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# The variable Planck's constant due to imaginary gravitational temperature

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**Abstract.** In a talk in PIRT-2015 we described how the Tolman-Thorne temperature of a general relativistic spinning star can be used to define a variable Planck-type constant where the temperature becomes imaginary and how this variable has a negligible value except in some region near a stationary gravitating particle of given mass and spin. In the present talk we recover the Lamb shift of the proton-electron system at thousands' place in MHz from the spread of this region. While this may suggest that the variable Planck's constant could be used with linear Fourier analysis to approximate gravitational effects in coupling gravity with non-gravitational interactions quite effectively, we also consider Einstein-Yang-Mills coupled equations for possible solutions that could be stable because the variable coupling implied by a variable Planck's constant could stop the solutions from blowing up. While no new significant result for these coupled equations has been found yet, we provide a progress report of an ongoing program in which eventually we would like to explain at least qualitatively every valid result in QM and QFT with the help of the variable Planck's constant suggested by GR (by which we mean here Einstein equations) and without using renormalization. We elaborate the first problem in more details. The remarkable coincidence in the earlier talk gave us a variable Planck's constant  $h_g$  of gravitational origin roughly at 0.2 Bohr radius for a gravitating particle having the mass and spin of a proton. In this talk we visit Welton's argument for the Lamb shift to see how far the Lamb shift can be explained with this  $h_g$ . The fluctuations of the

quantized EM field used in Welton's argument would take place only in the region where  $h_g$  is meaningful. If we consider the Fourier transform of a real-valued function of a single variable having nonzero value 1 only on an interval of the positive axis, we see that the outer endpoint is roughly inversely proportional to the lowest significant wave number and inner endpoint is roughly inversely proportional to the highest significant wave number. The omitted frequencies produce the Gibbs' phenomenon. Thus the ratio of the cutoff frequencies used in

Welton's argument  $\frac{k_{\max}}{k_{\min}}$  becomes in our discussion  $\frac{r_{\max}}{r_{\min}}$ . If we take  $r_{\min}$  to be the distance

from the center of mass of the proton-electron system where the gravitational temperature becomes imaginary and  $r_{\max}$  to be the distance where  $h_g \approx h$  ( $h_g$  decreasing as inverse square of  $r$  thereafter), we get values of our Lamb shift in the range 1019 to 1249 MHz. We do not expect to get the experimental value of 1057 MHz following our crude qualitative arguments. The discussion on the variation of values based on Welton's argument namely 667 to 1394 MHz (see for example the textbook of  $F. Schwabl$ ) makes one appreciative of our values and hence of our suspicion that gravity is behind the significant part of the Lamb shift. The next task is to look closely wherever Planck's constant is used in the equations of quantum



physics. Just naively replacing this Planck's constant by  $\hbar_g$  will not give the full or a consistent story. In particular one would like to know how the Dirac equation and Einstein-Yang-Mills coupled equations get modified with  $\hbar_g$ . A variable Planck's constant will make the coupling constant of the Einstein-Yang-Mills coupled system a varying function and the scale invariance of the system will be lost. We have done some initial calculation for the case of static Einstein-Yang-Mills equations. Spinning has not been added yet to keep the field equations manageable and so the justification behind using  $\hbar_g$  is somewhat missing. However the indications are that we cannot have SU(2) solutions, and we cannot have spherically symmetric solutions unless possibly when the gauge group is more complicated. Otherwise the coupling is forced to become constant and we get the known unstable solutions. A sufficiently complicated gauge group gives more equations. Looking at the phase space of the known solutions we then expect that the varying coupling field may stop the blowing up of some otherwise singular solutions. The global solution is expected to be a stable solution because its constant coupling analog is not the unstable separator of the classes of YM-potentials going to  $\pm\infty$ . Some related issues and how our results can be improved will be discussed.

## 1. Introduction

In [1] we have a function  $\hbar_g$  that plays the role of the Planck's constant in a Schrödinger-type equation guessed from the imaginary values of a gravitational temperature obtained by extrapolating the Tolman-Thorne temperature in some way in the exterior of a massive stationary spinning star or a star-like particle. This  $\hbar_g$  has values close to the Planck's constant only in a narrow region outside the particle. In other region  $\hbar_g$  either does not exist because the gravitational temperature (g-temperature) is real, or  $\hbar_g$  is small decaying rapidly far away from the axis. We suppose that the quantum phenomena will be observed only in the volume of importance where  $\hbar_g$  is defined and significant. We recall from [1]

$$\hbar_g \approx \frac{1,6R^{2+1,5\alpha}k_B}{M^{a/2}r^{1+a}}. \quad (1)$$

We assume that the Newtonian approximations used in deriving this equation crudely apply to a proton and take  $R = \text{proton radius } 0,88 \cdot 10^{-13} \text{ cm}$  and  $M = \text{proton mass } = 1,24 \cdot 10^{-52} \text{ cm}$ . We had another equation estimating the distance at which g-temperature becomes imaginary. This distance is of the order of  $\Omega^{-1}$  (in geometrized unit) where  $\Omega$  is the angular velocity of the star. The angular velocity (angular frequency) of the proton is related to its spin  $1/2$  by  $\Omega^{-1} = 2\hbar M^{-1}R^{-2}$  which equation we use to estimate this distance.

In this formula  $1s = 3 \cdot 10^{10} \text{ cm}$ . Thus for a proton the imaginary temperature comes out at about a distance of  $1,8 \cdot 10^{-13} \text{ cm}$ . With  $\alpha = 1$ , Eq.(1) gives  $\hbar_g \approx 2,87 \cdot 10^{-85} r^{-2} \text{ cm}^4$  at  $r \approx 0,33 \times 10^{-9} \text{ cm}$ .

We shall revisit Welton's argument in view of the variations in  $\hbar_g$ . In [1] we suspected that  $\hbar_g$  has something to do with the Planck's constant  $\hbar$  because for a nucleon we got  $\hbar_g \approx \hbar$  at  $r \approx 10^{-9} \text{ cm} \approx 0,2 \text{ Bohr radius}$ , while for a star it is at an enormously great distance. In this paper we show that the size of the region of importance gives the major part of the Lamb shift frequency for the hydrogen atom matching at thousands' place in MHz. Thus we get another strong suggestion that the Planck's constant is of gravitational origin.

## 2. Major part of the Lamb shift

We suppose that the fluctuations of the quantized EM field would take place only in the region where  $h_g$  is defined and appreciably nonzero. We are neglecting  $h_g$  for electron itself. An electron will be out of the region where  $h_g$  due to the electron is important. This is because of the inner cutoff and hence its effect could only be indirect. So we shall neglect it. For a proton the region of interest is from  $r \sim 1,8 \cdot 10^{-13}$  cm to  $r \sim 0,33 \cdot 10^{-9}$  cm as described in the previous two paragraphs.

### 2.1. Computation

We approximate  $h_g$  with  $\hat{h}\chi$  where  $\chi$  is the characteristic function of the interval  $(r_{\min}, r_{\max})$  and  $\hat{h}$  is some suitable average height. We write  $\hat{h}\chi$  as a difference of two rectangular functions on intervals  $(0, r_{\min})$  and  $(0, r_{\max})$ . We approximate the doubles of these two functions formed by reflecting the graphs on the vertical axis (the doubles being even functions) by their Fourier integrals. Let  $k_1, k_2$  denote the maximum wave numbers we would like to keep for each of the approximations. Such cutoffs produce some mismatch due to Gibb's phenomena but little mismatch will not matter much because we are dealing with approximations of  $h_g$  anyway. Now  $k_1$  is inversely proportional to  $r_{\min}$  ( $k_2$  is inversely proportional to  $r_{\max}$ ). This is because if we approximate the even function  $f(r) = \hat{h}\chi_{(-r_0, r_0)}(r)$  for  $r_0 > 0$ , where  $\chi_{(-r_0, r_0)}(r)$  is the characteristic function of the interval  $(-r_0, r_0)$ , by its Fourier cosine integral with frequency cutoff  $\omega_0$  then  $\omega_0$  is roughly inversely proportional to  $r_0$ :

$$f(r) = \frac{2\hat{h}}{\pi} \int_0^\infty \frac{\cos(\omega r) \sin(\omega r_0)}{\omega} d\omega \approx \frac{\hat{h}}{\pi} \int_0^{\omega_0} \frac{\sin(\omega(r+r_0))}{\omega} d\omega - \frac{\hat{h}}{\pi} \int_0^{\omega_0} \frac{\sin(\omega(r-r_0))}{\omega} d\omega.$$

The graph of  $\omega^{-1} \sin(\omega(r+r_0))$  as a function of  $\omega$  has the maximum height of  $(r+r_0)$  at  $\omega=0$  falling to zero at  $\omega = \pm \frac{\pi}{r+r_0}$ .  $\omega^{-1} \sin(\omega(r+r_0))$  has appreciable value only for  $|\omega| < \pm \frac{\pi}{r+r_0}$ . We see

that the cutoff frequency  $\omega_0$  is roughly inversely proportional to  $r_0$ . Hence we denote  $k_1$  by  $k_{\max}$  and  $k_2$  by  $k_{\min}$ . Then we have  $\frac{k_{\max}}{k_{\min}} \approx \frac{r_{\max}}{r_{\min}}$ .

Thus if we compute the energy due to fluctuations of the relative position of an electron caused by the fluctuations of the quantized EM field following Welton [2] (see also Bethe [3]) without using his argument for the upper and lower cutoff of frequencies we find the frequency shift

$$E \approx 135,64 \ln \frac{k_{\max}}{k_{\min}} \approx 135,64 \ln \frac{0,33 \cdot 10^{-9}}{1,8 \cdot 10^{-13}} = 1019 \text{ MHz} \quad (2)$$

or possibly

$$E \approx 135,64 \ln \frac{k_{\max}}{k_{\min}} \approx 135,64 \ln \frac{10^{-9}}{10^{-13}} = 1249 \text{ MHz}. \quad (3)$$

### 2.2. Discussion

The experimentally observed shift  $2S_{1/2} - 2P_{1/2}$  in the hydrogen atom is approximately 1057,86 MHz. Actually the estimate by Welton's method vary from about 667 MHz (see Schwabl [4] page 193) to 1394 (or 1413) MHz depending on the choice of the lower limit  $k_{\min}$ . Welton's value was roughly the average though Welton did not explicitly stated so in [2]. Schwabl notes it out of curiosity. Theoretical argument based on standard theory gives roughly about 1052. Our value  $E$  is supposed to be for the state  $2S_{1/2}$ . Contribution of  $2P_{1/2}$  is not much. So we shall not worry about it. Theoretical values in

text books may also differ because some other important corrections are added affecting up to tens' place. The factor of 135.64 in Eq.(2) was derived using several simplifying assumptions (see, for example, page 482 in [5]). One of these assumptions may need a closer look in view of our method. This factor has the value of the wave function at the origin at the center of mass of the proton-electron system and it comes out because of the convolution of the wave function with the delta function singularity of the Coulomb potential. Center of the proton charge will be outside the domain of definition of  $h_g$ . Thus this factor could be smaller and in that case the second estimate given in Eq.(3) which is more realistic in derivation but which is an overestimate will also be smaller.

### 2.3 Work remains to be done

Now we discuss the work remains to be done. Welton's method is based on the non-relativistic Schrödinger equation. The fluctuation of the EM potential is calculated only up to second order in the fluctuation of the position of the electron. We hope somebody recalculates what we did using the Dirac equation. At present we did not sort out what  $e^{-imc^2t/\hbar_g}$  means. Our values of  $h_g$  in the spacetime

show that an expression such as  $\frac{e^2}{\hbar c}$  (here  $e$  is the charge of the electron), should be avoided at both

low and high frequency ends. In certain quantum calculation an expansion in powers of  $\frac{e^2}{\hbar c}$  is used.

Bloch and Nordsieck [6] removed the divergence in the low frequency limit by considering alternative expansion procedure in which  $\hbar$  does not occur in the denominator. It is natural to demand that transition to classical limit  $\hbar \rightarrow 0$  is possible in the low frequency limit. High frequency limit is considered far from the classical regime and the need to eliminate high frequency divergence did not get similar attention.

Two other potential sources of corrections are as follows. One is the process involving the emission of virtual quanta by the field and their absorption by the electron. The other is the Uehling's vacuum polarization. For the Lamb shift  $2S_{1/2} - 2P_{1/2}$  in the hydrogen atom these corrections may affect only up to the tens' place in MHz while our heuristic method is crude at hundreds' place. So at present it is meaningless to include them in our calculation in terms of the renormalized charge or mass of the electron (see pages 157-159 in [7]). Assuming that these activities are important only in the region of the physical spacetime where the variable Planck's constant is important (and neglecting the completely disconnected Feynman graphs), one suspects that only  $h_g$  produced by the nucleus of the hydrogen atom is important. For this contribution to  $h_g$  both low and high frequency cutoffs as mentioned above would possibly remove divergences. As for the contribution to  $h_g$  from a quantum we remember the discussion of Feynman [8]. Feynman suggests that the electron wave function should be kept away from the light cone of the quanta to avoid a divergence. However for our purpose the light cone of the quanta will be inside the light cone of the radius where the  $g$ -temperature due to the quanta becomes imaginary. Hence one expects that the net effect of the contribution to  $h_g$  from the field quanta may not have much significance. How to calculate this contribution? Some naive calculation may be possible as in the case of the WKB analysis of electromagnetic waves giving the photon number conservation as well as considering a photon as a star-like particle of limiting zero rest mass in a co-moving frame of limiting velocity  $c$ , the star-like particle being a small region about the peak of the electromagnetic field where the energy density is momentarily maximum. Such calculation may also explain the meaning of the analog of the Einstein relation namely  $mc^2 = \omega \hbar_g$  and could be useful in clarifying the meaning of  $e^{-imc^2t/\hbar_g}$  as  $h_g \rightarrow 0$  if the expression occurs in a solution of the modified Dirac equation.

### 3. EYM equations

We now discuss the possible effect of the varying  $h_g$  on Einstein-Yang-Mills coupled equations. A variable Planck's constant possibly indicates that the coupling constant of the EYM equations may not be actually constant. While it is not totally clear how a varying  $h_g$  would affect the coupling we now report on an ongoing investigation and some ideas involving variable coupling for EYM equations.

#### 3.1. Variable coupling

For the electromagnetism (in short EM) the coupling constant is  $l_{em} = \frac{1}{4}\epsilon_0 c$  so that we get the Maxwell equations in SI units. We recall the Einstein equations  $R_{ab} - \frac{1}{2}Rg_{ab} = \left(\frac{8\pi G}{c^4}\right)T_{ab}$  coupled to EM field having the energy-momentum tensor

$$T_{ab} = \frac{1}{4}\epsilon_0 c^2 \left( F_{ac}F_{bd}g^{cd} - \frac{1}{4}g_{ab}F_{ij}F_{kl}g^{ik}g^{jl} \right).$$

There is no Planck's constant in the Lagrangian density of the Einstein-Maxwell coupled equations.

The Lagrangian density is  $L = -\kappa R - l_{em}\|F\|^2$  where  $\kappa = \frac{c^3}{16\pi G}$ . Planck's constant naturally enters in minimal coupling to make the Schrödinger equation or the Klein-Gordon equation for an electron in an EM field invariant under spacetime point-dependent U(1) gauge transformation. For example when introducing EM interaction to a free particle one takes

$$\partial_t \rightarrow \partial_t - i\left(\frac{e}{\hbar}\right)A^0, \quad \vec{\nabla} \rightarrow \vec{\nabla} + \frac{ie}{\hbar c}\vec{A}.$$

Since the coupling constant also enters in front of the gauge covariant derivative we have, with  $\beta = \frac{ie}{\hbar c}$ , curvature  $F = dA + \beta A \wedge A$ . For a general function  $\beta$ , usually for Yang-Mills (YM) potential one defines  $\tilde{A} = \beta A$  and  $\tilde{F} = \beta F$ . The Lagrangian density for  $\tilde{A}$  then modifies to

$$L = -\kappa R - l\beta^{-2}\|\tilde{F}\|^2$$

for some constant  $l$  which we shall henceforth absorb in  $\beta^{-2}$ . When  $\beta$  is constant,  $\tilde{F}$  satisfies  $\tilde{F} = d\tilde{A} + \tilde{A} \wedge \tilde{A}$ . It is well-known that the EYM equations is scale invariant when  $\beta$  is constant. However in case  $\beta$  is not constant  $\tilde{F}$  satisfies

$$\tilde{F} = d\tilde{A} + \tilde{A} \wedge \tilde{A} + A \wedge d\beta \quad (4)$$

and the scale invariance is gone. In view of our variable Planck's constant  $h_g$  and in case the coupling constant depends on the Planck constant, it is likely that  $\beta$  will not be constant.  $h_g$  suggests one possibility that  $\beta^{-1}$  is bounded going to zero at infinity and is not defined in an interior region. This region is actually cylindrical (see [1]) but for simplicity in this paper we assume it to be spherical. Without spinning we may not get stable solutions but still it could give some insight for modeling composite particles having spin zero. We therefore consider  $\beta$  to be an unknown function determined from the field equations.

#### 3.2. Coupling derived from an equation

EYM coupled equations with variable coupling has been considered by many authors. References can be found in Bekenstein [9] and in the volume edited by Solà [10]. Although the title of the latter work says time variations, the volume also contains some discussion of space variations. Bekenstein [9] discussed the problem of finding a "proper" dynamical equation for the coupling from a separate

action for it. We did not worry on this matter. Our variable coupling field is the result of the imaginary gravitational temperature of an EYM system. We do not even know that the equations we seek are derivable from an action integral in the usual way. We are interested only in field equations and we keep the option for matching constant coupling equations at two ends. We also do not know whether we need to add Higgs field because we already have a gravitational mass, and a non-constant coupling will behave like a scalar field. In general there would be more than one coupling for various components of the gauge group. If a Higgs-type field can be derived from the coupling by adding a separate action for the varying coupling or by some other modifications that is fine. There has been effort to derive Higgs potential from extended Brans–Dicke theory (see Solà, Karimkhani and Khodam-Mohammadi [11]). One also recalls the suggestive remark by Bartnik, Fisher and Oliynyk [12] in the introduction of their paper. Later we may add some structure mimicking the cosmological constant of appropriate sign. Variable coupling has some similarities with certain non-minimal coupling EYM solutions (see for example Müller-Hoissen [13], Balakin and Zayats [14–15]).

### 3.3. Forms of the metric and gauge potential considered

Now we specify the forms of the spacetime metric and the YM potential we use. Static spherically symmetric field will not produce imaginary g-temperature. However for the sake of simplicity we shall not consider stationary axisymmetric solution in this paper. Our spacetime is however a little bit more complicated than a spherically symmetric spacetime. This is an artificial situation but equations are much simpler than the stationary axisymmetric case. Our aim is to seek nontrivial solutions displaying qualitative features that may provide hope before we start a complicated program. We take a static spacetime metric of the form

$$g = -V^2 dt^2 + \gamma = -V^2 dt^2 + N^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

where  $V = V(r, \theta)$ ,  $N = N(r)$ . Here we consider only  $SU(2)$  YM fields. We assume that we can choose a basis of the Lie algebra so that the (modified) YM potential  $\tilde{A}$  is of the form

$$\tilde{A} = a\tau_3 dt + b\tau_3 dr + (C\tau_1 + D\tau_2)d\theta + (\cot\theta\tau_3 + C\tau_2 - D\tau_1)\sin\theta d\phi$$

where  $\tau_i$  are the Pauli spin matrices corresponding to the Lie algebra basis. Initially we suppose  $C, D$  are functions of  $r$ ,  $a, b$  are functions of  $r$  and  $\theta$ . Later we shall take  $D = 0 = b$ . The coupling  $\beta$  is assumed to be a function of  $r$  and  $\theta$ . We use bold indices as Lie algebra indices. Latin indices are spacetime indices. The components of the YM potential are

$$\begin{pmatrix} \tilde{A}_0^1 & \tilde{A}_r^1 & \tilde{A}_\theta^1 & \tilde{A}_\phi^1 \\ \tilde{A}_0^2 & \tilde{A}_r^2 & \tilde{A}_\theta^2 & \tilde{A}_\phi^2 \\ \tilde{A}_0^3 & \tilde{A}_r^3 & \tilde{A}_\theta^3 & \tilde{A}_\phi^3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & C & -D\sin\theta \\ 0 & 0 & D & C\sin\theta \\ a & b & 0 & \cos\theta \end{pmatrix}.$$

Components of the YM curvature are (using Eq.(4))

$$\tilde{F}_{ab}^c = \partial_a \tilde{A}_b^c - \partial_b \tilde{A}_a^c + f_{mn}^c \tilde{A}_a^m \tilde{A}_b^n + A_a^c \partial_b \beta - A_b^c \partial_a \beta.$$

Here  $f_{mn}^c$  is the totally antisymmetric tensor  $\varepsilon^{cmm}$ . Lie algebra indices are written in bold. A review of the EYM system can be found in the report by Volkov and Gal'tsov [16].

We shall now count the nontrivial EYM equations for the case  $D = 0 = b$ . In the following list the Greek index stands for  $r, \theta$  or  $\phi$ .

For  $\mathbf{c}=3$ ,  $\alpha = r$ , the YM equation is trivial if we choose  $D = 0 = b$ .

For  $\mathbf{c}=3$ ,  $\alpha = \theta$ , the YM equation is trivially satisfied.

For  $\mathbf{c}=3$ ,  $\alpha = \phi$ , the YM equation gives the 2-Laplacian of  $\beta(r, \theta)$ .

For  $\mathbf{c}=1$ ,  $\alpha = r$ , the YM equation gives an equation containing the mixed partial of  $\beta$ :

$$\partial_\theta \partial_r \beta^{-1} + \partial_r \beta^{-1} \cot\theta + (\ln C)' \partial_\theta \beta^{-1} + (\ln C) \beta^{-1} \partial_\theta \ln V + \partial_r \beta^{-1} \partial_\theta \ln V = 0.$$

For  $\mathbf{c}=1$ ,  $\alpha = \theta$ , the YM equation gives an equation for  $C(r)$ .

For  $\mathbf{c}=1$ ,  $\alpha = \varphi$ , the YM equation is trivially satisfied.

For  $\mathbf{c}=2$ ,  $\alpha = r$ , the YM equation is trivially satisfied.

For  $\mathbf{c}=2$ ,  $\alpha = \theta$ , the YM equation is trivially satisfied.

For  $\mathbf{c}=2$ ,  $\alpha = \varphi$ , the YM equation gives an equation containing  $C$  and  $\beta$ .

With  $a \neq 0$  there are two more equations:

For  $\mathbf{c}=2$ , spacetime index 0, the YM equation gives  $a^2 = V\beta^2\zeta(r)$  for some  $\zeta$ .

For  $\mathbf{c}=3$ , spacetime index 0, the YM equation gives an equation for  $\zeta(r)$ .

For  $\mathbf{c}=1$ , spacetime index 0, the YM equation is trivial.

With  $a = 0$  there are four nontrivial YM equations when  $\beta$  is not constant. Of these the following three equations threaten to render  $\beta$  constant. This is certainly the case if  $V$  is independent of  $\theta$  (spherically symmetric case) because of the  $\theta\theta$  and  $\varphi\varphi$  components of the Ricci tensor in the Einstein equations.  $\theta\theta$  and  $\varphi\varphi$  components of the Ricci tensor give two equations that are slightly different when  $\beta$  is not constant. The three equations are as follows: The equation for  $\mathbf{c}=3$ ,  $\alpha = \varphi$ . The equation for  $\mathbf{c}=2$ ,  $\alpha = \varphi$ . The equation for  $\mathbf{c}=1$ ,  $\alpha = r$ . The equation for  $\mathbf{c}=1$ ,  $\alpha = \theta$  gives the only surviving equation in the constant  $\beta$  case.

Einstein equations give six nontrivial equations. In the following 3-Ricci refers to the Ricci curvature of the 3-metric  $\gamma$ .  $\Delta_\gamma$  is the Laplacian of  $\gamma$ . The equations come from 0 $\varphi$ -component for 4-Ricci,  $rr$ -component of 3-Ricci,  $\theta\theta$ -component of 3-Ricci,  $\varphi\varphi$ -component of 3-Ricci,  $r\theta$ -component of 3-Ricci and 00-component for 4-Ricci. 00-component for 4-Ricci gives an equation for  $\partial_r^2 V$  coming from  $\Delta_\gamma V$ .

Equations for  $r\varphi$ -component and  $\theta\varphi$ -component of 3-Ricci, and 0 $r$ -component and 0 $\theta$ -component of 4-Ricci are trivial. So for  $SU(2)$  EYM system in the spacetime setting described above and for the chosen form of the YM potential, we have ten equations for four variables namely  $\beta$ ,  $C$ ,  $g_{rr}$  and  $V$ .  $\zeta$  satisfies a second order ODE involving  $C$ ,  $g_{rr}$  and  $V$  and we need not worry about it now. This is a very tight situation. The work is incomplete.

#### 4. Conclusion

Our concluding remarks on the work on the Lamb's shift is as follows. If our result is not due to mere coincidence, then it suggests that using the variable Planck's constant with the linear theory (Fourier analysis) one can get a way of calculating the gravitational effects on non-gravitational interaction that reflects the already unified nature of Einstein equations. It is likely that approximating this way one will not need renormalization because the divergence integrals will disappear. It seems we have gravitational justification for the cutoffs for both high and low frequencies. One therefore needs to study the cutoffs considered in the case of emission and absorption of virtual quanta in Feynman's 1948 model (or those in Bopp-Podolsky model). Our selective use of the results of QM and QFT in essentially classical arguments is not arbitrary. At present we are free to use any experimentally verified conclusion of QM and QFT in our analysis. In fact we can use them to get a better equation giving  $h_g$  from the rigorously defined Tolman-Thorne temperature in stationary spacetimes. Then we can guess how to extend the relevant concepts to the local charts of non-stationary spacetimes. Our eventual aim is to do the reverse, that is, use  $h_g$  to explain all the valid results of quantum physics. As for the second ongoing problem discussed at the end of the last subsection, one should not be disappointed if there are no nontrivial solutions for static  $SU(2)$  EYM system in the non-constant coupling case for the form of the metric and YM potential we considered. Because it would possibly suggest that the gauge group should be more complicated for nontrivial static solutions. In that case

there is also the added opportunity to use more than one coupling parameter. Still we expect that the pursuit would provide some useful hints for the search of static solutions modeling a spin zero composite particle with somewhat more complicated but realistic action. Ultimately one would like to explain the origin of the gauge group from the local symmetry of the Minkowski spacetime and hydrodynamic and thermodynamic considerations of Einstein equations.

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