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Thermodynamics and weak cosmic censorship conjecture in the charged RN-AdS black hole surrounded by quintessence under the scalar field

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Abstract

In this paper, we study the thermodynamics and the weak cosmic censorship conjecture of the RN-AdS black hole surrounded by the quintessence under the scattering of a charged complex scalar field. With scalar field scattering, the variation of the black hole is calculated in the extended and normal phase spaces. In the extended phase space, the cosmological constant and the normalization parameter are considered as thermodynamic variables, and the first law of thermodynamics is valid, and the entropy of a black hole decreases with time. In the normal phase space, the cosmological constant and the normalization parameter are fixed, and the first and second laws of thermodynamics can also be satisfied. Furthermore, the weak cosmic censorship conjecture is both valid in the extended and normal phase spaces.

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1. Introduction

Since Hawking and Bekenstein [1-3] proposed the black hole thermodynamics, the research on the thermodynamics of black holes has developed rapidly. Analogous to the laws of thermo-

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dynamics, the four laws of black hole thermodynamics were established in [4]. There are several ways to calculate the temperature and entropy of black holes [5–7]. A general argument gave that the Hawking temperature of a black hole is proportional to the surface gravity at the horizon, and the black hole entropy is proportional to the area of the horizon. The black hole's horizon is an important part for the study of the nature of black holes. According to Penrose's theory [8], all singularities caused by gravitational collapse must be hidden in the black hole. In other words, singularities need to be hidden from an observer at infinity by the event horizon of black hole, which is the weak cosmic censorship conjecture.

The thermodynamic laws and the weak cosmic censorship conjecture can be tested by the scattering of an external field or by the absorptions of a particle through a black hole. As the particles fall into the black hole, it has been proven that the first law and the second law of black hole thermodynamics are still established [9,10]. On the other hand, there are some controversies about the weak cosmic censorship conjecture. Wald innovatively proposed a method to test this conjecture in the extreme Kerr-Newman black hole by absorbing a particle, which showed that the conjecture was satisfied [11]. However, this conjecture is violated in the near-extreme Reissner-Nordström black hole [12] and the near-extreme Kerr black hole [13]. The validity of weak cosmic censorship conjecture in the context of various black holes via the absorption of a charged or rotating particle has been discussed by many work [14–32]. Although a lot of work has been done, no consistent conclusion has been reached.

In physics, the quintessence is a hypothetical form of dark energy, it is a scalar field, which is supposed to explain the accelerating expansion of the universe. The first example of this scenario was proposed by Ratra and Peebles [33]. Quintessence differs from the cosmological constant interpretation of dark energy because it is dynamic. Depending on the ratio of its kinetic energy to its potential energy, the quintessence can be attractive or repulsive [34–42].

In this paper, we study the thermodynamics and weak cosmic censorship conjecture of the RN-AdS black hole surrounded by the quintessence under the scattering of a charged complex scalar field. The first law of thermodynamics is always satisfied, but the second law of thermodynamics is not always satisfied. And the weak cosmic censorship conjecture does not violate. In addition, we compare our result with the situation of without the quintessence in reference [24]. The paper is organized as follows. In section 2, we investigate the dynamics of the charged complex scalar field, and calculated the variations of this black hole's energy and charge within the certain time interval. In section 3, the thermodynamics of the black hole are discussed in the extended and normal phase space. Moreover, we test the validity of the weak cosmic censorship conjecture in those case. We summarize our results in section 4.

2. Complex scalar field in RN-AdS black hole surrounded by quintessence

In this section, we first briefly review the RN-AdS black hole solution with quintessence and then investigate the complex scalar field in the black hole background.

2.1. Black hole solution

The bulk action for a RN-AdS black hole surrounded by quintessence dark energy in four dimensional curved space time could be described as follows [37,38]

$$S = \frac{1}{16\pi G} \int d^4 x \left(\sqrt{-g} \left[R - 2\Lambda - F^{\mu\nu} F_{\mu\nu} \right] + \mathcal{L}_q \right). \tag{1}$$

In the above action, the cosmological constant is related to the AdS space radius l by $\Lambda = -3/l^2$. The last term \mathcal{L}_q in the action is the Lagrangian of quintessence as a barotropic perfect fluid, which is given by [39]

$$\mathcal{L}_q = -\rho_q \left[1 + \omega_q \ln\left(\frac{\rho_q}{\rho_0}\right) \right],\tag{2}$$

where ρ_q is energy density, ρ_0 is the constant of integral, and the barotropic index ω_q . One has that $-1 < \omega_q < -1/3$ for the quintessence dark energy and $\omega_q < -1$ for the phantom dark energy. The metric of the charged RN-AdS black hole surrounded by quintessence is

$$ds^{2} = f(r)dt^{2} - \frac{1}{f(r)}dr^{2} - r^{2}(r)\left(d\theta^{2} + \sin^{2}\theta d\varphi^{2}\right),$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{a}{r^{3\omega_{q}+1}} + \frac{r^{2}}{l^{2}},$$
(3)

where M and Q are the mass and electric charge of the black hole, respectively, and a is the normalization factor related to the density of quintessence as [40,41]

$$\rho_q = -\frac{a}{2} \frac{3\omega_q}{r^3 \left(\omega_q + 1\right)}.\tag{4}$$

The electromagnetic potential of the black hole is

$$A_{\mu}(r) = (-\frac{Q}{r}, \ 0, \ 0, \ 0).$$
⁽⁵⁾

The potential of the black hole is

$$\varphi = -A_t \left(r_+ \right) = \frac{Q}{r_+}.\tag{6}$$

Moreover, the Hawking temperature can be derived as

$$T = \frac{f'(r_{+})}{4\pi} = \frac{1}{4\pi} \left[\left(3\omega_q + 1 \right) r_{+}^{-3\omega_q - 2} + \frac{2M}{r_{+}^2} + \frac{2r_{+}}{l^2} - \frac{2Q^2}{r_{+}^3} \right],\tag{7}$$

and the entropy of the black hole is

$$S = \pi r_+^2. \tag{8}$$

In the extended phase space where the cosmological constant is treated as a thermodynamic variable, the first law of thermodynamics via the RN-AdS black hole surrounded by quintessence have been studied in [42]. The mass of the black hole is then interpreted as the enthalpy

$$M = U + PV, \tag{9}$$

where U is internal energy.

2.2. Complex scalar field

The action of a complex scalar field in the fixed RN-AdS gravitational and electromagnetic fields is [24]

$$S = -\frac{1}{2} \int \sqrt{-g} \left[\left(\partial^{\mu} - iq A^{\mu} \right) \Psi^* \left(\partial_{\mu} + iq A_{\mu} \right) \Psi - m^2 \Psi^* \Psi \right] d^4 x, \tag{10}$$

where A_{μ} is the electromagnetic potential, *m* is the mass, *q* is the charge, Ψ denotes the wave function, and its conjugate is Ψ^* . Now we turn to investigate the dynamics of the charged complex scalar field. The field equation obtained from the action satisfies

$$\left(\nabla^{\mu} - iqA^{\mu}\right)\left(\nabla_{\mu} - iqA_{\mu}\right)\Psi - m^{2}\Psi = 0.$$
(11)

To solve this wave function, we use the method provided by references [24] and [25]. We carry out a separation of variables

$$\Psi = e^{-i\omega t} R(r) \Phi(\theta, \phi). \tag{12}$$

In the above equation, ω is the energy of the particle, and $\Phi(\theta, \phi)$ is the scalar spherical harmonics. We put Eq. (12) into Eq. (11) and obtain the radial wave function

$$R(r) = e^{\pm i(\omega - qQ/r)r_*},$$
(13)

where $dr_* = \frac{1}{f}dr$, r_* is a function of r, and +/- corresponds to the solution of the outgoing/ingoing radial wave. Since the thermodynamics and the validity of the weak cosmic censorship conjecture are discussed by the scattering of the ingoing wave at the event horizon in this paper, we focus our attention on the ingoing wave function.

From the action (10), the energy-momentum tensor is obtained as follows

$$T^{\mu}_{\nu} = \frac{1}{2} [(\partial^{\mu} - iqA^{\mu})\Psi^* \partial_{\nu}\Psi + (\partial^{\mu} + iqA^{\mu})\Psi \partial_{\nu}\Psi^*] + \delta^{\mu}_{\nu}\mathcal{L}.$$
 (14)

Combining the ingoing wave function and its conjugate with the energy-momentum tensor yields the energy flux

$$\frac{dE}{dt} = \int T_t^r \sqrt{-g} d\theta d\phi = \omega (\omega - q\varphi) r_+^2.$$
(15)

The electric current is obtained from the action (10)

$$j^{\mu} = \frac{\partial \mathcal{L}}{\partial A_{\mu}} = -\frac{1}{2} i q [\Psi^* (\partial^{\mu} + i q A^{\mu}) \Psi - \Psi (\partial^{\mu} - i q A^{\mu}) \Psi^*].$$
(16)

For the ingoing wave function (13), the charge flux is

$$\frac{dQ}{dt} = -\int j^r \sqrt{-g} d\theta d\varphi = q(\omega - q\varphi)r_+^2.$$
⁽¹⁷⁾

When the complex scalar field is scattered off the black hole, the decreases of the energy and charge of the scalar field are equal to the increases of these of the black hole due to the energy and charge conservation. From Eqs. (15) and (17), the transferred energy and charge within the certain time interval are

$$dU = dE = \omega(\omega - q\varphi)r_+^2 dt, \quad dQ = q(\omega - q\varphi)r_+^2 dt, \tag{18}$$

respectively. Since the transferred energy and charge are very small, the time dt must be also very small. The increase or decrease of dU and dQ depend on the relation between ω and $q\varphi$.

3. Thermodynamics and weak cosmic censorship conjecture

In this section, we test the thermodynamics laws and the weak cosmic censorship conjecture of the black hole by the scattering of the ingoing wave at the event horizon. And our derivation method is mentioned in these articles [24,25,30,31].

3.1. Extended phase space

In the extended phase space, the cosmological constant and normalization parameters are considered as thermodynamic variables. One can treat the cosmological constant as thermodynamic pressure and its conjugate quantity as thermodynamic volume [43–49]. The definitions are as follows

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi l^2}, V = \left(\frac{\partial M}{\partial P}\right)_{S,Q},\tag{19}$$

where M is the mass of the black hole. So, the metric function f(r) can be rewritten as

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{a}{r^{3\omega_q + 1}} + \frac{8}{3}\pi Pr^2.$$
 (20)

Solving the equation f(r) = 0 at the horizon radius $r = r_+$, one can obtain the mass of the black hole

$$M = \frac{1}{6} \left(-3ar_{+}^{-3\omega_{q}} + 8\pi Pr_{+}^{3} + \frac{3Q^{2}}{r_{+}} + 3r_{+} \right).$$
(21)

Using the Eqs. (19) and (21), we can obtain the thermodynamic volume as

$$V = \frac{4\pi r_+^3}{3}.$$
 (22)

The initial state of the black hole is represented by (M, Q, r_+, P, a) , and the final state is represented by $(M + dM, Q + dQ, r_+ + dr_+, P + dP, a + da)$. The variation of the horizon radius can be obtained from the variation of metric function $f(r_+)$. For the initial state (M, Q, r_+, P, a) satisfies

$$f(M, Q, r_+, P, a) = 0.$$
 (23)

When the black hole mass and charge are varied, we assume that the final state of the charged RN-AdS black hole surrounded by quintessence is still a black hole, which satisfies

$$f(M + dM, Q + dQ, r_{+} + dr_{+}, P + dP, a + da) = 0.$$
(24)

The functions $f(M+dM, Q+dQ, r_++dr_+, P+dP, a+da)$ and $f(M, Q, r_+, P, a)$ satisfy the following relation

$$f(M+dM, Q+dQ, r_{+}+dr_{+}, P+dP, a+da) = f(M, Q, r_{+}, P, a)$$

+ $\frac{\partial f}{\partial M}\Big|_{r=r_{+}} dM + \frac{\partial f}{\partial Q}\Big|_{r=r_{+}} dQ + \frac{\partial f}{\partial r_{+}}\Big|_{r=r_{+}} dr_{+} + \frac{\partial f}{\partial P}\Big|_{r=r_{+}} dP + \frac{\partial f}{\partial a}\Big|_{r=r_{+}} da, \quad (25)$

where

$$\frac{\partial f}{\partial M}\Big|_{r=r_{+}} = -\frac{2}{r_{+}}, \frac{\partial f}{\partial Q}\Big|_{r=r_{+}} = \frac{2Q}{r_{+}^{2}}, \frac{\partial f}{\partial r_{+}}\Big|_{r=r_{+}} = 4\pi T, \frac{\partial f}{\partial P}\Big|_{r=r_{+}} = \frac{8\pi r_{+}^{2}}{3}, \frac{\partial f}{\partial a}\Big|_{r=r_{+}} = -\frac{1}{r_{+}^{3\omega_{q}+1}}.$$
(26)

Bring Eqs. (23), (24) and (26) to Eq. (25) leads to



Fig. 1. The relation between dS and r_+ in extended phase space surrounded by quintessence. We fix Q = l = 1, a = 0.2, $\omega_q = -2/3$, $\omega = q = 0.1$ and dt = 0.001. In these curves, the curve all most on the r_+ -axis is corresponding to da = 0. Suppose the coordinate system is divided into four quadrants by the imaginary line and the r_+ -axis. Then, the curves of the first and third quadrants correspond to da = -0.5. And the curves of the second and fourth quadrants correspond to da = 0.5.

$$dM = 2\pi r_{+}Tdr_{+} + \frac{Q}{r_{+}}dQ + \frac{4}{3}\pi r_{+}^{3}dP - \frac{1}{2r_{+}^{3\omega_{q}}}da.$$
(27)

Using the Eqs. (6), (8) and (22), gives the first law of thermodynamics

$$dM = TdS + \varphi dQ + VdP + \eta da, \tag{28}$$

where η is the physical quantity conjugate to the parameter *a*

$$\eta = -\frac{1}{2r_{\perp}^{3\omega_q}}.$$
(29)

And the Smarr relation [50] can be written as

$$M = 2TS + \varphi Q - 2VP + (1 + 3\omega_q)\eta a.$$
(30)

For simplicity we use AdS radius l instead of pressure P do the next calculation. Inserting Eqs. (19) and (22) into Eq. (9) yields

$$dM = d(U + PV) = \omega(\omega - q\varphi)r_{+}^{2}dt + \frac{3r_{+}^{2}}{2l^{2}}dr_{+} - \frac{r_{+}^{3}}{l^{3}}dl.$$
(31)

Bring these results into the Eq. (27), we can obtain the variation of the radius in horizon

$$dr_{+} = \frac{2r_{+}l^{2}(\omega - q\varphi)^{2}}{4\pi l^{2}T - 3r_{+}}dt + \frac{l^{2}r_{+}^{-3\omega_{q}-1}}{4\pi l^{2}T - 3r_{+}}da.$$
(32)

The variation of the entropy then takes on the form

$$dS = 2\pi r_{+}dr_{+} = \frac{4\pi r_{+}^{2}l^{2}(\omega - q\varphi)^{2}}{4\pi l^{2}T - 3r_{+}}dt + \frac{2\pi l^{2}r_{+}^{-3\omega_{q}}}{4\pi l^{2}T - 3r_{+}}da.$$
(33)

In order to make our discussion more straightforward, we numerically analyze and plot the Eq. (33) as Fig. 1, where we fix Q = l = 1, a = 0.2, $\omega_q = -2/3$, $\omega = q = 0.1$ and dt = 0.001. The

imaginary line in FIG. 1 has a value of $r_+ = 0.9679$ is the critical black hole horizon radius. From this figure we can know that the second law of thermodynamics is not always valid. By compare our result with the situation of without the quintessence in [24], we find that no matter whether there is a quintessence around the black hole, the second law of thermodynamics can not be guaranteed, this is related to the initial conditions of the black hole. This conclusion is consistent with the reference [24].

Then we test the validity of the weak cosmic censorship conjecture in the extremal and nearextremal RN-AdS black hole surrounded by quintessence. When the charge absorbed by the black hole is more enough, the black hole is overcharged and the weak cosmic censorship conjecture is violated. Therefore, we just need check the existence of the event horizon after the scattering. A simple method to check this existence is to evaluate the solution of the equation f(r) = 0. If the solution exists, the metric function f(r) with a minimum negative value guarantees the existence of the event horizon.

We suppose that there exists one minimum point at $r = r_0$ for f(r), and the minimum value of f(r) is not greater than zero,

$$\delta \equiv f(r_0) \le 0,\tag{34}$$

where $\delta = 0$ corresponds to the extremal black hole. After the black hole scatters the scalar field, the minimum point would move to $r_0 + dr_0$, the other parameters of the black hole change from (M, Q, l, a) to (M + dM, Q + dQ, l + dl, a + da). The minimum value of f(r) at $r = r_0 + dr_0$ of the final state is given by

$$f(r_{0} + dr_{0}, M + dM, Q + dQ, l + dl, a + da) = \delta + \frac{\partial f}{\partial r_{0}}\Big|_{r=r_{0}} dr_{0} + \frac{\partial f}{\partial M}\Big|_{r=r_{0}} dM + \frac{\partial f}{\partial Q}\Big|_{r=r_{0}} dQ + \frac{\partial f}{\partial l}\Big|_{r=r_{0}} dl + \frac{\partial f}{\partial a}\Big|_{r=r_{0}} da$$
(35)
= $\delta + \delta_{1} + \delta_{2}$,

where

$$\frac{\partial f}{\partial M}\Big|_{r=r_0} = -\frac{2}{r_0}, \frac{\partial f}{\partial Q}\Big|_{r=r_0} = \frac{2Q}{r_0^2}, \frac{\partial f}{\partial r_0}\Big|_{r=r_0} = 0, \frac{\partial f}{\partial l}\Big|_{r=r_0} = -\frac{2r_0^2}{l^3}, \frac{\partial f}{\partial a}\Big|_{r=r_0}$$

$$= -\frac{1}{r_0^{3\omega_q+1}}.$$
(36)

Bring Eqs. (36) and (27) to Eq. (35) leads to

$$\delta = f(r_0, M, Q, l, a),$$

$$\delta_1 = -\frac{2T}{r_0} dS + \frac{2(r_+^3 - r_0^3)}{l^3 r_0} dl + \frac{\left(r_+^{-3\omega_q} - r_0^{-3\omega_q}\right)}{r_0} da,$$

$$\delta_2 = -\frac{2qQ(r_+ - r_0)(qQ - r_+\omega)}{r_0^2} dt.$$
(37)

When it is an extreme black hole, $r_0 = r_+$ and T = 0, we can obtain $\delta = 0$, $\delta_1 = 0$ and $\delta_2 = 0$. Hence Eq. (35) can be rewritten as

$$f(r_0 + dr_0, M + dM, Q + dQ, l + dl, a + da) = 0,$$
(38)

which shows that the scattering does not change the minimum value. This implies that the final state of the extremal black hole is still an extremal black hole, the weak cosmic censorship conjecture is valid in the extremal charged RN-AdS black hole surrounded by quintessence in the extended phase space.

When it is a near-extremal black hole, the order of the variables becomes important, $f'(r_+)$ is very close to zero. To evaluate the value of the above equation, we can let $r_+ = r_0 + \epsilon$, where $0 < \epsilon \ll 1$ and the relation $f(r_+) = 0$ and $f'(r_0) = 0$. And then, the Eq. (35) can be written as

$$\delta < 0,$$

$$\delta_{1} = -\frac{f''(r_{+})}{4(\pi r_{+})} \epsilon dS + \frac{6r_{+}}{l^{3}} \epsilon dl - 3\omega_{q} r_{+}^{-3\omega_{q}-2} \epsilon da + O\left(\epsilon^{2}\right),$$

$$\delta_{2} = \frac{2qQ\left(\omega - q\varphi\right)}{r_{+}} \epsilon dt + O\left(\epsilon^{2}\right),$$
(39)

where dt is an infinitesimal scale and is set as $dt \sim \epsilon$. If the initial black hole is near extremal, we have $dS \sim \epsilon$, $dl \sim \epsilon$ and $da \sim \epsilon$. So $\delta_1 + \delta_2 \ll \delta$, the final black hole has

$$f(r_0 + dr_0, M + dM, Q + dQ, l + dl, a + da) \approx \delta < 0.$$
⁽⁴⁰⁾

This indicates that the event horizon exists in the finial state. The black hole can not be overcharged by the scattering of the scalar field. Therefore, the weak cosmic censorship conjecture is valid in the near-extremal charged RN-AdS black hole surrounded by quintessence in the extended phase space.

3.2. Normal phase space

In the normal phase space, the cosmological constant and dimensional parameters are fixed. The initial state of the black hole is represented by (M, Q, r_+) , and the final state is represented by $(M + dM, Q + dQ, r_+ + dr_+)$. The variation of the radius can be obtained from the variation of the metric function $f(r_+)$. For the initial state (M, Q, r_+) , satisfies

$$f(M, Q, r_{+}) = 0. (41)$$

We assume that the final state of the charged RN-AdS black hole is still a black hole surrounded by quintessence, which satisfies

$$f(M + dM, Q + dQ, r_+ + dr_+) = 0.$$
(42)

The functions $f(M + dM, Q + dQ, r_+ + dr_+)$ and $f(M, Q, r_+)$ satisfy the following relation

$$f(M + dM, Q + dQ, r_{+} + dr_{+}) = f(M, Q, r_{+}) + \frac{\partial f}{\partial M} \Big|_{r=r_{+}} dM + \frac{\partial f}{\partial Q} \Big|_{r=r_{+}} dQ + \frac{\partial f}{\partial r_{+}} \Big|_{r=r_{+}} dr_{+}.$$
(43)

Bring Eqs. (42), (41) and (26) into Eq. (43) leads to

$$dM = \frac{Q}{r_{+}} dQ + 2\pi T r_{+} dr_{+}.$$
(44)

Using the Eqs. (6) and (8), gives the first law of thermodynamics

$$dM = TdS + \varphi dQ. \tag{45}$$

In the normal phase space, Eq. (18) gives the transferred energy and charge within the certain time interval

$$dM = \omega(\omega - q\varphi)r_+^2 dt, dQ = q(\omega - q\varphi)r_+^2 dt.$$
(46)

Bring these results into the Eq. (43), we can obtain the variation of the radius at horizon

$$dr_{+} = \frac{r_{+}}{2\pi T} (\omega - q\varphi)^{2} dt.$$
(47)

So the variation of the entropy takes on the form

$$dS = 2\pi r_{+}dr_{+} = \frac{r_{+}^{2}}{T}(\omega - q\varphi)^{2}dt > 0,$$
(48)

which shows that the entropy of the black hole increases. So the second law of black hole thermodynamics is satisfied for the black hole.

Then we test the validity of the weak cosmic censorship conjecture in the extremal and nearextremal via the black hole in this case. We suppose that there exists one minimum point at $r = r_0$ for f(r), and the minimum value of f(r) is not greater than zero

$$\delta \equiv f\left(r_0\right) \le 0,\tag{49}$$

where $\delta = 0$ corresponds to the extremal black hole. After the black hole scatters the scalar field, the minimum point would move to $r_0 + dr_0$. For the final black hole solution, if the minimum value of f(r) at $r = r_0 + dr_0$ is still not greater than zero, there exists an event horizon. Otherwise, the final black hole solution is over the extremal limit, and the weak cosmic censorship conjecture is violated. For the final state, the minimum value of f(r) at $r = r_0 + dr_0$ becomes

$$f(r_0 + dr_0, M + dM, Q + dQ) = \delta + \frac{\partial f}{\partial r} \Big|_{r=r_0} dr_0 + \frac{\partial f}{\partial M} \Big|_{r=r_0} dM + \frac{\partial f}{\partial Q} \Big|_{r=r_0} dQ.$$
(50)

Bring Eqs. (46) and (36) into Eq. (50), we can obtain

$$f(r_0 + dr_0, M + dM, Q + dQ) = \delta - \frac{2}{r_0} \left(\omega - \frac{qQ}{r_+} \right) \left(\omega - \frac{qQ}{r_0} \right) r_+^2 dt.$$
(51)

When it is an extreme black hole, $r_0 = r_+$ and $\delta = 0$. So the above equation becomes

$$f(r_0 + dr_0, M + dM, Q + dQ) = -2(\omega - q\varphi)^2 r_+ dt < 0,$$
(52)

which indicates that the horizon exists in the finial state. The black hole can not be overcharged by the scattering of the scalar field. The extremal black hole still extremal black hole if $\omega = q\varphi$, but the extremal black hole becomes non-extremal black hole if $\omega \neq q\varphi$.

When it is a near-extremal black hole, the Eq. (51) above equation can be regarded as a quadratic function of ω . When $\omega = 2qQ(\frac{1}{r_+} + \frac{1}{r_0})$, we find an maximum value on the function of $f(r_0 + dr_0, M + dM, Q + dQ)$

$$f(r_0 + dr_0, M + dM, Q + dQ)_{max} = \delta - \frac{dtq^2 Q^2 (r_+ - r_0)^2}{2r_0^3} < 0.$$
(53)

This implies metric function has a minimum negative value, which indicates that the event horizon also exists in the finial state. The black hole can not be overcharged by the scattering of the scalar field. Therefore, the weak cosmic censorship conjecture is valid in the near-extremal charged RN-AdS black hole surrounded by quintessence in this case.

4. Conclusion

In this paper, we first derived the RN-AdS black hole surrounded by quintessence via the scattering of a complex scalar field. The variations of this black hole's energy and charge within the certain time interval can be calculated. Then we investigated the validity of the thermodynamic laws in the extended and normal phase spaces with these variations. The first law of thermodynamics is always satisfied. However, the second law of thermodynamics is a little more complicated. When the PV term is not considered, the universe space-time is the normal phase space, and the second law of thermodynamics is valid. In the extended phase space, the cosmological constant and normalization parameters are considered as thermodynamic variables. We find that no matter whether there is a quintessence around the black hole, the second law of thermodynamics not always valid, this is related to the initial conditions of the black hole. Moreover, we test the validity of the weak cosmic censorship conjecture in the extremal and near-extremal RN-AdS black hole surrounded by quintessence. The weak cosmic censorship conjecture is valid both in the extended and normal phase spaces. In the extended phase space, the extremal black hole stays extremal under the scattering of the field. But in the normal phase space, the extremal black hole becomes non-extremal black hole if $\omega \neq q\varphi$, and the extremal black hole stays extremal if $\omega = q\varphi$. By compare our result with the situation of without the quintessence in [24], the weak cosmic censorship conjecture are both valid. The same conclusion indicates that the cosmological constant and the quintessence have similar physical effects. This conclusion is consistent with the previous work [37], which consider that both the cosmological constant and the quintessence are the reasons for the accelerated expansion of the universe.

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