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Comparison of the moments of the X_{max} distribution predicted by different cosmic ray shower simulation models

TODERO PEIXOTO, C. J. [1], DE SOUZA, V.[1], BELLIDO, J. A.[2]

[1] *Instituto de Física de So Carlos - Universidade de São Paulo, Brazil*
[2] *University of Adelaide, Adelaid S. A. 5005, Australia*

toderocj@ursa.ifsc.usp.br

Abstract: In this work we will present a study the depth at which a cosmic ray shower reaches its maximum (X_{max}) as predicted by Monte Carlo simulation.
The central idea is to study the differences between the available simulation models of the first and second moments of the X_{max} distribution using the CORSIKA and CONEX programs using different implementations of the hadronic interaction models: SIBILL2.1 and QGSJetII. We show that the predictions of the $< X_{max} >$ and $RMS_{X_{max}}$ depend slightly on the combination of simulation program and hadronic interaction model. Although these differences are small, they are not negligible in some cases (up to 5 g/cm^2 for the worse case) and they should be considered as a systematic uncertainty of the model predictions for $< X_{max} >$ and $RMS_{X_{max}}$. Finally, we present a parametrization of the $< X_{max} >$ distribution as a function of mass and energy, and showed an example of its application to obtain the predicted $< X_{max} >$ distributions from cosmic ray propagation models. The full work can be accessed in http://arxiv.org/abs/1301.5555v2

Keywords: Cosmic Ray, Slant Depth, Monte Carlo, Simulation.

1 Introduction

The most reliable technique to infer the mass composition of showers with energy above 10^{17} eV is the determination of the $X_{\rm max}$ and posterior comparison of the measured values with predictions from Monte Carlo simulation. This is because above 10^{17} eV fluorescence detectors can measure $X_{\rm max}$ with a resolution of 20 g/cm^2. The evolution of the detectors, the techniques used to measure the atmosphere, the advances in the understanding of the fluorescence emission and the development of innovative analysis procedures have resulted in a high precision measurement of $X_{\rm max}$ and its moments.

The dependency of $X_{\rm max}$ with primary energy and mass (A) has been analytically studied in a hadronic cascade model [1]. Monte Carlo programs can simulate the hadronic cascade in the atmosphere using extrapolation from the measured hadronic cross sections at somewhat lower energy. It has been shown before that different hadronic interaction models do not agree in the prediction of the $\langle X_{\rm max}\rangle$ and other parameters [2].

In this work we study in detail the dependence of $\langle X_{\rm max}\rangle$ and $\mathrm{RMS}(X_{\rm max})$ as a function of energy and primary mass. We compare the result of two hadronic interaction models. We have done a high statistics study and we show that the discrepancies between models and programs are at the same level of quoted systematic uncertainties of the experiments. The analysis done here points to the need of a better understanding of the interaction properties at the highest energies which can be achieved by ongoing analysis of the LHC data which already resulted in updates of the hadronic interaction models. At the same time the results presented in this paper point to discrepancies between different implementations of the same hadronic interaction model which need to be better understood.

We also present a parametrization of the $X_{\rm max}$ distribution as a function of mass and energy. Several theoretical models have predicted the mass abundance based on astrophysical arguments [3, 4, 5, 6]. In order to compare the predicted abundance with measurements, one has to convert the calculated flux for each particle into $X_{\rm max}$. Until now, this could only be done using full Monte Carlos simulations. We present here a parametrization of the $X_{\rm max}$ distribution to allow the conversion of astrophysical models into $X_{\rm max}$ measurements. Parametrizations of $\langle X_{\rm max}\rangle$ as a function of energy and mass have been already studied [1]. What we present here is a step forward, we show the parametrization of the $X_{\rm max}$ distribution which is good enough to calculate the first and second moments of the distribution.

2 Shower Simulation

We have used CONEX [7, 8] and CORSIKA [9] shower simulators. CONEX uses a one dimensional hybrid approach combining Monte Carlo simulation and numerical solutions of cascade equations. CORSIKA describes the interactions using a full three dimensional Monte Carlo algorithm. By using analytical solutions, CONEX saves computational time. On the other hand, CORSIKA makes use of the thinning algorithm [10, 11] to reduce simulation time and output size.

Both approaches have negative and positive features. CORSIKA offers a full description of the physics mechanisms and a three dimensional propagation of the particles in the atmosphere. However, it is very time consuming, limiting studies which depend on large number of events at the highest energies. The thinning algorithm introduces spurious fluctuations that have to be taken into account in the final analysis. CONEX is fast, but on the other hand it offers only a one dimensional description of the shower. The use of intermediate analytical solutions might also reduce the intrinsic fluctuation of the shower.

For each shower simulator many hadronic interaction models are available. We have used QGSJETII.v03 [12, 13] and SIBYLL2.1 [14] in this work. For the low energy hadronic interaction we have used GHEISHA [15] in all simulations.

Showers have been simulated with primary energy ranging from $10^{17.0}$ to $10^{20.4}$ eV in steps of $\log_{10}(E/eV) = 0.1$. We have simulated seven primary nuclei types with mass: 1, 5, 15, 25, 35, 45 and 55. For each primary particle, primary energy, and hadronic interaction model combination, a set of 1000 showers has been simulated. The zenith angle of the shower was set to 60° and the observation height was at sea level corresponding to a maximum slant depth of 2000 g/cm^2 allowing the simulation of the entire longitudinal profile of the showers. The longitudinal shower profile was sampled in steps of 5 g/cm^2. The energy thresholds in CORSIKA and CONEX were set to 1, 1, 0.001 and 0.001 GeV for hadron, muons, electrons and photons respectively.

Both programs will be compared in detail concerning the $X_{\rm max}$ calculations.

The $X_{\rm max}$ was calculated by fitting a Gaisser-Hillas [16] function to the energy deposited by the particle through the atmosphere. We chose a four parameter Gaisser-Hillas (GH4) function given by:

$$\frac{dE}{dX}(X) = \frac{dE}{dX}^{max} \left(\frac{X - X_0}{X_{\rm max} - X_0}\right)^{\frac{X_{\rm max}-X_0}{\lambda}} \exp\left(\frac{X_{\rm max} - X}{\lambda}\right) \quad (1)$$

in which $\frac{dE}{dx}^{max}$, X_0, λ and $X_{\rm max}$ are the four fitted parameters and X is the slant atmospheric depth.

3 Results

The Figures 1 show the $\langle X_{\rm max}\rangle$ and RMS$(X_{\rm max})$ using CONEX simulator for SIBYLL2.1.

Figure 2 summarizes the differences in $\langle X_{\rm max}\rangle$ and RMS$(X_{\rm max})$ between the simulation programs and between the hadronic interaction models. This figure shows simultaneously the $\langle X_{\rm max}\rangle$ and RMS$(X_{\rm max})$ values, where the corresponding $\langle X_{\rm max}\rangle$ and RMS$(X_{\rm max})$ for a nuclei with mass 55 has been taken as reference (as suggested in [17]). This figure illustrates the importance of taking into account the simulation program differences into the systematic uncertainty of the model predictions. Each blob corresponds to the $\langle X_{\rm max}\rangle$ and RMS$(X_{\rm max})$ predictions for one primary particle at different energies.

3.1 Parametrization of the $X_{\rm max}$ distributions

The $X_{\rm max}$ distributions can be described by a function which is a convolution of a Gaussian with an exponential [18]:

$$\frac{dX_{\rm max}}{dN} = N_f \exp\left(\frac{t_0 - t}{\lambda} + \frac{\sigma^2}{2\lambda^2}\right) Erfc\left(\frac{t_0 - t + \sigma^2/\lambda}{\sqrt{2}\sigma}\right) \quad (2)$$

This equation has four parameters. N_f is a normalization factor which gives the total number of events in the $X_{\rm max}$ distribution. λ, t_o and σ are parameters which are related to the decay factor of the exponential, the maximum of the distribution and the width of the distribution respectively. $Erfc$ is the error function.

We parametrized λ, t_o and σ as a function of primary mass and energy using the simulated showers. Given the degeneracy in shaping the width of the $X_{\rm max}$ distribution, the parameters σ and λ are inversely correlated. The parameters σ and λ compensate each other, fluctuations to higher values of σ are correlated to fluctuations to smaller values of λ.

We performed a fit to plots that was parametrized with a linear dependence on $log_{10}(A)$ and $log_{10}(E)$ following equation:

$$\left.\begin{array}{c} t_0 \\ \sigma \\ \lambda \end{array}\right\} = C_1 \times log_{10}(E/eV) + C_2 \times log_{10}(A) + C_3 \quad (3)$$

Tables 1 and 2 show the fitted parameters for CONEX and CORSIKA respectively. Despite the fluctuations of σ and λ a linear fit in $log_{10}(A)$ and $log_{10}(E)$ is reasonably good approximation to describe the $X_{\rm max}$ distribution. This can be seen in figures 3 and 4 where we show a comparison between the simulation, the direct fit of the $X_{\rm max}$ distribution using equation 2 and the calculation using equation 3 and table 1.

	Had. Model	C_1 ($\pm$ err)	C_2 ($\pm$ err)	C_3 ($\pm$ err)
t_o	QGSJETII	53.06 (0.05)	-28.74 (0.12)	-275.93 (1.18)
	SIBYLL2.1	60.48 (0.07)	-38.48 (0.13)	-402.80 (1.22)
σ	QGSJETII	-0.26 (0.06)	-5.63 (0.21)	31.68 (3.38)
	SIBYLL2.1	-1.09 (0.07)	-5.28 (0.19)	44.41 (1.54)
λ	QGSJETII	-2.68 (0.14)	-19.50 (0.43)	100.32 (2.63)
	SIBYLL2.1	-2.61 (0.11)	-17.89 (0.14)	96.28 (1.76)

Table 1: Fitted coefficients (equation 3)- CONEX. All values in g/cm^2.

	Had. Model	C_1 ($\pm$ err)	C_2 ($\pm$ err)	C_3 ($\pm$ err)
t_o	QGSJETII	53.32 (0.30)	-29.47 (0.52)	-283.93 (5.62)
	SIBYLL2.1	60.77 (0.23)	-38.88 (0.31)	-408.88 (4.67)
σ	QGSJETII	0.06 (0.002)	-5.06 (0.17)	35.99 (3.21)
	SIBYLL2.1	-0.56 (0.08)	-4.70 (0.21)	44.01 (2.03)
λ	QGSJETII	-1.73 (0.15)	-20.63 (0.34)	82.69 (3.54)
	SIBYLL2.1	-2.49 (0.22)	-19.54 (0.34)	96.04 (3.46)

Table 2: Fitted coefficients (equation 3) - CORSIKA. All values in g/cm^2.

4 Conclusions

We have studied the simulation programs CORSIKA and CONEX with the hadronic interaction models SIBYLL2.1 and QGSJETII. We have shown that the $\langle X_{\rm max}\rangle$ and the RMS$(X_{\rm max})$ depend slightly on the combination of program and hadronic interaction model chosen. It is widely known that $\langle X_{\rm max}\rangle$ and RMS$(X_{\rm max})$ predicted by SIBYLL2.1 and QGSJETII are different mainly due to the different extrapolations of the hadronic interaction properties to the highest energies. We have quantified here the differences between CORSIKA and CONEX by predicting the $\langle X_{\rm max}\rangle$ and the RMS$(X_{\rm max})$ using the same hadronic interaction model. These differences are small, but should be considered as systematic uncertainties of the model predictions.

Figure 5 shows the evolution of the $\langle X_{\rm max}\rangle$ and RMS$(X_{\rm max})$ with energy. No clear dependency of the difference between CORSIKA and CONEX with energy or primary particle type was seen. When using QGSJETII or SIBYLL2.1, CORSIKA and CONEX predict the $\langle X_{\rm max}\rangle$ with a difference smaller than 7 g/cm^2, and the RMS$(X_{\rm max})$ with a difference smaller than 5 g/cm^2. The differences in the slopes of a linear fit to the evolution of the $\langle X_{\max}\rangle$ and RMS$(X_{\max})$ with energy for CORSIKA and CONEX are quite small (< 3 %).

No assumption is made here for the cause of these differences. An investigation for the possible cause could be done, but in the meanwhile these differences between the programs should be considered as systematics error in the analysis of the $\langle X_{\max}\rangle$ and RMS$(X_{\max})$ when one tries to infer the composition abundance.

The curves that was used to estimate the first and second moments of the $X_{\max}$ distribution from abundance calculations was based on astrophysical arguments. As an example of the usage of this parametrization we have taken the astrophysical models developed by Allard et al. [3] (Model A) and used our paremetrization to transform the abundance curves predicted by the models into a $X_{\max}$ distribution. Figure 6 shows a $X_{\max}$ distributions predicted by the model in comparison to the data measured by the Pierre Auger Observatory [19]. We have convolved the model predictions with a Gaussian detector resolution of 20 g/cm^2. The utility of the parametrization is such that the models can be compared to the $X_{\max}$ distribution instead of only the $\langle X_{\max}\rangle$ and RMS$(X_{\max})$.

The complete work can be accessed in:
http://arxiv.org/abs/1301.5555v2

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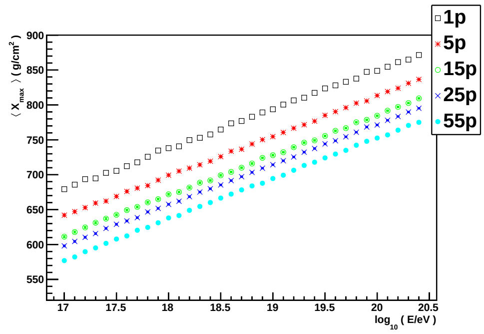
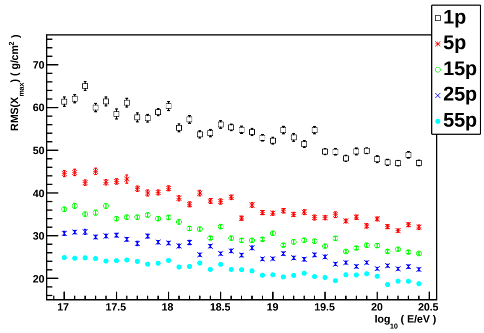


(a) CONEX- $\langle X_{\max}\rangle$ *versus* energy



(b) CONEX- RMS$(X_{\max})$ *versus* energy.

Fig. 1: $\langle X_{\max}\rangle$ as a function of energy as calculated by CONEX using SIBYLL2.1. Showers have been simulated with primary energy ranging from $10^{17.0}$ to $10^{20.4}$ eV in steps of $\log_{10}(E/eV) = 0.1$ and primary nuclei types with mass: 1, 5, 15, 25, 35, 45 and 55. A set of 1000 showers has been simulated for each combination. Not all energies and primaries are shown for clarity.

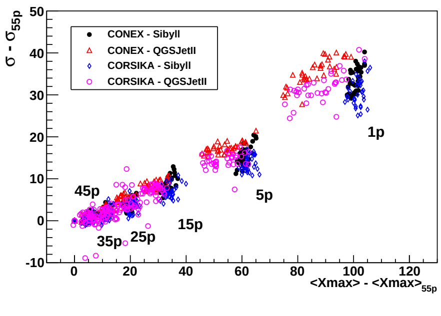
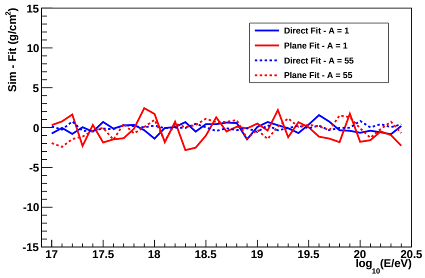
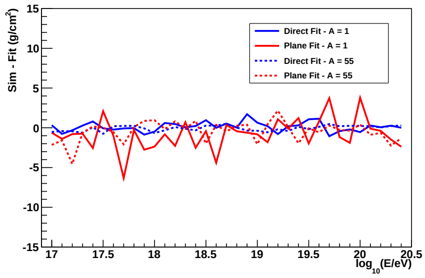


Fig. 2: RMS$(X_{\max})$ *versus* $\langle X_{\max}\rangle$ for all primary particles used in this work. The corresponding RMS$(X_{\max})$ and $\langle X_{\max}\rangle$ for a nuclei with mass 55 has been taken as reference. Each blob corresponds to the $\langle X_{\max}\rangle$ and RMS$(X_{\max})$ predictions for one primary particle at different energies.

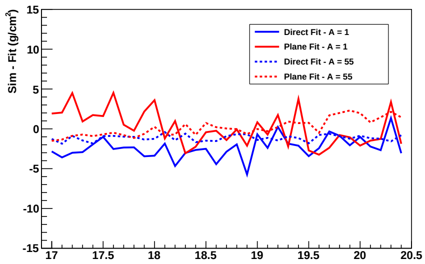


(a) CONEX- SIBYLL2.1

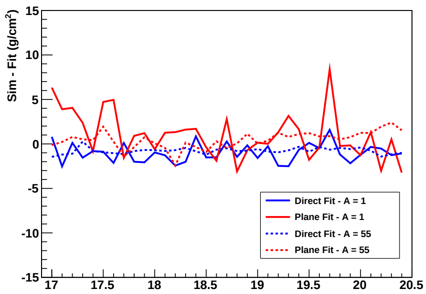


(b) CORSIKA- SIBYLL2.1

Fig. 3: Differences in $\langle X_{\max} \rangle$ *versus* energy. Comparison to the simulation, the direct fit of the $X_{\max}$ distribution using equation 2 and the calculation using equation 3 and table 1

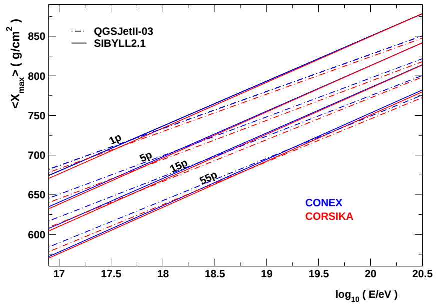


(a) CONEX- SIBYLL2.1

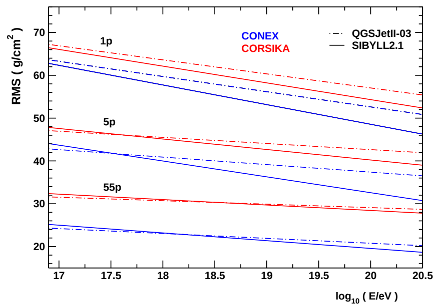


(b) CORSIKA- SIBYLL2.1

Fig. 4: Differences in RMS$(X_{\max})$ *versus* energy. Comparison to the simulation, the direct fit of the $X_{\max}$ distribution using equation 2 and the calculation using equation 3 and table 1

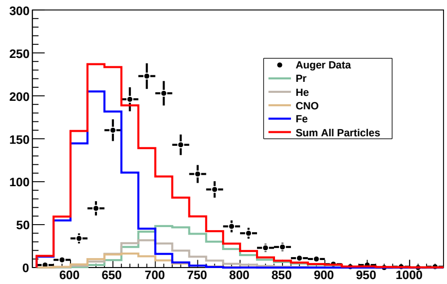


(a) $\langle X_{\max} \rangle$.



(b) RMS$(X_{\max})$.

Fig. 5: Comparison of $\langle X_{\max} \rangle$ and RMS$(X_{\max})$ for CORSIKA and CONEX as a function of energy.



(a) Allard et al. - CONEX- SIBYLL2.1.

Fig. 6: $X_{\max}$ distributions. Data measured by the Pierre Auger Observatory with energy $10^{18.0} < E < 10^{18.1}$ eV [19]. Astrophysical model extracted from [3]. The model have been calculated at $E = 10^{18.05}$ eV. The curves have been calculated using the parametrizations proposed above.