Beamline to MICE Alignment and Matching Tolerance

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I describe the tolerance of MICE for alignment and matching to a beam from the muon beamline. I consider alignment in three variables: position in x, momentum in x and energy. I consider matching in terms of correlations between these three variables. In addition, tolerance to beam angular momentum is examined and some non-linear effects are studied.

Introduction

The MICE muon beamline is designed to produce a beam that is matched to the MICE cooling channel at a variety of momenta and emittances [TRD]. The MICE muon beamline will be commissioned during MICE Steps 1, 2 and 3. It is desirable to know when the beamline is sufficiently well aligned and matched to MICE; or if some form of offline analysis is required for final "matching".

The method used in this note is to simulate beams passing through MICE and examine the cooling performance as a function of misalignment or mismatch. Each variable is changed independently while all others are held constant. Misalignment and mismatch are considered in transverse variables and energy. I take advantage of the cylindrical symmetry of the cooling channel in order to examine the cooling performance in x only, rather than looking at x and y. Also some consideration is given to longitudinal-transverse correlations, angular momentum and non-linear effects.

Simulation Details

Simulations were performed using icool simulation code [ICOOL]. A simplified icool deck was used. Only the absorber and RF apertures were represented, and absorbers were taken to be cylindrical. Each simulation run was performed with 10,000 events. Where I consider means (first moments) and covariances (second moments), input beams were in general Gaussian multivariate. Nonlinear matching terms were also considered; the beams for these simulations will be described in more detail in the appropriate section.

The baseline beam was aligned to the axis with initial mean energy of 232 MeV, which gives a mean beam momentum of about 200 MeV/c for the large transverse momenta present in the beam. The baseline beam had 6 mm transverse emittance and 0.04 ns longitudinal emittance. Input beta function was 333 mm and alpha was 0.

Magnets were set up for MICE Stage 6 in flip mode and 200 MeV/c beam momentum. RF cavities were phased at 90^{o} with a peak field of 7.7 MeV. Output mean energy is about 219 MeV. The beam was inserted at -4.7 metres and output emittance measured at +4.7 metres relative to the MICE centre. The effect of alignment and matching on beam loss was not studied.

Beam Alignment

I changed the mean beam x-position, x-momentum and energy and examined the effect on the transverse cooling performance. The resulting change is shown in Figure 1.

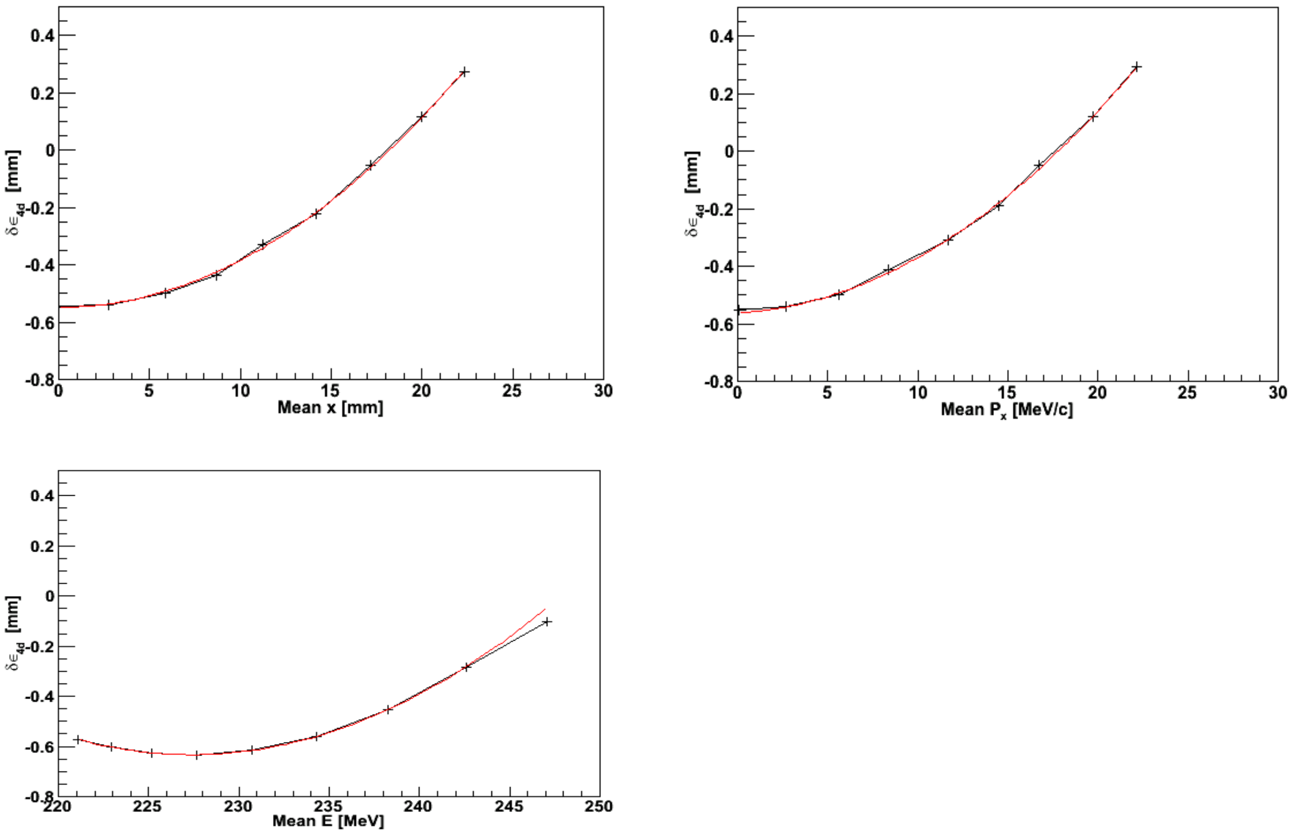

Figure 1: Effect of beam misalignment on cooling performance

Beam Mismatch

Transverse mismatch

A transverse mismatch was introduced to the beam while the beam emittance was kept constant. In this case it was desirable to introduce a mismatch in only one dimension at a time; the covariance matrix was generated in the rotating Larmor frame and then a rotation applied to transform to kinetic coordinates. In this case it is possible to define β and α functions in x and y; and hence to vary the functions in x independently of y, while keeping transverse emittance constant. The resulting transverse cooling performance is shown in Figure 2.

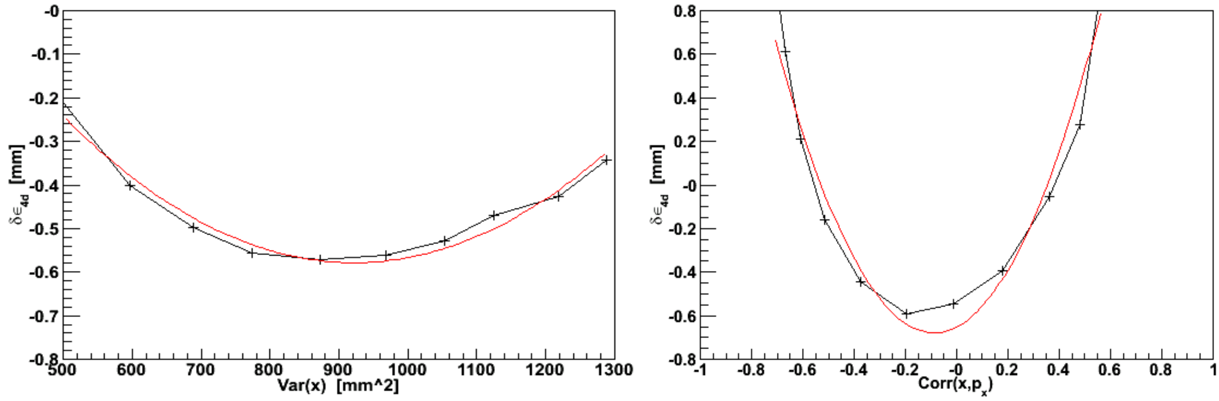

Figure 2: Effect of beam mismatch on cooling performance.

Requirements on Alignment and Matching

The results of the simulations can be used to place a requirement on the output of the beamline. For the purposes of this note, two such requirements are considered.

In an ideal situation, misalignment would introduce an effect such that is small compared to the intended sensitivity of the MICE detector systems. The MICE detectors are designed to be sensitive to a change in emittance of about 0.1% at the nominal 6 mm transverse emittance, which is 1% of the cooling performance.

It is possible that this case may be extremely challenging for the beamline to achieve. In this case, a weaker requirement is considered that the beamline misalignment should not change the cooling performance by more than 10%. This still enables the direct measurement of cooling in MICE without any offline beam selection, albeit with a reduced performance in the cooling channel due to the misalignment. In this case any fine tuning of the analysis will require offline beam selection.

It should be noted that, as the energy distribution of the input beam is large and the beamline offers no control over input timing distributions, which is expected to be flat, some offline beam selection will be necessary in any case.

The requirements this places on alignment and matching are shown in Table 1. These requirements are found by fitting polynomial functions to the figures and interpolating the data based on this fit. The functions used in the fit are listed in Appendix A.

Variable	***1% Cooling Requirement***	***10% Cooling Requirement***
$<x>$	2 mm	6 mm
$<p_x>$	2 MeV/c	6 MeV/c
$<E>$	2 MeV	7 MeV
$<x^2>$	50 mm^2	200 mm
Corr(x,p_x)	0.04	0.1

Table 1: Tolerance on matching and alignment at the tracker reference plane for different cooling measurement accuracies.

Other Second Moments

I also consider beam correlations that the MICE muon beamline is not designed to control. These are longitudinal-transverse correlations and angular momentum.

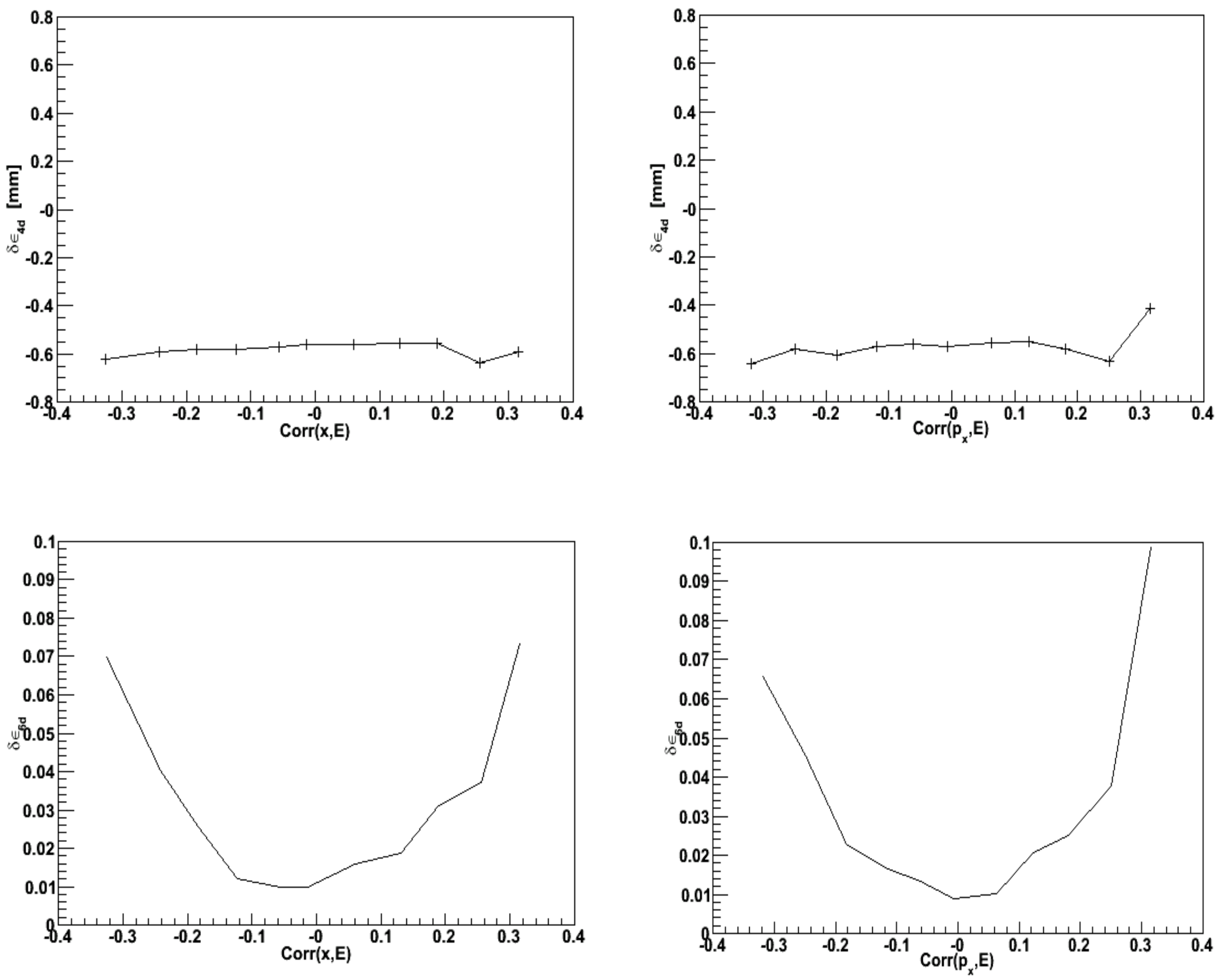

Figure 3: Effect of longitudinal-transverse correlations on cooling performance. (Top) 4D cooling performance (Bottom) 6D cooling performance (Left) correlation between x and E (Right) correlation between px and E

Longitudinal-Transverse Correlations

Due to the nature of the muon beamline, it is likely that there will be correlations between longitudinal and transverse coordinates. Two such correlations were introduced in the beam; firstly a correlation between energy and position; and secondly a correlation between energy and momentum. The effect of these correlations on transverse cooling and 6D emittance change is shown in Figure 3.

In examining the change in 6D emittance, it should be noted that the input 6D emittance was not held constant as the correlations were introduced. Also it should be noted that a multivariate Gaussian distribution was used so that third moments were 0; as I show elsewhere, control of third moments and other non-linear effects can make significant changes in 6D the cooling performance.

Canonical Angular Momentum

When a beam enters a solenoid, the fringe field of the solenoid introduces kinetic angular momentum into the beam, while conserving canonical angular momentum. The presence of a lead diffuser in the solenoidal field of the upstream tracker solenoid will reduce the kinetic angular momentum of the beam, leading to a non-zero canonical angular momentum [Diffuser].

The fractional change in kinetic angular momentum on passing through a diffuser is approximately equal to the fractional change in momentum on passing through the material. The maximum fractional change in kinetic angular momentum is expected to be about 10% [Diffuser]. In Figure 4, the transverse β function and cooling performance of a beam with a 10% reduction in angular momentum is shown.

Three examples are shown. Firstly a nominal matched beam with no canonical angular momentum is passed through the simulation. Secondly a beam with canonical angular momentum is passed through the simulation. Here the canonical angular momentum was generated by reducing the x-p_y and y-px covariances by 10%, and making no other changes to the beam covariance matrix. Thirdly, a 10% reduction in kinetic angular momentum was introduced but the beam was rematched to account for the loss in kinetic angular momentum by changing the p_x variance. Here, unlike other simulations, MICE was simulated using G4MICE and a monochromatic gaussian beam.

The mismatched beam with canonical angular momentum actually shows the greatest cooling performance; however, it should be noted that the introduction of canonical angular momentum also led to an increase in emittance such that the comparison is unfair. The rematching procedure is, however, successful and leads to a more closely matched beam.

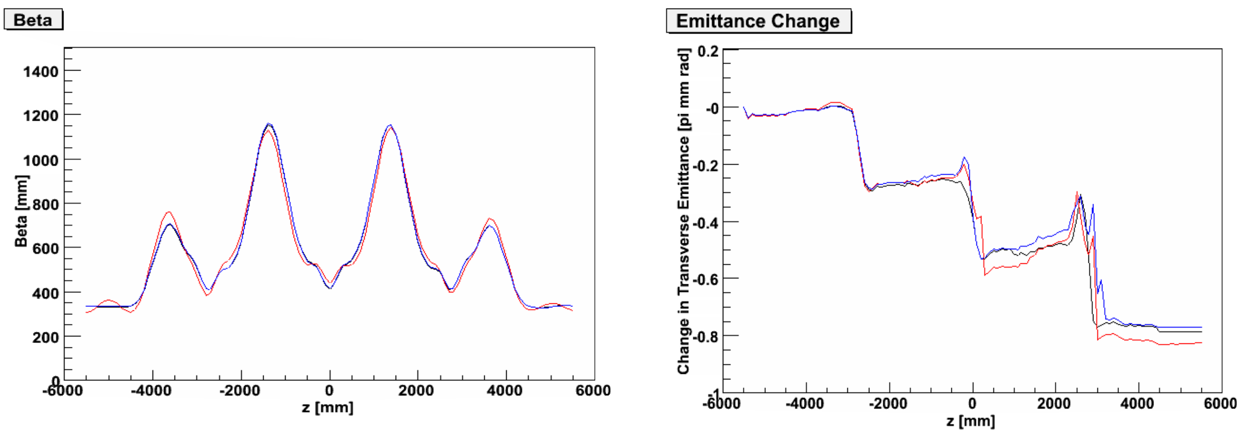

Figure 4: beta function and transverse emittance evolution for a muon beam in the presence of angular momentum (black) 0 canonical angular momentum (red) 10% reduction in kinetic angular momentum (blue) 10% reduction in kinetic angular momentum, rematched.

Non-Linear Matching

I study the effect on 6D cooling of two non-linear matching techniques. Firstly I examine the effects of amplitude-momentum correlation and secondly the effects of introducing a matched β function for different momenta.

Amplitude-Momentum Correlation

As a high-emittance beam travels down an accelerating structure, a correlation emerges between momentum and particle amplitude [FS2]. Particles with high amplitude have a longer time-of-flight than those with low amplitude leading to a longitudinal-transverse correlation [A-P Corr]. This emergence of significant third moments in the beam is a non-linear effect.

In Figure 5 I show the effect of this correlation on the 6D cooling performance of MICE. I introduce a correlation *C* by first generating a gaussian multivariate beam and then adding *dE* to the beam energy, where *dE* is given by

$$dE = C(A^2 - \langle A^2 \rangle)$$

and A is the particle's transverse amplitude. This transformation introduces the desired correlation while leaving the mean energy unchanged. In Figure 5, the actual covariance between amplitude and energy is shown, rather than the parameter *C* used to generate that covariance.

It is observed that without introducing such a correlation, MICE does not cool; in fact it heats in six dimensions. It should be noted that the Amplitude-Momentum correlation quoted in the Feasibility Study 2 beam on entrance into the cooling channel is 0.7 GeV/c [FS2].

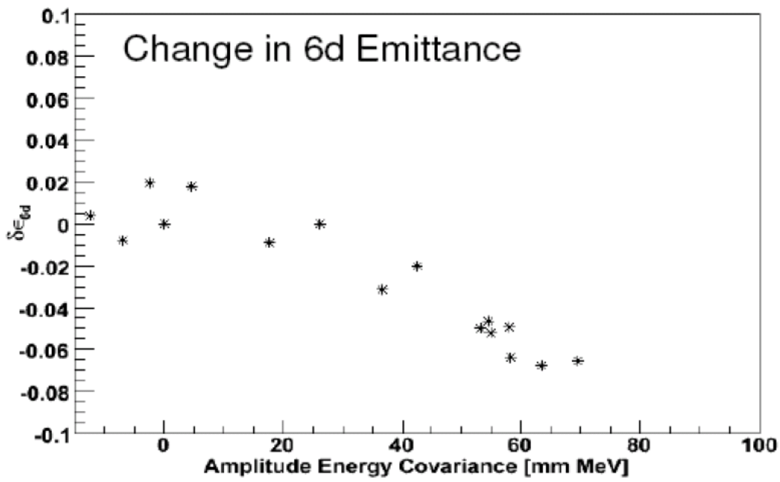

Figure 5: Change in 6d emittance as a function of Amplitude Energy Covariance.

Momentum-Dependent Beta

The other non-linear matching term that can be introduced is to make the beam matched even for off-momenta. The longitudinal emittance of the MICE beam is sufficiently large that chromatic aberrations cause significant heating and are quite detrimental to the MICE performance. One way to avoid this is to match the beta function in the beam for a variety of momenta [β-P Corr]. This feature develops naturally in a long cooling channel.

In Figure 6 I show the effect of matching the beam at different momenta. Here, the matched β function (β periodic between the centre of the absorbers) is calculated at various momenta. Particle energies are chosen from a Gaussian distribution and then the matched transverse covariance matrix is calculated by interpolation from the periodic β functions for different momenta. Finally, an amplitude momentum smearing is introduced as described above.

It is observed that such an arrangement leads to an improvement in the transverse cooling performance by approximately 20 %.

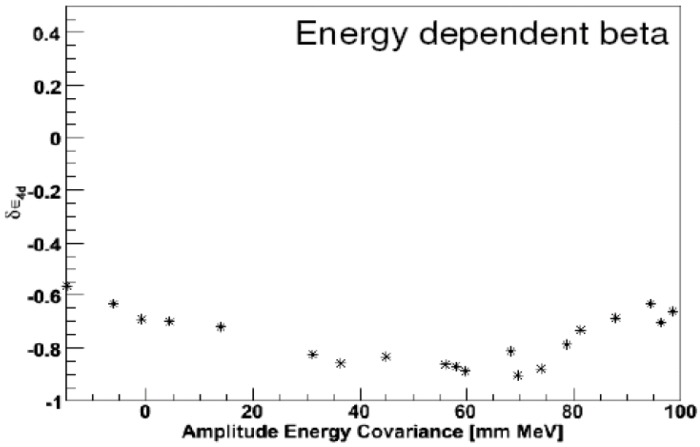

Figure 6: Effect of applying beam matching at different momenta on the transverse cooling performance together with an amplitude energy correlation.

Final Comments

I have described several issues for the input beam in MICE and examined how they effect the cooling performance. First and second moment matching can be achieved by careful beamline design in transverse phase space. However, practical constraints may limit what can be achieved even here. It is expected that the beam will have canonical angular momentum, longitudinal-transverse correlations and the time distribution will be flat, making beam selection necessary for most analyses.

The necessity of non-linear matching makes off-line selection even more vital. I again emphasise that from these simulations it appears that without appropriate amplitude-momentum correlation, MICE does not reduce 6D emittance and hence does not cool. In Feasibility Study 2, this issue was resolved by calculating the emittance using a combination of second and third moments [ECALC9].

Bibliography

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[A-P Corr] J. Scott Berg, Amplitude Dependence of Time of Flight and its Connection to Chromaticity, NIM A 570 (1), p.15-21, Jan 2007.

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Appendix A - Fit Parameters

Fit using a function like f(x) = a_0 + a_1 x + a_2 x^2 +...

x fit parameters

NO.	VALUE	ERROR
0	-0.547782	0.811399
1	-0.000506	0.168562
2	0.001678	0.007221

p_x fit parameters

NO.	VALUE	ERROR
0	-0.561759	1.369885
1	0.003356	0.252755
2	0.001585	0.009916

E fit parameters

NO.	VALUE	ERROR
0	78.055630	555.847956
1	-0.691985	4.781118
2	0.001521	0.010274

beta parameters

NO.	VALUE	ERROR
0	1.023245	4.290123
1	-0.003474	0.010102
2	0.000002	0.000006

alpha parameters

NO.	VALUE	ERROR
0	-0.652085	0.539235
1	0.610249	0.759699
2	3.485336	2.274950

Appendix 2 - Alignment Outside of Solenoid

Due to difficulties in simulating the spectrometer solenoid fringe field, it was requested that alignment conditions be provided upstream of the solenoid fringe field. Two requirements can be formulated; firstly it is required that the transverse momentum at the tracker reference plane lie within the tolerances outline above; and secondly it is required that the transverse position at the tracker reference plane lie within the tolerances outlined above the two conditions are shown in Figure 7. Here, test particles were tracked from -6650 mm to the upstream tracker reference plane and the misalignment was examined at this point. It is observed that the alignment tolerances are somewhat more generous in position but quite similar in momentum. This might be expected as the beam is focussed in the entrance to the spectrometer solenoid.

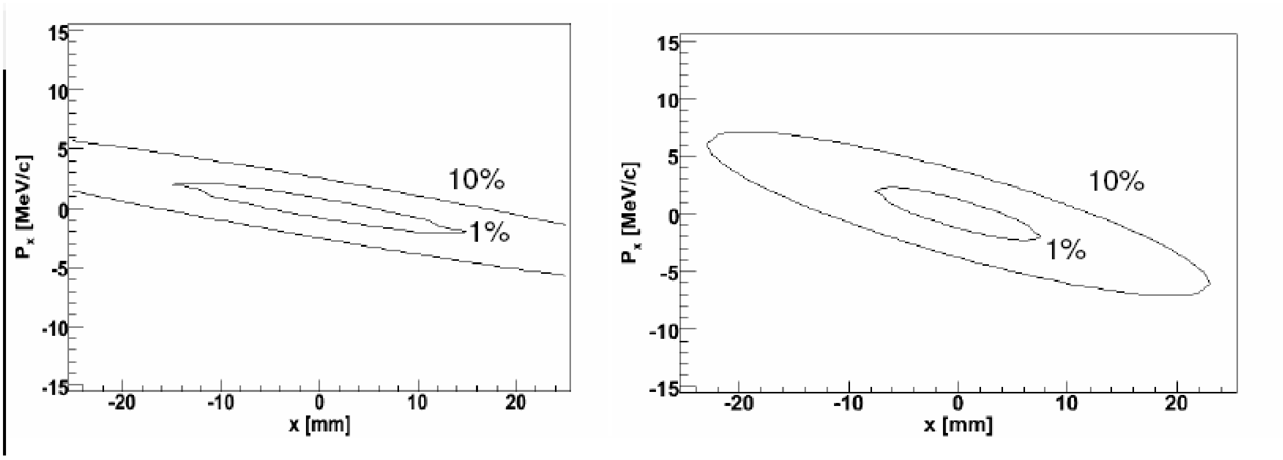

Figure 7: Alignment tolerances at a position somewhere upstream of the spectrometer solenoid. On the left, tolerances are shown for the condition that r *is within tolerance at the tracker reference plane. On the right, tolerances are shown for the condition that* p_t *is within tolerance at the tracker reference plane.*