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Erratum: Systematic classification of three-loop realizations of the Weinberg operator

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The strategy used in our original publication to classify the 3-loop realizations of the Weinberg operator admits a loophole which was overlooked; it enlarges the set of *genuine* topologies. We have found that by means of this loophole, the 26 topologies depicted in figure 1 categorized as *non-genuine* in our original publication actually have to be classified as *special genuine*. Recall that *special genuine topologies* are those associated to neutrino mass diagrams which under normal circumstances can be redrawn with less loops, unless some particular quantum numbers are assigned to some of the particles in the internal lines. More specifically, these special diagrams contain fermion-fermion-scalar, $(\text{scalar})^3$ and/or $(\text{scalar})^4$ effective interactions generated through loops which cannot be compressed to a point, as is ordinarily the case. That is because these effective couplings involve derivatives of the fields, making them non-renormalizable, so there exist exceptional cases in which there is no corresponding tree level realization of the effective vertex.

For the *special genuine* topologies discussed in the paper, the existence of derivatives can be traced to the antisymmetric nature of some $\mathrm{SU}(2)_L$ contractions, which makes some loop interactions non-compressible for appropriate choices of the quantum numbers of the diagram lines. However, when writing the paper we overlooked a second possible reason why such derivative terms might be unavoidable: fermion-fermion-scalar couplings may contain a derivative due to the chirality of the standard model fermions. In particular, two left-handed Weyl fermions ψ and ψ' may interact with a scalar S through a $\partial\psi^\dagger\psi' S$ effective coupling (the number of derivatives can be higher, as long as it is an odd number).[1] However, if ψ is a vector-like field, its left-handed partner $\overline{\psi}$ has the same gauge quantum numbers as $\psi^\dagger$ (the conjugate of ψ) and opposite chirality, hence there is no symmetry forbidding the renormalizable interaction $\overline{\psi}\psi' S$, with no derivatives.[2] Using this coupling and a mass insertion $\overline{\psi}\psi$ one can then always generate $\partial\psi^\dagger\psi' S$ without loops, as depicted in figure 2. However, if neither ψ nor ψ' has a vector-like partner, this argument fails and the vertex $\partial\psi^\dagger\psi' S$ may not be realizable at tree-level. Thus, if ψ and ψ' are fixed to be Standard Model fermions, the loop might not be compressible and our general arguments fail. Let us discuss this with one particular example.

Take for instance topology 89 in figure 1 and one of its diagrams as an example (shown in figure 3). This diagram contains a one-loop realization of the vertex $\partial L\psi^\dagger S$, with L the SM lepton doublet and ψ and S an arbitrary left-handed fermion and scalar, respectively. If ψ is not a Standard Model fermion, it is necessary to add its vector-like partner $\overline{\psi}$ to the model, in order to generate a bare mass term $M\overline{\psi}\psi$. From the argument in figure 2, it is

[1]From a symmetry argument, one can see that the number of derivatives is odd, and therefore there is at least one of them. It goes as follows: the complexified algebra of the Lorentz group is the same as the complexified algebra of $\mathrm{SU}(2)\times\mathrm{SU}(2)$, so it's representations can be labeled by a pair of spins (j_L, j_R). Left-handed fermions ψ and their conjugates $\psi^\dagger$ transform as $(1/2, 0)$ and $(0, 1/2)$ (conjugation flips j_L with j_R) respectively, while vector-like fields, such as derivatives, are bi-doublets $(1/2, 1/2)$. Therefore, the bilinear $\psi\psi^\dagger$ transforms as a vector, and Lorentz invariance can only be obtained by adding to this fermion combination an odd number of derivatives.

[2]The same argument follows interchanging ψ with ψ'.

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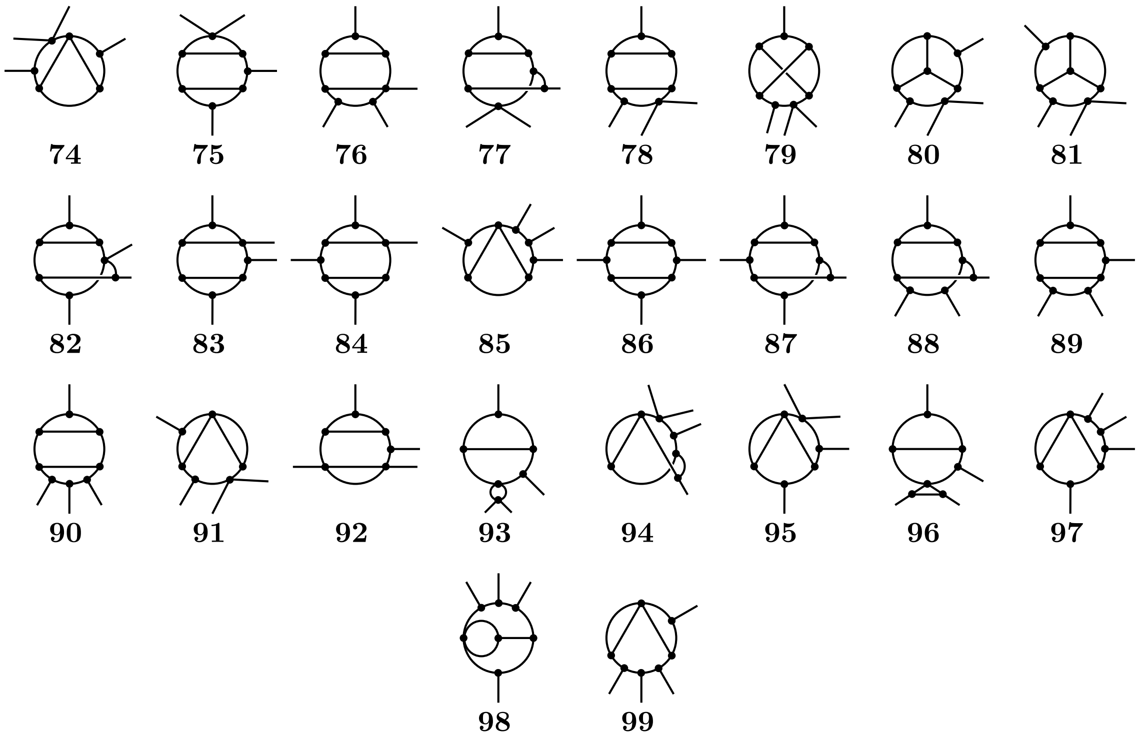

Figure 1. List of new topologies associated to *special* genuine diagrams. We follow the numbering of the paper and this list should be understood as a continuation of figure 15 in the appendix A of the paper. See text for details.

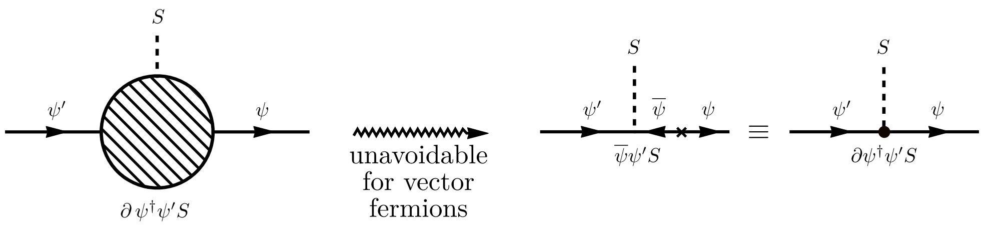

Figure 2. Consider a loop induced coupling of the left-handed fermions ψ and ψ' to a scalar S as indicated on the left. Given the chirality of the fermions, the effective coupling is $\partial\psi^\dagger\psi' S$. If ψ has a vector-like partner $\overline{\psi}$ (which we may consider to be left-handed as well), then the tree-level coupling $\overline{\psi}\psi' S$ exists, and together with a mass insertion $\overline{\psi}\psi$ it can be used to generate the effective interaction $\partial\psi^\dagger\psi' S$ without loops (this is the leading order interaction; extra pairs of derivatives appear at higher order). This argument fails if both ψ and ψ' are Standard Model fermions.

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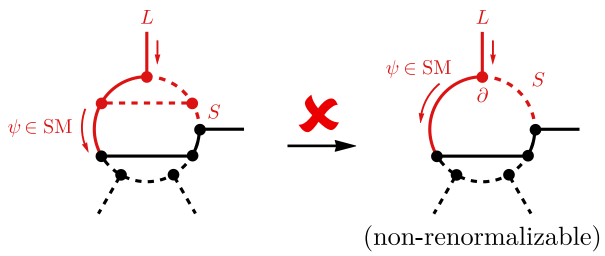

Figure 3. Example of a diagram with topology 89, containing a one-loop fermion-fermion-scalar effective interaction (in red). This loop is removable unless ψ is a SM fermion.

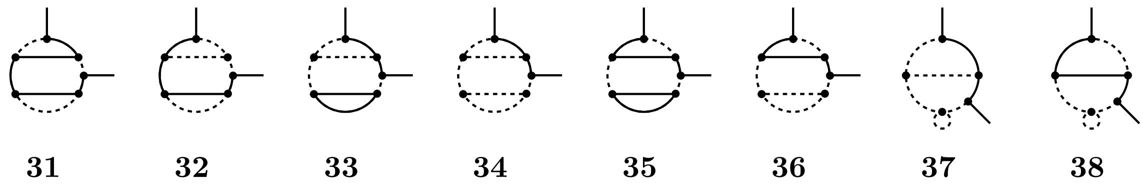

Figure 4. List of new *special* genuine diagrams in the mass basis, i.e. with the external Higgs lines removed. The numbering follows that of figure 8 of the paper and should be understood as a continuation of that figure.

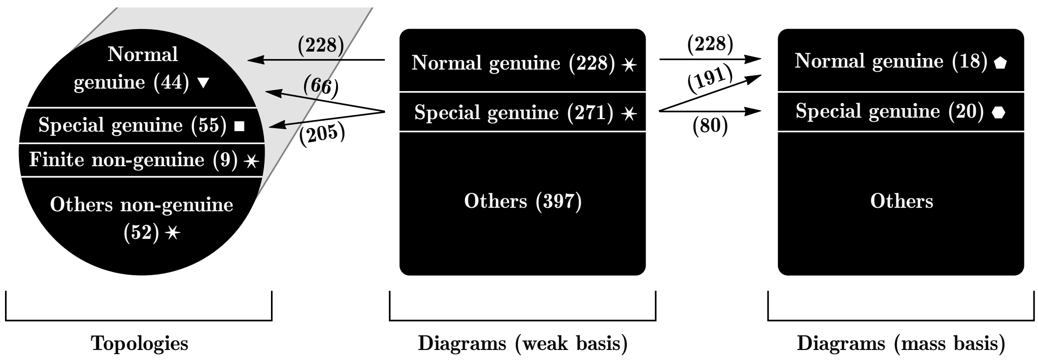

Figure 5. Corrected counting for topologies and diagrams.

then possible to rewrite the diagram with one less loop. On the other hand, it might not be possible to do so if ψ is a Standard Model fermion, in which case the diagram is genuine.

Note that, since this loophole to our general argument exists only for standard model fermions, the list of all possible genuine models generated from the diagram in figure 3 will be quite constrained, due to the limited number of choices of $\psi \in \{L, e^c, Q, u^c, d^c\}$.

In total, there are 125 genuine diagrams associated to the 26 topologies in figure 1. They yield 8 new diagrams in the mass basis, after electroweak symmetry breaking (see figure 4). The complete lists of topologies and diagrams are given in [1].[3]

[3] See supplementary material, available also at http://renatofonseca.net/3loop-Weinberg-operator.php, accessed 05 June 2019.

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The newly re-classified topologies, see figure 1, and diagrams, figure 4, affect some numbers quoted in our original paper. In figure 5 we give the updated numbers for each type of topology and each type of diagram.

References

[1] R. Cepedello, R.M. Fonseca and M. Hirsch, *Systematic classification of three-loop realizations of the Weinberg operator*, *JHEP* **10** (2018) 197 [arXiv:1807.00629] [INSPIRE].