PHOTONIC PORTAL TO THE STERILE WORLD OF COLD DARK MATTER

WOJCIECH KRÓLIKOWSKI

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00-681 Warszawa, Poland

We assume that the cold dark matter consists of spin-1/2 and spin-0 particles described by a bispinor field ψ and a scalar field φ, sterile from all Standard Model (SM) charges (in contrast, neutralinos, supersymmetric candidates for cold dark matter, are not sterile from weak SM charges). We propose, however, that such a sterile world can contact with our SM world not only through gravity but also through a portal provided by photons coupled to sterile particles by means of two very weak effective interactions $-(f/M^2)\varphi F^{\mu\nu}\varphi F_{\mu\nu}$ and $-(f'/M^2)\bar{\psi}\sigma^{\mu\nu}\psi\varphi F_{\mu\nu}$, where M is a very large mass scale and f and f' are dimensionless coupling constants. Thus, in our picture, the electromagnetic field $F_{\mu\nu}$ — as the only SM field — participates in both worlds, providing a nongravitational link between them (other than the popular supersymmetric weak interaction, active in the case of neutralinos). In consequence, there appears a tiny quasi-magnetic correction to the conventional electromagnetic current (described in Appendix A).

PACS numbers: 14.80.–j, 04.50.+h, 95.35.+d

1. Introduction: content of dark matter

All presently known fundamental fermions *i.e.*, leptons and quarks, carry the Standard Model charges and so, are coupled to the Standard Model gauge bosons. Leptons differ from quarks by not participating in SU(3) strong interactions, though both display SU(2)× U(1) electroweak interactions. Besides, leptons and quarks (as well as the Standard Model gauge and Higgs bosons) are coupled also to the gravitational field that, if successfully quantized, is represented by gravitons (collaborating, perhaps, with their dilaton-like and/or axion-like partners).

In this situation, one may ask a so-called good question, if in Nature there is a place for a sort of fundamental spin-1/2 fermions interacting only gravitationally. Such fundamental fermions are by definition sterile from all Standard Model charges and do not mix with active neutrinos and so, they

are blind to all Standard Model gauge interactions and do not co-oscillate with active neutrinos. These sterile spin-1/2 particles will be called here *sterinos* (a shortening from sterilinos) to get a short name emphasizing both their sterility and their half-integer spin.

It is very natural to wonder, if just the sterinos — rather than neutralinos of the supersymmetric extension of the Standard Model — can be responsible for the fermion component of cosmic cold dark matter which dominates globally in all matter of our Universe and, at present, is one of the most important problems of today's particle physics and astrophysics [1].

The experimental search for physical effects caused by possible interactions of cold dark matter other than gravity (extended, perhaps, by the action of hypothetic spin-0 partners of gravitons) is presently in the centre of attention. This search can be classified either as direct detection experiments [2,3], where one hopes to observe recoils of nuclei scattered elastically from dark matter particles, or indirect experiments [2,4], where one tries to identify annihilation or decay products of dark matter as *e.g.* positrons possibly created by dark matter in the centre of our Galaxy and subsequently annihilated at rest with the emission of 511 keV line observed since 1970s.

Sterinos require a considerable mass in order to participate in the cold dark matter. One may speculate that in Nature there are fundamental scalar bosons, also sterile from all Standard Model charges [5], whose field — like the neutral component of Standard Model Higgs boson field — develops a nonzero vacuum expectation value. It will be convenient to use for these sterile scalar particles the name *sterons* (a shortening from sterilons).

The terms in the Lagrangian needed to generate the sterino Dirac mass may be taken in the form

$$-\bar{\psi}_{\text{sto}}\, y\, \psi_{\text{sto}}\varphi_{\text{stn}} + \frac{1}{2}\mu^2\varphi_{\text{stn}}^2 - \frac{1}{4}\lambda\varphi_{\text{stn}}^4 \tag{1}$$

with

$$\langle\varphi_{\text{stn}}\rangle_{\text{vac}} = \frac{\mu}{\sqrt{\lambda}} \neq 0\,, \tag{2}$$

(in the tree approximation), where y is an unknown Yukawa coupling constant (being a matrix in the case of sterinos developing more generations). The constants $\mu > 0$ and $\lambda > 0$ appearing in the steron potential $V(\varphi_{\text{stn}}) = -(1/2)\mu^2\varphi_{\text{stn}}^2 + (1/4)\lambda\varphi_{\text{stn}}^4$ are also unknown. Then, the sterino Dirac mass becomes

$$m_{\text{sto}}^{(\text{D})} = y\langle\varphi_{\text{stn}}\rangle_{\text{vac}} = y\frac{\mu}{\sqrt{\lambda}}\;. \tag{3}$$

The physical steron field is given by the difference $\varphi_{\text{stn}}^{(\text{ph})} \equiv \varphi_{\text{stn}} - \langle\varphi_{\text{stn}}\rangle_{\text{vac}}$. Then, the mass of physical sterons is generated as

$$m_{\text{stn}} = \mu\sqrt{2} = \sqrt{2\lambda}\langle\varphi_{\text{stn}}\rangle_{\text{vac}}\,, \tag{4}$$

since $V(\langle\varphi_{\mathrm{stn}}\rangle_{\mathrm{vac}}+\varphi_{\mathrm{stn}}^{(\mathrm{ph})})=\mu^2\varphi_{\mathrm{stn}}^{(\mathrm{ph})2}+\mu\sqrt{\lambda}\varphi_{\mathrm{stn}}^{(\mathrm{ph})3}+(1/4)\lambda\varphi_{\mathrm{stn}}^{(\mathrm{ph})4}-(1/4)\mu^4/\lambda$. The physical sterons may exist in Nature as the (probably unstable) boson component of cold dark matter.

Thus, the nonzero vacuum expectation value of the steron field φ_{stn} breaks spontaneously the scale symmetry of the sterile world consisting of sterinos and sterons. The righthanded neutrinos, as being sterile, should also belong to this world. *A priori*, they may be either different or identical with the righthanded components of sterinos. But, in both options their Majorana mass $m_\nu^{(\mathrm{R})}$, usually considered as very large, may be generated by the same vacuum expectation value of the steron field φ_{stn} which generates also the sterino Dirac mass $m_{\mathrm{sto}}^{(\mathrm{D})}$. Then, this expectation value breaks spontaneously once more the scale symmetry of the sterile world. Restricting oneself to a minimal picture of sterile world, one may try to imagine that the second option is true *i.e.*, the righthanded neutrino field is identical with the righthanded component of sterino field,

$$\nu_{\mathrm{R}}\equiv\psi_{\mathrm{sto\,R}}\,. \tag{5}$$

Then, the combined fields $\nu=\nu_{\mathrm{L}}+\nu_{\mathrm{R}}$ and $\psi_{\mathrm{sto}}=\psi_{\mathrm{sto\,L}}+\psi_{\mathrm{sto\,R}}$ get the common righthanded component ν_{R}. We contest this option in the footnote number 1 (the next section).

2. Generations in dark matter

The sterino Dirac mass (3) is the only existing kind of sterino mass, if sterinos are Dirac fermions. If, however, they are Majorana fermions, then one can define for them a more general mass term in the Lagrangian, namely

$$-\frac{1}{2}\left(\overline{\psi_{\mathrm{sto\,L}}},\overline{(\psi_{\mathrm{sto\,R}})^c}\right)\left(\begin{array}{cc} m_{\mathrm{sto}}^{(\mathrm{L})} & m_{\mathrm{sto}}^{(\mathrm{D})} \\ m_{\mathrm{sto}}^{(\mathrm{D})T} & m_{\mathrm{sto}}^{(\mathrm{R})} \end{array}\right)\left(\begin{array}{c} (\psi_{\mathrm{sto\,L}})^c \\ \psi_{\mathrm{sto\,R}} \end{array}\right)+\mathrm{h.c.}\,, \tag{6}$$

where $\psi=\psi_{\mathrm{L}}+\psi_{\mathrm{R}}$, $\psi^c=C\bar\psi^T=-\beta C\psi^*$ and $\overline{\psi^c}=\psi^{c\,\dagger}\beta=-\psi^T C^{-1}$ ($\dagger=*T$). The Dirac and Majorana righthanded parts of this mass term are

$$-\overline{\psi_{\mathrm{sto}}}\,m_{\mathrm{sto}}^{(\mathrm{D})}\,\psi_{\mathrm{sto}} \tag{7}$$

and

$$\frac{1}{2}\left[(\psi_{\mathrm{sto\,R}})^T C^{-1}m_{\mathrm{sto}}^{(\mathrm{R})}\psi_{\mathrm{sto}R}+(\psi_{\mathrm{sto\,R}})^{T*}Cm_{\mathrm{sto}}^{(\mathrm{R})}(\psi_{\mathrm{sto\,R}})^*\right]\,, \tag{8}$$

respectively. A similar form can be written down for the Majorana left-handed part.

In Eqs. (6), (7) and (8), the masses $m_{\text{sto}}^{(\text{L})}, m_{\text{sto}}^{(\text{R})}$ and $m_{\text{sto}}^{(\text{D})}$ (as well as the Yukawa coupling constant y in Eqs. (1) and (3)) are actually 2×2 or 3×3 matrices, when two or three generations of sterinos exist in Nature *i.e.*, when ψ_{sto} is a doublet or triplet of sterino fields[1]. If, in contrast to the footnote, the identification (5) holds for three generations of ν_{R}, then $m_{\text{sto}}^{(\text{R})}$ is identical with the 3×3 Majorana mass matrix for righthanded neutrinos of three generations, $m_{\nu}^{(\text{R})}$, involved in the general neutrino mass matrix in the Lagrangian

$$-\frac{1}{2} \left(\overline{\nu_{\text{L}}}, \overline{(\nu_{\text{R}})^c} \right) \begin{pmatrix} 0 & m_{\nu}^{(\text{D})} \\ m_{\nu}^{(\text{D})T} & m_{\nu}^{(\text{R})} \end{pmatrix} \begin{pmatrix} (\nu_{\text{L}})^c \\ \nu_{\text{R}} \end{pmatrix} + \text{h.c.}, \qquad (9)$$

so that

$$m_{\nu}^{(\text{R})} \equiv m_{\text{sto}}^{(\text{R})}. \qquad (10)$$

If in the mass term (6) $m_{\text{sto}}^{(\text{R})}$ dominates over $m_{\text{sto}}^{(\text{L})}$ and $m_{\text{sto}}^{(\text{D})}$, then two mass states, formed in the diagonalization procedure in Eq. (6), get approximately the masses

$$m_{\text{sto}}^{(\text{L})} - m_{\text{sto}}^{(\text{D})} m_{\text{sto}}^{(\text{R})-1} m_{\text{sto}}^{(\text{D})T}, m_{\text{sto}}^{(\text{R})} + m_{\text{sto}}^{(\text{D})} m_{\text{sto}}^{(\text{R})-1} m_{\text{sto}}^{(\text{D})T} \simeq m_{\text{sto}}^{(\text{R})}, \qquad (11)$$

analogical to those in the case of seesaw mechanism for neutrinos [7].

If, on the contrary, $m_{\text{sto}}^{(\text{D})}$ dominates over $m_{\text{sto}}^{(\text{L})}$ and $m_{\text{sto}}^{(\text{R})}$, then two sterino mass states, constructed for sterinos in the diagonalization procedure in Eq. (6), obtain approximately the masses

$$\mp m_{\text{sto}}^{(\text{D})} + \frac{1}{2} \left(m_{\text{sto}}^{(\text{L})} + m_{\text{sto}}^{(\text{R})} \right) \simeq \mp m_{\text{sto}}^{(\text{D})}, \qquad (12)$$

similar to those in the case of pseudo-Dirac neutrinos.

When $m_{\text{sto}}^{(\text{L})}$, $m_{\text{sto}}^{(\text{R})}$ and $m_{\text{sto}}^{(\text{D})}$ in Eqs. (11) and (12) are 2×2 or 3×3 sterino mass matrices, then these equations, resulting in the first step of diagonalization in Eq. (6), still require the second step which leads eventually to two doublets or triplets of sterino mass corresponding to two doublets or triplets of sterino mass states.

---

[1] In a series of papers, we have formulated an "intrinsic interpretation" of three lepton and quark generations, based on a generalization of Dirac's square root procedure [6]. In addition, we have found there that for hypothetic Standard Model fundamental spin-0 particles (Higgs bosons) as well as for hypothetic fundamental spin-1/2 and spin-0 particles, sterile from all Standard Model charges, there should exist two generations rather than one or three. Of course, in such an approach, the identity (5) cannot work for three generations of righthanded neutrinos ν_{R}. However, a peculiar option still exists, where ν_{R} and $\psi_{\text{sto}R}$ appear in two generations and so, the identity (5) may hold for two generations in spite of the fact that there are three generations for ν_{L}.

3. Photonic portal to dark matter

In our picture, therefore, the sterile world consists of sterinos and sterons and, in addition, right-handed neutrinos if the latter are different from right-handed sterinos. Such a sterile world contacts with our Standard Model world through the exchange of gravitons (collaborating with their hypothetic spin-0 partners) and, possibly, also by means of Standard Model Higgs bosons playing the role of a *Higgs portal* to the sterile world [5,8], if *Higgs bosons interact directly with sterons* through an additional term in the Lagrangian. This renders the physical scalar bosons of both sorts mixed.

An alternative to this Higgs portal may be a *photonic portal* to the sterile world provided by the Standard Model electromagnetic field $F_{\mu\nu}$ acting as *the pair $F^{\mu\nu}F_{\mu\nu}$ interacting directly with the pair $\varphi_{\text{stn}}\varphi_{\text{stn}}$ of the steron field* φ_{stn} through an additional term in the Lagrangian

$$-\frac{f}{M^2}\,\varphi_{\text{stn}}^2 F^{\mu\nu}F_{\mu\nu}\,, \tag{13}$$

where M denotes a very large mass scale and $f > 0$ is an unknown dimensionless coupling constant[2]. Here, $\varphi_{\text{stn}}^2 = \varphi_{\text{stn}}^{(\text{ph})\,2} + 2(\mu/\sqrt{\lambda})\varphi_{\text{stn}}^{(\text{ph})} + \mu^2/\lambda$. Due to the very large M, the photonic portal is very narrow at low energies. Note that the bilinear form $\varphi_{\text{stn}}F^{\mu\nu}$ in the effective interaction (13) of dimension six plays the role of an antisymmetric tensor current, coupled to itself.

One can speculate that — on a more fundamental level — the antisymmetric tensor current $\varphi_{\text{stn}}F^{\mu\nu}$ is coupled to a very massive antisymmetric tensor field $A_{\mu\nu}$ of dimension one: $\propto \varphi_{\text{stn}}F^{\mu\nu}A_{\mu\nu}$ (the field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is of dimension two). The new field $A_{\mu\nu}$ comprises two simpler component fields: the three-dimensional vector $\vec{A}_E = (A_{01},\, A_{02},\, A_{03})$ and axial $\vec{A}_H = (A_{23},\, A_{31},\, A_{12})$, so it describes two kinds of sterile spin-1 particles with the parity -1 and $+1$, respectively, and with a very large mass M. The exchange of these bosons leads to the current×current effective interaction (13), when momentum transfers through the field $A_{\mu\nu}$ are negligible in comparison with M. One may speculate further that the conventional tensor current formed of sterinos, $\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}}$, is also coupled to $A_{\mu\nu} : \propto (\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}})A_{\mu\nu}$, leading to two extra effective interactions

$$-\frac{f'}{M^2}\,(\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}})\varphi_{\text{stn}}F_{\mu\nu}\,, \qquad -\frac{f''}{M^2}\,(\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}})(\bar{\psi}_{\text{sto}}\sigma_{\mu\nu}\psi_{\text{sto}})\,, \tag{14}$$

with unknown dimensionless coupling constants $f' > 0$ and $f'' > 0$, when momentum transfers *via* the field $A_{\mu\nu}$ can be neglected *versus* M.

---

[2] It may be convenient to replace the constant f in Eq. (13) by $f/4$ because of the normalization of scalar $F^{\mu\nu}F_{\mu\nu}$ in the Lagrangian (see Appendix A).

If the field $A_{\mu\nu}$ is coupled universally to the bilinear form $\varphi_{\text{stn}}F^{\mu\nu} + \zeta\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}}$ playing the role of total antisymmetric tensor current where $\zeta > 0$ is an unknown constant: $\propto (\varphi_{\text{stn}}F^{\mu\nu} + \zeta\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}})A_{\mu\nu}{}^3$, then in the case of vanishing momentum transfers through the field $A_{\mu\nu}$ the universal effective interaction

$$-\frac{f}{M^2}(\varphi_{\text{stn}}F^{\mu\nu} + \zeta\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}})(\varphi_{\text{stn}}F_{\mu\nu} + \zeta\bar{\psi}_{\text{sto}}\sigma_{\mu\nu}\psi_{\text{sto}}) \qquad (15)$$

follows. This implies that

$$f : f' : f'' = 1 : 2\zeta : \zeta^2 \,, \qquad (16)$$

when compared with Eqs. (13) and (14). Eventually, the pair $A^{\mu\nu}A_{\mu\nu}$ may be coupled to the pair $\varphi_{\text{stn}}\varphi_{\text{stn}} : \propto \varphi_{\text{stn}}^2 A^{\mu\nu}A_{\mu\nu}$, generating the mass $M \propto \langle\varphi_{\text{stn}}\rangle_{\text{vac}}$ for the field $A_{\mu\nu}$.

4. Examples of annihilation and decay of dark matter

The interaction (13) and the first interaction (14) might be considered as responsible for the phenomenon of low energy positrons, boldly presumed by Boehm and collaborators [9] to be created in the centre of our Galaxy in process of dark matter annihilation and subsequently annihilated at rest with the emission of $511\,\text{keV}$ line observed since 1970s (in this case, the cold dark matter is argued to be considerably light, in fact, it is called MeV dark matter).

If steron and sterino masses are appropriate, the hypothetic process initiated in our case by the interaction (13) and the first interaction (14) should run as follows:

$$\left.\begin{array}{c}\text{(steron)(steron)}\\ \text{or}\\ \text{(steron)}\end{array}\right\} \to \gamma^*\gamma \to e^+e^-\gamma\,, \qquad (17)$$

and

$$\text{(antisterino)(sterino)} \to \left\{\begin{array}{c}\gamma^*\,\text{(steron)} \to e^+e^-\,\text{(steron)}\\ \text{or}\\ \gamma^* \to e^+e^-\end{array}\right.\,, \qquad (18)$$

respectively, and subsequently

$$e^+e^- \to \gamma_{511}\,\gamma_{511}\,. \qquad (19)$$

Here, the created positron e^+ is annihilated afterwards at rest in pair with an electron e^-, other than the primarily created e^-. In the first processes

---

[3] It is not clear for us, if such a coupling may follow from an even more fundamental mechanism, as *e.g.* the extra dimensions.

(17) and (18) steron is obviously the physical steron, while in the second processes (17) and (18) one steron field has been replaced by its vacuum expectation value, $\langle\text{steron}\rangle_{\text{vac}}$, the remaining steron being physical.

Making use of our effective interactions (13) and (14) (as well as the Standard Model electromagnetic coupling $-e\bar{\psi}_e\gamma^\mu\psi_e A_\mu$ for electrons), we can calculate the probabilities for processes (17) and (18), respectively.

For instance, the total cross-section for the first process (17) (the annihilation of a physical steron pair into an electron–positron pair and a photon) multiplied by the steron relative velocity is given in the steron centre-of-mass frame as follows:

$$\sigma(\text{stn stn} \to e^+ e^- \gamma)2v_{\text{stn}} = \frac{1}{(2\pi)^3}\left(\frac{ef}{M^2}\right)^2\frac{32}{3}\,\omega_{\text{stn}}^2\,, \qquad (20)$$

if the electron mass m_e can be neglected. Here, m_{stn} and $\omega_{\text{stn}} = \sqrt{\vec{p}_{\text{stn}}^2 + m_{\text{stn}}^2}$ is the steron mass and the steron energy, respectively, while the steron velocity $v_{\text{stn}} = |\vec{p}_{\text{stn}}|/\omega_{\text{stn}} = \sqrt{1 - m_{\text{stn}}^2/\omega_{\text{stn}}^2}$ may be replaced by an average value of v_{stn}, implying (through Eq. (20)) an average cross-section $\sigma(\text{stn stn} \to e^+ e^- \gamma)$ dependent on an average energy squared ω_{stn}^2. In the centre-of-mass frame, the relative velocity of colliding sterons is $2v_{\text{stn}}$.

Similarly, the total rate for the second process (17) (the decay of a physical steron into an electron–positron pair and a photon) is at rest equal to

$$\begin{aligned}\Gamma(\text{stn} \to e^+ e^- \gamma) &= \frac{1}{(2\pi)^3}\left(\frac{ef\langle\varphi_{\text{stn}}\rangle_{\text{vac}}}{M^2}\right)^2\frac{4}{3}\,\omega_{\text{stn}}^3\\[2mm] &= \frac{1}{6\pi^3}\left(\frac{ef/\sqrt{2\lambda}}{M^2}\right)^2 m_{\text{stn}}^5\,,\qquad (21)\end{aligned}$$

if the electron mass is negligible. Here, at rest $\omega_{\text{stn}} = m_{\text{stn}}$, while $\langle\varphi_{\text{stn}}\rangle_{\text{vac}} = m_{\text{stn}}/\sqrt{2\lambda}$ (Eq. (4)) denotes the vacuum expectation value of the steron field. With $(1/4)G_{\text{eff}}/\sqrt{2} \equiv (ef/\sqrt{2\lambda})/M^2$ (see the footnote number 4), the steron rate (21) can be rewritten as $\Gamma(\text{stn} \to e^+ e^- \gamma) = G_{\text{eff}}^2 m_{\text{stn}}^5/(192\pi^3)$, where the rhs reminds formally of the total rate for muon decay.

However, the simplest annihilation channel for a physical steron pair and decay channel for a physical steron is:

$$(\text{steron})(\text{steron}) \to \gamma\,\gamma \qquad (22)$$

and

$$(\text{steron}) \to \gamma\,\gamma\,, \qquad (23)$$

respectively. In this case, one gets respectively the following formulae for the total cross-section multiplied by the steron relative velocity:

$$\sigma(\text{stn stn} \to \gamma\gamma)2v_{\text{stn}} = \frac{1}{\pi}\left(\frac{f}{M^2}\right)^2 \omega_{\text{stn}}^2 \qquad (24)$$

in the steron centre-of-mass frame, and the total rate:

$$\Gamma(\text{stn} \to \gamma\gamma) = \frac{1}{2\pi}\left(\frac{f\langle\varphi_{\text{stn}}\rangle_{\text{vac}}}{M^2}\right)^2 \frac{1}{4}\omega_{\text{stn}}^3 = \frac{1}{8\pi}\left(\frac{f/\sqrt{2\lambda}}{M^2}\right)^2 m_{\text{stn}}^5 \qquad (25)$$

at rest (where $\omega_{\text{stn}} = m_{\text{stn}}$).

The large number of produced photons provided by the steron mechanism (17) and by the annihilation (22) and decay (23) may be inconsistent with observations, when the mechanism is fitted to the required positron production of approximately 3×10^{42} positrons appearing per second in the inner kiloparsecs of our Galaxy [9]. In contrast, the sterino mechanism (18), when considered for the positron production, is in a better situation, since in this case the simplest annihilation channel contains only one photon,

$$(\text{antisterino})(\text{sterino}) \to \gamma\,(\text{steron})\,, \qquad (26)$$

and, first of all, the single-sterino state is stable under interactions of our photonic portal giving — contrarily to the unstable single-steron state — no additional photons. But, it seems that in the natural case of thermal sterinos, they are too heavy to explain the positronium-annihilation $511\,\text{keV}$ line. For the form of total cross-sections in the second process (18) and in the channel (26) see the end of Appendix B (also the elastic scattering of electrons on sterinos is calculated in some detail in this Appendix).

The steron decay channels open through the photonic portal cause that only sterinos remain as our candidates for stable dark matter (at least, sterinos of the lowest generation; farther on, we will consider for simplicity one-generation sterinos).

In this paper, we leave open the question, whether sterinos can form the cold dark matter as a result of thermal-equilibrium freeze-out processes in the early Universe (what is usually assumed for neutralinos, supersymmetric candidates for cold dark matter). However, some comments on the subject are due. The comments presented in the next section are not inserted in the earlier electronic version `0712.0505 [hep-ph]` of the paper.

5. Sterinos and the thermal freeze-aut

The normalized density of cold dark matter $\Omega_{\text{DM}} \equiv \rho_{\text{DM}}/\rho_{\text{c}}$, where $\rho_{\text{c}} \equiv 3H_0^2/8\pi G_N \simeq 10^{-29}\text{g}\,\text{cm}^3$ [2,10] is the cosmological critical density, depends on the decoupling mechanism of dark-matter candidates in the early

Universe. This mechanism is not necessarily based on thermal-equilibrium freeze-out processes usually assumed to work for neutralinos, supersymmetric candidates for cold dark matter (or, more generally, for any weak-interacting massive candidates, so-called WIMPs).

In the case of thermal freeze-out processes in the early Universe, the order-of-magnitude theoretical estimation for the relic dark-matter abundance is [2]

$$\Omega_{\mathrm{DM}}h^2 \simeq \frac{3 \times 10^{-27}\,\mathrm{cm^3s^{-1}}}{\langle \sigma_{\mathrm{ann}}v_{\mathrm{DM}}\rangle}\,, \tag{27}$$

where $\langle \sigma_{\mathrm{ann}}v_{\mathrm{DM}\rangle}$ denotes the thermal average of the dark-matter total annihilation cross-section multiplied by relative velocity, while h stands for the today's value of scaled Hubble parameter, $100h = 72 \pm 3(\mathrm{stat}) \pm 7(\mathrm{syst})$ and $H_0 \equiv 100h$ km s^{-1}Mpc^{-1} . The recent WAMP experiments imply the following figures for the analogical abundances of all matter and baryonic matter [2]:

$$\Omega_{\mathrm{M}}h^2 = 0.127^{+0.007}_{-0.013}\,, \qquad \Omega_{\mathrm{B}}h^2 = 0.0223^{+0.0007}_{-0.0009}\,, \tag{28}$$

respectively. Hence, the experimental estimate for relic dark-matter abundance is

$$\Omega_{\mathrm{DM}}h^2 \simeq 0.1\,. \tag{29}$$

Thus, Eq. (27) and the experimental estimate (29) lead to

$$\langle \sigma_{\mathrm{ann}}v_{\mathrm{DM}}\rangle \simeq 3 \times 10^{-26}\mathrm{cm^3s^{-1}} \simeq \mathrm{pb} \simeq \frac{8}{\pi}\frac{10^{-3}}{\mathrm{TeV}^2} \tag{30}$$

in the units where $c = 1$ and $\hbar c = 1$ (pb $= 10^{-36}$cm^2). The thermal-equilibrium experimental value (30) happens to be consistent with the typical size of weak-interaction cross-sections, providing therefore a strong numerical argument for WIMPs as candidates for cold dark matter (as well as for the thermal-equilibrium mechanism of their decoupling in the early Universe).

In the case of our model of cold dark matter consisting of sterinos interacting through the photonic portal, we can put approximately in the centre-of-mass frame

$$\begin{aligned}\sigma_{\mathrm{ann}}v_{\mathrm{DM}} &\simeq \sigma(\mathrm{asto\,sto} \to \gamma\,\mathrm{stn})2v_{\mathrm{sto}} \\ &= \frac{8}{3\pi}\left(\frac{\zeta f}{M^2}\right)^2 \left(E_{\mathrm{sto}}^2 + 2m_{\mathrm{sto}}^2\right)\left(1 - \frac{m_{\mathrm{stn}}^2}{4E_{\mathrm{sto}}^2}\right),\end{aligned} \tag{31}$$

when we make use of the leading sterino–antisterino annihilation cross-section (B22). We will assume for simplicity that in the Universe there is no asymmetry between sterinos and antisterinos (no excess of either).

If $E_{\mathrm{sto}}/m_{\mathrm{sto}} \simeq 1$ (*i.e.*, $|\vec{p}_{\mathrm{sto}}|/m_{\mathrm{sto}} \ll 1$) and tentatively $m_{\mathrm{stn}} \sim m_{\mathrm{sto}}$, then Eq. (31) gives

$$\sigma_{\mathrm{ann}} v_{\mathrm{DM}} \simeq \frac{8}{\pi} \left(\frac{\zeta f}{M^2} \right)^2 m_{\mathrm{sto}}^2 \left(1 - \frac{m_{\mathrm{stn}}^2}{4 m_{\mathrm{sto}}^2} \right) \sim \frac{6}{\pi} \left(\frac{\zeta f}{M^2} \right)^2 m_{\mathrm{sto}}^2 \,. \qquad (32)$$

Thus, when the thermal-equilibrium experimental value (30) is accepted[4], the formula (32) implies

$$m_{\mathrm{stn}} \sim m_{\mathrm{sto}} \sim \frac{2 \times 10^{-3/2}}{\sqrt{3}} \frac{M^2}{\zeta f \, \mathrm{TeV}} \simeq \frac{1}{27} \frac{M^2}{\zeta f \, \mathrm{TeV}} \,, \qquad (33)$$

since here $\langle \sigma_{\mathrm{ann}} v_{\mathrm{DM}} \rangle \simeq \sigma_{\mathrm{ann}} v_{\mathrm{DM}}$. From Eq. (33) we can obtain the possible mass estimation

$$m_{\mathrm{stn}} \sim m_{\mathrm{sto}} \sim 27 \, \zeta f \, \mathrm{TeV} \sim 0.6 \, \mathrm{TeV} \,, \qquad (34)$$

if we put tentatively $m_{\mathrm{sto}} \sim M$ and $\zeta f \sim e^2/4 = \pi \, \alpha = 1/43.6$, where $e^2 = 4\pi \, \alpha = 1/10.9$ (with $f \sim e^2/4$, the estimate $f' = 2\zeta f \sim e^2/2$ *i.e.*, $\zeta f \sim e^2/4$ would be consistent with the sterile universality (16) when $\zeta \sim 1$; see also the footnote[2]). Then, $M \sim 27 \, \zeta f \, \mathrm{TeV} \sim 0.6 \, \mathrm{TeV}$. We will tentatively accept the TeV range for the mass scale M of our quasi-electromagnetic interaction (see Appendix A). Note from Eq. (32) that now

$$\langle \sigma_{\mathrm{ann}} v_{\mathrm{DM}} \rangle \sim \frac{6}{\pi} \left(\frac{\zeta f}{m_{\mathrm{sto}}} \right)^2 \sim \frac{3 \times 10^{-3}}{\pi} \frac{1}{m_{\mathrm{sto}}^2} \,. \qquad (35)$$

The maximum of $\langle \sigma_{\mathrm{ann}} v_{\mathrm{DM}} \rangle$ with respect to m_{sto} is here $\langle \sigma_{\mathrm{ann}} v_{\mathrm{DM}} \rangle_{m_{\mathrm{stn}}=0} \simeq (8/\pi)(\zeta f/m_{\mathrm{sto}})^2 \sim (4 \times 10^{-3}/\pi)(1/m_{\mathrm{sto}}^2)$, as it follows from Eq. (32) (with $m_{\mathrm{sto}} \sim M$ and $\zeta f \sim e^2/4$).

---

[4] Then, $\langle \sigma_{\mathrm{ann}} v_{\mathrm{DM}} \rangle_{\mathrm{sto}} \simeq \mathrm{pb} \simeq \langle \sigma_{\mathrm{ann}} v_{\mathrm{DM}} \rangle_{\mathrm{WIMP}}$ as well as $(\Omega_{\mathrm{DM}} h^2)_{\mathrm{sto}} \simeq 0.1 \simeq (\Omega_{\mathrm{DM}} h^2)_{\mathrm{WIMP}}$. This implies the necessary condition $x_{f\,\mathrm{sto}} \simeq x_{f\,\mathrm{WIMP}}$ with $x_f \equiv m_{\mathrm{DM}}/T_f$, in consequence of the basic formula for the relic dark-matter abundance (less approximate than (27)): $\Omega_{\mathrm{DM}} h^2 \simeq 1.07 \times 10^9 x_f \, \mathrm{GeV}^{-1}/(g_*^{1/2} M_{\mathrm{Pl}} \langle \sigma_{\mathrm{ann}} v_{\mathrm{DM}} \rangle)$, being valid in this form when $\langle \sigma_{\mathrm{ann}} v_{\mathrm{DM}} \rangle$ contains approximately only S wave (as in Eq. (32)) [2]. Here, T_f is the freeze-out temperature, g_* denotes the total number of effectively relativistic degrees of freedom in the Standard Model thermal plasma at the time of freeze-out and $M_{\mathrm{Pl}} = 1.22 \times 10^{19}$ GeV stands for the Planck mass. From the equation $x_f = \ln \left[0.038 \, g_{\mathrm{DM}} \, M_{\mathrm{Pl}} \, m_{\mathrm{DM}} \langle \sigma_{\mathrm{ann}} v_{\mathrm{DM}} \rangle/(g_* x_f)^{1/2} \right]$ defining x_f [2], we infer that $x_{f\,\mathrm{sto}}/x_{f\,\mathrm{WIMP}} - 1 + (1/2) \ln(x_{f\,\mathrm{sto}}/x_{f\,\mathrm{WIMP}})/x_{f\,\mathrm{WIMP}} \simeq \ln(g_{\mathrm{sto}} m_{\mathrm{sto}}/g_{\mathrm{WIMP}} m_{\mathrm{WIMP}})/x_{f\,\mathrm{WIMP}}$. Here, $g_{\mathrm{DM}} = g_{\mathrm{sto}}$ or g_{WIMP} counts internal degrees of freedom of sterino or WIMP. Then, from $x_{f\,\mathrm{sto}}/x_{f\,\mathrm{WIMP}} \simeq 1$ we obtain the necessary condition $|\ln(g_{\mathrm{sto}} m_{\mathrm{sto}}/g_{\mathrm{WIMP}} m_{\mathrm{WIMP}})/x_{f\,\mathrm{WIMP}}| \ll 1$ for the applicability of WIMP freeze-out formula (27) to our sterinos as the cold dark matter. Here, $x_{f\,\mathrm{WIMP}} \simeq 25$. Thus, for *e.g.* $g_{\mathrm{sto}} m_{\mathrm{sto}}/g_{\mathrm{WIMP}} m_{\mathrm{WIMP}} = m_{\mathrm{sto}}/m_{\mathrm{WIMP}} \sim 1$ to 6, we get $\ln(g_{\mathrm{sto}} m_{\mathrm{sto}}/g_{\mathrm{WIMP}} m_{\mathrm{WIMP}})/x_{f\,\mathrm{WIMP}} \sim 0$ to 0.07, what is a not-so-bad result.

We can see that — with and only with the sterino-mass m_{sto} as given in Eq. (33) or, possibly, Eq. (34) — the thermal-equilibrium decoupling mechanism (leading to the formula (27)) can work in the case of sterinos annihilating according to our cross-section (31) (this statement is valid under the tentative assumptions of $m_{\text{stn}} \sim m_{\text{sto}}$ and, possibly, $m_{\text{sto}} \sim M$ and $\zeta f \sim e^2/4$).

When instead of the thermal condition (33) we required that

$$ m_{\text{stn}} \sim m_{\text{sto}} \gg \frac{1}{27} \frac{M^2}{\zeta f \, \text{TeV}} \,, \tag{36} $$

where M was kept the same as in the thermal option, we would get a contradiction with the thermal-equilibrium freeze-out mechanism. This shows that in the case of inequality (36), the thermal-equilibrium decoupling mechanism cannot work in our model of cold dark matter. From Eq. (36) we infer that

$$ m_{\text{stn}} \sim m_{\text{sto}} \gg 27 \zeta f \, \text{TeV} \sim 0.6 \, \text{TeV} \,, \tag{37} $$

if we insert tentatively $M \sim 27 \zeta f \, \text{TeV} \sim 0.6 \, \text{TeV}$, the same M as in the thermal option.

In the case of inequality (37), the sterino mass may be as large as *e.g.* the Majorana mass of righthanded neutrinos in the conventional seesaw mechanism [7]. Such very heavy sterinos would behave as the so-called (fermionic) wimpzillas [2], whose non-thermal decoupling mechanism in the early Universe might be of gravitational nature [11], related to the generally nonadiabatic expansion of physical spacetime collaborating with the vacuum quantum fluctuations.

Finally, we will show that, in the case of $M \sim 27 \, \zeta f \, \text{TeV} \sim 0.6 \, \text{TeV}$, one-steron states are unstable on the Universe time-scale already for considerably small steron mass m_{stn} (*e.g.* lying in the MeV range). However, our possible estimation (34) or (37) for m_{stn} is much higher.

In fact, in our model, the steron total decay rate at rest is given approximately as

$$ \Gamma_{\text{decay}} \simeq \Gamma(\text{stn} \to \gamma \, \gamma) = \frac{1}{8\pi} \left(\frac{(f/\sqrt{2\lambda})}{M^2} \right)^2 m_{\text{stn}}^5 \,, \tag{38} $$

when the leading steron decay rate (25) is used. The steron unstability condition on the Universe time-scale reads

$$ \Gamma_{\text{decay}}^{-1} < \text{age of the Universe} \,, \tag{39} $$

where (the present) age of the Universe $= 13.7^{+0.1}_{-0.2}$ Gyr $= 4.3 \times 10^{17}$ s [10]. This condition implies the lower bound

$$m_{\text{stn}} > 5.7 \times 10^{-4} \left[\left(\frac{M^2}{(f/\sqrt{2\lambda})} \right)^2 \frac{1}{\text{s}} \right]^{1/5} = 33 \left[\frac{M^2}{(f/\sqrt{2\lambda}) \, \text{TeV}^2} \right]^{2/5} \text{keV} \tag{40}$$

($c = 1$ and $\hbar c = 1$). Inserting $M \sim 27 \zeta f$ TeV ~ 0.6 TeV, we obtain from the inequality (40) the following lower bound for the mass of unstable steron on the Universe time-scale:

$$m_{\text{stn}} > 0.5 \left(\zeta^2 f \sqrt{2\lambda} \right)^{2/5} \text{MeV} \sim 0.1 \text{ MeV} , \tag{41}$$

if we put tentatively $\zeta\sqrt{2\lambda} \sim 1$ (beside $\zeta f \sim e^2/4$).

With our tentative assumption $m_{\text{stn}} \sim m_{\text{sto}}$ that gives $m_{\text{stn}} \sim M \sim 0.6$ TeV or $\gg M \sim 0.6$ TeV (if $m_{\text{stn}} \sim M$ or $\gg M$ and $\zeta f \sim e^2/4$), the lower bound (41) shows that then sterons are certainly unstable on the Universe time-scale.

For sterino Dirac mass, it follows from Eqs. (3) and (4) that $m_{\text{sto}}^{(\text{D})}/m_{\text{stn}} = y/\sqrt{2\lambda}$. Thus, in the Dirac case, our assumption $m_{\text{sto}}^{(\text{D})} \sim m_{\text{stn}}$ implies $y \sim \sqrt{2\lambda}$. We incline to treat sterinos as Dirac fermions, since their interactions and mass terms are expected to make no difference between lefthanded and righthanded components (in contrast to the situation in the neutrino case). The special option of $m_{\text{sto}}^{(\text{L})} = m_{\text{sto}}^{(\text{R})}$ ($\neq 0$) in Eq. (6), where also Majorana sterinos could have such a property, seems less natural.

In conclusion, the picture emerging from our model (proposing sterinos and sterons as particles responsible for cold dark matter) looks as follows. Sterinos are stable under interactions of our photonic portal, while sterons appear as unstable, also on the Universe time-scale if only $m_{\text{stn}} > 0.1$ MeV (with $M \sim 0.6$ TeV, $\zeta f \sim e^2/4$ and $\zeta\sqrt{2\lambda} \sim 1$). For sterinos, the thermal-equilibrium freeze-out mechanism can work, if and only if $m_{\text{sto}} \sim 0.6$ TeV (with $m_{\text{stn}} \sim m_{\text{sto}}$, $m_{\text{sto}} \sim M$ and $\zeta f \sim e^2/4$). If $m_{\text{sto}} \gg M \sim 0.6$ TeV instead of $m_{\text{sto}} \sim M \sim 0.6$ TeV, the sterino decoupling mechanism in the early Universe must be different in order that sterinos may constitute cold dark matter.

To obtain quantitative conclusions, we have made in our argument three *tentative* assumptions

$$m_{\text{stn}} \sim m_{\text{sto}} , \qquad m_{\text{sto}} \sim M , \qquad \zeta f \sim \frac{e^2}{4} , \tag{42}$$

leading to the necessary and sufficient condition $m_{\text{sto}} \sim 0.6$ TeV for the applicability of thermal-equilibrium freeze-out to our model. Then, with $M \sim 0.6$ TeV, the fourth *tentative* assumption

$$\zeta\sqrt{2\lambda} \sim 1 \tag{43}$$

gives the lower bound $m_{\text{stn}} > 0.1\,\text{MeV}$ for sterons to be unstable on the Universe time-scale. Non-dramatic variations of these assumptions do not change qualitative features of the emerging picture.

The particle models, where beside the Standard Model sector there exists a sterile sector interacting through new forces with itself as well as with the Standard Model sector, have been called Hidden Valley models [12]. Our model of sterinos and sterons interacting very weakly through the photonic portal provided by a new quasi-electromagnetic force (see Appendix A) is a natural, specific realization within such a class of models, where photons are common elements which link both sectors: Hidden Valley and Standard Model. This happens, of course, after the electroweak symmetry is broken and photons emerge.

6. Final remarks

We would like to stress finally that it is still possible that — in reality — the direct coupling exists neither between Higgs bosons and sterons nor between photons, sterons and sterino–antisterino pairs, so that in the Standard Model world there is no Higgs nor photonic portal to the sterile world. Then, only gravitons and, perhaps, also dilaton-like scalar and/or axion-like pseudoscalar partners of gravitons [13,14] can mediate between both worlds as well as within the sterile world itself (of course, they can mediate also within the Standard Model world itself, but there they are dominated at the atomic scale by Standard Model media). Such a puristic picture still may explain the fundamental phenomenon of cold dark matter and, perhaps, provide a gravitational interpretation of the equally fundamental phenomenon of dark energy.

Appendix A

Quasi-electromagnetic current induced by dark matter

In connection with the footnote number 2, it is worthwhile to observe that the hypothetic effective interaction (13) implies the free electromagnetic term in the Lagrangian being supplemented to the form

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \to -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\left(1 + \frac{4f}{M^2}\varphi_{\text{stn}}^2\right) . \tag{A.1}$$

Then, the electromagnetic Lagrangian, supplemented as well by the hypothetic effective interaction (14) (with $f' = 2\zeta f$), is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\left(1 + \frac{4f}{M^2}\varphi_{\text{stn}}^2\right) - \frac{2\zeta f}{M^2}\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}}F_{\mu\nu}\varphi_{\text{stn}} - j^\mu A_\mu . \tag{A.2}$$

This leads to the following electromagnetic field equation:

$$\partial_\nu F^{\mu\nu} = -(j^\mu + \delta j^\mu) \tag{A.3}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where the additional current

$$\delta j^\mu \equiv \frac{4f}{M^2}\partial_\nu\left[\varphi_{\text{stn}}\left(\varphi_{\text{stn}}F^{\mu\nu} + \zeta\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}}\right)\right] \tag{A.4}$$

is a quasi-magnetic correction induced by the photonic effective interactions (13) and (14) of the cold dark matter. Here, evidently,

$$\partial_\mu\left(j^\mu + \delta j^\mu\right) = 0\,, \tag{A.5}$$

with the additional part $\partial_\mu\delta j^\mu$ being zero identically like the anomalous magnetic part of the conventional $\partial_\mu j^\mu$ in an effective presentation. Thus, $\partial_\mu j^\mu$ is zero dynamically for the conventional electromagnetic current j^μ, while δj^μ *supplements effectively the anomalous magnetic part of the conventional* j^μ.

We can see from the effective electromagnetic Lagrangian (A.2) that the bare electric charge e in the Standard Model current $j^\mu \propto e$ undergoes an extra finite renormalization by the factor $Z^{1/2} = \left[1 + (4f/M^2)\langle\phi_{\text{stn}}\rangle^2_{\text{vac}}\right]^{-1/2} < 1$, when $\phi_{\text{stn}} = \langle\phi_{\text{stn}}\rangle_{\text{vac}} + \phi^{(\text{ph})}_{\text{stn}}$ with $\langle\phi_{\text{stn}}\rangle_{\text{vac}} \neq 0$. In fact, if the finite renormalization

$$F_{\mu\nu}Z^{-1/2} \to F_{\mu\nu}\,, \quad A_\mu Z^{-1/2} \to A_\mu\,, \quad \psi_{\text{sto}}Z^{-1/4} \to \psi_{\text{sto}}\,, \quad \varphi_{\text{stn}} \to \varphi_{\text{stn}} \tag{A.6}$$

and

$$eZ^{1/2} \to e\,, \qquad fZ \to f \tag{A.7}$$

is performed, then from Eq. (A.2), where the first term is

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\left(1 + \frac{4f}{M^2}\varphi^2_{\text{stn}}\right) = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$
$$\times\left[Z^{-1} + \frac{4f}{M^2}\left(2\langle\varphi_{\text{stn}}\rangle_{\text{vac}}\varphi^{(\text{ph})}_{\text{stn}} + \varphi^{(\text{ph})\,2}_{\text{stn}}\right)\right] \tag{A.8}$$

we obtain

$$\begin{aligned}
\mathcal{L} = {}& -\frac{1}{4}F^{\mu\nu}Z^{-1/2}F_{\mu\nu}Z^{-1/2}\left[1 + \frac{4f\,Z}{M^2}\left(2\langle\varphi_{\text{stn}}\rangle_{\text{vac}}\varphi^{\text{ph}}_{\text{stn}} + \varphi^{\text{ph}\,2}_{\text{stn}}\right)\right] \\
& -\frac{2\zeta\,f\,Z}{M^2}\left(\bar{\psi}_{\text{sto}}Z^{-1/4}\sigma^{\mu\nu}\psi_{\text{sto}}Z^{-1/4}\right) \\
& \times F_{\mu\nu}Z^{-1/2}\left(\langle\varphi_{\text{stn}}\rangle_{\text{vac}} + \varphi^{\text{ph}}_{\text{stn}}\right) - j^\mu Z^{1/2}A_\mu Z^{-1/2} \\
& \to -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\left[1 + \frac{4f}{M^2}\left(2\langle\varphi_{\text{stn}}\rangle_{\text{vac}}\varphi^{\text{ph}}_{\text{stn}} + \varphi^{\text{ph}\,2}_{\text{stn}}\right)\right] \\
& -\frac{2\zeta\,f}{M^2}\bar{\psi}_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}}F_{\mu\nu}\left(\langle\varphi_{\text{stn}}\rangle_{\text{vac}} + \varphi^{\text{ph}}_{\text{stn}}\right) - j^\mu A_\mu\,.
\end{aligned} \tag{A.9}$$

This is a new, *spontaneous renormalization* leading to a new bare electric charge e in the Standard Model current $j^\mu \propto e$, spontaneously transformed. Under this transformation, the coupling constant f behaves formally as e^2 (while the fields A_μ and ψ_{sto} follow the pattern of e^{-1} and $e^{-1/2}$, respectively). Consistently, in Sec. 5 it was tentatively assumed that $f \sim e^2/4$ and $f' \equiv 2\zeta f \sim e^2/2$ when $\zeta \sim 1$ (then, $f'' \equiv \zeta^2 f \sim e^2/4$). Notice from Eq. (A.9) that then sterinos, though they are sterile, display the effective quasi-magnetic interaction $-\mu_{\text{eff}}\bar\psi_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}}F_{\mu\nu}$ proportional to their quasi-magnetic moment $\mu_{\text{eff}} \equiv 2\zeta f \langle\varphi_{\text{stn}}\rangle_{\text{vac}}/M^2 \sim e^2 \langle\varphi_{\text{stn}}\rangle_{\text{vac}}/(2M^2)$. In addition, they interact quasi-magnetically with photons and physical sterons. Thus, the cold dark matter composed of sterinos interacts effectively with the cosmic magnetic fields!

At the same time, for the sterino kinetic Lagrangian we get

$$\bar\psi_{\text{sto}}\left(\gamma p - m_{\text{sto}}\right)\psi_{\text{sto}} = \bar\psi_{\text{sto}}Z^{-1/4}\left(\gamma p\, Z^{1/2} - m_{\text{sto}}Z^{1/2}\right)\psi_{\text{sto}}Z^{-1/4}$$
$$\to \bar\psi_{\text{sto}}\left(\gamma p - m_{\text{sto}}\right)\psi_{\text{sto}} \tag{A.10}$$

with the finite renormalization

$$m_{\text{sto}}Z^{1/2} \to m_{\text{sto}}\,,\ pZ^{1/2} \to p\,, \tag{A.11}$$

where the primary sterino bare mass $m_{\text{sto}} = y\langle\varphi_{\text{stn}}\rangle_{\text{vac}}$ multiplied by $Z^{1/2}$ goes over into a new sterino bare mass $m_{\text{sto}} = y\langle\varphi_{\text{stn}}\rangle_{\text{vac}}$. Thus,

$$yZ^{1/2} \to y \tag{A.12}$$

as $\langle\varphi_{\text{stn}}\rangle_{\text{vac}} \to \langle\varphi_{\text{stn}}\rangle_{\text{vac}}$ (see the last Eq. (A.6)) The mass change is $\delta m_{\text{sto}} = (Z^{1/2} - 1)m_{\text{sto}} < 0$ with m_{sto} being the primary bare mass.

Added in proof

One can note that the hypothetic mediating field $A_{\mu\nu}$, discussed briefly in Section 3, does not change, $A_{\mu\nu} \to A_{\mu\nu}$, during the renormalization (A.6) and (A.7), since — on a more fundamental level — it is coupled in the Lagrangian to the non-changing "tensor current", $-\sqrt{f}(\varphi_{\text{stn}}F^{\mu\nu} + \zeta\bar\psi_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}})A_{\mu\nu}$. This implies two gauge invariant field equations:

$$(\Box - M^2)A^{\mu\nu} = -2\sqrt{f}(\varphi_{\text{stn}}F^{\mu\nu} + \zeta\bar\psi_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}}) \tag{A.13}$$

and Eq. (A.3) with $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, where now

$$\delta j^\mu \equiv 2\sqrt{f}\,\partial_\nu(\varphi_{\text{stn}}A^{\mu\nu}) \simeq (4f/M^2)\partial_\nu\left[\varphi_{\text{stn}}(\varphi_{\text{stn}}F^{\mu\nu} + \zeta\bar\psi_{\text{sto}}\sigma^{\mu\nu}\psi_{\text{sto}})\right]. \tag{A.14}$$

They may be called the *supplemented Maxwell equations*. In Eq. (A.14), the second step holds if M dominates over $-\Box$, giving Eq. (A.4). Here, $\partial_\mu\delta j^\mu \equiv 0$ identically, while $\partial_\mu j^\mu = 0$ dynamically.

Appendix B

Scattering of electrons on dark matter through photonic portal

Due to our photonic portal the Standard Model world can interact quasi-electromagnetically with the cold dark matter. Consider for illustration the elastic scattering of electrons on sterinos, making use of the first interaction (14) with $f' = 2\zeta f$ (and the Standard Model electromagnetic coupling $-e\,\bar{\psi}_e\gamma^\mu\psi_e A_\mu$ for electrons with $e = -|e|$). The corresponding S matrix element is

$$S_{fi} = \frac{2e\zeta f\langle\varphi_{\mathrm{stn}}\rangle_{\mathrm{vac}}}{M^2}\left[\frac{1}{(2\pi)^{12}}\frac{m_e^2 m_{\mathrm{sto}}^2}{E_e' E_e E_{\mathrm{sto}}' E_{\mathrm{sto}}}\right]^{1/2}(2\pi)^4\,\delta^4\left(p_e' + p_{\mathrm{sto}}' - p_e - p_{\mathrm{sto}}\right)$$

$$\times\,\bar{u}_e'(p_e')\frac{1}{i}\left(k_\mu\gamma_\nu - k_\nu\gamma_\mu\right)u_e(p_e)\frac{-i}{k^2}\bar{u}_{\mathrm{sto}}'(p_{\mathrm{sto}}')\sigma^{\mu\nu}u_{\mathrm{sto}}(p_{\mathrm{sto}}) \qquad \text{(B.1)}$$

with the obvious notation. Here,

$$k = p_e - p_e' = p_{\mathrm{sto}}' - p_{\mathrm{sto}}\,. \qquad \text{(B.2)}$$

The factor appearing in the second line of Eq. (B.1) can be obviously rewritten as

$$-2i\bar{u}_e'(p_e')\gamma_\nu u_e(p_e)\frac{1}{k^2}\bar{u}_{\mathrm{sto}}'(p_{\mathrm{sto}}')\left[2m_{\mathrm{sto}}\gamma^\nu - (p_{\mathrm{sto}}' + p_{\mathrm{sto}})^\nu\right]u_{\mathrm{sto}}(p_{\mathrm{sto}})\,, \quad \text{(B.3)}$$

when the Gordon identity

$$\bar{u}'(p')\gamma^\nu u(p) = \bar{u}'(p')\left[\frac{(p'+p)^\nu}{2m} + \frac{i\sigma^{\nu\mu}(p'-p)_\mu}{2m}\right]u(p) \qquad \text{(B.4)}$$

is applied.

Hence, we can calculate the fully differential cross-section

$$\frac{d^6\sigma}{d^3\vec{p}_e'\,d^3\vec{p}_{\mathrm{sto}}'} = \frac{(2\pi)^6}{v_{\mathrm{flux}}}\frac{1}{4}\sum_{u_e' u_{\mathrm{sto}}'}\sum_{u_e u_{\mathrm{sto}}}\frac{|S_{fi}|^2}{(2\pi)^4\delta^4(0)}$$

$$= \frac{1}{v_{\mathrm{flux}}}\left(\frac{2e\zeta f\langle\varphi_{\mathrm{stn}}\rangle_{\mathrm{vac}}}{2\pi M^2}\right)^2\frac{m_e^2 m_{\mathrm{sto}}^2}{E_e' E_e E_{\mathrm{sto}}' E_{\mathrm{sto}}}\delta^4\left(p_e' + p_{\mathrm{sto}}' - p_e - p_{\mathrm{sto}}\right)$$

$$\times\sum_{u_e' u_e}\sum_{u_{\mathrm{sto}}' u_{\mathrm{sto}}}\left|\bar{u}_e'(p_e')\gamma_\nu u_e(p_e)\frac{1}{k^2}\bar{u}_{\mathrm{sto}}'(p_{\mathrm{sto}}')\left[2m_{\mathrm{sto}}\gamma^\nu - (p_{\mathrm{sto}}' + p_{\mathrm{sto}})^\nu\right]u_{\mathrm{sto}}(p_{\mathrm{sto}})\right|^2\,,$$

$$\text{(B.5)}$$

where we get

$$m_e^2 \sum_{u_e' u_e u_{\mathrm{sto}}' u_{\mathrm{sto}}} | \,|^2 = 4 \left(1 - 2\frac{p_e \cdot p_{\mathrm{sto}}}{m_{\mathrm{sto}}^2} \right) + 16 \left[m_e^2 - \left(\frac{p_e \cdot p_{\mathrm{sto}}}{m_{\mathrm{sto}}} \right)^2 \right] \frac{1}{k^2}, \quad \text{(B.6)}$$

evaluating traces in Dirac bispinor indices (and treating sterinos as Dirac fermions). Here, in the sterino rest frame, where $\vec{p}_{\mathrm{sto}} = 0$, the collision relative velocity is $v_{\mathrm{flux}} = v_e = |\vec{p}_e|/E_e$. In this Appendix, σ denotes $\sigma(e^- \, \mathrm{sto} \to e^- \, \mathrm{sto})$.

Finally, we can evaluate the electron differential cross-section on sterinos:

$$\begin{aligned} \frac{d\sigma}{d\Omega_e} &= \int_0^\infty \vec{p}_e'^{\,2} d|\vec{p}_e'| \int d^3\vec{p}_{\mathrm{sto}}' \frac{d^6\sigma}{d^3\vec{p}_e' \, d^3\vec{p}_{\mathrm{sto}}'} \\ &= \frac{1}{v_{\mathrm{flux}}} \left(\frac{2e\zeta f \langle \varphi_{\mathrm{stn}} \rangle_{\mathrm{vac}}}{\pi M^2} \right)^2 \frac{\vec{p}_e'^{\,2}}{(E_e + E_{\mathrm{sto}}')|\vec{p}_e'| - E_e'|\vec{p}_e| \cos\theta_e} \frac{m_{\mathrm{sto}}^2}{E_e E_{\mathrm{sto}}} \\ &\quad \times \left\{ 1 - 2\frac{p_e \cdot p_{\mathrm{sto}}}{m_{\mathrm{sto}}^2} + 4 \left[m_e^2 - \left(\frac{p_e \cdot p_{\mathrm{sto}}}{m_{\mathrm{sto}}} \right)^2 \right] \frac{1}{k^2} \right\}, \quad \text{(B.7)} \end{aligned}$$

where $d\Omega_e = 2\pi \sin\theta_e d\theta_e$ and $\cos\theta_e = \vec{p}_e' \cdot \vec{p}_e / (|\vec{p}_e'||\vec{p}_e|)$. Here, $\vec{p}_e' + \vec{p}_{\mathrm{sto}}' = p_e + p_{\mathrm{sto}}$ giving for $k = \vec{p}_e - \vec{p}_e'$ the first of relations

$$k^2 = -2k \cdot p_{\mathrm{sto}}, \qquad k^2 = 2 \left(m_e^2 - E_e' E_e + |\vec{p}_e'||\vec{p}_e| \cos\theta_e \right), \quad \text{(B.8)}$$

the second following from the definition of k.

In the sterino rest frame, where $\vec{p}_{\mathrm{sto}} = 0$, Eq. (B.7) takes the form

$$\begin{aligned} \frac{d\sigma}{d\Omega_e} &= \left(\frac{2e\zeta f \langle \varphi_{\mathrm{stn}} \rangle_{\mathrm{vac}}}{\pi M^2} \right)^2 \frac{|\vec{p}_e'|}{|\vec{p}_e|} \frac{m_{\mathrm{sto}}}{E_e + m_{\mathrm{sto}} - (E_e'|\vec{p}_e|/|\vec{p}_e'|) \cos\theta_e} \\ &\quad \times \left(1 - 2\frac{E_e}{m_{\mathrm{sto}}} + 4\frac{m_e^2 - E_e^2}{k^2} \right). \quad \text{(B.9)} \end{aligned}$$

Here, the first relation (B.8) gives

$$k^2 = 2 \left(E_e' - E_e \right) m_{\mathrm{sto}} \quad \text{(B.10)}$$

and then, together with the second relation (B.8) implies

$$(E_e + m_{\mathrm{sto}})E_e' - |\vec{p}_e'||\vec{p}_e| \cos\theta_e = m_e^2 + E_e m_{\mathrm{sto}}. \quad \text{(B.11)}$$

If $m_e/E_e \ll 1$ *i.e.*, the electron mass is negligible, then from the second Eq. (B.8)

$$k^2 \simeq -4E_e' E_e \sin^2 \frac{\theta_e}{2} \quad \text{(B.12)}$$

and Eq. (B.11) gives

$$\frac{E_e}{E'_e} \simeq 1 + \frac{2E_e}{m_{\text{sto}}} \sin^2 \frac{\theta_e}{2} \,, \tag{B.13}$$

while the denominator in Eq. (B.9) becomes

$$E_e + m_{\text{sto}} - (E'_e |\vec{p}_e|/|\vec{p}'_e|) \cos \theta_e \simeq m_{\text{sto}} \left(1 + \frac{2E_e}{m_{\text{sto}}} \sin^2 \frac{\theta_e}{2} \right) \simeq \frac{E_e m_{\text{sto}}}{E'_e} \,. \tag{B.14}$$

Thus, if $m_e/E_e \ll 1$, we get from Eq. (B.9)

$$\frac{d\sigma}{d\Omega_e} \simeq \left(\frac{2e\zeta f \langle \varphi_{\text{stn}} \rangle_{\text{vac}}}{\pi M^2} \right)^2 \frac{1 + \sin^2 \frac{\theta_e}{2}}{\sin^2 \frac{\theta_e}{2} \left[1 + (2E_e/m_{\text{sto}}) \sin^2 \frac{\theta_e}{2} \right]} \,. \tag{B.15}$$

We can see that for our quasi-electromagnetic interactions between electrons and sterinos the forward singularity still appears, though it is softer than for the Standard Model electromagnetic interaction of electrons and, say, point-like protons, where the differential electron cross-section on protons takes the form [15]

$$\frac{d\sigma}{d\Omega_e} \simeq \left(\frac{e^2}{4\pi} \right)^2 \frac{1}{4E_e^2} \frac{\cos^2 \frac{\theta_e}{2} - (k^2/2m_p) \sin^2 \frac{\theta_e}{2}}{\sin^4 \frac{\theta_e}{2} \left[1 + (2E_e/m_p) \sin^2 \frac{\theta_e}{2} \right]} \,, \tag{B.16}$$

valid if $m_e/E_e \ll 1$ in the proton rest frame. Here,

$$k^2 \simeq -4E'_e E_e \sin^2 \frac{\theta_e}{2} \simeq -4E_e^2 \frac{\sin^2 \frac{\theta_e}{2}}{1 + (2E_e/m_p) \sin^2 \frac{\theta_e}{2}} \,. \tag{B.17}$$

On the contrary, if $E_e/m_{\text{sto}} \ll 1$ in the sterino rest frame *i.e.*, the sterino recoils are negligible, then $|\vec{p}'_e| \simeq |\vec{p}_e|$, $E'_{\text{sto}} \simeq E_{\text{sto}} = m_{\text{sto}}$ and so, Eq. (B.7) gives

$$\frac{d\sigma}{d\Omega_e} \simeq \left(\frac{2e\zeta f \langle \varphi_{\text{stn}} \rangle_{\text{vac}}}{\pi M^2} \right)^2 \frac{1 + \sin^2 \frac{\theta_e}{2}}{\sin^2 \frac{\theta_e}{2}} \,, \tag{B.18}$$

since in this case

$$k^2 = (p_e - p'_e)^2 \simeq 2(m_e^2 - E_e^2 + \vec{p}_e^2 \cos^2 \theta_e) = -4\vec{p}_e^2 \sin^2 \frac{\theta_e}{2} \,. \tag{B.19}$$

The forward singularity in Eq. (B.18) is softer than in the Standard Model electron differential cross-section on, say, point-like protons, valid if $E_e/m_p \ll 1$ in the proton rest frame (Mott cross-section). This is of the form [15]

$$\frac{d\sigma}{d\Omega_e} \simeq \left(\frac{e^2}{4\pi}\right)^2 \frac{E_e^2}{4\vec{p}_e^4} \frac{1 - (\vec{p}_e^2/E_e^2)\sin^2\frac{\theta_e}{2}}{\sin^4\frac{\theta_e}{2}} \qquad \text{(B.20)}$$

with $|\vec{p}'_e| \simeq |\vec{p}_e|$ and $k^2 \simeq -4\vec{p}_e^2 \sin^2(\theta/2)$.

Mutatis mutandis, the same formula (B.7) or (B.9) as in the case of electrons scattered on sterinos holds for the elastic scattering of point-like protons on sterinos. This scattering is the simplest interaction between nuclei and cold dark matter composed of heavy sterinos, subject to possible direct experiments on the cold dark matter.

In contrast to the elastic scattering of electrons on sterinos (e^-(sterino)$\rightarrow$ e^-(sterino)), for the crossed process (18) of annihilation of a sterino–anti-sterino pair into an electron–positron pair ((antisterino)(sterino) $\rightarrow e^+e^-$) the corresponding differential cross-section $d\sigma(\text{asto}\,\text{sto} \rightarrow e^+e^-)/d\Omega_{e^+}$ calculated in our photonic portal can be integrated. Then, in the sterino–antisterino centre-of-mass frame, where the relative velocity of the colliding sterino–antisterino pair is $2v_{\text{sto}}$ with $v_{\text{sto}} = |\vec{p}_{\text{sto}}|/E_{\text{sto}} = \sqrt{1 - m_{\text{sto}}^2/E_{\text{sto}}^2}$, we obtain the following formula for total cross-section multiplied by sterino relative velocity:

$$\sigma(\text{asto}\,\text{sto} \rightarrow e^+e^-)\, 2v_{\text{sto}} \simeq \frac{e^2}{4\pi} \left(\frac{2\zeta f \langle\varphi_{\text{stn}}\rangle_{\text{vac}}}{M^2}\right)^2 \frac{16}{3} \frac{E_{\text{sto}}^2 + 2m_{\text{sto}}^2}{E_{\text{sto}}^2} \, , \quad \text{(B.21)}$$

if $m_e/E_{\text{sto}} \ll 1$ *i.e.*, the electron mass is negligible.

However, the simplest annihilation channel of a sterino–antisterino pair is that leading into a photon and a physical steron (see the process (26)). The corresponding total cross-section multiplied by the sterino relative velocity gets in our photonic portal the following form:

$$\sigma(\text{asto}\,\text{sto} \rightarrow \gamma\,\text{stn})\, 2v_{\text{sto}} = \frac{1}{4\pi} \left(\frac{2\zeta f}{M^2}\right)^2 \frac{8}{3} \left(E_{\text{sto}}^2 + 2m_{\text{sto}}^2\right) \left(1 - \frac{m_{\text{stn}}^2}{4E_{\text{sto}}^2}\right)$$
$$\text{(B.22)}$$

in the sterino–antisterino centre-of-mass frame (of course, if masses allow *i.e.*, if $m_{\text{stn}} < 2\sqrt{\vec{p}_{\text{sto}}^2 - m_{\text{sto}}^2}$ for a given sterino momentum $\vec{p}_{\text{sto}}$; this inequality is always satisfied with $m_{\text{stn}} < 2m_{\text{sto}}$, in particular with $m_{\text{stn}} \simeq m_{\text{sto}}$).

REFERENCES

[1] For recent comments *cf.* M.E. Peskin, `arXiv: 0707.1536` [hep-ph]; F. Wilczek, `arXiv: 0708.4036` [hep-ph].

[2] For recent reviews *cf.* G. Bartone, D. Hooper, J. Silk, *Phys. Rept.* **405**, 279 (2005); M. Taoso, G. Bartone, A. Masiero, `arXiv: 0711.4996` [astro-ph]; *cf.*

also E.W. Kolb, S. Turner, *Early Universe*, Addison-Wesley, Reading, Mass., 1994; K. Griest, D. Seckel, *Phys. Rev.* **D43**, 3191 (1991).

[3] For a recent review *cf.* R. Essig, `arXiv: 0710.1668` [hep-ph].

[4] For a recent review *cf.* D. Hooper, `arXiv: 0710.2062` [hep-ph].

[5] J.J. van der Bij, *Phys. Lett.* **B636**, 56 (2006) [`arXiv: 0707.0359` [hep-ph]]; D. O'Connell, M.J. Ramsay-Musolf, M.B. Wise, *Phys. Rev.* **D75**, 037701 (2007) and references therein.

[6] For a recent presentation *cf.* W. Królikowski, *Acta Phys. Pol. B* **38**, 3133 (2007); W. Królikowski, `hep-ph/0604148`, `hep-ph/0504256` and references therein.

[7] P. Minkowski, *Phys. Lett.* **B67**, 421 (1977); M. Gell-Mann, P. Ramond, R. Slansky, in *Supergravity*, ed. F. van Nieuvenhuizen and D. Freedman, North Holland, Amsterdam 1979; T. Yanagida, Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe, ed. O. Sawada and A. Sugamoto, KEK, Sukuba, 1979; S.L. Glashow, in *Quarks and Leptons*, ed. M. Lévy *et al.*, Plenum, New York 1980; R.N. Mohapatra, G. Sanjanovic, *Phys. Rev. Lett.* **44**, 912 (1980).

[8] R. Schabinger, J.D. Wells, *Phys. Rev.* **D72**, 093007 (2005); B. Patt, F. Wilczek, `hep-ph/0605188`; O. Bartolomi, R. Rosenfeld, `arXiv: 0708.1784` [hep-ph]; J. March-Russell, S.M. West, D. Comberbatch, D. Hooper, `arXiv: 0801.3440v2` [hep-ph].

[9] For a recent discussion *cf.* C. Boehm *et al.*, *Phys. Rev. Lett.* **92**, 101301 (2004); Y. Ascasibar, P. Jean, C. Boehm, J. Knödlseder, `astro-ph/0507142v2`; D. Hooper, F.M. Zurek, `arXiv: 0801.3686` [hep-ph] and references therein.

[10] Particle Data Group, *Review of Particle Physics, J. Phys*, **G33**, 1 (2006).

[11] D.J.H. Chang, P. Crotty, E.W. Kolb, A. Riotto, *Phys. Rev.* **D64**, 043503 (2001) and references therein.

[12] M.J. Strassler, K.M. Zurek, *Phys. Lett.* **B651**, 374 (2007); *cf.* also J. Kummer, J. Wells, *Phys. Rev.* **D74**, 115017 (2006); T. Han, Z. Si, K.M. Zurek, M.J. Strassler, `arXiv: 0712.2041` [hep-ph].

[13] P. Jordan, *Schwerkraft und Weltall*, Vieweg, Braunschweig, 1955; M. Fierz, *Helv. Phys. Acta* **29**, 128 (1956); C. Brans, R.H. Dicke, *Phys. Rev.* **124**, 925 (1961); T. Damour, K. Nordtvadt, *Phys. Rev. Lett.* **70**, 2217 (1993); R. Catena, M. Pietroni, L. Scarabello, `astro-ph/0604492`.

[14] X. Calmet, `arXiv: 0708.2767` [hep-ph]; R. Catena, J. Möller, `arXiv: 0709.1931` [hep-ph].

[15] J.D. Bjorken, S.D. Drell, *Relativistic Quantum Mechanics*, McGraw-Hill, New York 1964.