b_\mu \mathcal{D}c^\mu \mathcal{D}\bar{c}_\mu \mathcal{D}u^{mn} \Psi_f^* \Psi_i \\ &\quad \prod_x \left[(\tilde{g}^{00})^{N/2} (\sqrt{-g})^D\right] \exp i \int \mathrm{d}^D x \left(\mathcal{L} + \sqrt{-g} u^{mn} \Phi_{mn}\right). \end{aligned} \tag{30}$$

7. Induction of gravity

To get the effective Lagrangian $\mathcal{L}_{\text{eff}}$ for induced gravity we perform the integration by ϕ in T_{fi}

$$\int \mathcal{D}\phi^I \exp i \int \mathrm{d}^D x \left(\mathcal{L} + \sqrt{-g} u^{mn} \Phi_{mn}\right) = \exp i \int \mathrm{d}^D x \mathcal{L}_{eff}. \tag{31}$$

We use the stationary phase approximation, i. e. in the integration by ϕ, we neglect $\mathrm{O}(\phi^3)$ terms in $\mathcal{L} + \sqrt{-g} u^{mn} \Phi_{mn}$. After a lengthy calculation, we finally obtain the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \sqrt{-g}\Bigg[\frac{1}{16\pi G_{\mathrm{N}}} R + \frac{1}{\sqrt{-g}} b_\mu \partial_\nu (\sqrt{-g} g^{\mu\nu}) + i g^{\mu\nu} \partial_\mu \bar{c}_\lambda \partial_\nu c^\lambda$$

$$
\begin{aligned}
&+\frac{1}{2}\Big\{\frac{1}{2}(\mathcal{D}_\lambda U^{mn})^2-(\mathcal{D}_m U^{mn})^2+\mathcal{D}_m U^{mn}\mathcal{D}_n U+\frac{1}{4}(\mathcal{D}_m U)^2\Big\}\\
&+\frac{1}{2}M^2\Big\{(U^{mn})^2+\frac{1}{2}U^2\Big\}+k_1 U+k_2\left(U^{mn}R_{mn}-\frac{1}{2}UR\right)\\
&+k_3 U^{mn}\left(-\partial_m b_n+i\partial_m\bar{c}_\lambda\partial_n c^\lambda\right)\Bigg]
\end{aligned}
\tag{32}
$$

with the divergent coefficients

$$
\begin{aligned}
&\frac{1}{16\pi G_{\rm N}}=\rho^2\Lambda^{D-2},\quad M=\sqrt{\frac{3(D-2)}{D}}\Lambda,\\
&k_1=3\rho\Lambda^{D/2+1}\ \ k_2=\rho\Lambda^{D/2-1},\ \ k_3=\frac{1}{\rho}\left(1+\frac{D}{2}\right)\Lambda^{-D/2+1},\\
&\left(\rho=\sqrt{N/6(D-2)(D/2)!(4\pi)^{D/2}}\right),
\end{aligned}
\tag{33}
$$

where Λ is the ultraviolet cut off (Pauli-Villars mass), and the cosmological constant λ is fine-tuned to be 0. Since $G_{\rm N}>0$, the induced gravity is attractive. Roles of the field U is not yet fully investigated.

8. Conclusions and discussions

If the number N of the fundamental scalar fields coincides with the number of dimensions D, the scalar pregeometry becomes topological [4]. In this case, we can show that no local physical mode exists. Only the topological invariant quantities are observable. It is interesting to see that the gravity is still induced. This is because the fundamental scale breaks the topological symmetry.

In summary, if the Einstein gravity is an induced effect (pregeometry), we have first to quantize the pregeometric matters rather than the gravity itself. To make a first step towards the quantization we applied the BRST formalism to the scalar pregeometry. We derived the Dirac bracket algebra, but the problem of the operator ordering is not solved. We derived the precise expression for induced gravity. It depends on the ultraviolet cutoff Λ, which we take as the physical fundamental scale.

Finally, we would like to emphasize that, the quantum gravity has another possibility that the gravity is not fundamental, and we should first quantize the pregeometric matter.

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