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ALGEBRAIC PROPERTIES OF ELECTROMAGNETIC AND WEAK CURRENTS AND THEIR REPRESENTATION ON HADRON STATES*

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I. INTRODUCTION

For almost a decade quarks have been used as the basis of a successful description of hadron spectroscopy. As originally introduced by Gell-Mann (1) and Zweig (2), meson and baryon states are to be constructed from "constituent quark" building blocks as quark-antiquark and three quark states, respectively.

With internal angular momentum between the quarks, a simple non-relativistic prescription for building the hadron states gives both the SU(3) and spin-parity quantum numbers to be expected. Particularly for baryons, where the physical states which exist are best determined, such a scheme is very successful (3). While there are a number of meson states which are expected in such a quark model but have yet to be found, the relevant experiments are sufficiently difficult that these states could well exist and not have been observed as yet (4).

At the same time, quarks have been used in another distinct way as the basis of constructing algebras from current components. Here one writes the relevant currents in terms of quark fields and the appropriate SU(3) and Dirac matrices, commutes them using the naive free field rules, and then throws away the quarks as a set of basis states — abstracting just the algebraic properties of the current commutators themselves.

In this way one obtains, for example, the chiral $SU(3) \times SU(3)$ algebra of the vector and axial-vector currents at equal times proposed by Gell-Mann (5). This "algebra of currents" has also been very successful, being the basis of the Adler-Weisberger sum rule (6) and many other calculations (7).

When extended from equal time commutators to those of current densities whose coordinates are on the light cone with respect to one another, one generates the quark light cone algebra (8, 9). A powerful description of the scaling behavior of the structure functions measured in deep inelastic scattering follows, with local relations as well as global relations or sum rules between different structure functions.

However, these two uses of quarks are not identical. In fact, identifying the three quarks which "constitute" a nucleon with the "current quark" fields which generate the vector and axial-vector currents at $q^2=0$ yields results like $g_A=5/3$ and $\mu_A(N)=0$, in clear contradiction with experiment. Similarly, as $q^2 \to \infty$ in deep inelastic scattering a description of the current-nucleon interaction in terms of just three quarks with a symmetrical wave function, as in a naive parton model, is known to fail.

The relation between these two uses (10) of quarks will be the main subject of the first several lectures. We will formulate the relation in terms of a transformation between basis states of the algebra of currents, going from a set of irreducible representations to those complicated mixtures of representations characteristic of the physical hadron under the action of the algebra of vector and axial-vector charges. The theoretical framework we will discuss will enable us to treat as a first step, vector and axial-vector transitions between hadron states at $q^2=0$. With the use of PCAC, all real photon and pion transitions fall within this domain.

Up to the present, the main evidence for the constituent quark model has been in the observed hadron spectrum and its mass pattern — the existence of identifiable SU(6) supermultiplets (3). More convincing evidence of the constituent quark makeup of the observed hadron spectrum would follow if we could demonstrate that the algebraic relations among the state vectors in the quark model also held true in the real world. Unfortunately the state vectors themselves are inaccessible to us — one only measures matrix elements or amplitudes. Therefore we must know the algebraic properties of the transition operators acting between the states, as well as the algebraic relations among the states, if one is to extract any information of this type. In other words, we have to solve two problems at once: what is the spectrum of hadron states and their relationship to one another; and what is the character of the operators which induce transitions between them.

The new development that we discuss in the first several lectures is that we now have a hypothesis, abstracted from the free quark model, for the algebraic properties of the photon and pion transition operators (11). Following work of Gilman, Kugler, and Meshkov (12, 13), we shall conduct a fairly detailed analysis, exploring the basic algebraic structure and seeing how it is translated into matrix elements and widths for photon and pion transitions at $q^2=0$. We shall systematically go through meson and baryon decays, reviewing the present situation with respect to the comparison of theory and experiment. In general we find a very encouraging situation, and shall see that there now is good evidence for both the algebraic properties of the transition operators which we abstract and for the algebraic relations between hadron state vectors predicted by the constituent quark model.

In the last few lectures we turn from $q^2=0$ to the opposite regime of $q^2 \to \infty$ and deep inelastic scattering, extending the current algebra to that of the vector and axial-vector current densities on the light cone (8, 9). The resulting quark light cone algebra, and its concrete realization in the quark parton model, yields a concise description of the scaling behavior of the structure functions in inelastic electron-, neutrino-, and anti-neutrino-nucleon scattering. We shall review the experimental situation in this regard, with emphasis on recent data from SLAC, NAL, and CEA and its implications for the quark light cone algebra or quark parton model.

Finally, with the coming advent of experiments with polarized lepton beams and targets, we devote the last lecture to this subject. Again with emphasis on the application of quark light cone algebra or of parton model ideas, the scaling behavior of two additional (spin dependent) structure functions and the sum rules they satisfy are discussed in some detail.

II. CONSTITUENT QUARKS AND HADRON STATES

In order that we have the same hadron states in mind when we go to apply the theory of current matrix elements to actual transitions between hadrons, let us briefly review the constituent quark model and its comparison with the observed spectroscopy. In such a model, mesons are simply constructed as quark-antiquark ($q\bar{q}$) states and baryons as three quark (qqq) states.

Since the quark is by assumption in the $\underline{3}$ representation of SU(3), in the constituent quark model all mesons will be in representations contained in $\underline{3} \times \underline{\bar{3}} = \underline{1} + \underline{8}$. Similarly baryons must lie in representations contained in $\underline{3} \times \underline{3} \times \underline{3} = (\underline{\bar{3}} + \underline{6}) \times \underline{3} = \underline{1} + \underline{8} + \underline{8} + \underline{10}$ when constructed as qqq. By definition, states of either mesons or baryons which can not be obtained in this way are called exotic. Up to now, probably the strongest spectroscopic evidence for the constituent quark model lies not so much in the filling of every slot by an established particle, but in the continued absence of confirmed exotic states. While there are a couple of candidates for exotic Z* baryons, there are literally hundreds of established or candidate states which are non-exotic (14).

The rules for constructing the constituent quark model states involve taking $q\bar{q}$ for mesons and qqq for baryons and proceeding in an essentially non-relativistic manner by adding a total quark angular momentum, L, to the net quark spin, S, to form the total J corresponding to a given state. At this stage we assume exact SU(3) and "forces" between the quarks which do not depend on $\vec{L} \cdot \vec{S}$ or $\vec{S} \cdot \vec{S}$ or couple quark spin and SU(3), e.g., purely harmonic forces between quark and quark or quark and antiquark. As a result we may classify the resulting states in terms of the combined SU(3) and SU(2) (spin) representations of the quarks and the net 0(3) (internal angular momentum, L) representation. The states are therefore discussed in terms of representations of SU(6) $\times$ 0(3). As usual, one treats the quarks as far as statistics is concerned as if they were parafermions of order three (15). Alternately, if one deals with colored quarks (16) they are fermions, but with all low lying states as singlets of the color SU(3).

In Table I the baryon states expected (15, 17) in the constituent quark model are given in terms of SU(6) supermultiplets of increasing values of L (and increasing mass). Each SU(6) supermultiplet is broken down into its SU(3) and quark spin, S, components. Combination of S with L gives the various J^P states expected. A candidate for the non-strange baryon member of each SU(3) and J^P multiplet is listed in each case using the standard notation: $(\pi N \text{ partial wave})_{2I,\,2J}$ (Mass in MeV). In several cases possible mixing between different states with the same J^P and isospin (I) in a given supermultiplet is indicated. In these cases the candidate member of a multiplet is chosen as the physical state which likely has a dominant component coming from a given quark model state. Also expected at the same mass as the $\underline{56}$ L=2 level in a model with purely harmonic forces are $\underline{70}$ L=2, $\underline{56}$ L=0, $\underline{70}$ L=0, and $\underline{20}$ L=1 positive parity states (15, 17).

As is immediately seen, good candidates exist for all the non-strange baryon states in the 56 L=0, 70 L=1, and 56 L=2, with the possible exception of a P_{33} state to complete the 56 L=2. Furthermore, all non-strange baryon states known below 2.0 GeV in mass are thereby classified, except for P_{11} states at 1470 and 1780 MeV. At least one of these could well be assigned to an expected 56 L=0, although it requires finding still another P_{33} state as its non-strange partner! The other P_{11} might possibly be assigned to a 70 L=0 or 70 L=2. Evidence for the existence of the latter multiplet has been recently summarized by Faiman, Rosner and Weyers (18).

The meson states, which lie in either the 35 or 1 representations of SU(6), are summarized in Table II. Again the SU(3), quark spin S, and J^P for states which make up a given SU(6) $\times$ 0(3) multiplet are listed, as well as non-strange mesons which are candidates among the physically observed mesons for identification with the quark model states (14). In the case of mesons there is no trouble with extra states — many "slots" remain to be filled. The lack of isoscalar companions to the now established B meson, a decent A_1 candidate, and the isoscalar members of the L=2 supermultiplet stand out as particularly needed discoveries (4).

However, particularly for baryons where detailed phase shift analysis has permitted the establishment of states with dominantly inelastic decay modes, the fact remains that the expected quark model states exist and even fall in very well identifiable supermultiplets. Even for mesons, the I=1 mesons with L=0, 1, and 2 are almost (except for the A_1 and A_3??)

all observed and are found in easily identified supermultiplets. Although we clearly are not dealing with a true symmetry and the supermultiplets are not even nearly degenerate, very few meson or baryon resonances do not fit into the clear pattern of $SU(6) \times 0(3)$ supermultiplets.

III. THE TRANSFORMATION FROM CURRENT TO CONSTITUENT QUARKS

Examination of the algebraic properties of hadron states and of the operators for current induced transitions between them naturally involves classifying transformation properties in a group-theoretical manner, i.e., in terms of irreducible representations of an appropriate Lie algebra or Lie group. For this purpose, consider the algebra formed by the 16 vector and axial-vector charges, $Q^{\alpha}(t)$ and $Q_5^{\alpha}(t)$, which are simply integrals over all space of the time components of the corresponding currents measurable in weak and electromagnetic interactions:

$$Q^{\alpha}(t) = \int d^3x\, V_0^{\alpha}(\vec{x},t) \quad , \tag{1a}$$

$$Q_5^{\alpha}(t) = \int d^3x\, A_0^{\alpha}(\vec{x},t) \quad . \tag{1b}$$

Here α is an SU(3) index which runs from 1 to 8. At equal times these charges commute to form the algebra proposed by Gell-Mann (5),

$$\left[Q^{\alpha}(t), Q^{\beta}(t)\right] = i\, f^{\alpha\beta\gamma}\, Q^{\gamma}(t) \tag{2a}$$

$$\left[Q^{\alpha}(t), Q_5^{\beta}(t)\right] = i\, f^{\alpha\beta\gamma}\, Q_5^{\gamma}(t) \tag{2b}$$

$$\left[Q_5^{\alpha}(t), Q_5^{\beta}(t)\right] = i\, f^{\alpha\beta\gamma}\, Q^{\gamma}(t) \quad . \tag{2c}$$

This is the algebra of chiral SU(3)$\times$ SU(3), for it can be easily shown that Eqs. (2) are equivalent to the statement that the right-handed charges, $Q^{\alpha} + Q_5^{\alpha}$, and the left-handed charges, $Q^{\alpha} - Q_5^{\alpha}$, each form an SU(3), and that they commute with each other — hence, chiral SU(3) $\times$ SU(3). For α=1, 2, 3 the Q^{α}'s are the generators of isospin rotations: for α=1, . . . , 8, they are the generators of SU(3). The last of Eqs. (2), sandwiched between nucleon states moving at infinite momentum in the z direction, yields the Adler-Weisberger sum rule (6).

Taken between states at infinite momentum (19), the Q^{α}'s and Q_5^{α}'s are "good" operators, i.e., they have finite (generally non-vanishing) values as $p_z \to \infty$. These values are the same as those of space integrals over the z components of the respective currents. If we adjoin to the integrals of the time component of the vector currents and the z-component of the axial-vector currents, integrals over certain "good" tensor current densities, the $SU(3) \times SU(3)$ algebra between states at infinite momentum can be enlarged still further. Letting the index α correspond to an SU(3) singlet when $\alpha=0$, we have the operator $\lambda^{o}/2$ (proportional to the identity) plus an $SU(6)_W$ algebra of 35 generators whose elements commute like the products of SU(3) and Dirac matrices: $\lambda^{\alpha}/2$, $(\lambda^{\alpha}/2)\beta\sigma_x$, $(\lambda^{\alpha}/2)\beta\sigma_y$, and $(\lambda^{\alpha}/2)\sigma_z$. We refer to this algebra, introduced by Dashen and Gell-Mann (20) in 1965 as the $SU(6)_W$ of currents. We denote these generators collectively by F^i, and use them to label the transformation properties of our states and operators. Note that $\beta\sigma_x$, $\beta\sigma_y$ and σ_z, which commute with z boosts and are "good" operators, are <u>not</u> the same as the spin components σ_x, σ_y and σ_z. The appropriate algebra to use is that of $SU(6)_W$ and not SU(6). For quarks, $\beta=+1$ and quark spin and "W-spin" are the same, but for antiquarks, $\beta=-1$, and we have $-\sigma_x$, $-\sigma_y$ and σ_z instead of the antiquark spin components σ_x, σ_y, and σ_z.

In what follows we will label states or operators by their transformation properties under this $SU(6)_W$ algebra of currents. For this purpose we shall often use just the $SU(3) \times SU(3)$ subalgebra of the whole $SU(6)_W$ algebra of currents, as this subalgebra has elements which are directly measurable in weak and electromagnetic interactions. The overall $SU(6)_W$ representation will either be obvious or be made explicit. We will write

$$\left\{(A, B)_{S_z}, L_z\right\} ,$$

where A is the SU(3) representation under $Q^\alpha + Q_5^\alpha$, B the representation under $Q^\alpha - Q_5^\alpha$, and S_z is the eigenvalue of Q_5^0, the singlet axial-vector charge (21). The quantity L_z is then <u>defined</u> in terms of the z component of the total angular momentum J, as $L_z = J_z - S_z$. The "ordinary" SU(3) content (under Q^α) of such a representation is just that of the direct product $A \times B$.

With such a labelling it is clear that, for example, the generator

$$Q_5^\alpha = \frac{Q^\alpha + Q_5^\alpha}{2} - \frac{Q^\alpha - Q_5^\alpha}{2}$$

transforms as $\left\{(8,1)_0, 0\right\} - \left\{(1,8)_0, 0\right\}$, while Q^α transforms as $\left\{(8,1)_0, 0\right\} + \left\{(1,8)_0, 0\right\}$. Being generators, they are in a <u>35</u> of the full $SU(6)_W$ of currents.

Representations of $SU(3) \times SU(3)$ can be built up from $(3,1)_{1/2}$, $(1,3)_{-1/2}$, $(1,\bar{3})_{1/2}$, and $(\bar{3},1)_{-1/2}$ which we <u>define</u> to be the <u>current quark</u> and <u>current antiquark</u> states with spin projection $\pm 1/2$ in the z direction. The quarks form a <u>6</u> and the antiquarks a $\bar{6}$ in $SU(6)_W$. As an example, consider the states of $q\bar{q}$ which can be formed with $L_z=0$. The three states with quark spin $S=1$ and $S_z = J_z = 1$, 0, and -1 are respectively labelled

$$S=1,\ S_z=1: \quad \left\{(3,\bar{3})_1, 0\right\} \tag{3a}$$

$$S=1,\ S_z=0: \quad \left\{(8,1)_0, 0\right\} + \left\{(1,8)_0, 0\right\} \text{ and } \left\{(1,1)_0, 0\right\} + \left\{(1,1)_0, 0\right\} \tag{3b}$$

$$S=1,\ S_z=-1: \quad \left\{(\bar{3},3)_{-1}, 0\right\}, \tag{3c}$$

while the $S=0$, $S_z = J_z = 0$ state is written:

$$S=0,\ S_z=0: \quad \left\{(8,1)_0, 0\right\} - \left\{(1,8)_0, 0\right\} \text{ and } \left\{(1,1)_0, 0\right\} - \left\{(1,1)_0, 0\right\}. \tag{4}$$

All these states lie in a $\underline{35}$ plus a $\underline{1}$ representation of the full $SU(6)_W$ of currents.

As another example, consider combining three current quarks to form a baryon. If we again take $L_z=0$ and a symmetrical quark wave function, then we find the states with $S=1/2$ and $S=3/2$ transform as

$$\begin{aligned}
S=3/2,\ S_z=3/2 &: \quad \left\{(10,1)_{3/2}, 0\right\} \\
S=3/2,\ S_z=1/2 &: \quad \left\{(6,3)_{1/2}, 0\right\} \\
S=1/2,\ S_z=1/2 &: \quad \left\{(6,3)_{1/2}, 0\right\} \\
S=1/2,\ S_z=-1/2 &: \quad \left\{(3,6)_{-1/2}, 0\right\} \\
S=3/2,\ S_z=-1/2 &: \quad \left\{(3,6)_{-1/2}, 0\right\} \\
S=3/2,\ S_z=-3/2 &: \quad \left\{(1,10)_{-3/2}, 0\right\},
\end{aligned} \tag{5}$$

and they all lie in a $\underline{56}$ of the full $SU(6)_W$ of currents. In particular, if a nucleon at infinite momentum with $J_z=1/2$ acted under the algebra of currents as if it were simply composed of two current quarks with $S_z=1/2$ and one quark with $S_z=-1/2$ in a symmetrical wave function, we would have

$$|N> = |\underline{56}, \left\{(6,3)_{1/2}, 0\right\}> \quad . \tag{6}$$

However, the SU(3) content of $(6,3)_{1/2}$ is just that of an octet (including the nucleon) and a decuplet (including the $\Delta(1236)$). Since Q_5^α is a generator of SU(3) $\times$ SU(3), it can only connect this representation to itself, i.e., for $\alpha=1,2,3$ it can only connect the nucleon to the nucleon or to the $\Delta(1236)$. Furthermore, such a classification of the nucleon gives $g_A=5/3$. Both these results are in glaring contradiction with experiment. The nucleon cannot be in such a simple representation. This is already apparent from the Adler-Weisberger sum rule (6) itself, for it shows that the nucleon is connected by a generator of the algebra, the axial-vector charge Q_5^α (in the frame of the pion field through the use of PCAC), to many higher mass N*'s. Thus the nucleon and these N*'s must be in the same representation of SU(3) $\times$ SU(3). Conversely, the nucleon state must span many different representations of the SU(3) $\times$ SU(3) of currents.

An attempt to describe approximately the nucleon state in terms of a sum of irreducible representations of SU(3) $\times$ SU(3) yields (22)

$$\begin{aligned}|N> = \cos\,\theta\,|\left\{(6,3)_{1/2}, 0\right\}> + \sin\theta\left\{\sin\phi\,|\left\{(\bar{3},3)_{1/2}, 0\right\}>\right. \\ \left. + \cos\phi\left[\cos\,\psi\,|\left\{(8,1)_{3/2}, -1\right\}> + \sin\,\psi\,|\left\{(3,\bar{3})_{-1/2}, 1\right\}>\right]\right\},\end{aligned} \tag{7}$$

where θ, ϕ and ψ are parameters to be fitted phenomenologically. It is clear that parametrizing states in a manner resembling the complicated nucleon wave function in Eq. (7) is not the way to proceed in order to understand systematically the classification of higher resonances. The number of phenomenological parameters would increase so as to render the approach essentially useless.

Instead, one may assume (10, 11, 23, 24) that there exists a unitary operator, V, which transforms an irreducible representation (I.R.) of the algebra of currents into the physical state:

$$|\text{Hadron}\rangle = V \; |\text{I.R., currents}\rangle \quad . \tag{8}$$

The state $|\text{I.R., currents}\rangle$ is chosen as that irreducible representation of the algebra of currents which corresponds to baryons being built from just three current quarks and mesons from quark-antiquark. Thus, for example, the complicated nucleon state in Eq. (7) is rewritten as

$$|N\rangle = V \; |\underline{56}, \left\{(6,3)_{1/2}, 0\right\}\rangle \quad . \tag{9}$$

All the complicated mixing of the real hadron states has been subsumed in the operator V.

In the following we will be interested in evaluating the hadronic matrix elements of a charge or current, say Q_5^α. Using Eq. (8) we have

$$\langle\text{Hadron}'|Q_5^\alpha|\text{Hadron}\rangle = \langle\text{I.R.}', \text{ currents}|V^{-1}Q_5^\alpha V|\text{I.R., currents}\rangle \, . \tag{10}$$

The complications of hadronic states under the algebra of currents have now been transferred to the effective operator $V^{-1}Q_5^\alpha V$ which may be studied as an independent object. Moreover, if the operator $V^{-1}Q_5^\alpha V$ has simple transformation properties under the algebra of currents, the way is now open to systematically evaluate the matrix elements of Q_5^α between any two hadronic states.

The operator V serves another useful purpose. It is easy to see that if we define a new set of generators

$$W^i = VF^i V^{-i} \quad , \tag{11}$$

then the W^i also form an $SU(6)_W$ algebra and furthermore, from the definition of V in Eq. (8), hadron states transform as irreducible representations under the W^i corresponding to the naive constituent quark model of hadrons. We therefore call the basis states of this new $SU(6)_W$ "constituent quarks" and identify the algebra with that of the $SU(6)_W$ of strong interactions (25). Equation (8) can therefore be rewritten as

$$|\text{Hadron}\rangle = |\text{I.R., constituents}\rangle = V|\text{I.R., currents}\rangle \quad , \tag{12}$$

while Eq. (10) becomes

$$\begin{aligned} &\langle \text{Hadron}'|Q_5^\alpha|\text{Hadron}\rangle \\ &\qquad = \langle \text{I.R.}', \text{constituents}|Q_5^\alpha|\text{I.R., constituents}\rangle \\ &\qquad = \langle \text{I.R.}', \text{currents}|V^{-1}Q_5^\alpha V|\text{I.R., currents}\rangle \quad . \end{aligned} \tag{13}$$

From this standpoint the operator V just takes one from one set of basis states to another, or alternately, from one set of generators to another.

In the free quark model, the $SU(6)_W$ of strong interactions would be identical with the $SU(6)_W$ of currents if the quarks were restricted to have momentum purely in the z direction ($p_\perp = 0$). It is the transverse momentum of quarks which is the reason for breaking the identity of the two algebras. This is intuitive if we keep in mind that the $SU(6)_W$ of strong interactions (25) was conceived of as a collinear "symmetry". As we will see shortly, it is not a symmetry respected in strong interaction transitions — its conservation is badly violated in pion and photon decays.

As we are interested at the moment in limiting our attention to transitions between hadrons at $q^2=0$, we will be primarily concerned with the algebraic properties of the transformed axial-vector charge, $V^{-1}Q_5^\alpha V$, where Q_5^α is defined in Eq. (1b), and the transformed first moment of the vector current, $V^{-1}D_\pm^\alpha V$, where $D_\pm^\alpha$ is defined as

$$D_\pm^\alpha = \int d^3x \left[\frac{\mp(x \pm iy)}{\sqrt{2}}\right] V_0(\vec{x},t) \quad . \tag{14}$$

Taken between states at infinite momentum, commutators of Q_5^α lead to Adler-Weisberger sum rules (6), while commutators of $D_\pm^\alpha$ lead to Cabibbo-Radicati sum rules (26). Matrix elements of these operators are proportional to pion and photon transition amplitudes respectively. Their properties under the algebra of currents are that

$$Q_5^\alpha \text{ transforms as } \left\{(8,1)_0 - (1,8)_0, 0\right\} \tag{15a}$$

$$D_\pm^\alpha \text{ transforms as } \left\{(8,1)_0 + (1,8)_0, \pm 1\right\} \ . \tag{15b}$$

For guidance on what might be the algebraic properties of $V^{-1}Q_5^\alpha V$ and $V^{-1}D_\pm^\alpha V$ we turn to the free quark model. There Melosh has been able to construct an explicit form of the operator V as

$$V = \exp\left[\left(\frac{i}{2}\right)\int d^3x\, q^+(x) \arctan\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m}\right) q(x)\right] , \tag{16}$$

where m is the quark mass. The transformation V bears a strong similarity to the Foldy-Wouthuysen transformation, only restricted to transverse directions. As expected, if there was no transverse motion of the quarks $(\vec{p}_\perp \propto \vec{\partial}_\perp = 0)$, or if $m \to \infty$, $V \to 1$ and "current" and "constituent" quarks coincide.

In a free quark model at $p_z = \infty$, either $V^{-1} Q_5^\alpha V$ or $V^{-1} D_\pm^\alpha V$ must connect only single quark states to single quark states; they thus have the general form:

$$V^{-1} Q_5^\alpha V \quad \text{or} \quad V^{-1} D_\pm^\alpha V = \int d^3x \, q^+(x) \, \mathscr{O}(\partial_\perp, \gamma_i) \, \frac{\lambda^\alpha}{2} \, q(x) \quad , \tag{17}$$

where $\mathscr{O}$ is some function of the transverse derivatives $(\partial_\perp)$ and the gamma matrices (γ_i). An explicit form of $\mathscr{O}$ was determined by Melosh (1) using the transformation in Eq. (16), while Eichten et al. (27) argue that a large class of such functions exist. Without having a detailed dynamical formalism we are unable to make use of an explicit form, even if it were given to us. What is important here is that the operator is a "single quark" operator; i.e., it depends only on the coordinates of a single quark and it does not create connected $q\bar{q}$ pairs.

It is this property that we abstract from the free quark model and assume to hold in Nature. In general, we assume that: The operators $V^{-1} Q_5^\alpha V$ and $V^{-1} D_\pm^\alpha V$ have the transformation properties of the most general linear combination of single quark operators consistent with SU(3) and Lorentz invariance.

This is verified in the explicit free quark model calculations (11, 27). As SU(3) is assumed conserved there, we have $V^{-1} Q^\alpha V = Q^\alpha$. The operator $V^{-1} Q_5^\alpha V$, with $J_z=0$, contains two terms which transform under $SU(3) \times SU(3)$ as $\left\{(8,1)_0 - (1,8)_0, 0\right\}$ and $\left\{(3,\bar{3})_1, -1\right\} - \left\{(\bar{3},3)_{-1}, 1\right\}$ and behave as components of $\underline{35}$'s of the full $SU(6)_W$ of currents.

The operator $V^{-1} D_+^\alpha V$, with $J_z=1$, is slightly more complicated (28). In general, there are four possible terms (29): $\left\{(8,1)_0 + (1,8)_0, 1\right\}$, $\left\{(3,\bar{3})_1, 0\right\}$, $\left\{(\bar{3},3)_{-1}, 2\right\}$, and $\left\{(8,1)_0 - (1,8)_0, 1\right\}$. It appears that all four occur in the operator $V^{-1} D_+^\alpha V$ in the free quark model (11, 27). However, the last term, which corresponds to $q\bar{q}$ in a net quark spin S=0, unnatural spin-parity state in the non-relativistic model (see Eq. (4)), has no analogue with any natural spin-parity (in particular, vector meson) state of the quark model. Moreover, under a generalized parity transformation, $P e^{-i\pi J_y}$, which takes (22) $\left\{(A,B)_{S_z}, L_z\right\} \rightarrow \left\{(B,A)_{-S_z}, -L_z\right\}$, the first three terms do not change sign while the last one does. For the longitudinal ($J_z=0$) component of the current this would eliminate the possibility of such a term. Therefore the $\left\{(8,1)_0 - (1,8)_0, 1\right\}$ term in D_+^α has no correspondence with any natural spin-parity meson state and can not occur in the longitudinal component of the vector current. We drop it in the following discussion and assume that $V^{-1} D_+^\alpha V$ only has terms which transform as (12,13) $\left\{(8,1)_0 + (1,8)_0, 1\right\}$, $\left\{(3,\bar{3})_1, 0\right\}$, and $\left\{(\bar{3},3)_{-1}, 2\right\}$, again in $\underline{35}$'s of the $SU(6)_W$ of currents. Thus, in spite of the enormous complication of V itself, we abstract these remarkably simple algebraic properties of $V^{-1} Q_5^\alpha V$ and $V^{-1} D_+^\alpha V$ from the free quark model and postulate them to hold in the real world. We now proceed to apply this hypothesis to transitions between hadrons.

IV. CALCULATION OF PHOTON AND PION DECAYS OF HADRONS

We have now set up the basic apparatus necessary to carry out the application of the algebraic structure of the transformed currents to hadron transitions. First we have described how to classify states and operators in terms of the $SU(6)_W$ of currents and its subalgebra, chiral $SU(3)\times SU(3)$. Having seen that physical hadron states are not in irreducible representations of this algebra of currents, we defined an operator V, assumed unitary, which carries that irreducible representation of the algebra of currents characteristic of the quark model for any given hadron into the physical hadron state. Applying this to matrix elements of the axial-vector charge, Q_5^{α}, or the first moment of the vector current, D_+^{α}, and moving the transformation V from the state to the operator, shows that one must study the properties of the operators $V^{-1}Q_5^{\alpha}V$ and $V^{-1}D_+^{\alpha}V$. Appealing to the free quark model and the work of Melosh (11), we abstract the algebraic properties of these operators, namely, we assume that in Nature $V^{-1}Q_5^{\alpha}V$ transforms as $\left\{(8,1)_0-(1,8)_0,0\right\}$ and $\left\{(3,\bar{3})_1,-1\right\}-\left\{(\bar{3},3)_{-1},1\right\}$ while $V^{-1}D_+^{\alpha}V$ transforms as $\left\{(8,1)_0+(1,8)_0,1\right\}$, $\left\{(3,\bar{3})_1,0\right.$, and $\left\{(\bar{3},3)_{-1},2\right\}$, all components of $\underline{35}$'s of the full $SU(6)_W$ of currents.

To carry through this application of the algebraic structure of transformed currents to decays of real hadrons, we need several additional physical assumptions. Unfortunately very few weak axial-vector transitions have been measured. Until such data exist, we must content ourselves with a comparison of the theory for $q^2=0$ axial-vector matrix elements with amplitudes for pion decays. To relate matrix elements of Q_5 between states at infinite momentum to matrix elements of the pion field we need the PCAC hypothesis. Explicitly, for α=1, 2, 3 we assume (30):

$$\partial_\mu A_\mu^\alpha(x) = \frac{1}{\sqrt{2}} f_\pi \phi_\pi^\alpha(x) \quad , \tag{18}$$

where $A_\mu^\alpha(x)$ is the axial-vector current and $f_\pi \simeq 135$ MeV is a constant related to the charged pion decay rate. The decay rate for Hadron' $\rightarrow$ Hadron + π^- can then be computed in narrow resonance approximation in terms of matrix elements of $1/\sqrt{2}\left(Q_5^1 - iQ_5^2\right)$ between states at infinite momentum as

$$\Gamma(\text{Hadron}' \rightarrow \text{Hadron} + \pi^-)$$
$$= \frac{1}{\left(4\pi f_\pi^2\right)} \frac{p_\pi}{2J'+1} \frac{(M'^2 - M^2)^2}{M'^2} \sum_\lambda |\langle \text{Hadron}', \lambda| \frac{1}{\sqrt{2}} \left(Q_5^1 - iQ_5^2\right) |\text{Hadron}, \lambda\rangle|^2 , \tag{19}$$

where p_π is the pion momentum and the sum extends over all the possible common helicities, λ of the hadrons. The total width, Γ(Hadron' $\rightarrow$ Hadron + π), may be obtained from Eq. (19) by adding the π^+ and π^0 widths, which are related by isospin Clebsch-Gordan coefficients. Equation (19) may also be obtained in a more clearly covariant way by considering the narrow resonance approximation to the Hadron' intermediate state contribution to the Adler-Weisberger sum rule (6) obtained by taking Eq. (2c) between Hadron states at infinite momentum. In either case, we see that the width for Hadron' $\rightarrow$ Hadron + π is directly fixed by matrix elements of Q_5, up to the validity (31) of PCAC. As a result, there are <u>no arbitrary phase space factors</u> in the calculation.

For photon decays we need no additional assumption to relate the width to the matrix element of the D_+^α operator of Eq. (14) taken between states at infinite momentum. We have directly that in narrow resonance approximation

$\Gamma(\text{Hadron}' \rightarrow \text{Hadron} + \gamma)$

$$= \frac{e^2}{\pi} \frac{p_\gamma^3}{2J'+1} \sum_\lambda |\langle \text{Hadron}', \lambda | D_+^3 + \frac{1}{\sqrt{3}} D_+^8 | \text{Hadron}, \lambda-1 \rangle|^2 \quad , \qquad (20)$$

where e is the proton charge, p_γ the photon momentum, and the sum extends over all possible helicities λ. Note that although the definition of D_+^α in Eq. (14) involves only a first moment of the current, between states at infinite momentum all multipole amplitudes consistent with the spin and parity of the states enter matrix elements of D_+^α. Equation (20) may also be obtained from consideration of the narrow resonance approximation to the Hadron' contribution to the Cabibbo-Radicati sum rule (26) on Hadron states. Again we have no arbitrary phase space factors.

For the present we shall use the narrow resonance approximation expressions, Eqs. (20) and (21), for pion and photon decay widths in order to make a comparison of the theory with experiment. For broad resonances in the initial and/or final state, or for decays of resonances where the physically available phase space is small, such an approximation introduces non-negligible errors (32). However, we view the present comparison as being sufficiently accurate as a first test of the theory, particularly in view of the experimental errors on values for pion or photon decay widths. When the situation warrants it, the values of $|\langle \text{Hadron}'|Q_5^\alpha|\text{Hadron}\rangle|^2$ and $|\langle \text{Hadron}'|D_+^\alpha|\text{Hadron}\rangle|^2$ should be determined irrespective of any approximation in terms of contributions to Adler-Weisberger and Cabbibo-Radicati sum rules, respectively.

Recalling that, for example, $\langle \text{I.R.}', \text{constituents}|Q_5^\alpha|\text{I.R.}, \text{constituents}\rangle = \langle \text{I.R.}', \text{currents}|V^{-1}Q_5^\alpha V|\text{I.R.}, \text{currents}\rangle$, we see that with the assumed algebraic properties of $V^{-1}Q_5^\alpha V$ (as abstracted from the free quark model), we know the transformation properties under the $SU(6)_W$ of currents of all quantities in a given matrix element. To make contact with experiment we make a physical assumption. Namely, we assume that we can identify the observed (non-exotic) hadrons with constituent quark states. In other words, we assume that there is a portion of the physical Hilbert space which is well approximated by the single particle states of the constituent quark model. As we saw in Section II, for baryons, composed of qqq, we do have candidates which fit very well into the $SU(6)\times 0(3)$ representations $\underline{56}$ L=0, $\underline{70}$ L=1, and $\underline{56}$ L=2. For mesons we have correspondingly the $q\bar{q}$ states $\underline{35}$ L=0, $\underline{1}$ L=0, $\underline{35}$ L=1, etc. Moreover, we shall assume that states with different values of the quark spin as well as L_z and S_z are related as in the constituent quark model, i.e., by the $SU(6)_W$ of strong interactions. This will allow us to relate different helicity states of a given hadron to each other.

We then know the algebraic properties (under the algebra of currents) of all terms of a transformed matrix element of the physically observed states. Therefore we may use the Wigner-Eckart theorem and tables of

Clebsch-Gordan coefficients to carry out the calculation from this point onward. Note that $\underline{SU(6)_W}$ $\underline{\text{invariance}}$ of the transition operator under either the algebra of currents or of strong interactions $\underline{\text{is not assumed}}$ — only the transformation properties of the various terms are needed in the calculation.

More explicitly, for a given matrix element of Q_5^α we write the initial and final hadron states with $J_z = \lambda$ in terms of states with definite quark L_z and S_z. This involves coupling internal L and S to form total J for each hadron. After transforming to an $SU(6)_W$ of currents basis, the matrix element of the (33) $(8,1)_0 - (1,8)_0$ or $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term in $V^{-1} Q_5^\alpha V$ can then be written, using the Wigner-Eckart theorem applied to representations of the $SU(6)_W$ of currents, as a reduced matrix element times the produce of quark angular momentum, $SU(6)_W$, SU(3), and W-spin Clebsch-Gordan coefficients (34, 35, 36). For example, suppose we were calculating the matrix element of the $(8,1)_0 - (1,8)_0$ piece of $V^{-1} Q_5^\alpha V$ between initial and final states with common helicity λ, total angular momentum J and J', internal quark angular momentum L and L', quark spins S and S', $SU(6)_W$ representations R and R', and SU(3) representations A and A' respectively. Then we have that

$$\langle R', A', L', S', J', \lambda, \text{currents} \,|\, \left\{(8,1)_0 - (1,8)_0, \, 0\right\} \,|\, R, A, L, S, J, \lambda, \text{currents} \rangle$$
$$= \sum_{S_z S'_z} \underbrace{(L'L'_z S'S'_z \,|\, J'\lambda)\,(LL_z SS_z \,|\, J\lambda)}_{\substack{\text{quark angular momentum} \\ \text{Clebsch-Gordan coefficient}}} \quad \underbrace{(R' \,|\, \underline{35} \,|\, R)}_{\substack{SU(6)_W \text{ Clebsch-} \\ \text{Gordan coefficient}}}$$
$$\underbrace{\left(A' \,|\, \underline{8} \,|\, A\right)}_{\substack{SU(3) \text{ Clebsch-} \\ \text{Gordan coefficient}}} \quad \underbrace{(1\;0\;WW_z \,|\, W'W'_z)}_{\substack{\text{W-spin Clebsch-} \\ \text{Gordan coefficient}}} \quad \underbrace{\langle R', L', L'_z \,\|\, (8,1)_0 - (1,8)_0 \,\|\, R, L, L_z \rangle}_{\text{Reduced matrix element}}. \tag{21}$$

The W-spin Clebsch-Gordan coefficient follows since the $(8,1)_0 - (1,8)_0$ operator has W=1 and $W_z=0$. For any state, $W_z = S_z$. For baryons, $\vec{W}=\vec{S}$, while for mesons we have the conventional correspondence (25) (W-S flip),

$$
\begin{aligned}
|W=1,\ W_z=1\rangle &= |S=1,\ S_z=1\rangle \\
|W=1,\ W_z=0\rangle &= -|S=0,\ S_z=0\rangle \\
|W=1,\ W_z=-1\rangle &= -|S=1,\ S_z=-1\rangle \\
|W=0,\ W_z=0\rangle &= -|S=1,\ S_z=0\rangle \quad .
\end{aligned}
\tag{22}
$$

The signs which result from using Eq. (22) to convert from quark spin to W-spin are understood to be included in Eq. (21) in the $SU(6)_W$ Clebsch-Gordan coefficient.

The reduction of the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ piece of $V^{-1} Q_5^{\alpha} V$ proceeds just as above, except that from Eq. (22) it transforms under W-spin as $|W=1, W_z=1\rangle + |W=1, W_z=-1\rangle$. As a result, the sum in Eq. (21) is replaced by two sums involving the W-spin Clebsch-Gordan coefficients $(1 1 W W_z | W' W_z')$ and $(1 -1 W W_z | W' W_z')$. For photon decays we need only recall that $(8,1)_0 + (1,8)_0$ is a W=0, W_z=0 object, while $(3,\bar{3})_1$ and $(\bar{3},3)_{-1}$ transform as $|W=1, W_z=1\rangle$ and $-|W=1, W_z=-1\rangle$. Since the net J_z initially and finally must be the same for either Hadron' $\rightarrow$ Hadron + π or Hadron' $\rightarrow$ Hadron + γ decays, and since the net value of $W_z=S_z$ must also be the same by the W-spin Clebsch-Gordan coefficient in Eq. (21) and its analogues, it follows that $L_z = J_z - S_z$ must also be additively conserved between the initial and final state (including the pion or photon operator).

The general algebraic structure of the results is now apparent (12, 13, 37). All the Q_5^α matrix elements taken between hadron states in two given SU(6) multiplets with given L_z and L_z' are related to at most one non-zero independent $SU(6)_W$ reduced matrix element, corresponding to the $\left\{(8,1)_0+(1,8)_0, 0\right\}$, $\left\{(3,\bar{3})_1, -1\right\}$, or $-\left\{(\bar{3},3)_{-1}, 1\right\}$ pieces of $V^{-1}Q_5^\alpha V$. Similarly, there is at most one independent $SU(6)_W$ reduced matrix element for photon decays between states in two given SU(6) multiplets with given values of L_z and L_z'. If L is zero, as is the case in essentially all cases of physical interest at the present, then of course $L_z=0$ and the L_z' dependence of the $SU(6)_W$ reduced matrix element becomes trivial (in particular, the $\left\{(3,\bar{3})_1, -1\right\}$ and $-\left\{(\bar{3},3)_{-1}, 1\right\}$ pieces of $V^{-1}Q_5^\alpha V$, with $L_z'=-1$ and $+1$ respectively, have the same reduced matrix element). In such a case (L=0) there are at most two independent reduced matrix elements of Q_5^α,

$$\langle R', L' \| (8,1)_0 - (1,8)_0 \| R, 0 \rangle \tag{23a}$$

and

$$\langle R', L' \| (3,\bar{3})_1 - (\bar{3},3)_{-1} \| R, 0 \rangle \quad , \tag{23b}$$

and three independent reduced matrix elements of D_+^α taken between two given SU(6) multiplets,

$$\langle R', L' \| (8,1)_0 + (1,8)_0 \| R, 0 \rangle \tag{24a}$$

$$\langle R', L' \| (3,\bar{3})_1 \| R, 0 \rangle \tag{24b}$$

and

$$\langle R', L' \| (\bar{3},3)_{-1} \| R, 0 \rangle \ , \tag{24c}$$

given our assumption on the algebraic properties of $V^{-1}Q_5^\alpha V$ and $V^{-1}D_+^\alpha V$.

The algebraic structure of the theory presented here has much in common with relativistic quark model calculations, such as those of Ref. 38. In fact, the results of Ref. 38 may be cast into a form which permits the complete identification of certain parameters there with the reduced matrix elements discussed here. However, the assumption of a "potential" in the quark model calculations yields definite predictions of the reduced matrix elements themselves as they depend on masses and other parameters of the model. This is something we do not obtain using purely the algebraic structure discussed in Section III. Also very similar in algebraic structure, at least for decays to L=0 hadrons, are some broken $SU(6)_W$ schemes (39). The relation of such schemes, and in particular ℓ-broken $SU(6)_W$, to the present theory is discussed in detail in Ref. 37. The results of assuming $SU(6)_W$ conservation for pion transitions are reproduced in the present theory by retaining only the $\left\{(8,1)_0 - (1,8)_0, 0\right\}$ term in $V^{-1}Q_5^{\alpha}V$ and using PCAC. The assumption of $SU(6)_W$ conservation plus vector dominance is equivalent to keeping only the $\left\{(3,\bar{3})_1, 0\right\}$ term in $V^{-1}D_+^{\alpha}V$.

V. EXPERIMENTAL TESTS OF THE TRANSFORMATION BETWEEN CURRENT AND CONSTITUENT QUARKS

With the basic features and assumptions of the theory described in the previous sections, we are in a position to apply it. We begin with the pionic decays of mesons. Only non-strange meson decays will be discussed in detail as all the corresponding strange meson decay rates are related to those we calculate by SU(3). At the present time they add little to the experimental tests of the theory.

Consider the pionic transitions from $\underline{35}$ $L'=1 \rightarrow \underline{35}$ $L=0$. From the previous section, there are two independent reduced matrix elements, $\langle L'=1 \| (8,1)_0 - (1,8)_0 \| L=0 \rangle$ and $\langle L'=1 \| (3,\bar{3})_1 - (\bar{3},3)_{-1} \| L=0 \rangle$. Rather than try to make a best fit in terms of all the measured decays, we simply use two inputs (40): $\Gamma(A_2 \rightarrow \pi\rho) = 77$ MeV and $\Gamma_{\lambda=0}(B \rightarrow \pi\omega) = 0$. This latter condition is in agreement with experiments (41) which see a dominantly transverse decay, and corresponds to setting $\langle L'=1 \| (8,1)_0 - (1,8)_0 \| L=0 \rangle = 0$. All amplitudes are then multiples of $\langle L'=1 \| (3,\bar{3})_1 - (\bar{3},3)_{-1} \| L=0 \rangle$. The results are gathered in Table III. Predictions of particular interest are

(1) $\Gamma(B \rightarrow \pi\omega)$ agrees within errors with $\pi\omega$ being the dominant (and so far, only observed) mode out of a total B width (40) of 100 ± 20 MeV.

(2) $\Gamma(f \rightarrow \pi\pi)$ is in excellent agreement with experiment. Use of a d-wave phase space factor instead of the PCAC dictated factor changes the prediction by more than a factor of 2, destroying the agreement.

(3) $\Gamma(A_2 \rightarrow \pi\eta)$ is in excellent agreement with experiment.

(4) We predict a relatively narrow $A_1 \to \pi\rho$ with a dominantly longitudinal character. This is obviously not the non-resonance observed (42) in $\pi^{\pm}p \to (3\pi)^{\pm}p$, and there is no established state with which to compare our prediction.

(5) $\Gamma(\delta \to \pi\eta)$ agrees with the roughly known (40) total width.

(6) We have somewhat arbitrarily assigned the σ a mass of 760 MeV. While it is gratifying that the resulting $\Gamma(\sigma \to \pi\pi)$ is broad, the uncertainties in identifying the non-strange quark state with a particular observed $J^{PC} = 0^{++}$ hadron are very large.

Overall we find that experiment and theory compare quite favorably for $L'=1 \to L=0$ pionic decays of mesons. What is known about $L'=0 \to L=0$ and $L'=2 \to L=0$ pionic matric elements between meson states also is quite consistent with the theory (12, 12), although present data on these transitions does not provide a very restrictive test (43, 44).

Encouraged by this we turn to baryons. As in the meson case, we discuss mainly the non-strange baryon decays. Our choice of multiplets discussed is motivated by the fact that they are the only ones where a fairly complete experimental comparison can be made.

For transitions of the type $\underline{56}$ $L'=0 \to \underline{56}$ $L=0$ only the $\left\{(8,1)_0 - (1,8)_0, 0\right\}$ term in $V^{-1}Q_5^{\alpha}V$ contributes. The two predictions made by the theory are

(1) $F/D = 2/3$ for the baryon decays. This is tested directly by the axial-vector contribution to the weak leptonic decays of the baryon octet, without need for PCAC. This prediction agrees quite well with experiment (40).

(2)

$$< \Delta^{o}, \lambda{=}1/2 \mid \frac{1}{\sqrt{2}}\left(Q_5^1 - iQ_5^2\right) \mid p, \lambda{=}1/2 > = \left(\frac{2}{5}\sqrt{2}\right)$$

$$\times < n, \lambda{=}1/2 \mid \frac{1}{\sqrt{2}}\left(Q_5^1 - iQ_5^2\right) \mid p, \lambda{=}1/2 > \quad . \tag{25}$$

This prediction also agrees with experiment (45).

For $\underline{70}$ L'=1 → $\underline{56}$ L=0 and $\underline{56}$ L'=2 → $\underline{56}$ L=0 pionic decays there are two reduced matrix elements for each set of decays of the form $N^* \to \pi N$ and $N^* \to \pi\Delta$ where the N^* is a resonance in the $\underline{70}$ L'=1 or $\underline{56}$ L'=2 (see Table I).

Table IV compares the experimental partial widths (40, 46, 47) of the $\underline{70}$ L'=1 and $\underline{56}$ L'=2 baryons with theory (12, 13). We have again chosen to fit the reduced matrix elements to certain decays rather than doing an overall least squares fit. We observe that the agreement of experiment with theory is only qualitative, and that large experimental errors in the matrix elements are involved. One of the strongest disagreements is in the decays of the $D_{15}(1670)$, which cannot be mixed within the $\underline{70}$ L=1 multiplet. The disagreement is in fact sharper than is apparent in Table IV, since the errors quoted on the πN and $\pi\Delta$ widths are correlated by the reasonably well-known inelasticity. While the theory predicts that less than 20% of the width is due to the πN decay, experiment indicates a 40% branching ratio (40).

We must emphasize at this point that a large experimental ambiguity exists in evaluating the partial widths of resonances even when phase shift analysis results are known. In the case of strongly inelastic resonances, different ways of extracting resonance couplings may be used such as: extrapolations to the pole, K-matrix fits, Breit-Wigner fits, etc. These give widely varying estimates of partial widths. For example, the width of the $D_{13}(1520)$ decay to $\pi\Delta$ changes from 24 to 53 MeV depending on whether one uses coupling estimates from the Argand diagram or a T-matrix pole fit (47). There is also a theoretical error in our use of the narrow resonance approximation. This is particularly so for $\pi\Delta$ decays, where some of the larger discrepancies between theory and experiment occur. With this in mind, the overall situation for pionic widths of non-strange baryons is not unreasonable.

For real photon transitions, all helicity amplitudes can be obtained by taking matrix elements of $D_+^3 + (1/\sqrt{3})\, D_+^8$ between states at infinite momentum. Such matrix elements of D_+^α are equal to the sum of those of the three possible terms in $V^{-1} D_+^\alpha V$ discussed in Section III. At the present time only non-strange baryon transitions have been sufficiently investigated experimentally so as to provide a test of the theory.

For $\underline{56}$ L'=0 $\rightarrow$ $\underline{56}$ L=0 photon transitions, only the term in $V^{-1} D_+^\alpha V$ which transforms as $\left\{(3,3)_1, 0\right\}$ can make a non-zero contribution, because $L_z=0$ in both the initial and final state. All matrix elements are therefore proportional to the single reduced matrix element $<\underline{56}\ L'=0 \,\|\, (3,\bar{3})_1 \,\|\, \underline{56}\ L=0>$. For transitions between two octet members of the $\underline{56}$, this term is characterized by an F/D value of 2/3.

Between J=1/2 baryon states the matrix elements of $V^{-1} D_+^\alpha V$ are interpretable as proportional to the total magnetic moment of the baryon (11).

As a result the theory gives (11)

$$\mu_T(n)/\mu_T(p) = -\,2/3 \quad , \tag{26}$$

the old SU(6) result (48) which is rather close to experiment (40). Furthermore, the ratio of $\sqrt{3}$ between the λ=3/2 and 1/2 amplitudes for $\Delta \to \gamma N$ corresponds to a pure magnetic dipole transition with

$$\mu^*/\mu_T(p) = \frac{2}{3}\sqrt{2} \quad , \tag{27}$$

if we use the relation

$$< \Delta, \lambda{=}1/2 \,|D_+|\, N, \lambda{=}-1/2 > = \mu^*/\sqrt{2} \quad . \tag{28}$$

A phenomenological analysis (49) of the data for pion photoproduction gives a value for $\mu^*/\mu_T(p)$ which is 1.28 ± 0.03 times the right-hand side of Eq. (27). However, this is the result of finding the residue at the Δ pole in $\gamma N \to \pi N$. In our approach one should evaluate μ^* by taking the Δ contribution to the Cabibbo-Radicati sum rule (see Section IV). This results in a value (50) of $\mu^*/\mu_T(p)$ which is 0.9 ± 0.1 times the right-hand side of Eq. (27), i.e., in quite satisfactory agreement with the theory. Equations (26) and (27) are standard SU(6) results, as is to be expected since the $(3,\bar{3})_1$ term in $V^{-1} D_+^{\alpha} V$ has the same transformation properties as the magnetic moment operator (48) used in SU(6).

For $\underline{70}$ L'=1 $\to$ $\underline{56}$ L=0 and $\underline{56}$ L'=2 $\to$ $\underline{56}$ L=0 photon decays (13, 29, 51, 52) the situation is more complicated because of the presence of more independent reduced matrix elements than in the $\underline{56}$ L'=0 $\to$ $\underline{56}$ L=0 case. However, if we disregard the $\left\{(3,3)_{-1}, 2\right\}$ term (as well as the $\left\{(8,1)_0 - (1,8)_0, 1\right\}$ term which we discarded before) in $V^{-1} D_+^{\alpha} V$, only two terms are left and there is a one to one correspondence with the results of quark model calculations (53): the $(8,1)_0 + (1,8)_0$ term in $V^{-1} D_+^{\alpha} V$ corresponds to the photon interacting with the quark convection current, while the $(3,\bar{3})_1$ term corresponds to the interaction with the quark magnetic moments. Of course, explicit quark model calculations with, say, harmonic potentials give the reduced matrix elements as well, something we do not obtain at all with the theory under discussion. Since the more restrictive quark model predictions are roughly consistent (38) with the experimental data on photon decay amplitudes (54), so is the theory discussed here.

A more crucial test of the theory comes in predicting the relative signs of amplitudes for inelastic scattering. For, even if some mixing occurs or the identification of a given hadron with a particular state of the constituent quark model is only approximate, resulting in say a 30 or 40% error in the magnitude of a matrix element and a factor of two error in a width, the sign of an amplitude should still be preserved if the mixing is not too large.

For the reaction $\pi N \rightarrow N^* \rightarrow \pi\Delta$ we can compare our predictions to recent isobar model phase shift analyses (46, 47, 55). Table V lists the theoretically predicted phases (12, 13, 56, 57) coming from the $(8,1)_0 - (1,8)_0$ and $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ pieces of $V^{-1} Q_5^{\alpha} V$ and the experimental results (58). The theoretical predictions are of two kinds. First are those involving amplitudes with the same (ℓ) partial wave in both the incoming and outgoing channel and which are therefore proportional to squares of matrix elements. These have well-defined signs regardless of the relative magnitudes of the reduced matrix elements of the $(8,1)_0 - (1,8)_0$ and $(3,3)_1 - (3,3)_{-1}$ terms in $V^{-1} Q_5^{\alpha} V$. The second kind of sign prediction depends on this relative magnitude, and may help us in deducing which term is dominant for pion decays from one SU(6) multiplet to another.

We see that the signs in the original solution, A, of the experimental analysis disagrees with the theory even for predictions of the first kind as seen in Table V. We note, however, that the only disagreement is between the $D_{13}(1520)$ couplings and all other couplings. This sign cannot be changed by mixing the two D_{13} states. If the signs of this resonance can be reversed, one would have complete agreement between theory and experiment. We note that the analysis on which we base our comparison suffers from a lack of data between 1540 and 1650 MeV, i.e., between the $D_{13}(1520)$ and the other resonances in the $\underline{70}$ L=1 and $\underline{56}$ L=2. The relative phases of amplitudes above and below the gap are determined by continuity and K matrix fits. While a complete K matrix fit has yet to be done, it appears (55) that a second solution, B, of the phase shifts exists with the signs of the $D_{13}(1520)$ reversed with respect to solution A. This removes the most serious experimental failure among the many predictions of the theory as reviewed at Purdue (56).

In solution B, the signs of the amplitudes are such as to indicate dominance of the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term in $V^{-1} Q_5^{\alpha} V$ for $\underline{70}$ $L'=1 \rightarrow \underline{56}$ L=0 decays, but dominance of the $(8,1)_0 - (1,8)_0$ term in $\underline{56}$ $L'=2 \rightarrow \underline{56}$ L=0 decays. An analysis (59) of the signs of amplitudes for $\gamma N \rightarrow N^* \rightarrow \pi N$

shows that the measured signs are consistent with the theory at <u>both</u> the pion and photon vertex for $\underline{70}$ L=1 and $\underline{56}$ L=2 intermediate N* resonances if the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term dominates in $\underline{70}$ L'=1 → $\underline{56}$ L=0 pion transitions. Up to this time the experimental analysis of $\gamma N \rightarrow \pi N$ is insufficient to give confirmatory evidence on the dominance of the $(8,1)_0 - (1,8)_0$ term in $\underline{56}$ L'=2 → $\underline{56}$ L=0 pion decays. An interesting sidelight to the situation with regard to signs is that both P_{11} resonances appear in $\pi N \rightarrow \pi\Delta$ solution B to behave like members of $\underline{56}$ supermultiplets, rather than one being a member of a $\underline{70}$.

Note that both our analysis of signs for $\underline{70}$ L'=1 → $\underline{56}$ L=0 pionic transitions of baryons and of widths for $\underline{35}$ L'=1 → $\underline{35}$ L=0 meson decays show that the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term in $V^{-1}Q_5^{\alpha}V$ is dominant. This is completely contradictory to the assumption of $SU(6)_W$ conservation, which only allows the $(8,1)_0 - (1,8)_0$ term to be present. Also for photon decays from $\underline{70}$ L'=1 → $\underline{56}$ L=0 and $\underline{56}$ L'=2 → $\underline{56}$ L=0, the experimental amplitudes (57) indicate the importance of both the $(8,1)_0 + (1,8)_0$ and $(3,\bar{3})_1$ terms in $V^{-1}D_+^{\alpha}V$, while $SU(6)_W$ conservation plus vector dominance would allow only $(3,\bar{3})_1$.

Taking solution B for $\pi N \rightarrow \pi\Delta$, there are more than 20 signs in $\gamma N \rightarrow \pi N$ and $\pi N \rightarrow \pi\Delta$ which agree with experiment. Further, the domination of the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ term in $V^{-1}Q_5V$ for $\underline{70}$ L'=1 → $\underline{56}$ L=0 pion decays is obtained consistently in both reactions. With that many signs correct, one begins to have the feeling that there is something right about the theory we have been employing.

Such a theory can be regarded as one more step in a program of abstracting algebraic properties from the free quark model, without necessitating the reality of free quarks themselves. Abstraction from the free quark model assures us that the assumed algebraic properties could be exact, and are at least consistent with relativity, invariance principles, etc. They are presumably the least complicated that one might expect to hold in the real world.

As an elegant and beautiful theory of the algebraic structure of weak and electromagnetic current induced transitions between hadrons at $q^2=0$, the present theory greatly unifies the treatment of weak and electromagnetic transitions with the systematics of hadron spectroscopy. With the identification of the observed hadrons to good approximation with constituent quark states and the use of PCAC, a powerful approximate theory of all pion and photon decay results.

As a theory of pion transitions, the present theory has much in common as far as general algebraic structure is concerned with both previous relativistic quark model calculations (38) and certain broken $SU(6)_W$ schemes (39). We regard this theory of current-induced transitions, when supplemented by PCAC and/or vector meson dominance, as in fact providing a method of constructing a phenomenology of purely hadronic vertices (60) and providing justification for some aspects of these other theoretical schemes.

An important aspect of the present theory is that the comparison with experiment is in terms of amplitudes which are related in a straightforward way by Clebsch-Gordan coefficients, and decay widths are in turn related to these amplitudes in a non-arbitrary, known way. The agreement found, particularly in regard to signs, provides strong support for the theory as a valid description of both photon and pion transitions, as well as for the identification of the observed hadron states with those of the constituent quark model.

VI. DEEP INELASTIC SCATTERING AND ANNIHILATION

A. INTRODUCTION

We now turn from the study of current induced transitions between hadrons at $q^2=0$ to the opposite regime of $q^2 \to \infty$. While it would be very interesting to extend our treatment of the transformed vector and axial-vector currents to $q^2 \neq 0$, little experimental information exists on matrix elements between specific hadrons. Instead we sum over all final hadrons obtainable in the interaction of a current with a given hadron and obtain quantities measured in the experimental investigation of inelastic electron-nucleon, neutrino-nucleon, and antineutrino-nucleon scattering for (61) space-like q^2 ($q^2 > 0$) and in electron-positron annihilation into hadrons for time-like q^2 ($q^2 < 0$).

B. INELASTIC ELECTRON-NUCLEON SCATTERING

For the case of inelastic electron scattering, the diagram of interest (62) is indicated in Fig. 1, where k and k' are the initial and final electron four-momenta, q is the four-momentum transfer carried by the virtual photon, and p is the target nucleon's four-momentum. The final hadronic state n then has four-momentum $p_n = p + q$ and invariant mass squared $W^2 = -(p+q)^2$. In the laboratory frame (initial nucleon at rest) with E and E' the energies of the initial and final electrons, the Lorentz scalar variable

$$\nu = -p \cdot q / M_N = E - E' \tag{29}$$

is the virtual photon's energy, and the invariant momentum transfer squared is

$$q^2 = 4EE' \sin^2 \theta/2 \tag{30}$$

where θ is the scattering angle and the electron mass has been neglected compared to its energy. Knowing ν and q^2 from measuring the incident and scattered electron, the invariant mass W of the final hadrons is fixed by

$$W^2 = 2M_N\nu + M_N^2 - q^2 \quad . \tag{31}$$

The S-matrix element for the process in Fig. 1 may be written using the rules of quantum electrodynamics at the photon-electron vertex as

$$S_{fi} = \delta_{fi} + (2\pi)^4 \, i\delta^{(4)}(p_n+k'-p-k) \sqrt{\frac{m_e^2}{EE'}} \left(-e\bar{u}(k') \, i\gamma_\mu \, u(k)\right)$$

$$\times \left(\delta_{\mu\nu}/q^2\right) \langle p_n | J_\nu(0) | p\rangle \quad , \tag{32}$$

where

$$J_\nu = e\left(V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8\right) \tag{33}$$

is the (hadronic) electromagnetic current operator. Averaging over initial and summing over final electron and nucleon spins, we are led to an expression for the double differential cross section in the laboratory for detection of only the final electron of the form

$$\frac{d^2\sigma}{d\Omega' dE'} = \frac{1}{(2\pi)^2} \left(\frac{E'}{E}\right)\left(\frac{e^2}{q^2}\right)^2 L_{\mu\nu} W_{\mu\nu} \quad , \tag{34}$$

where the factor $L_{\mu\nu}$ arises from the trace of the gamma matrices due to the electron (neglecting the electron mass),

$$L_{\mu\nu} = \frac{1}{2}\left[k_\mu k'_\nu + k'_\mu k_\nu + \left(\frac{q^2}{2}\right)\delta_{\mu\nu}\right] \quad , \tag{35}$$

and the structure of the nucleon is summarized in

$$\begin{aligned}
W_{\mu\nu} &= \frac{1}{2} \sum_{\substack{\text{nucleon}\\ \text{spin}}} \sum_n \left(\frac{1}{e^2}\right) <p\,|J_\mu(0)|\,n><n\,|J_\nu(0)|\,p> \\
&\quad \times (2\pi)^3 \delta^{(4)}(p_n - p - q) \\
&= \frac{1}{2} \sum_{\substack{\text{nucleon}\\ \text{spin}}} \frac{1}{(2\pi e^2)} \int d^4x\, e^{-iq\cdot x} <p\,|\left[J_\mu(x), J_\nu(0)\right]|\,p> \\
&= \frac{1}{2} \sum_{\substack{\text{nucleon}\\ \text{spin}}} \left(\frac{1}{2\pi}\right) \int d^4x\, e^{-iq\cdot x} <p\,|\left[V_\mu(x), V_\nu(0)\right]|p> \,,
\end{aligned} \tag{36}$$

where the second term in the commutator is zero by energy conservation for $\nu > 0$.

By Lorentz and gauge invariance the tensor $W_{\mu\nu}$ may be written as

$$\begin{aligned}
W_{\mu\nu} &= W_1(\nu, q^2)\left(\delta_{\mu\nu} - q_\mu q_\nu/q^2\right) \\
&\quad + W_2(\nu, q^2)\left(p_\mu - p\cdot q\, q_\mu/q^2\right)\left(p_\nu - p\cdot q\, q_\nu/q^2\right)/M_N^2 \quad .
\end{aligned} \tag{37}$$

The quantity $W_{\mu\nu}$ is just $(1/4\pi^2\alpha)$ times the imaginary part of the Feynman amplitude for forward Compton scattering of virtual photons of mass$^2 = -q^2$. In terms of W_1 and W_2 the experimentally measured double differential cross section resulting from combining Eqs. (34), (35) and (37) is

$$\frac{d^2\sigma}{d\Omega' dE'} = \frac{4\alpha^2 E'^2}{q^4} \left[2W_1(\nu, q^2) \sin^2 \theta/2 + W_2(\nu, q^2) \cos^2 \theta/2 \right], \tag{38}$$

so that the structure functions W_1 and W_2, as they depend on ν and q^2, summarize the results of inelastic electron-nucleon scattering.

Now suppose that ν and q^2 are large, with ν/q^2 fixed. In the expression, Eq. (36), for $W_{\mu\nu}$ in terms of the Fourier transform of a commutator of two currents the exponential is

$$\begin{aligned} e^{-iq\cdot x} &= e^{-i(q_z z - q_0 t)} = e^{-i(\sqrt{q^2+\nu^2}\, z - \nu t)} \\ &\simeq e^{-i\nu(z-t)}\, e^{-i(q^2/2\nu)z} \end{aligned} \tag{39}$$

in this domain of ν and q^2 with $\vec{q}$ in the z direction. In order that the argument of the exponential not become large and produce cancelling oscillations of the integrand, the region of integration in configuration space must satisfy

$$\begin{aligned} z-t &\lesssim 0(1/\nu) \\ z &\lesssim 0(2\nu/q^2) \quad . \end{aligned} \tag{40}$$

Furthermore, since we want the commutator to be causal, it should vanish unless

$$x^2 = x_\perp^2 + z^2 - t^2 \leq 0 \quad , \tag{41}$$

which together with Eqs. (40) yields

$$x_\perp^2 \leq (t-z)(t+z) \lesssim 0(1/q^2) \quad . \tag{42}$$

Therefore, the important region of integration over the current commutator is $x^2 = x_\mu x_\mu \simeq 0$, i.e., along the light-cone (63).

To gain theoretical insight into the commutator of two currents on the light-cone we again turn to the quark model and abstract certain properties, particularly algebraic ones, of such commutators from the free field case. In the free quark model one finds (8,9)

$$
\begin{aligned}
\left[V_\mu^\alpha(x), V_\nu^\beta(0)\right] \underset{x^2 \to 0}{\sim} \Big\{ & i f^{\alpha\beta\gamma}\Big[\left(V_\nu^\gamma(x,0) + V_\nu^\gamma(0,x)\right) \delta_{\mu\lambda} \\
& + \left(V_\mu^\gamma(x,0) + V_\mu^\gamma(0,x)\right) \delta_{\nu\lambda} \\
& - \left(V_\lambda^\gamma(x,0) + V_\lambda^\gamma(0,x)\right) \delta_{\mu\nu} \\
& + i\, \epsilon_{\mu\nu\lambda\sigma}\left(A_\sigma^\gamma(x,0) - A_\sigma^\gamma(0,x)\right)\Big] \\
& + d^{\alpha\beta\gamma}\Big[\left(V_\nu^\gamma(x,0) - V_\nu^\gamma(0,x)\right) \delta_{\mu\lambda} \\
& + \left(V_\mu^\gamma(x,0) - V_\mu^\gamma(0,x)\right) \delta_{\nu\lambda} \\
& - \left(V_\lambda^\gamma(x,0) - V_\lambda^\gamma(0,x)\right) \delta_{\mu\nu} \\
& - i\, \epsilon_{\mu\nu\rho\sigma}\left(A_\sigma^\gamma(x,0) + A_\sigma^\gamma(0,x)\right)\Big]\Big\} \\
& \frac{1}{4\pi}\, \partial_\lambda \left[\epsilon(x_0)\, \delta(x^2)\right] ,
\end{aligned}
\tag{43}
$$

where $V_\mu^\alpha(x)$ and $A_\mu^\alpha(x)$ are the vector and axial vector currents and

$$V_\mu^\alpha(x,0) = :\bar{\psi}(x)\, (\lambda^\alpha/2)\, i\gamma_\mu\, \psi(0):$$

and (44)

$$A_\mu(x,0) = :\bar{\psi}(x)\, (\lambda^\alpha/2)\, i\gamma_\mu \gamma_5\, \psi(0):$$

are bilocal operators defined so that

$$V_\mu^\alpha(x, x) = V_\mu^\alpha(x)$$

and (45)

$$A_\mu^\alpha(x, x) = A_\mu^\alpha(x) \quad .$$

All the SU(3) properties are again summarized by the SU(3) structure constants $f^{\alpha\beta\gamma}$ and $d^{\alpha\beta\gamma}$. In fact, if we specialize to the time components of the currents and go to the tip of the light cone ($x_0=0$) then we recover the old equal time algebra of Gell-Mann (5).

An important property of Eq. (43) is that it factorizes into a product of a c-number singularity which contains no masses or other dimensional parameters and a bilocal operator which carries the SU(3) indices. When Eq. (43), as the leading light-cone singularity in the limit, ν, $q^2 \to \infty$, is inserted back in Eq. (36) for $W_{\mu\nu}$, it is seen that Fourier transforms of matrix elements of the bilocal turn out to be the structure functions and that W_1 and νW_2 scale, i.e., are functions of $\omega = 2M_N \nu / q^2 = -2p \cdot q/q^2$. Because of the structure of the tensor indices in Eq. (43), one also obtains the relation:

$$\frac{2M_N}{\omega} W_1(\omega) = \nu W_2(\omega) \quad , \tag{46}$$

which corresponds to the vanishing of the longitudinal relative to transverse photon-nucleon total cross sections ($\sigma_L/\sigma_T = 0$) in the indicated limit.

Another, more mundane, way to see that it is νW_2 and W_1 which should scale if there is no dimensionless parameter or scale in the virtual photon-nucleon interaction is to rewrite the double differential cross section in Eq. (38) in terms of the dimensionless variables

$$x = \frac{1}{\omega} = -q^2/2p\cdot q = q^2/2M_N\nu \tag{47}$$

and

$$y = \nu/E = p\cdot q/p\cdot k \quad . \tag{48}$$

Then in the high energy regime where ν and q^2 are both large, we can write

$$\frac{d^2\sigma}{dxdy} = \left(\frac{4\pi\alpha^2}{q^4}\right)\left(2M_N E\right)\left[(1-y)\,\nu W_2 + \frac{1}{2}y^2\, 2M_N x W_1\right] \quad , \tag{49}$$

where terms of order M_N/ν and M_N/E have been neglected. The factor $(4\pi\alpha^2/q^4)$ is just the elastic electron scattering cross section for a point particle. We see immediately that if the quantity in brackets, which involves the photon-nucleon interaction, is not to depend on some internal scale then νW_2 and $2M_N x W_1$ should only depend on a dimensionless variable involving the photon-nucleon vertex, i.e., they should depend on $x = -q^2/2p\cdot q = 1/\omega$.

The absence of any scale in the interaction of a virtual photon with a nucleon is realized explicitly in the parton model (64,65), which might also be regarded as a concrete representation of the light cone algebra. In the parton model one regards the nucleon as composed of point constituents. In an infinite momentum frame, each type (i) of parton, with charge Q_i (in units of e), is taken to have a distribution $f_i(x)$ in the fractional longitudinal momentum $x = p_z^{(\mathrm{parton})}/p_z^{(\mathrm{nucleon})}$. A straightforward calculation then shows that for spin 1/2 partons (64,65)

$$\nu W_2(x) = \sum_i Q_i^2\, x f_i(x) = 2M_N\, x W_1(x) \ , \tag{50}$$

where x is both the fractional longitudinal momentum of the struck parton and the value of the scaling variable $q^2/2M_N\nu$. Taking the partons to be quarks (and more generally, also antiquarks) one has a concrete representation of the quark light cone algebra. As with any particular representation of a given algebra, certain results may hold which do not follow necessarily in the general case.

The scaling behavior exhibited by the data (66,67) outside the region of prominent resonances for νW_2 and $2M_N W_1$ is shown in Fig. 2. There the values of the structure functions are plotted versus $\omega' = 1+W^2/q^2 = \omega + M_N^2/q^2$, which is the same as ω is the limit $\nu, q^2 \to \infty$. Clearly values of νW_2 and $2M_N W_1$ at the same ω' but different q^2 coincide, i.e., νW_2 and $2M_N W_1$ for the proton are functions of ω' to within the accuracy of the data for $q^2 > 1$ GeV^2 and $1 < \omega' < 10$.

The validity of the relation $\nu W_2 = 2M_N\, x W_1$ as $\nu, q^2 \to \infty$ is more clearly examined in terms of the quantity $R = \sigma_L/\sigma_T$, which should then vanish as $\nu, q^2 \to \infty$ at fixed ω. A previous global average of R for the proton gave the value (67) $R_p = 0.18 \pm 0.10$. Newer data, but over essentially the same kinematic range, has recently been analyzed and yields (68) a global average in agreement with this. More interestingly, there is some indication (68) that R_p is vanishing as $1/\nu$ for fixed values of $\omega \lesssim 5$. The analysis of the deuteron data taken in the same experiment shows (68) that $R_p = R_d = R_n$ to within the statistical errors of the measurement (± 0.04).

The ratio of neutron to proton inelastic cross sections, as extracted from deuterium data, shows consistency with the neutron structure functions scaling also (69,70). The ratio of n/p decreases from values near unity at small x (and large $\omega = 1/x$) to values (70) definitely below 1/2 for $x > 0.65$.

For values of x near one, where it appears that the n/p ratio is the smallest, one could hope to challenge the bounds from the quark light-cone algebra (71):

$$\frac{1}{4} \leq n/p \leq 4 \quad . \tag{51}$$

Although present data extends to $x \simeq 0.8$ and doesn't indicate any violation of the lower bound, experiments are under way to investigate the region near x=1 in considerable detail.

Also of interest is the region of small x or large ω where the n/p ratio is expected to eventually approach unity on the basis of the dominance of the Pomeranchuk singularity in the photon-nucleon amplitude at large values of ω. Some recent data (72) on this ratio is shown in Fig. 3 where it is seen that even for values of ω between 10 and 20 the value of n/p is still only ~0.85 and only slowly approaching unity. This bears directly on the convergence of the sum rule (73)

$$\int_1^\infty \frac{d\omega}{\omega} \left[\nu W_{2p}(\omega) - \nu W_{2n}(\omega)\right] = \frac{1}{3} \quad . \tag{52}$$

This can be derived in parton models where the nucleon is composed of three "valence" quarks plus an isoscalar "sea" of $q\bar{q}$ pairs (plus neutrals), or it can be derived using exchange degeneracy arguments (74), but it does not follow generally from the quark light cone algebra. Earlier evaluations of the sum rule using then existing data and Regge extrapolations for $\nu W_{2p}(\omega) - \nu W_{2n}(\omega)$ as $\omega \to \infty$ gave the estimate 0.19 ± 0.06 for the left hand side. The data shown in Fig. 3, however, gives (72)

$$\int_1^{20} \frac{d\omega}{\omega} \left[\nu W_{2p}(\omega) - \nu W_{2n}(\omega)\right] = 0.18 \pm 0.04 \quad , \tag{53}$$

and a rough estimate of the contribution from $\omega = 20$ to ∞ is 0.09. There is no longer an experimental basis for worrying about the sum rule's validity.

C. INELASTIC NEUTRINO- AND ANTINEUTRINO-NUCLEON SCATTERING

The extension of our discussion to inelastic neutrino and antineutrino scattering is easily made. Again neglecting lepton masses and averaging over nucleon spins, the double differential cross section is (75)

$$\frac{d^2\sigma(\nu/\bar{\nu})}{d\Omega' \, dE'} = \frac{G^2 E'^2}{2\pi^2} \Big[2 \sin^2\theta/2 \; W_1^{(\nu/\bar{\nu})}(\nu, q^2) + \cos^2\theta/2 \; W_2^{(\nu/\bar{\nu})}(\nu, q^2) \mp \frac{E+E'}{M_N} \sin^2\theta/2 \; W_3^{(\nu/\bar{\nu})}(\nu, q^2) \Big] \, , \tag{54}$$

with $G \simeq 1.0 \times 10^{-5}/M_N$ the weak coupling constant. In the high energy limit, using the same variables x and y in Eqs. (47) and (48) as before, this becomes

$$\frac{d^2\sigma^{(\nu/\bar{\nu})}}{dxdy} = \left(\frac{G^2 M_N E}{\pi}\right) \left[(1-y) \, \nu W_2 + \frac{y^2}{2} (2M_N x) \, W_1 \mp y\left(1 - \frac{1}{2}y\right) x \, \nu W_3\right] . \tag{55}$$

The structure functions W_1 and W_2 now involve Fourier transforms of both vector-vector and axial vector-axial vector current commutators, while the new structure function W_3 involves only vector-axial vector commutators. Neglecting strangeness changing currents, the restriction of the weak, strangeness non-changing current to have isospin one implies that

$$
\begin{aligned}
W_i^{\nu p} &= W_i^{\bar{\nu} n} \\
W_i^{\nu n} &= W_i^{\bar{\nu} p} \quad .
\end{aligned}
\tag{56}
$$

The quark light cone algebra in Eq. (43) is also simply extended (8, 9) to include commutators of two axial-vector currents or a vector and axial-vector current. As an immediate consequence it follows that $2M_N\, x W_1$, νW_2, and νW_3 should scale, as can also be seen directly from Eq. (55) using the argument that if the weak current-nucleon interaction doesn't depend on parameters with dimensions, then the quantity in brackets on the right-hand side should only be dependent on the dimensionless quantity x. Scaling of $W_1 = F_1(x)$, $\nu W_2 = F_2(x)$, and $\nu W_3 = F_3(x)$ implies on integrating Eq. (55) first over y and then over x that

$$
\sigma_{TOT}^{(\nu/\bar{\nu})} = \frac{G^2 M_N E}{\pi} \int_0^1 dx \left[\frac{F_2(x)}{2} + \frac{2Mx\, F_1(x)}{6} \mp \frac{xF_3(x)}{6} \right] , \tag{57}
$$

i.e., that the total neutrino or antineutrino cross section rises linearly with the incident beam energy, E.

From the quark light cone algebra we have furthermore that (8, 9, 75)

$$
2M_N x\, F_1(x) = F_2(x) \quad , \tag{58}
$$

and the local relation between inelastic electron and neutrino scattering:

$$
6 \left[F_2^{ep}(x) - F_2^{en}(x) \right] = x \left[F_3^{\nu p}(x) - F_3^{\nu n}(x) \right] \; . \tag{59}
$$

Various sum rules follow as well, including

$$\int_0^1 \frac{dx}{x} \left[F_2^{\nu n}(x) - F_2^{\nu p}(x)\right] = 2 \quad , \tag{60}$$

the Adler sum rule (76), which actually is supposed to hold for all q^2, and the sum rule (77)

$$\int_0^1 dx \left[F_3^{\nu p}(x) + F_3^{\nu n}(x)\right] = -6 \quad . \tag{61}$$

In a parton model with only fermion constituents which interact with the current (no antifermions) one has as well that

$$F_2(x) = 2M_N x\, F_1(x) = -x\, F_3(x) \quad , \tag{62}$$

i.e., maximal V-A interference.

The simplest quantity with which to compare theory and experiment is the total cross section summed over neutrino and antineutrino beams. From Eq. (57) we find that the F_3 term cancels in the sum and using $F_2(x) = 2M_N\, x\, F_1(x)$ yields

$$\sigma_{TOT}^{\nu N}(E) + \sigma_{TOT}^{\bar{\nu} N}(E) = \frac{G^2 M_N E}{\pi}\, \frac{4}{3} \left[\frac{1}{2} \int_0^1 dx \left(F_2^{\nu N}(x) + F_2^{\bar{\nu} N}(x)\right)\right] \quad . \tag{63}$$

We write νN ($\bar{\nu} N$) to denote an average over neutrino (antineutrino) cross sections on protons and neutrons. Of course,

$$F_2^{\nu N}(x) = \frac{1}{2}\left(F_2^{\nu p}(x) + F_2^{\nu n}\right) = \frac{1}{2}\left(F_2^{\bar{\nu} p}(x) + F_2^{\bar{\nu} n}\right) = F_2^{\bar{\nu} N}(x) \quad . \tag{64}$$

The data from the Gargamelle experiment (78) are quite consistent with a linear rise with E of $\sigma_{TOT}(E)$ for both neutrinos and antineutrinos. The coefficients of $G^2 M_N E/\pi$ yield (78)

$$\frac{1}{2}\int_0^1 dx \left(F_2^{\nu N}(x) + F_2^{\bar{\nu} N}(x)\right) = 0.47 \pm 0.07 \ . \tag{65}$$

To relate this to electron scattering we must make some additional assumption beyond just the light cone algebra. We assume that in the x region which gives the most important contribution to $\int_0^1 dx\, F_2(x)$, one has only quark partons (no antiquarks). As we will see in a moment there is independent support for this from the ratio $\sigma_{TOT}^{\bar{\nu}N}/\sigma_{TOT}^{\nu N}$. With no antiquarks, one has the relation (79)

$$\frac{1}{2}\left(F_2^{\nu N}(x) + F_2^{\bar{\nu} N}(x)\right) = \frac{18}{5}\left(\frac{1}{2}\right)\left(F_2^{ep}(x) + F_2^{en}(x)\right) \ , \tag{66}$$

which together with the result from SLAC data (with a minor extrapolation) (66,67),

$$\frac{1}{2}\int_0^1 dx \left(F_2^{ep}(x) + F_2^{en}(x)\right) = 0.15 \pm 0.01 \quad , \tag{67}$$

predicts that

$$\frac{1}{2}\int dx \left(F_2^{\nu N}(x) + F_2^{\bar{\nu} N}(x)\right) = 0.54 \pm 0.04 \quad . \tag{68}$$

The agreement with the direct measurement, Eq. (65), is obviously very good (80).

Now let us return to $\sigma_{TOT}(E)$ for neutrinos and antineutrinos separately. Rewriting Eq. (57) we have

$$\sigma_{TOT}^{(\nu/\bar{\nu})}(E) = \frac{G^2 ME}{\pi}\int_0^1 dx\, F_2(x)\left[\frac{1}{2} + \frac{1}{6} \pm \frac{B}{3}\right] \ , \tag{69}$$

and therefore

$$\sigma_{TOT}^{\bar{\nu}N}/\sigma_{TOT}^{\nu N} = (2-B)/(2+B) \qquad (70)$$

where

$$B = -\int_0^1 dx\, xF_3(x)/\int_0^1 dx\, F_2(x) \quad . \qquad (71)$$

Purely from kinematic inequalities $|xF_3| \leq F_2(x)$ or $|B| \leq 1$. The extreme values of B correspond to maximal V-A interference and are met for purely fermion partons (B = +1) or purely antifermion partons (B = -1). The experimental value of (78)

$$\sigma_{TOT}^{\bar{\nu}N}/\sigma_{TOT}^{\nu N} = 0.38 \pm 0.02 \qquad (72)$$

gives (81)

$$B = 0.90 \pm 0.04 \quad , \qquad (73)$$

and indicates almost purely fermion constituents in the region accessible to the Gargamelle experiment. Everything is quite consistent with the quark light cone algebra or the even more restrictive quark parton model with only a small component of antiquarks.

In the past few months the first data on inelastic neutrino scattering at NAL have been reported. The Caltech experiment (82) uses a "narrow band" beam with neutrinos of average energies of 50 and 145 GeV arising from decays of 160 GeV pions and kaons, respectively. While there are only 112 neutrino events from a steel target reported, they already allow some tentative conclusions (82). If one assumes $2M_N xF_1(x) = F_2(x) = -xF_3(x)$, then Eq. (55) becomes

$$\frac{d^2\sigma^{\nu N}}{dxdy} = \frac{G^2 M_N E}{\pi} F_2^{\nu N}(x) \tag{74}$$

which is independent of y. Within large errors, the data are consistent with y independence. Although the flux is not known accurately, one can integrate over y and <u>compare the shape</u> of $d\sigma^{\nu N}/dx$ with what is seen at Gargamelle, or the better determined $F_2^{ep} + F_2^{en} \simeq F_2^{ed}$. This is shown in Fig. 4, with consistency seen between the x distribution measured with high energy neutrinos and that measured with electrons at SLAC.

A more stringent test of scaling is provided by the quantity

$$\langle q^2 \rangle = \langle xy\, 2M_N E \rangle = 2M_N E \langle xy \rangle \quad , \tag{75}$$

where

$$\langle f(x,y) \rangle = \frac{\int_0^1 \int_0^1 dx\, dy\, f(x,y) \frac{d^2\sigma}{dxdy}}{\int_0^1 \int_0^1 dx\, dy \frac{d^2\sigma}{dxdy}} \quad . \tag{76}$$

$\langle q^2 \rangle$ should therefore rise linearly with E if there is scaling. The comparison with the results from the Caltech experiment for $\langle q^2 \rangle$ are shown in Fig. 5, where the curves are computed using the acceptance of the apparatus and assuming

$$\frac{d^2\sigma}{dxdy} = \frac{G^2 M_N E}{\pi} \frac{F_2(x)}{(1 + q^2/\Lambda^2)^2} \tag{77}$$

Again the data are consistent with $\Lambda = \infty$, i.e., scaling. Note the large values of $\langle q^2 \rangle$ seen at NAL energies — in itself an indication of a "point-like" interaction.

A second experiment with a "broad band" beam of mean energy ~ 50 GeV has been carried out by a Harvard-Pennsylvania-Wisconsin collaboration (83,84). With about 300 neutrino and antineutrino events they are able to state that (84) $\sigma_{TOT}^{\nu N} + \sigma_{TOT}^{\bar{\nu} N}$ is very roughly ten times larger at a mean energy of 50 GeV than it is at 5 GeV in the Gargamelle experiment, just as expected from scaling (Eq. (57)). The ratio $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ lies between 1/3 and 1/2 and is therefore consistent with the Gargamelle result of 0.38 ± 0.02. Noting that if

$$v = xy = 2E' \sin^2 \theta/2 / M_N \quad , \tag{78}$$

then (85) $\frac{1}{N}\frac{dN}{dv}$ is independent of flux and is a function of v alone if the structure functions scale. Analysis of their data shows that (84) both their neutrino and antineutrino data are consistent with scaling, and more particularly, with the $\frac{1}{N}\frac{dN}{dv}$ curves calculated on the basis of the SLAC electron scattering data and the quark parton model.

Thus both high energy neutrino experiments seem to show that while we have increased the beam energies by an order of magnitude or better, nothing striking has changed from what was learned with Gargamelle. Everything seems remarkably consistent with the rather simple picture of scaling embodied in the quark light cone algebra and the quark parton model.

D. ELECTRON-POSITRON ANNIHILATION INTO HADRONS

There is one major indication of trouble with this simple picture: electron-positron annihilation into hadrons via one photon. The cross section for $e^+e^- \to$ hadrons is directly proportional to $\int d^4x\, e^{-iq\cdot x} <0|\left[J_\mu(x), J_\nu(0)\right]|0>$. As a result, in the e^+e^- center-of-mass where $\vec{q}=0$ and $q_0 = \sqrt{|q^2|}$, the limit $q_0 \to \infty$ or $|q^2| \to \infty$ implies that $x_0 \approx 0$ and $\vec{x} \approx 0$, i.e., the tip of the light cone, is the important region of integration in that limit. As a result, assuming the quark model also gives the disconnected part of the commutator on the light cone, one finds

$$\sigma(e^+e^- \to \text{hadrons}) = \left(\frac{4\pi\alpha^2}{3|q^2|}\right) \sum_i Q_i^2 \quad , \tag{79}$$

where the sum is over the charged quark pairs creatable by the electromagnetic current. The quantity $(4\pi\alpha^2/3|q^2|)$ is just the cross section for $e^+e^- \to \mu^+\mu^-$. Three quark constituents with charges 2/3, -1/3, and -1/3 yield $\sum_i Q_i^2 = 2/3$, while colored quarks (16) give $\sum_i Q_i^2 = 2$, and the Han-Nambu integrally charged quark scheme (86) has $\sum_i Q_i^2 = 4$. The present experimental situation is seen (87) in Fig. 6, including the most recent CEA results (88) for $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ of 4.8 ± 1.1 and 6.3 ± 1.5 at $|q^2| = 16$ and 25 GeV^2, respectively.

The seeming disagreement with our expectations leads one to ask whether there is a breakdown in the scaling behavior (89), i.e., whether $\sigma(e^+e^- \to \text{hadrons})$ does not behave as $1/q^2$ as $|q^2| \to \infty$. A careful reexamination of the electron and neutrino data is also called for, particularly to look for the behavior (90) of the moments $\int_0^1 dx\, x^n\, F_2(x)$ as powers of $(1/\ln q^2)$, as suggested in asymptotically free gauge theories (91). Most important experimentally, one awaits the results from SPEAR and DORIS on the magnitude of $\sigma(e^+e^- \rightarrow \text{hadrons})$ at the same as well as still higher values of $|q^2|$.

VII. INELASTIC LEPTON SCATTERING WITH POLARIZED BEAMS AND TARGETS

A. INTRODUCTION

In the previous lectures we discussed theoretical ideas and phenomenology which were mostly related to experiments already completed or in the process of being carried out. In this lecture we turn to a subject where no experiments have yet been accomplished, but where much of the previously discussed theoretical superstructure has immediate application.

In the near future the possibility of performing experiments with both polarized lepton beams and polarized targets will become a reality. The muon beams at BNL, CERN, or NAL automatically possess a longitudinal polarization because of their origin in the weak decays of pions. At SLAC a polarized electron source is being installed. The planned experiment (92) on deep inelastic scattering will be a technological tour de force involving production of a polarized lithium atomic beam, an ultraviolet flash lamp operating at 180 pps to knock off the electrons, injection into the accelerator and acceleration without loss of polarization, magnetic bending of the beam into the end station at which point g-2 of the electron demands the energy be a multiple of 3.22 GeV so that the spin is rotated by a multiple of 180°, and finally the beam striking a longitudinally polarized proton target operating at 1°K in the field of a superconducting magnet.

With this exciting background of approaching experiments, in this lecture I would like to discuss some of the theory and phenomenology related to inelastic lepton scattering with polarized beams and targets. As we shall see, a number of interesting questions having to do with amplitude analysis, scaling, parton models or light cone algebra, sum rules, quark models for resonance excitation, and even exotic J-plane singularity structure are subject to direct experimental answer with polarized beams and targets.

B. POLARIZED BEAMS

The scattering of a longitudinally polarized lepton beam on an unpolarized target should be independent of the lepton polarization if no hadrons are detected in the final state. This follows from parity conservation, and while this is well checked at small q^2 in the electromagnetic interactions of hadrons, an experiment (93) is planned at SLAC to verify this at large q^2 in deep inelastic scattering. Of course, if such a violation is found it is of enormous interest (due to weak interactions at an unexpected level?) and the theoretical questions discussed in the remainder of this talk become of secondary interest.

From this point on let us assume one photon exchange, together with the C, P and T invariance of electromagnetism. We use the kinematics and notation (61, 62) already given (see Fig. 1).

Suppose a final hadron (four-momentum p'_μ) is detected. Then $\vec{k}$ and $\vec{k}'$ define a plane (the lepton plane), as do $\vec{q}$ and $\vec{p}'$ (the hadron plane), with an angle ϕ between their normals.

Now recall for a moment the case of an unpolarized beam. There, to calculate a cross section we must evaluate a lepton trace, $L_{\mu\nu}$, which is essentially the virtual photon density matrix. If we take the direction $\vec{q}$ as a z-axis, with x-axis in the lepton plane (so that the positive x direction is toward the leptons) then $L_{\mu\nu} = (1/2)(k_\mu k'_\nu + k'_\mu k_\nu + (q^2/2)\,\delta_{\mu\nu})$ can be written as

$$L_{\mu\nu} = \left(\frac{2}{1-\epsilon}\right)\left(\frac{q^2}{4}\right)\left\{\begin{pmatrix}\sqrt{\frac{1}{2}(1+\epsilon)}\\ 0\\ -\sqrt{q^2\epsilon/\nu^2}\\ 0\end{pmatrix}\begin{pmatrix}\sqrt{\frac{1}{2}(1+\epsilon)} & 0 & -\sqrt{q^2\epsilon/\nu^2} & 0\end{pmatrix} + \begin{pmatrix}0\\ \sqrt{\frac{1}{2}(1-\epsilon)}\\ 0\\ 0\end{pmatrix}\begin{pmatrix}0 & \sqrt{\frac{1}{2}(1-\epsilon)} & 0 & 0\end{pmatrix}\right\} \tag{80}$$

where

$$\epsilon = \frac{1}{1 + 2\left(1 + \frac{\nu^2}{q^2}\right)\tan^2\theta/2} \tag{81}$$

with θ = the lepton scattering angle in the laboratory. We have subtracted multiples (94) of q_μ from the virtual photon polarization vector so as to make $L_{\mu\nu} = 0$ when μ or $\nu = 4$. As is seen from Eq. (80), the virtual photon is the incoherent sum of a piece with linear polarization in the x-z plane and a piece with linear polarization in the y-direction.

If $\epsilon_\mu^{(\gamma)} f_\mu$ is the amplitude for

$$\text{"}\gamma(q)\text{"} + N(p) \rightarrow \text{hadron}'\,(p') + \ldots \quad , \tag{82}$$

with the property

$$q_\mu f_\mu = 0 \quad , \tag{83}$$

then from Eq. (80) we have that

$$\frac{d\sigma}{d^3p'} \propto \left| \sqrt{\frac{1}{2}(1+\epsilon)}\, f_x - \sqrt{q^2\epsilon/\nu^2}\, f_z \right|^2 + \left| \sqrt{\frac{1}{2}(1-\epsilon)}\, f_y \right|^2$$

$$\propto \frac{1}{2}\left(|f_x|^2 + |f_y|^2\right) + \frac{\epsilon}{2}\left(|f_x|^2 - |f_y|^2\right)$$

$$+ \epsilon\left(q^2 |f_z|^2/\nu^2\right) - \sqrt{\frac{1}{2}\epsilon(1+\epsilon)}\; 2\,\mathrm{Re}\left(f_x \sqrt{q^2/\nu^2}\, f_z^*\right) . \qquad (84)$$

The four terms in Eq. (84) are in the form of the standard equation for, e.g., "γ" + p $\to \pi^+$ + n, where they are often labelled (95) $d\sigma_U/dt$, $d\sigma_P/dt$, $d\sigma_L/dt$, and $d\sigma_I/dt$ respectively, after their ϕ dependence is explicitly exhibited. This ϕ dependence is easily read off if f_x, f_y and f_z are rewritten in terms of helicity amplitudes (of the virtual photon). The four terms behave respectively as constant, cos 2ϕ, constant, and cos ϕ.

With this background, consider a longitudinally polarized beam where the leptons have helicity $\pm 1/2$. The lepton trace now reads

$$L_{\mu\nu}^{(\pm)} = \frac{1}{2}\left(k_\mu k'_\nu + k'_\mu k_\nu + (q^2/2)\,\delta_{\mu\nu} \pm \epsilon_{\mu\nu\lambda\sigma} k_\lambda k'_\sigma\right) , \qquad (85)$$

which can be rewritten with $\vec{q}$ as a z-axis, using the same manipulations as above, in the form (94)

$$L_{\mu\nu}^{(\pm)} = \left(\frac{2}{1-\epsilon}\right)\left(\frac{q^2}{4}\right)\left\{\begin{pmatrix} \sqrt{\frac{1}{2}(1+\epsilon)} \\ \mp i\sqrt{\frac{1}{2}(1-\epsilon)} \\ -\sqrt{q^2\,\epsilon/\nu^2} \\ 0 \end{pmatrix} \overbrace{\underbrace{\begin{pmatrix} \sqrt{\frac{1}{2}(1+\epsilon)} & \pm i\sqrt{\frac{1}{2}(1-\epsilon)} & -\sqrt{q^2\,\epsilon/\nu^2} & 0 \end{pmatrix}}}\right\} .$$

(86)

The virtual photon now can be represented in terms of a single elliptical polarization state. For incident leptons with helicity ±1/2, the analogue of Eq. (84) for detection of a final hadron is (summed over the helicities of all hadrons):

$$
\begin{aligned}
\frac{d\sigma}{d^3p'} &\propto \left| \sqrt{\frac{1}{2}(1+\epsilon)}\ f_x \pm i\ \sqrt{\frac{1}{2}(1-\epsilon)}\ f_y - \sqrt{q^2\epsilon/\nu^2}\ f_z \right|^2 \\
&\propto \frac{1}{2}\left(|f_x|^2 + |f_y|^2\right) \\
&\quad + \frac{\epsilon}{2}\left(|f_x|^2 - |f_y|^2\right) + \epsilon\left(q^2|f_z|^2/\nu^2\right) \\
&\quad - \sqrt{\frac{1}{2}\epsilon(1+\epsilon)}\ 2\,\mathrm{Re}\left(f_x\sqrt{q^2/\nu^2}\ f_z^*\right) \\
&\quad \pm \sqrt{\frac{1}{2}\epsilon(1-\epsilon)}\ 2\,\mathrm{Im}\left(f_y\sqrt{q^2/\nu^2}\ f_z^*\right) \quad ,
\end{aligned}
\tag{87}
$$

where we have gained the last term when compared with Eq. (84). An apparent term of the form $\sqrt{(1/4)\,(1-\epsilon^2)}\ 2\,\mathrm{Im}\ (f_x f_y^*)$ vanishes because of parity conservation, which becomes obvious when the amplitudes are rewritten in terms of (photon) helicity amplitudes (96). In this form the ϕ dependence of the last term in Eq. (87) is also easily seen to be that of $\sin\phi$.

Since the (complex) amplitudes f_x, f_y, and f_z are characterized by six real quantities, with one overall phase, there are five real numbers which give all possible physical information on the amplitude f_μ. As there are five independent quantities in Eq. (87), we see that <u>all the physics contained in f_μ is obtainable with a polarized beam</u>.

Particular cases of interest in the literature are (97)

$$"\gamma" + N \rightarrow \pi + \text{anything} \quad , \tag{88}$$

and (98, 99)

$$"\gamma" + N \rightarrow \rho^o + N \quad , \tag{89}$$

where one also has information on the ρ^o density matrix coming from its two pion decay mode. It has recently been shown (98) explicitly that all possible joint density matrix elements of the virtual photon and rho meson are determinable through use of a polarized lepton beam and examination of the rho decay angular distribution. In short, the use of a polarized lepton beam alone allows one to perform complete amplitude analyses of the photon segment of virtual photon induced processes.

C. POLARIZED BEAM AND TARGET KINEMATICS

As in the unpolarized case, there are two equivalent ways to proceed, each particularly useful for certain purposes. One method is to define virtual photon-nucleon total cross sections; the other is to define structure functions. We begin with the definition of total cross sections.

First we recall the unpolarized case (62), where there are two independent photon-nucleon total cross sections, σ_T and σ_L. By the optical theorem they are proportional to the imaginary part of the forward photon-nucleon scattering amplitude, $T_{\lambda'_\gamma \lambda'_N, \lambda_\gamma \lambda_N}(\nu, q^2, t{=}0)$:

$$\sigma_T(\nu, q^2) \propto \frac{1}{2}\left(\mathrm{Im}\, T_{1\frac{1}{2}, 1\frac{1}{2}} + \mathrm{Im}\, T_{1-\frac{1}{2}, 1-\frac{1}{2}}\right) \tag{100a}$$

and

$$\sigma_L(\nu, q^2) \propto \mathrm{Im}\, T_{0\frac{1}{2},0\frac{1}{2}} \quad . \tag{100b}$$

In fact, the usual tensor, $W_{\mu\nu}$, is also proportional to Im $T_{\mu\nu}$, and the $\mu, \nu = 1, 2, 3$ (z axis along $\vec{q}$) components are (100)

$$W_{ij} = \frac{K}{4\pi^2 \alpha} \begin{pmatrix} \sigma_T & 0 & 0 \\ 0 & \sigma_T & 0 \\ 0 & 0 & \nu^2 \sigma_L / q^2 \end{pmatrix} , \tag{101}$$

where $K = \nu - q^2/2M_N$ is the real photon (laboratory) energy necessary to produce the same center-of-mass energy. The double differential cross section for detecting a final lepton of energy E' if the incident beam has energy E is given by Eq. (34) as

$$\frac{d^2\sigma}{d\Omega' dE'} = \left(\frac{1}{2\pi}\right)^2 \left(\frac{E'}{E}\right) \left(\frac{4\pi\alpha}{q^2}\right)^2 L_{\mu\nu} W_{\mu\nu} \quad ,$$

so that using the lepton trace in Eq. (80) we have

$$\begin{aligned} \frac{d^2\sigma}{d\Omega' dE'} &= \frac{\alpha}{4\pi^2} \frac{K}{q^2} \left(\frac{E'}{E}\right)\left(\frac{2}{1-\epsilon}\right) (\sigma_T + \epsilon\sigma_L) \\ &= \Gamma\, (\sigma_T + \epsilon\sigma_L) \quad , \end{aligned} \tag{102}$$

which is the standard result.

In the polarized beam and target case, one is able to measure separately the total cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ (net photon plus nucleon value of J_z equal to 1/2 and 3/2 respectively) and σ_{TL} defined by (101)

$$\sigma_{1/2}(\nu, q^2) \propto \mathrm{Im}\ T_{1\frac{1}{2},1\frac{1}{2}} \quad ,$$

$$\sigma_{3/2}(\nu, q^2) \propto \mathrm{Im}\ T_{1-\frac{1}{2},1-\frac{1}{2}} \quad , \tag{103}$$

$$\sqrt{2}\sigma_{TL}(\nu, q^2) \propto \mathrm{Im}\ T_{1\frac{1}{2},0-\frac{1}{2}} \quad ,$$

with the same proportionality constant as in Eq. (100), so that

$$\sigma_T = \frac{1}{2}(\sigma_{1/2} + \sigma_{3/2}) \quad . \tag{104}$$

Therefore, there are two new measurable quantities, which we take to be $(1/2)(\sigma_{1/2} - \sigma_{3/2})$ or the asymmetry,

$$A(\nu, q^2) = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{\frac{1}{2}(\sigma_{1/2} - \sigma_{3/2})}{\sigma_T} \quad , \tag{105}$$

and σ_{TL}. For these quantities we have the bounds (102)

$$\left|\frac{1}{2}(\sigma_{1/2} - \sigma_{3/2})\right| \leq \sigma_T \quad \text{or} \quad |A| \leq 1 \quad , \tag{106}$$

and

$$\sqrt{2}\,\sigma_{TL} \leq \sqrt{\sigma_{1/2}\sigma_L} \quad \text{or} \quad \sigma_{TL} \leq \sqrt{\sigma_T\sigma_L} = \sqrt{R}\,\sigma_T \quad . \tag{107}$$

It is not difficult to see from our definitions that if the nucleon target was polarized along $\pm\vec{q}$, only $\sigma_{1/2} - \sigma_{3/2}$ would contribute to the difference, while with polarization perpendicular to $\vec{q}$ (but still in the lepton plane), only σ_{TL} contributes to a difference between target spin orientations. However, in real life it is far easier to orient the target relative to the lepton beam, and not $\vec{q}$. For a nucleon target polarized antiparallel or parallel to the beam we find (103),

$$\frac{d^2\sigma\uparrow\downarrow}{d\Omega' dE'} - \frac{d^2\sigma\uparrow\uparrow}{d\Omega' dE'} = 2\Gamma\left\{\left(\frac{E-E'\epsilon}{E}\right) A\sigma_T + \frac{\sqrt{q^2}}{E}\epsilon\sigma_{TL}\right\}, \tag{108}$$

to be compared with the unpolarized result,

$$\frac{d^2\sigma\uparrow\downarrow}{d\Omega' dE'} + \frac{d^2\sigma\uparrow\uparrow}{d\Omega' dE'} = 2\Gamma\left(\sigma_T + \epsilon\sigma_L\right) . \tag{109}$$

If we neglect σ_L and σ_{TL} as being small (104), then we have that

$$\frac{\dfrac{d^2\sigma\uparrow\downarrow}{d\Omega' dE'} - \dfrac{d^2\sigma\uparrow\uparrow}{d\Omega' dE'}}{\dfrac{d^2\sigma\uparrow\downarrow}{d\Omega' dE'} + \dfrac{d^2\sigma\uparrow\uparrow}{d\Omega' dE'}} = \left(\frac{E-E'\epsilon}{E}\right) A(\nu, q^2) = \left(\frac{E-E'\epsilon}{E}\right)\frac{\sigma_{1/2}-\sigma_{3/2}}{\sigma_{1/2}+\sigma_{3/2}} . \tag{110}$$

This simple result shows that the virtual photon-nucleon asymmetry is "degraded" by the factor $(E-E'\epsilon)/E$ in order to obtain the lepton beam-target asymmetry which is actually measured.

It is also possible to polarize the target perpendicular to the beam, but in the lepton plane. In this case we find (105)

$$\frac{d^2\sigma\uparrow\leftarrow}{d\Omega' dE'} - \frac{d^2\sigma\uparrow\rightarrow}{d\Omega' dE'} = 2\Gamma\left\{\sqrt{\frac{1}{2}}\,\epsilon(1+\epsilon)\,\frac{\sqrt{q^2}}{E}\,A\sigma_T - \sqrt{\frac{2\epsilon}{1+\epsilon}}\,\frac{E-E'\epsilon}{E}\,\sigma_{TL}\right\} . \tag{111}$$

Under presently contemplated experimental conditions, the $A\sigma_T$ term has an order of magnitude greater weight than the σ_{TL} term in Eq. (108), while the situation is reversed in Eq. (111). Thus, while it is possible in principle to separate $A\sigma_T$ and σ_{TL} by only running with the target polarized parallel or antiparallel to the beam at different values of E, E', and θ (but the same ν and q^2), in practice this is very difficult.

The use of perpendicular polarization (Eq. (111)) makes the task of separation much easier (106, 107).

Isolation of σ_{TL} may be of some importance in obtaining information on $R = \sigma_L/\sigma_T$. As R is known to be small in deep inelastic scattering, it is difficult to measure accurately. Equation (107), however, shows that measurement of σ_T and σ_{TL} will give a lower bound on R. For the particular case of elastic scattering, where $\sigma_{TL} \propto G_M G_E$, measurement of σ_{TL} may provide the only way to study G_E at large q^2.

Just as there were two new cross sections to define in the spin dependent case, there are two additional structure functions besides the familiar W_1 and W_2. We write (108),

$$
\begin{aligned}
W_{\mu\nu} = {} & W_1(\nu, q^2)\left(\delta_{\mu\nu} - q_\mu q_\nu/q^2\right) \\
& + W_2(\nu, q^2)\left(p_\mu - p\cdot q\, q_\mu/q^2\right)\left(p_\nu - p\cdot q\, q_\nu/q^2\right)/M_N^2 \\
& - d(\nu, q^2)\ \epsilon_{\mu\nu\lambda\sigma} q_\lambda s_\sigma/4\pi M_N \\
& + g(\nu, q^2)\ s\cdot q\ \epsilon_{\mu\nu\lambda\sigma}\, q_\lambda p_\sigma/4\pi M_N \quad ,
\end{aligned}
\tag{112}
$$

where s_μ is a covariant spin four-vector associated with the nucleon target and possessing the properties

$$
s\cdot p = 0, \qquad s\cdot s = +1 \quad . \tag{113}
$$

In terms of the structure functions defined in Eq. (112), the measured difference between double differential cross sections with lepton and nucleon spins antiparallel and parallel is (103)

$$\frac{d^2\sigma\uparrow\downarrow}{d\Omega' dE'} - \frac{d^2\sigma\uparrow}{d\Omega' dE'} = \frac{4\alpha^2 E'}{q^2 E}\left\{(E+E'\cos\theta)\, d(\nu, q^2) + (E-E'\cos\theta)(E+E')\, M_N g(\nu, q^2)\right\}/(4\pi M_N) \quad . \tag{114}$$

The catalogue is completed by giving the relationship of the structure functions to the previously defined total cross sections:

$$\frac{1}{4\pi M_N}\left[\nu\, d(\nu, q^2) + M_N(\nu^2+q^2)\, g(\nu, q^2)\right] = \frac{K}{4\pi^2\alpha}\, \frac{\sigma_{1/2}-\sigma_{3/2}}{2}$$

$$= \frac{K}{4\pi^2\alpha} A\sigma_T \quad , \tag{115a}$$

and

$$\frac{\sqrt{q^2}}{4\pi M_N}\, d(\nu, q^2) = \frac{K}{4\pi^2\alpha}\sigma_{TL} \quad . \tag{115b}$$

These allow one, for example, to go directly from Eq. (108) to (114).

The particular choice of structure functions given in Eq. (112) is of course not a unique one. In fact, each paper on this subject seems to define a new set, which can only be linear combinations of the ones given here. As will be seen in the next section, $d(\nu, q^2)$ and $g(\nu, q^2)$ are slightly more convenient when one wishes to use the inequalities for total cross sections, Eqs. (106) and (107), to bound the integrands in certain sum rules.

D. SCALING AND SUM RULES

In Section VI we discussed the scaling behavior (109) of the spin averaged structure functions: that as ν and q^2 become infinite, $W_1(\nu, q^2)$ and $\nu W_2(\nu, q^2)$ become nontrivial functions of $\omega = 2M_N\nu/q^2$. It comes as no surprise that a similar behavior is expected on the same basis for the spin-dependent structure functions: one expects (110, 111) $\nu d(\nu, q^2)$ and $\nu^2 g(\nu, q^2)$ to scale. Neglecting q^2/ν^2 as ν and $q^2 \rightarrow \infty$ with ω fixed, $\nu d + M_N \nu^2 g \propto A(\nu, q^2) W_1$ from Eq. (115a), and we see that $A(\nu, q^2)$ should also scale.

What magnitude and sign might we expect for these new structure functions in the scaling domain? For purposes of a very rough orientation and guide to our intuition, let us consider the most naive parton model — something which can not possibly be completely correct and which I will consequently call "dumb" quarks. Namely, let the nucleon consist of three point quarks: the proton is ppn and the neutron is nnp, i.e., just the three "valence" quarks with no $q\bar{q}$ sea, gluons, etc. Then in the unpolarized case, the usual parton manipulations yield that νW_2 and W_1 scale with $\nu W_2 = (2M_N/\omega)W_1$ and $W_{1n}/W_{1p} = 2/3$. The same calculations show, as expected, that νd and $\nu^2 g$ scale with

$$\nu^2 g = 0 \quad , \tag{116}$$

for either the neutron or proton, and

$$\frac{\nu d + M_N \nu^2 g}{4\pi M_N} = \frac{5}{9} W_1 \quad \text{or} \quad A(\omega) = \frac{5}{9} \tag{117a}$$

for the proton and

$$\frac{\nu d + M_N \nu^2 g}{4\pi M_N} = 0 \quad \text{or} \quad A(\omega) = 0 \tag{117b}$$

for the neutron. Equation (116) is seen to follow in the detailed computation from the (assumed) Dirac point particle nature of the partons.

The positive, or at least nonnegative, values of A found in Eqs. (117) are to be expected on naive grounds. For A in a parton model is just a measure of the extent to which the parton and nucleon spins are aligned, weighted by the parton charges squared. Therefore, since we don't expect the parton spins on the average to tend to line up opposite to the nucleon's spin, we expect $A \geq 0$.

In general, the use of a polarized target and beam could be thought of as weighting the individual parton contributions to the scattering by an additional factor: their spin alignment (112). For example, in the naive three quark model the p quark in the proton is weighted more heavily in $\sigma_{1/2}$ and less heavily in $\sigma_{3/2}$ than in the spin averaged cross section $\sigma_T = \frac{1}{2}(\sigma_{1/2}+\sigma_{3/2})$. If there is a connection of the parton charges to the excess (113) of positive hadrons projected forward in the hadronic final state at large q^2, then the excess should be different in $\sigma_{1/2}$ and $\sigma_{3/2}$.

Now let us turn from "dumb" quarks to what might be called "sophisticated" quarks. By this I mean the algebra of currents at equal time, and more generally the quark light cone algebra (8). Here one abstracts certain algebraic properties from the free quark model; properties which might be exactly true in Nature. But one discards the quark themselves. These algebraic relations can often be converted into sum rules. For example, the equal time commutator of two space components of the vector current as abstracted from the quark model (see Eq. (43)),

$$\left[V_i^{\alpha}(\vec{x},t), V_j^{\beta}(\vec{y},t)\right] = -i\epsilon_{ijk}\, d^{\alpha\beta\gamma} A_k^{\gamma}\; (\vec{x},t)\; \delta^{(3)}(\vec{x}-\vec{y})$$

$$+ \text{ terms symmetric in } i, j \tag{118}$$

leads to the sum rule (114)

$$\lim_{q^2 \to \infty} q^2 \int_0^{\infty} \frac{d\nu}{\nu} \left(\frac{\nu d + M_N \nu^2 g}{4\pi M_N} \right) = Z \quad . \tag{119}$$

Rewriting this in the scaling limit gives (115)

$$\int_1^{\infty} \frac{d\omega}{\omega} A(\omega) \left(\frac{2M}{\omega} W_1(\omega) \right) = Z \quad . \tag{120}$$

The quantity Z on the right-hand side of Eqs. (119) and (120) arises from the one nucleon matrix elements of the axial-vector current, A_k^{γ}, on the right-hand side of Eq. (118). With the indices α and β chosen to correspond to the electromagnetic current, the index γ takes the values 3, 8, or 0, so that one has for protons and neutrons

$$Z_p = Z^{(3)} + Z^{(8)} + Z^{(0)}$$

and (121)

$$Z_n = -Z^{(3)} + Z^{(8)} + Z^{(0)} \quad .$$

$Z^{(3)} = g_A/6$ is directly measurable in weak interactions. Thus by taking the difference between Eq. (120) for protons and for neutrons, one obtains a sum rule with the known quantity $Z_p - Z_n = g_A/3$ on the right-hand side. $Z^{(8)}$, arising from the same octet of currents as $Z^{(3)}$ is calculable if we know the F/D ratio of the axial-vector currents. If $(F/D)_{axial} = 2/3$, then $Z^{(8)}/Z^{(3)} = 1/5$.

It is instructive to examine the sum rule, Eq. (120), in the naive quark parton model discussed previously. There $A = 5/9$ and $\int_1^\infty \frac{d\omega}{\omega}(2M_N/\omega)W_1(\omega) = 1$ for the proton, while in the same naive quark model $Z^{(3)} = 5/18$ (since $g_A = 5/3$), $Z^{(8)} = 1/18$, and $Z^{(0)} = 4/18$. Therefore $Z_p = (5+1+4)/18 = 5/9$ and the left- and right-hand sides of Eq. (120) are both 5/9 for the proton. Similarly, since $A=0$ and $Z_n = (-5+1+4)/18 = 0$, both sides are zero for the neutron. Therefore we might regard Eq. (120) as being the correct "sophisticated" quark version of the "dumb" quark results in Eqs. (117).

Independent of any particular model recall that we expect $A \geq 0$. If this is to be correct for both proton and neutron (116), $Z^{(8)} + Z^{(0)} > |Z^{(3)}|$. As a result $Z_p \geq 2Z^{(3)} = g_A/3 \simeq 0.41$, which, as we will see in a moment, demands large asymmetries over a considerable ω range for the proton.

A second sum rule for the spin dependent structure functions (117),

$$\int_0^\infty d\nu \; g(\nu, q^2) = 0 \quad , \tag{122}$$

or (115, 118)

$$\int_1^\infty d\omega \; \nu^2 g(\omega) = 0 \tag{123}$$

in the scaling limit, can be derived as a superconvergence relation or from the quark light cone algebra. Regge arguments indicate (117) that both Eqs. (122) and (123), where the amplitudes involved receive no contribution (119) from a Pomeranchuk pole, should converge on the basis of known Regge pole intercepts. In particular, one expects $A(\omega) \to 0$ as $\omega \to \infty$.

Can anything rigorous be said about the validity of these sum rules in the absence of actual data from polarized beam experiments? An examination of the left-hand side of Eq. (120) shows that if we replace $A(\omega)$ by its upper bound (of 1), then we have

$$Z \leq \int_1^\infty \frac{d\omega}{\omega} \frac{2M_N}{\omega} W_1(\omega) \quad . \tag{124}$$

However, $(2M_N/\omega)\, W_1(\omega) \to$ constant as $\omega \to \infty$ if the usual assumption of dominance of the high ω behavior by the Pomeranchuk singularity is made. The right-hand side of Eq. (124) is then logarithmically divergent, and we have no rigorous bound which is useful in this case.

We can use bounds on $A(\omega)$ and presently existing data though to answer a lesser question. For example, suppose $A_p \leq 0.5$ (remember that $A = 0.5$ is a large asymmetry — it is equivalent to $\sigma_{1/2}/\sigma_{3/2} = 3$). How far in ω do we have to carry the integration in Eq. (120) to get $Z = 5/9$, as in the naive parton model discussed above? Present data on $W_1(\omega)$ shows that the upper limit of integration must be $\gtrsim 50$. Even for $Z_p \geq g_A/3$, which we expect from the argument that A_p and A_n should be nonnegative, the upper limit of integration must be $\gtrsim 25$. Thus if Z_p obeys our expectations, present data on deep inelastic scattering and the sum rule in Eq. (120) indicate that we must have large asymmetries over a very considerable range of ω values (120). Put another way, if the sum rule in Eq. (120) is correct, it converges slowly and receives important contributions from the large ω region. In this regard it is similar to the Adler sum rule (121), and the parton model sum rule (discussed in Section VI)

$$\int_1^{\infty} \frac{d\omega}{\omega} \left[\nu W_{2p}(\omega) - \nu W_{2n}(\omega)\right] = 1/3 \quad .$$

One might ask in this regard if the coefficient of A_k^{γ} in the commutation relation, Eq. (118), and consequently the Z on the right-hand side of Eq. (120), could be reduced by a factor K from the quark model value, making the sum rule easier to satisfy without large asymmetries. Here we are constrained by the Crewther relation (122)

$$3S = KR' \quad , \tag{125}$$

where K is defined as above (K = 1 in the quark model), S is the coefficient of the Adler anomaly (123), and R' is the ratio $\sigma(e^+e^- \to \gamma^{(3)} \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. Since S = 1/6 and R' = 1/2 for ordinary quarks, and S = 1/2 and R' = 3/2 for colored quarks, we see that the relation is satisfied in both these cases. Note that if S is fixed (say by PCAC and $\Gamma(\pi^0 \to \gamma\gamma)$) then large values of R' force K to be small and vice versa. One therefore has an interesting relation between the results of e^+e^- colliding beam experiments and those involving polarized lepton beams.

Finally, we note that if we subtract the second sum rule, Eq. (122), from the first, Eq. (119), and use Eqs. (107) and (115), we have

$$|Z| = \left| \lim_{q^2 \to \infty} q^2 \int_0^\infty \frac{d\nu}{\nu} \frac{\nu d}{4 M_N} \right| \leq \lim_{q^2 \to \infty} \int_0^\infty \frac{d\nu}{\nu} \sqrt{q^2 R}\, W_1$$

$$\leq \lim_{q^2 \to \infty} \int_0^\infty \frac{d\omega}{\omega} \left[\frac{2M_N}{\omega} W_1(\omega) \right] \sqrt{\nu^2 R/q^2} \ . \qquad (126)$$

Therefore, if $Z \neq 0$, R must not vanish faster than $q^2/\nu^2 = \frac{2M}{\omega} \frac{1}{\nu}$ at fixed ω as $\nu, q^2 \to \infty$. Of course exactly the behavior (124) $R = (q^2/\nu^2) f(\omega)$ is what is expected from the quark light cone algebra from which both sum sum rules were derived, so that everything is consistent in this regard.

E. DUALITY AND THE POLARIZATION ASYMMETRY IN DEEP INELASTIC SCATTERING

By now it is rather well accepted that there is a substantial non-diffractive component of the structure functions, at least for $1 \leq \omega \leq 10$, and that the distinguishable s-channel nucleon resonances exhibit a behavior which is closely connected (125) with that of νW_2 near $\omega = 1$. Inasmuch as there is no Pomeranchuk pole contribution to the spin dependent structure functions, one might look here also for a connection of deep inelastic behavior with that of s-channel resonances. To examine this question further, it is useful to construct a sum of resonances, in the spirit of the Veneziano model, which duplicates parton model results, and in particular the results of the naive three quark model for the structure functions which we discussed earlier.

For this purpose, we consider the states generated by three quarks with harmonic potentials between them. We let the electromagnetic current interact with only one quark at a time, considering only interactions with the quark magnetic moments. This immediately leads to $R = \sigma_L/\sigma_T = 0$.

The ground and excited states of such a set of three quarks are classifiable in terms of SU(6) representations, with the nucleon itself being in the ground state, the totally symmetric $\underline{56}$ representation. Since only one quark at a time is to be excited, only resonances in the totally symmetric $\underline{56}$ and mixed symmetry $\underline{70}$ representations can be produced. A little calculation (126) shows that summing over all possible excitations of resonances lying in a $\underline{56}$ at some given ν and q^2:

$$W_{1n}/W_{1p} = 12/17, \qquad A_p = 5/17, \qquad A_n = 0, \tag{127}$$

while for excitation of a $\underline{70}$,

$$W_{1n}/W_{1p} = 3/5, \qquad A_p = 1, \qquad A_n = 0 \ . \tag{128}$$

In particular there exists a combination of $\underline{56}$ and $\underline{70}$ s-channel nucleon resonances excited in photon-nucleon collisions which yield (126)

$$W_{1n}/W_{1p} = 2/3, \qquad A_p = 5/9, \quad A_n = 0, \tag{129}$$

exactly the naive three quark parton model results. Therefore it is possible to construct (126) a sum of s-channel resonances which for some, or for that matter all, ν and q^2 duplicates the naive parton results. Varying the weights of different $\underline{56}$'s and $\underline{70}$'s with ν and q^2 then yields an infinity of models of the nondiffractive part of the structure functions in terms of an infinite sum of s-channel nucleon resonances.

Does this s-channel model make sense when compared to the behavior of the known individual resonances which have been classified in $\underline{56}$'s or $\underline{70}$'s of SU(6)? In particular, we have the well known $P_{33}(1236)$ in a $\underline{56}$ (with the nucleon), the $D_{13}(1520)$ in a $\underline{70}$, and the $F_{15}(1688)$ in another $\underline{56}$.

Now in photoproduction (on a proton target) all these resonances have $\sigma_{3/2}(\text{resonance}) \geq \sigma_{1/2}(\text{resonance})$, i.e., $A_p(\text{resonance}) < 0$. Quark models (127) with harmonic forces (128) provide a neat explanation of this for the D_{13} and F_{15}. Both magnetic and convection current terms enter $\sigma_{1/2}$, where they cancel, while $\sigma_{3/2}$ comes from the convection current alone. Thus $\sigma_{3/2} >> \sigma_{1/2}$ or $A \simeq -1$ for these two resonances. However, the same quark models also make a prediction. As $q^2 \to \infty$, the magnetic term dominates, just as we assumed in the model discussed above, and there is no longer any cancellation in $\sigma_{1/2}$ for these resonances. In fact $\sigma_{1/2} >> \sigma_{3/2}$ or $A \simeq +1$ is predicted for these resonances as $q^2 \to \infty$. Detailed calculations (126) show that this change from $A(\text{resonance}) \simeq -1$ to $A(\text{resonance}) \simeq +1$ should occur rapidly as q^2 goes from zero to a few tenths of a GeV^2. These resonances would then have the same sign (positive) of A at large q^2 as is expected for deep inelastic scattering.

Experimentally there are no results from using polarized beams and targets, but the relevant information on the D_{13} and F_{15} is obtainable from observation of the angular distribution of their πN decays. Up to $q^2 \simeq 0.6\ \text{GeV}^2$, where large effects are expected, no appropriate change is observed in the $\pi^o p$ angular distributions (129). It appears that the quark model with harmonic forces predicts the wrong result, as the D_{13} and F_{15} still seem to have the opposite sign of A at $q^2 \simeq 0.6\ \mathrm{GeV}^2$ from that expected in deep inelastic scattering and in the quark models of resonance excitation.

What then happens to duality for the spin dependent structure functions near $\omega = 1$? To discuss this one must first say what behavior the deep inelastic scattering will have. On the basis of the sum rules we have discussed here and the ratio $\nu W_{2n}/\nu W_{2p}$ observed near $\omega = 1$, I expect (126) that both A_p and A_n will be $\simeq +1$ at $\omega = 1$ with A_n dropping rapidly to zero with increasing ω, but A_p staying large and going to zero only at very large values of ω. There are then two possibilities for the resonances: (1) all resonances eventually have $A_p > 0$ for large q^2 and one has a kind of "local duality" as in the case of νW_2, with each resonance mimicking the deep inelastic behavior; or (2) only the nucleon (with $A = +1$), of the "prominent" resonances, has $A > 0$. A "global duality" situation holds, as in πN charge exchange at $t = 0$, and the average over many contributions of different sign must be taken to give the deep inelastic behavior (130). Experimentally, a few sweeps through the region of prominent nucleon resonances with a polarized beam and target should answer this. Perhaps next year at this time we will know the answer to whether possibility (1) or (2) is realized in Nature.

F. SUMMARY

We have now seen some of the many and varied theoretical or phenomenological questions which are susceptible to experimental answer with a polarized lepton beam and target. Among the possibilities we have examined are that:

(1) A detailed, and in principle complete, amplitude analysis of virtual photon induced processes can be carried out;

(2) The scaling behavior of two new spin-dependent structure functions can be tested;

(3) The size of the asymmetry, A, which indicates in parton models the alignment of the charged constituent and nucleon spins can be measured. The various constituents are weighted in their contribution to the deep inelastic scattering in a different way from the spin averaged case;

(4) Rigorous sum rules exist which have right- and left-hand sides that both can be directly measured, providing a test of the quark light cone algebra;

(5) The asymmetries of individual resonances can be measured, providing a very restrictive check of quark models of resonances and their electromagnetic excitation, as well as of the validity and form duality takes in spin-dependent scattering near $\omega = 1$;

(6) The large ν or ω behavior of the structure functions is measurable, yielding additional information on Regge singularities which

are essentially inaccessible in other processes, and in particular the position of the leading singularity of odd signature and even parity. An interesting period of the beginning of experimental answers to these and other questions lies immediately ahead.

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79. The same result is also obtainable by assuming that the isoscalar electromagnetic current is responsible for 10% of $F_2^{ep}(x) + F_2^{en}(x)$ and there are equal vector and axial-vector contributions to $F_2^{\nu N}(x)$ or $F_2^{\bar{\nu}N}(x)$. See Perkins, D. H. (1972), Ref. 78.

80. A more careful treatment makes (small) corrections for the non-zero value of the Cabibbo angle and for the excess of target neutrons over protons in nuclei. See Perkins, D. H. (1972), Ref. 78.

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99. The case of both a polarized beam and target has been considered by Fraas, H. (1973). University of Wurzburg preprint (unpublished).

100. We need only consider μ, $\nu = 1, 2, 3$ since we have previously put $L_{\mu\nu}$ in a form where $L_{\mu 4} = L_{4\nu} = 0$.

101. While $\sigma_{1/2}$, $\sigma_{3/2}$, σ_L, and σ_T are all sums of squares of amplitudes, and hence nonnegative, σ_{TL} involves an interference between transverse and longitudinal amplitudes and has no positivity properties.

102. In this particular case, having assumed C, P, and T invariance, these inequalities follow simply from the positivity of $\sigma_{1/2}$ and

$\sigma_{3/2}$ and the Schwartz inequality. The general case, which requires more powerful techniques, is discussed by Doncel, M. G. and deRafael, E. (1971). Nuovo Cimento 4A, 363

103. We use the notation $d^2\sigma^{\uparrow\downarrow}/d\Omega' dE'$ ($d^2\sigma^{\uparrow\uparrow}/d\Omega' dE'$) to denote the double differential cross section for scattering a helicity +1/2 lepton on a nucleon target with spin direction antiparallel (parallel) to the beam. In all our equations we neglect the lepton mass.

104. Since σ_{TL} involves an interference between the transverse and longitudinal amplitudes, σ_{TL}/σ_T may not be small even though σ_L/σ_T is.

105. We denote by $d^2\sigma^{\uparrow\leftarrow}/d\Omega' dE'$ ($d^2\sigma^{\uparrow\rightarrow}/d\Omega' dE'$) the cross section for helicity +1/2 leptons scattering on a nucleon target with spin direction perpendicular to the beam and toward (away from) the direction of the final lepton.

106. This was emphasized by Carlson, C. E. and Tung, W. K. (1972). Phys. Rev. D 5, 721 and also in Ref. 107.

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108. We use the structure functions defined by Kuti, J. and Weisskopf, V. F. (1971). Phys. Rev. D 4, 3418

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Carlson, C. E. and Tung, W. K. (1972). Ref. 106

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111. The general scaling behavior of structure functions is treated by Cheng, T. P. and Zee, A. (1972). Phys. Rev. D 6, 3182

112. More detailed results of parton models can be found in Kuti, J. Weisskopf, V. F. (1971). Ref. 108

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Dakin, J. T., et al. (1973). Stanford Linear Accelerator Center Report No. SLAC-PUB-1269 (unpublished).

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115. This sum rule has also been discussed in the scaling regime by

Carlson, C. E. and Tung, W. K. (1972). Ref. 106

Hey, A. J. G. and Mandula, J. E. (1972). Ref. 107

Nash, C. (1971). Ref. 110

Dicus, D. A., Jackiw, R. and Teplitz, V. L. (1971). Phys. Rev. D 4, 1733

Wray, D. (1972). Nuovo Cimento 9A, 463

Ward, B. (1973). Princeton preprint (unpublished)

have also analyzed the sum rules for neutrinos.

116. Ellis, J. and Jaffe, R. (unpublished) have noted that the assumption that in a parton model of the nucleon the presence of λ and $\bar{\lambda}$ quarks is to be associated with the Pomeron, together with the absence of a

Pomeron contribution to the spin dependent structure functions, allows one to relate $Z^{(0)}$ and $Z^{(8)}$. With an F/D value of 2/3 this implies $Z^{(3)} = Z^{(8)} + Z^{(0)}$, so that $Z_p = 2Z^{(3)}$ and $Z_n = 0$.

117. Burkhardt, H. and Cottingham, W. N. (1971). Ann. Phys. (N.Y.) 56, 453

118. The sensitivity of this sum rule to model dependent assumptions in the light cone approach has been emphasized by Cheng, T. P. and Zee, A. (1972). Ref. 111.

119. A detailed Regge analysis of the spin dependent structure functions is presented in Refs. 94 and 117. The leading contributions correspond to an odd signature, positive parity (pole ?) in one amplitude and an even signature, positive parity cut in the other.

120. Large asymmetries were previously argued on the basis of the sum rule in Bjorken, J. D. (1970). Phys. Rev. D 1, 1376

121. For recent discussions of the saturation of the Adler sum rule see Paschos, E. A. (1973). In "Proceedings of the XVI International

Conference on High Energy Physics" (J. D. Jackson and A. Roberts, eds.) Vol 2, p. 166. National Accelerator Laboratory, Batavia.

Tuan, S. F. (1973). Phys. Rev. D 7, 2092 and references therein.

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124. This is spelled out explicitly in Mandula, J. E. (1973). Phys. Rev. D 8, 328

125. See Gilman, F. J. (1972). Ref. 62 for a review of duality in electroproduction and further references.

126. Close, F. E., Gilman, F. J. and Karliner, I. (1972). Phys. Rev. D 6 2533

127. Photoproduction or electroproduction of the $P_{33}(1236)$ is predicted to be pure magnetic dipole in character by such quark models, which is equivalent to $\sigma_{3/2} = 3\sigma_{1/2}$ for this resonance. This agrees with the experimental data (see Ref. 129).

128. Copley, L. A., et al. (1969). Ref. 53

Faiman, D. and Hendry, A. W. (1969). Ref. 53

Feynman, R. P., et al. (1971). Ref. 38

129. Close, F. E. and Gilman, F. J. (1972). Phys. Letters 38B, 541 and references to experiments therein.

Very recent data at $q^2 = 0.45$ GeV^2 on $\pi^+ n$ electroproduction at forward angles, as presented by Meaburn, R., et al. (1973). Daresbury Nuclear Physics Laboratory Report DNPL/P163 (unpublished), do show an indication of a possible change in helicity structure of the $D_{13}(1520)$ and $F_{15}(1688)$. Further data and analysis is needed to clarify this situation.

130. See also the s-channel resonance structure and predictions for the asymmetry of Chaichian, M. and Kitakado, S. (1973). CERN preprint TH.1640 (unpublished)

and the previous work of Domokos, G., et al. (1971). Phys. Rev. D 3, 1191

Cleymans, J. (1972). Phys. Rev. D 6, 814

Table I

Some baryon states expected in the constituent quark model and candidates for the SU(3) singlets and for the nonstrange member of each other SU(3) and J^P multiplet (see text).

SU(6) × 0(3) multiplet	SU(3) multiplet	Total quark spin, S	J^P	Candidate members of the multiplet
	$\underline{8}$	1/2	$1/2^+$	$P_{11}(940), \ldots$
$\underline{56}$ L=0	$\underline{10}$	1/2	$3/2^+$	$P_{33}(1236), \ldots$
	$\underline{1}$	1/2	$1/2^-$	$Y_0^*(1405)$
	$\underline{1}$	1/2	$3/2^-$	$Y_1^*(1520)$
	$\underline{8}$	1/2	$1/2^-$	$S_{11}(1550), \ldots$
possible mixing	$\underline{8}$	1/2	$3/2^-$	$D_{13}(1520), \ldots$
$\underline{70}$ L=1	$\underline{10}$	1/2	$1/2^-$	$S_{31}(1640), \ldots$
	$\underline{10}$	1/2	$3/2^-$	$D_{33}(1690), \ldots$
	$\underline{8}$	3/2	$1/2^-$	$S_{11}(1715), \ldots$
	$\underline{8}$	3/2	$3/2^-$	$D_{13}(1700), \ldots$
	$\underline{8}$	3/2	$5/2^-$	$D_{15}(1670), \ldots$
	$\underline{8}$	1/2	$5/2^+$	$F_{15}(1688), \ldots$
	$\underline{8}$	1/2	$3/2^+$	$P_{13}(1860), \ldots$
	$\underline{10}$	3/2	$7/2^+$	$F_{37}(1950), \ldots$
$\underline{56}$ L=2	$\underline{10}$	3/2	$5/2^+$	$F_{35}(1890), \ldots$
	$\underline{10}$	3/2	$3/2^+$	$P_{33}(?), \ldots$
	$\underline{10}$	3/2	$1/2^+$	$P_{31}(1860), \ldots$

Table II

Some meson states expected in the constituent quark model and possible candidates for the isospin one and zero members of each SU(3) and J^P multiplet.

SU(6) × 0(3) multiplet		SU(3) multiplet	Total quark spin, S	J^{PC}	Candidate members of the multiplet
$\underline{35}$ L=0		$\underline{8}+\underline{1}$	1	1^{--}	$\rho, \omega, \phi, \ldots$
	possible mixing	$\underline{8}$	0	0^{-+}	$\pi, \eta, \ldots$
$\underline{1}$ L=0		$\underline{1}$	0	0^{-+}	η'
$\underline{35}$ L=1		$\underline{8}+\underline{1}$	1	2^{++}	$A_2, f, f', \ldots$
		$\underline{8}+\underline{1}$	1	1^{++}	$A_1 ?, D, \ldots$
		$\underline{8}+\underline{1}$	1	0^{++}	$\delta, \sigma ?, \ldots$
		$\underline{8}$	0	1^{+-}	B, ?, ...
$\underline{35}$ L=2		$\underline{8}+\underline{1}$	1	3^{--}	$g, \omega_3, \ldots$
		$\underline{8}+\underline{1}$	1	2^{--}	$F_1 ?, ?, \ldots$
		$\underline{8}+\underline{1}$	1	1^{--}	$\rho' ?, ?, \ldots$
		$\underline{8}$	0	2^{-+}	$A_3 ?, ?, \ldots$

Table III

Decays of nonstrange $\underline{35}$ L=1 mesons into $\underline{35}$ L=0 mesons by pion emission.[a,b] All decay rates are fixed in terms of $\Gamma_{\lambda=0}(B\rightarrow\pi\omega)=0$ and $\Gamma(A_2\rightarrow\pi\rho)=77$ MeV.

		Γ(predicted) (MeV)	Γ(experimental) (40) (MeV)
$A_2(1310)\rightarrow\pi\rho$		77 (input)	77 ± 20
$B(1235)\rightarrow\pi\omega$	λ=0	0 (input)	dominantly λ=1 100 ± 20 total width
$B(1235)\rightarrow\pi\omega$	λ=1	76	
$A_1(1070)\rightarrow\pi\rho$	λ=0	52	?
$A_1(1070)\rightarrow\pi\rho$	λ=1	26	
$A_2(1310)\rightarrow\pi$		17	16 ± 4
$\sigma(975)\rightarrow\pi$		35	~60 total width
$f(1260)\rightarrow\pi\pi$		118	125 ± 25
$\sigma(760?)\rightarrow\pi\pi$		234	Broad ?

a. Decay rates of the corresponding K* states are simply obtained using SU(3) for the matrix elements of Q_5.

b. The ω, f and σ mesons are taken as ideal mixtures of singlets and octets, so as to be purely constituted by nonstrange quarks. Zweig's rule (2) is used to relate decay amplitudes involving the $SU(6)_W$ $\underline{35}$ and $\underline{1}$ parts of the λ=0 ω, σ and f.

Table IV

Decays of 70 L=1 and 56 L=2 baryons into 56 L=0 baryons by pion emission. All rates are fixed by the D_{13} and S_{11} decays to πN for the 70 L=1 decays, and by the F_{15} and P_{31} decays to πN for the 56 L=2 decays. For two states which may be mixed, a combination of widths which is independent of mixing is used and listed under Γ(predicted).

Decay	Γ(predicted) (MeV)	Γ(experimental)(40, 47) (MeV)
$D_{13}(1520) \to (\pi N)_d$	$\Gamma(1520) + 0.50\ \Gamma(1700)$	
$D_{13}(1700) \to (\pi N)_d$	= 79 MeV (input)	79 ± 20
$D_{13}(1520) \to (\pi\Delta)_d$	$\Gamma(1520) + 0.243\ \Gamma(1700)$	
$D_{13}(1700) \to (\pi\Delta)_d$	= 30 MeV	10 ± 6
$S_{11}(1535) \to (\pi\Delta)_d$	$\Gamma(1535) + 0.264\ \Gamma(1715)$	
$S_{11}(1715) \to (\pi\Delta)_d$	= 35 MeV	not seen
$D_{15}(1670) \to (\pi N)_d$	21 MeV	56 ± 14
$D_{15}(1670) \to (\pi\Delta)_d$	82 MeV	84 ± 21
$S_{31}(1640) \to (\pi\Delta)_d$	81	52 ± 20
$D_{33}(1690) \to (\pi N)_d$	19	32 ± 9
$D_{33}(1690) \to (\pi\Delta)_d$	55	not seen
$S_{11}(1535) \to (\pi N)_s$	$\Gamma(1535) + 0.505\ \Gamma(1715)$	
$S_{11}(1715) \to (\pi N)_s$	= 116 (input)	116 ± 55
$D_{13}(1520) \to (\pi\Delta)_s$	$\Gamma(1520) + 0.243\ \Gamma(1700)$	
$D_{13}(1700) \to (\pi\Delta)_s$	= 46	19 ± 10
$S_{31}(1640) \to (\pi N)_s$	18	48 ± 9
$D_{33}(1690) \to (\pi\Delta)_s$	61	172 ± 60

Table IV (continued)

Decay	Γ(predicted) (MeV)	Γ(experimental)(40, 47) (MeV)
$F_{15}(1688) \rightarrow (\pi N)_f$	84 (input)	84 ± 25
$F_{37}(1950) \rightarrow (\pi N)_f$	74	92 ± 20
$F_{37}(1950) \rightarrow (\pi \Delta)_f$	65	37 ± 18
$F_{35}(1880) \rightarrow (\pi N)_f$	14	36 ± 18
$F_{35}(1880) \rightarrow (\pi \Delta)_f$	77	16 ± 16
$P_{33}(\quad) \rightarrow (\pi \Delta)_f$		?
$F_{15}(1688) \rightarrow (\pi \Delta)_f$	12	not seen
$P_{13}(1860) \rightarrow (\pi \Delta)_f$	57	not seen
$P_{31}(1860) \rightarrow (\pi N)_p$	75 (input)	75 ± 25
$P_{31}(1860) \rightarrow (\pi \Delta)_p$	8	not seen
$P_{33}(\quad) \rightarrow (\pi N)_p$		?
$P_{33}(\quad) \rightarrow (\pi \Delta)_p$		?
$F_{35}(1880) \rightarrow (\pi \Delta)_p$	44	not seen
$P_{13}(1860) \rightarrow (\pi N)_p$	118	75 ± 25
$P_{13}(1860) \rightarrow (\pi \Delta)_p$	5	not seen
$F_{15}(1688) \rightarrow (\pi \Delta)_p$	15	22 ± 7

Table V

Signs of resonant amplitudes in $\pi N \rightarrow N^* \rightarrow \pi\Delta$ for N^*'s in the 70 L=1 and 56 L=2. The arbitrary overall phase is chosen so that the $DD_{15}(1670)$ amplitude is negative.

	Amplitude in $\pi N \rightarrow \pi\Delta$	Theoretical sign from $(8,1)_0-(1,8)_0$	Theoretical sign from $(3,\bar{3})_1-(\bar{3},3)_{-1}$	Experimental Sign Solution A(46,47)	Experimental Sign Solution B(55)
70 L=1	$DS_{13}(1520)$	-	+	-	+
	$DD_{13}(1520)$	+	+	-	+
	$SD_{31}(1640)$	+	-	-	-
	$DS_{33}(1690)$	-	+	+	+
	$DD_{15}(1670)$	-	-	-	-
	$DS_{13}(1700)$	-	+	+	+
56 L=2	$FP_{15}(1688)$	+	-	+	+
	$FF_{37}(1950)$	+	+	+	+
	$FF_{35}(1880)$	+	+	+	+

FIGURE CAPTIONS

1. Kinematics of inelastic electron-nucleon scattering.

2. The structure functions νW_2 and $2M_N W_1$ versus ω' for various q^2 ranges (67).

3. The ratio of neutron to proton inelastic electron scattering cross sections (72) for large values of ω'.

4. The x distribution of inelastic neutrino scattering from the Caltech experiment (82) at NAL compared to $F_2^{ed}(x)$ measured at SLAC.

5. Values of $\langle q^2 \rangle$ plotted versus the incident neutrino energy, E, from the Caltech experiment (82) at NAL.

6. Experimental results (87) for $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$.

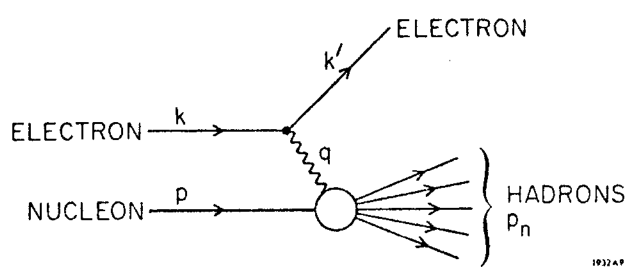
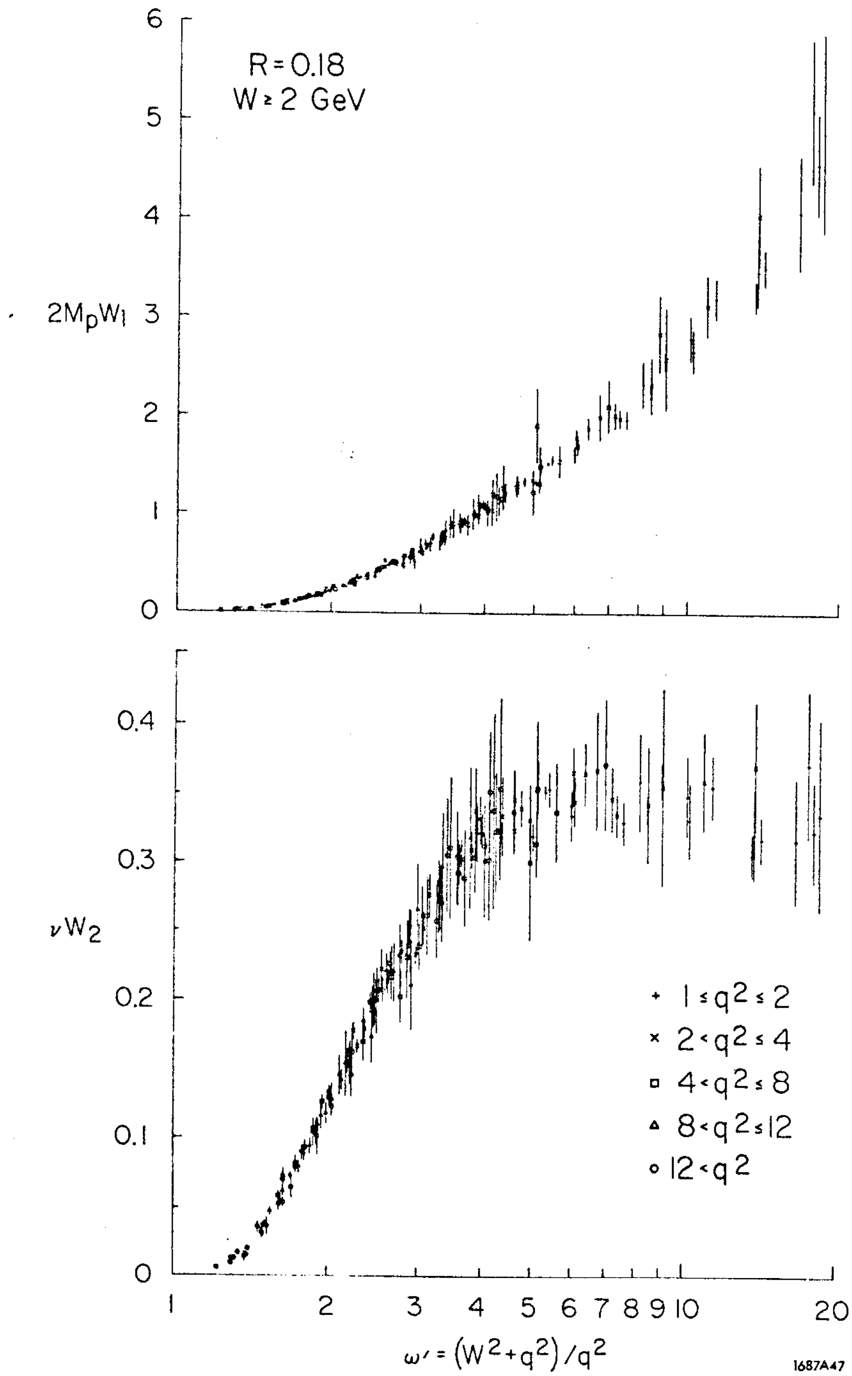


FIG. 1



R = 0.18
W ≥ 2 GeV
2MpW1
νW2
1 ≤ q2 ≤ 2
2 < q2 ≤ 4
4 < q2 ≤ 8
8 < q2 ≤ 12
12 < q2
ω′ = (W2+q2)/q2
1687A47

FIG. 2

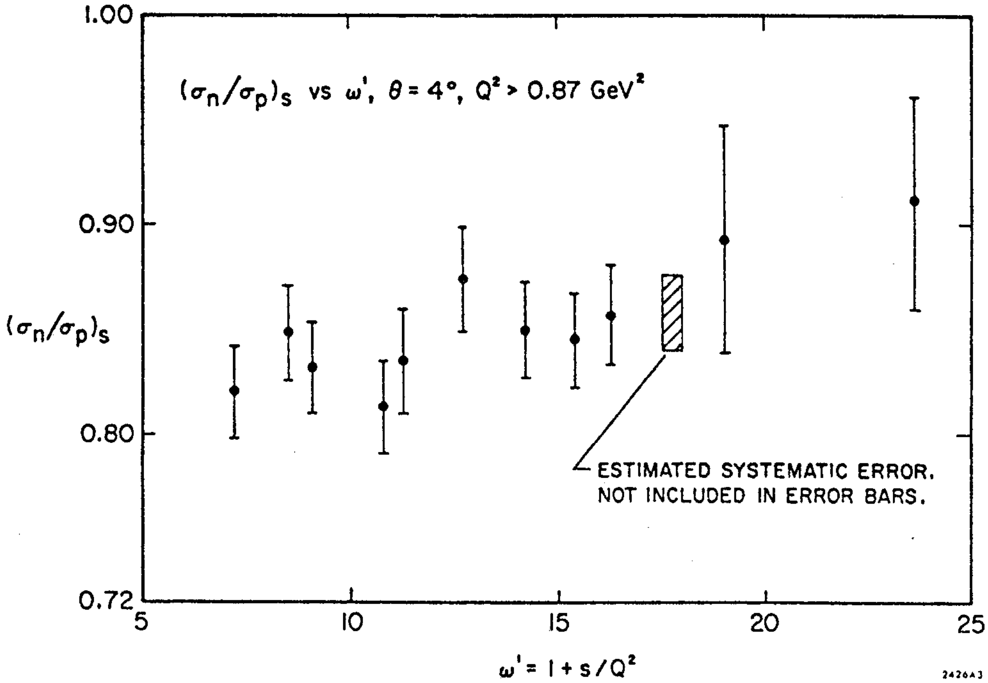

$(\sigma_n/\sigma_p)_s$ vs ω', $\theta = 4°$, $Q^2 > 0.87$ GeV^2
$(\sigma_n/\sigma_p)_s$
1.00
0.90
0.80
0.72
5
10
15
20
25
ESTIMATED SYSTEMATIC ERROR.
NOT INCLUDED IN ERROR BARS.
$\omega' = 1 + s/Q^2$
2426A3

FIG. 3

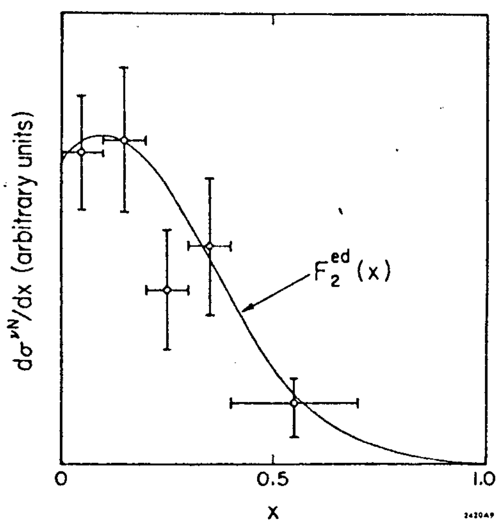
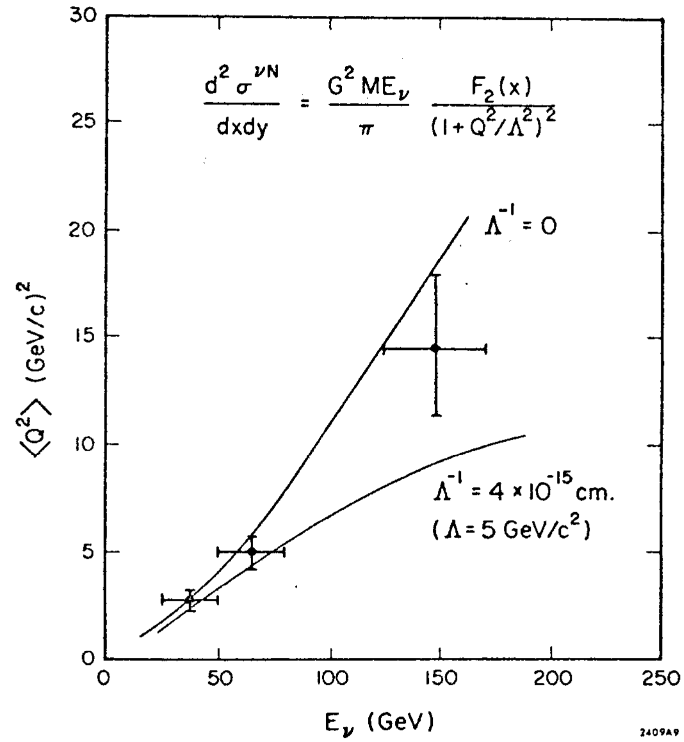
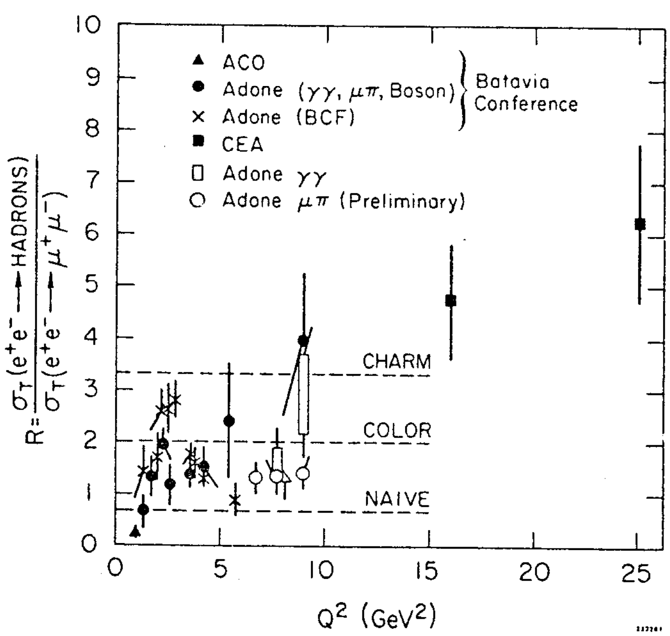

$d\sigma^{\nu N}/dx$ (arbitrary units)
$F_2^{ed}(x)$
0
0.5
1.0
x

FIG. 4



$\frac{d^2\sigma^{\nu N}}{dxdy} = \frac{G^2 ME_\nu}{\pi} \frac{F_2(x)}{(1+Q^2/\Lambda^2)^2}$
$\Lambda^{-1} = 0$
$\Lambda^{-1} = 4 \times 10^{-15}$ cm.
($\Lambda = 5$ GeV/c^2)
$\langle Q^2 \rangle$ (GeV/c)^2
E_ν (GeV)
2409A9

FIG. 5



ACO
Adone ($\gamma\gamma$, $\mu\pi$, Boson)
Adone (BCF)
CEA
Adone $\gamma\gamma$
Adone $\mu\pi$ (Preliminary)
Batavia Conference
CHARM
COLOR
NAIVE
$R = \frac{\sigma_T(e^+e^- \rightarrow \text{HADRONS})}{\sigma_T(e^+e^- \rightarrow \mu^+\mu^-)}$
Q^2 (GeV^2)

FIG. 6