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2 Relation between Finestructure Constants at the Planck Scale from Multiple Point Principle *

D.L. Bennett[1], L.V. Laperashvili [2] and H.B. Nielsen [3]

[1] Brookes Institute for Advanced Studies, Copenhagen, Denmark
email:bennett@nbi.dk
[2] The Institute of Theoretical and Experimental Physics, Moscow, Russia
email:laper@itep.ru
[3] The Niels Bohr Institute, Copenhagen, Denmark
email:hbech@nbi.dk

Abstract. We derive a relation between the three finestructure constants in the Standard Model from the assumptions of what we call “multiple point principle” (MPP) and “AntiGUT”. By the first assumption we mean that we require coupling constants and mass parameters to be adjusted - by our multiple point principle - to be just so as to make several vacua have the same cosmological constants (from our point of view, basically zero). By AntiGUT we refer to our assumption of a more fundamental precursor to the usual Standard Model Group (SMG) consisting of the N_{gen}-fold cartesian product of the usual SMG such that each of the three families of quarks and leptons has its own set of gauge fields. The usual SMG comes about when SMG^3 breaks down to the diagonal subgroup at roughly a factor 10 below the Planck scale. Up to this scale we assume the absence of new physics. Relative to earlier work where the multiple point principle was used to get predictions for the gauge couplings independently of one another, the point here is to increase accuracy by considering a relation between all the gauge couplings (i.e., for U(1), and SU(N) with N=2 or 3)as a function of a N-dependent parameter d_N that is a characteristic of SU(N) groups. In doing this, the parameter d_N that initially only takes discrete values corresponding to the “N in SU(N) is promoted to being a continuous variable (corresponding to fantasy groups for $N \notin \mathbf{Z}$). By an approprite extrapolation in the variable d_N to a fantasy group for which the β-function for the magnetic coupling $\tilde{g}^2$ vanishes we avoid the problem of our ignorance of the ratio of the monopole mass scale to the fundamental scale. In addition to increasing the accuracy of our predictions for the gauge couplings by circumventing the uncertainty in our knowledge of this ratio, we interpret our results as being very supportive of the multiple point principle and AntiGut.

2.1 Introduction

In earlier work [1] we invented our Multiple Point Principle / AntiGUT (MPP / AntiGUT) gauge group model for the purpose of predicting the Planck scale values of the three Standard Model Group gauge couplings. These predictions were made independently for the three gauge couplings. In this work we test an

* Invited talk by D.L. Bennett

alternative method of treatment of MPP/AntiGUT in which we seek a relation that would put a rather severe constraint on the values of the SMG couplings.

An important ingredient for the calculational technique in this paper is the Higgs monopole model description in which magnetic monopoles are thought of as particles described by a scalar field ϕ with an effective potential V_{eff} of the Weinberg-Coleman type [2],[3]. The MPP is implemented by requiring that the two minima of V_{eff} are degenerate. This requirement results in a relation between the square of the monopole charge $\tilde{g}^2$ and the self-coupling λ that defines a phase transition boundary between a Coulomb-like phase (with $<\phi>=0$) and a phase with a monopole condensate (with $<\phi>\neq 0$). For Abelian monopoles this phase boundary has been presented in earlier work [4],[5]. This phase boundary condition which has a term quadratic in $\tilde{g}^2$ (as well as terms linear and quadratic in λ) consists of tuples $(\lambda, \tilde{g}^2)$ of critical values of λ and $\tilde{g}^2$. A characteristic feature of this boundary is that it has negative curvature as a function of λ and hence a maximum value $\tilde{g}^2_{U(1)\ crit\, max}$ of $\tilde{g}^2$. It is important to keep in mind that this phase transition that we find by applying MPP to V_{eff} is assumed to be at the scale of the monopole mass. In the present work, the V_{eff} is generalized in such a way that it also embodies nonAbelian $SU(N)$ monopoles using the technique of Chan & Tsou in which a $SU(N)$ monopole is described as a $\underline{\mathbf{N}}$ under a dual Yang-Mills vector potential $\tilde{A}_\mu$ [6]. The corresponding phase boundary now has coefficients that depend on N (i.e., "N" as in $SU(N)$). But otherwise the phase boundary for a $SU(N)$ monopole is qualitatively the same as that for the Abelian monopole the important difference being that the maximum value of $\tilde{g}^2$ in the Abelian theory is different than in the nonAbelian $SU(N)$ theory: $\tilde{g}^2_{U(1)\ crit\ max} \neq \tilde{g}^2_{SU(N)\ crit\ max}$.

In fact a major trick in the present article is to consider a correspondance between Abelian and non-Abelian monopoles (the latter in the understanding of Chan-Tsou to be explained in section 2 below). This correspondence is defined using a N-dependent parameter $C = C(N)$ defined by

$$C \hat{=} \frac{\tilde{g}^2_{SU(N)\ crit\ max}}{\tilde{g}^2_{U(1)\ crit\ max}}. \tag{2.1}$$

We can think of this definition of parameter C as a definition of the Abelian theory with $\tilde{g}^2_{U(1)\ crit\ max}$ that corresponds to the nonAbelian $SU(N)$ theory with $\tilde{g}^2_{SU(N)\ crit\ max}$.

As shall be seen soon we need an unambiguous way to define the Abelian magnetic charge $\tilde{g}^2_{U(1)\ corresp.\ SU(N)}$ corresponding to *any* nonAbelian $SU(N)$ magnetic charge $\tilde{g}^2_{SU(N)}$. Our definition of such a correspondence is simply:

$$C \hat{=} \frac{\tilde{g}^2_{SU(N)}}{\tilde{g}^2_{U(1)\ corresp\ SU(N)}}, \tag{2.2}$$

where $\tilde{g}^2_{SU(N)}$ can have any value (not necessarily critical or critical maximum). We can thereby think of say the fundamental scale nonAbelian dual - i.e. monopole - coupling also as an Abelian one.

We now go to the Planck scale where the MPP/AntiGUT model was originally invented as a simple way of relating the experimental values of the SMG gauge couplings $g_{U(1)}$, $g_{SU(2)}$and $g_{SU(3)}$ (extrapolated to the Planck scale in the absence of new physics underway) to the critical values of these three coupling as determined using lattice gauge theory. This MPP/AntiGUT relation in terms of the (critical values of the) gauge couplings g_i ($i \in \{U(1), SU(2), SU(3)\}$) is in this work reformulated in terms of the dual (critical values of the) of the magnetic charges using the Dirac relations $g\tilde{g} = 2\pi$ and $g\tilde{g} = 4\pi$ for respectively Abelian and nonAbelian monopole theories. Recall that we already have a convention for calculating the (squared) Abelian magnetic charge that corresponds to a (squared) nonAbelian magnetic charge $\tilde{g}^2_{SU(N)}$ using the parameter C:

$$\tilde{g}^2_{U(1)\ \text{corresp.}\ SU(N)} = \frac{\tilde{g}^2_{SU(N)}}{C}. \tag{2.3}$$

Now assuming for the moment that our MMP/AntiGUT model is in fact a law of Nature it would not be unreasonable to discuss whether the Abelian correspondent couplings $\tilde{g}^2_{U(1)\ \text{corresp.}SU(2)}$ coupling is smooth or not as a function of a gauge group characteristica such as N for SU(N) groups. Also the U(1) gauge group can be taken into consideration using our nonAbelian to Abelian correspondence relation in the special case

$$\tilde{g}^2_{U(1)\ \text{corresp.}U(1)} \equiv \frac{\tilde{g}^2_{U(1)}}{C=1} \tag{2.4}$$

which will be seen below to correspond to $d_N = 0$

Actually for later convenience we shall instead of N use an N-dependent parameter d_N as our independent variable and the quantitiy $\frac{3\tilde{g}^2_{U(1)\ \text{corresp.}SU(2)}}{\pi}$ instead of $\tilde{g}^2_{U(1)\ \text{corresp.}SU(2)}$) as our on d_N analytically dependent variable. We have now three points $(d_N, \frac{3\tilde{g}^2_{U(1)\ \text{corresp.}\ SU(N)}}{\pi})$ belonging to our hypothesized in d_N analytic function namely the points

$$(0, \frac{3\tilde{g}^2_{U(1)}}{\pi}),\ \ (d_2, \frac{3\tilde{g}^2_{U(1)\ \text{corresp.}SU(2)}}{\pi}),\ \ (d_3, \frac{3\tilde{g}^2_{U(1)\ \text{corresp.}SU(3)}}{\pi}). \tag{2.5}$$

Now we make the guess that the analytic function in d_N that we seek is the parabolic fit obtained using these three points. It must be emphasized that our hypothesized function analytic in d_N lives at the Planck scale while the phase transition boundary discussed above lives at the (unknown) scale of the monopole mass and consists of critical λ and $\tilde{g}^2$ values that both run with scale. So the problem is how to connect hypothesized Planck scale physics in the form of our in d_N analytic function $\frac{3\tilde{g}^2_{U(1)\ \text{corresp.}SU(2)}}{\pi}$ with say critical values of the function $\tilde{g}^2_{U(1)\ \text{corresp.}\ SU(N)\ \text{crit}}$ at the unknown scale of the monopole mass.

There is one value of $\tilde{g}^2$ for which this connection would be trivial namely the special point $(d_{N_{spec}}, \frac{3\tilde{g}^2_{spec}}{\pi})$ lying at the intersection of our in d_N analytically continued function with the function defined by requiring that the β-function for $\tilde{g}^2$ vanishes. I.e., $\beta_{\tilde{g}^2} = 0$. Just this value of $\frac{3\tilde{g}^2_{spec}}{\pi}$ is the same at the Planck scale and at the (unknown) scale of the monopole mass. We describe now briefly how we find the "fantasy" (and nonexistent!) gauge group "'$SU(N_{spec})$"' for which the corresponding $\tilde{g}^2$ does not run with scale. But first a little digression on how $\beta_{\tilde{g}^2}$ depends on d_N

Starting from the definition of the Abelian magnetic charge correspondent to a nonAbelian magnetic charge we use the nonAbelian Dirac relation to obtain

$$\tilde{g}^2_{U(1)\ corresp.SU(N)} \hat{=} \frac{\tilde{g}^2_{SU(N)}}{C} = \frac{1}{C}\frac{16\pi^2}{g^2_{SU(N)}} \tag{2.6}$$

We now require that the Dirac relation remain intact under scaling; i.e.,

$$\frac{d}{dt}\tilde{g}^2_{U(1)\ corresp.SU(N)} = \frac{16\pi^2}{C}\frac{d}{dt}\left(\frac{1}{g^2_{SU(N)}}\right) = \frac{16\pi^2}{C}\frac{1}{4\pi}\frac{d}{dt}(\alpha^{-1}_{SU(N)}). \tag{2.7}$$

Using that $\frac{d}{dt}(\alpha^{-1}_{SU(N)}) = \frac{11N}{12\pi}$ (which is just the usual Yang-Mills contribution to the β-function for $\alpha^{-1}_{SU(N)}$ using t=lnμ^2) we get the Yang-Mills contribution to the running of $\tilde{g}^2_{U(1)\ corresp.\ SU(N)}$:

$$\beta_{\tilde{g}^2_{U(1)\ corresp.\ SU(N)}}|_{Y.-M.\ contrib} = \frac{16\pi^2}{C}\frac{1}{4\pi}\frac{11N}{12\pi} = \frac{11N}{3C}\hat{=}d_N. \tag{2.8}$$

So even though there is of course no Yang-Mills contribution to $\beta_{U(1)}$ we see that $\beta_{U(1)\ corresp.\ SU(N)}$ inherits a dependence on $\beta_{g^2_{SU(N)}}$ through the requirement that the Dirac relation remain intact under scale changes. Using the known β-function for $\tilde{g}^2_{U(1)}$ (to 2-loops):

$$\beta_{\tilde{g}^2_{U(1)}} = \frac{\tilde{g}^4}{48\pi^2} + \frac{\tilde{g}^6}{(16\pi^2)^2} \tag{2.9}$$

which leads to the β-function for $\tilde{g}^2_{U(1)\ corresp,\ SU(N)}$:

$$\beta_{\tilde{g}^2_{U(1)\ corresp,\ SU(N)}} = \beta_{\tilde{g}^2_{U(1)}} + d_N = \frac{\tilde{g}^4}{48\pi^2} + \frac{\tilde{g}^6}{(16\pi)^2} + d_N. \tag{2.10}$$

The intersection point (with no running of $\tilde{g}^2$)is readily found as the intersection of

$$\beta_{\tilde{g}^2_{U(1)\ corresp,\ SU(N)}} = 0 \tag{2.11}$$

and our in d_N analytically extrapolated function $\frac{3\tilde{g}^2_{U(1)\ corresp.\ SU(N)}}{\pi}$. At the intersection point we have

$$(d_N, \frac{3\tilde{g}^2}{\pi}) = (-0.57, 14.54). \tag{2.12}$$

So any intersection point would have a $\tilde{g}^2$ that necessarily has a RG trajectory parallel to the λ axis in the space spanned by $(\lambda, \tilde{g}^2)$ where the phase transition boundary lives. But only one of the horizontal (i.e., parallel to the λ axis) RG trajectories can be tangent to the phase transition boundary and the point of tangency must neccessarily be at the top point of the phase transition curve. The interesting result of this work is that our intersection point (which depends of course on the experimental values of the SMG finestructure constants) to high accuracy is the value of $\tilde{g}^2$ with the RG trajectory that is tangent to the phase boundary at its top point where $\tilde{g}^2 = \tilde{g}^2_{U(1)\,crit\,max} = 15.11$. Had our intersection point singled out any other RG trajectory our MPP would have been falsified (see Figures 2.1 and 2.2).

2.2 The Chan-Tsou duality and monopole critical coupling calculation

Investigating nonAbelian theories, we have used the quantum Yang-Mills theory by Chan-Tsou [6] for a system of fields with chromo-electric charge g and chromo-magnetic charge $\tilde{g}$ (monopoles). This theory describes symmetrically the non-dual and the dual sectors of theory with nonAbelian vector potentials A_μ and $\tilde{A}_\mu$ covariantly interacting with chromo-electric j_μ and chromo-magnetic $\tilde{j}_\mu$ currents respectively. As a result, the Chan-Tsou nonAbelian theory has a doubling of symmetry from $SU(N)$ to

$$SU(N) \times \widetilde{SU(N)}$$

and reveals the generalized dual symmetry which reduces to the well-known electromagnetic (Hodge star) duality in the Abelian case.

We want in principle to consider three phase transitions connected with a single nonAbelian monopole which in the philosophy of the Chan-Tsou-theory to be described below is an $\underline{N}$-plet under the by Chan-Tsou introduced dual Yang Mills four vector field $\tilde{A}_\mu$, namely 1) a confining phase, 2) a Coulomb phase, and 3) a phase with monopole condensate. According to the Multiple Point Principle the coupling constants and mass parameters should then be adjusted in Nature to just make these phase degenerate (i.e. same cosmological constant). Earlier we have used two loop calculations to obtain the Abelian gauge group a phase transition between the monopole-condensate phase and the Coulomb phase using the Coleman-Weinberg effective potential technique, which led to a relation between the self coupling λ for the monopole Higgs field and the monopole charge $\tilde{g}$. To determine the monopole mass we should, however, in principle involve one more phase, i.e. the monopole confining one, but that would need a string description in the language used for the two other phases and doing that sufficiently accurately for the fit towards which we aim in this article is not undertaken here. Hence we shall assume only that the ratio of the mass scale of the monopole condensate or approximately equivalently the monopole mass to fundamental scale, taken here to be the Planck scale, is an analytical function of some group characteristic, which we shall take to be the quantity d_N that we shall return to shortly. The basic point is that we derive by the Coleman-Weinberg effective potential an a priori scale independent phase transition curve in as far as the monopole mass drops out of the relation describing the phase border between the Coulomb phase and the monopole condensate one, so that the only scale dependence of this relation comes in via the renormalization group. The lack of a good technology for calculating the mass scale of the monopole therefore means that we have troubles in calculating the renormalization group correction of the by the Coleman-Weinberg-technique calculated relation between λ and $\tilde{g}^2$ to run it from the monopole mass scale to the fundamental scale. The major idea of the present article now is that this would be no problem if the beta-function for the monopole coupling $\tilde{g}$ had been zero. The trick now is to effectively achieve that zero beta function by extrapolating in the gauge group so to speak to a "fantasy" group having zero beta-function.

A priori magnetic monopole couplings for different gauge groups cannot be compared, and so to make the statement that the phase transition coupling is analytic as a function of some group characteristic, call it d_N say, is a priori not meaningfull. This is so because in principle one could vary notation from group to group, and such a choice of notation would not a priori be analytical. We shall primarily be interested in the phase transitions from a Coulomb phase to the monopole condensed phase as obtained from studying the effective potential as a function of the norm of the vacuum expectation value of the monopole scalar field in the manner of Coleman-Weinberg. We decide to take the a priori arbitrary ratio between the ratio of a gauge coupling for an $SU(N)$ gauge group and the coupling for the corresponding Abelian $U(1)$ theory to be the same as that for the critical values of these couplings. We take the Lagrangian densities for a $U(1)$ theory and an $SU(N)$ Higgs Yang Mills theory respectively as

$$\mathcal{L} = -\frac{1}{4\tilde{g}^2}\tilde{F}^2_{\mu\nu} + |\tilde{D}_\mu\phi|^2 - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}|\phi|^4 \tag{2.13}$$

and

$$\mathcal{L} = -\frac{1}{4\tilde{g}^2}\tilde{F}^{j2}_{\mu\nu} + |\tilde{D}_\mu\phi^a|^2 - \frac{1}{2}\mu^2|\phi^a|^2 - \frac{\lambda}{4}(|\phi^a|^2)^2 \tag{2.14}$$

where ϕ^a is a monopole $\underline{N}$-plet, $\tilde{D}_\mu$ is the covariant derivative for dual gauge field $\tilde{A}_\mu$ and $\tilde{g}$ is magnetic charge, such that the meaning of the mass μ and the self coupling λ becomes related in as far as we decide to identify as having corresponding meaning of the length squares of the fields; i.e. we identify

$$|\phi|^2 = \sum_{a=1}^{N} |\phi^a|^2 \tag{2.15}$$

as is natural, since from the derivative part in the kinetic term respectively $\frac{1}{2}|\partial_\mu\phi|^2$ and $\frac{1}{2}|\partial_\mu\phi^a|^2$ we can claim that a given size of $|\phi|^2$ and $|\phi^a|^2$ corresponds to a given density of Higgs particles, a number of particles per unit volume being the same in both theories. Accepting (2.15) as a physically meaningfull identification we can also claim that the λ and the μ in (2.13) and (2.14) are naturally identified i an N-independent way (i.e., N as in SU(N)).

Denoting (2.15) by just $|\phi|^2$ one can write - as is seen by a significant amount of calculation or by using Coleman-Weinberg [2] and Sher [3] - the one-loop effective potential for $U(1)$ and $SU(N)$ gauge groups as

$$V_{eff} = -\frac{1}{2}\mu^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4 + \frac{|\phi|^4}{64\pi^2}$$
$$[3B\tilde{g}^4 \ln\frac{|\phi|^2}{M^2} + (-\mu^2 + 3\lambda|\phi|^2)^2 \ln\frac{-\mu^2 + 3\lambda|\phi|^2}{M^2}$$
$$+ A(-\mu^2 + \lambda|\phi|^2)^2 \ln\frac{-\mu^2 + \lambda|\phi|^2}{M^2}], \tag{2.16}$$

where

$$A = B = 1 \qquad \text{for Abelian case,} \tag{2.17}$$

and

$$A = 2N - 1, \tag{2.18}$$
$$B = \frac{(N-1)(N^2+2N-2)}{8N^2} \qquad \text{for SU(N) gauge group.} \tag{2.19}$$

The $SU(N)$ formula we used here were derived using an $\underline{N}$-plet monopole and with a convention for the covariant derivative

$$\tilde{D}_\mu = \partial_\mu - i\tilde{A}^j_\mu \lambda^j/2 \tag{2.20}$$

in the convention with absorbed coupling where the generators $\lambda^j/2$ were normalized to

$$Tr(\frac{\lambda^j}{2}\frac{\lambda^k}{2}) = \frac{1}{2}\delta_{jk}, \tag{2.21}$$

while for the Abelian theory we used the convention

$$\tilde{D}_\mu = \partial_\mu - i\tilde{A}_\mu. \tag{2.22}$$

We have stable or meta-stable vacua when we have minima in the effective potential (2.16) which of course then means that the derivatives of it are zero there:

$$\frac{\partial V_{eff}(|\phi|^2)}{\partial|\phi|^2}|_{min\ i} = 0 \tag{2.23}$$

where i enumerates the various minima.
Now our multiple point principle asserts that there should be as many degenerate vacua as possible - i.e., the more degenerate vacua the more intensive parameters that become finetuned by the requirement of being at the multiple point (in parameter space). In the case we consider here there are just two degenerate vacua at say $\phi = \phi_{min1}$ and $\phi = \phi_{min2}$. This means that if we take the degenerate minima to have zero energy density (cosmological constant) that

$$V_{eff}(|\phi|^2_{min1}) = V_{eff}(|\phi|^2_{min2}) = 0. \tag{2.24}$$

The joint solution of equations (2.24) and (2.23) for the effective potential (2.16) gives the phase transition border curve between a Coulomb phase and monopole condesced phase:

$$3B\tilde{g}^4_{p.t.} + (5+A)\lambda^2_{p.t.} + 16\pi^2\lambda_{p.t.} = 0. \tag{2.25}$$

All of the combinations $(\lambda, \tilde{g}^2)$ satisfying (2.25) are critical in the sense of separating phases. The maximum value of $\tilde{g}^2_{U(1)\ crit}$ - we have called it $\tilde{g}^2_{U(1)\ crit\ max}$ turns out to be interesting for us. Let us now find this top point of the phase boundary curve (2.25) (see also [4] and [5]).

$$\frac{d\tilde{g}^4}{d\lambda}|_{crit} = 0, \tag{2.26}$$

which gives

$$\tilde{g}^4_{crit} = \frac{(16\pi^2)^2}{12(5+A)B}, \tag{2.27}$$

and

$$\lambda_{crit} = -\frac{16\pi^2}{2(5+A)}. \tag{2.28}$$

From Eq. (2.27) we obtain:

for U(1) group:

A=1, B=1,

$$\tilde{g}^2_{crit,U(1)} = \frac{8\pi^2}{3\sqrt{2}} \approx 18.61, \tag{2.29}$$

for N=2:

A=3, $B = \frac{3}{16}$,

$$\tilde{g}^2_{crit,SU(2)} = \frac{8\pi^2}{3\sqrt{2}} \approx 37.22, \tag{2.30}$$

for N=3:

A=5, $B = \frac{13}{36}$,

$$\tilde{g}^2_{crit,SU(3)} = \sqrt{\frac{108}{65}} \cdot \frac{8\pi^2}{3\sqrt{2}} \approx 11.99. \tag{2.31}$$

In general we define the parameter C such that

$$\tilde{g}^2_{crit,SU(N)} = C\tilde{g}^2_{crit,U(1)}, \quad \text{where} \quad C = \sqrt{\frac{6}{(5+A)B}}. \tag{2.32}$$

We shall assume that this relationship between the Abelian and nonAbelian couplings is also valid when the couplings are not critical. These results are given at the scale of monopole mass or VEV.

2.3 Compilation and correction of finestructure constants in AntiGut model

Recall that MPP is to be applied to the N_{gen}-fold replication of SMG. For $N_{gen} = 3$ we have

$$(SMG)^3 = U(1)^3 \times SU(2)^3 \times SU(3)^3 \tag{2.33}$$

that breaks down to the diagonal subgroup at roughly the Planck scale. It is the couplings for the diagonal subgroup that are predicted to coincide with experimental gauge group couplings at the Planck scale [1]:

$$\begin{aligned} \alpha_{1,exp}^{-1} &= 6\alpha_{1,crit}^{-1}, \\ \alpha_{2,exp}^{-1} &= 3\alpha_{2,crit}^{-1}, \\ \alpha_{3,exp}^{-1} &= 3\alpha_{3,crit}^{-1}. \end{aligned} \tag{2.34}$$

According to the Particle Data Group results [7], we have:

$$\alpha_{1,exp}^{-1}(\mu_{Pl}) \approx 55.4; \qquad \alpha_{2,exp}^{-1}(\mu_{Pl}) \approx 49.0_3; \qquad \alpha_{3,exp}^{-1}(\mu_{Pl}) \approx 53.00. \tag{2.35}$$

In the Abelian theory the Dirac relation is

$$g\tilde{g} = 2\pi n \text{ where } n \in \mathbf{Z} \tag{2.36}$$

which leads to

$$\alpha\tilde{\alpha} = \frac{1}{4} \tag{2.37}$$

or

$$\alpha^{-1} = 4\tilde{\alpha} = \frac{\tilde{g}^2}{\pi}. \tag{2.38}$$

In the nonAbelian case the Dirac relation is

$$g\tilde{g} = 4\pi n \text{ where } n \in \mathbf{Z}. \tag{2.39}$$

This leads to

$$\alpha\tilde{\alpha} = 1 \tag{2.40}$$

or

$$\alpha^{-1} = \tilde{\alpha} = \frac{\tilde{g}^2}{4\pi} \tag{2.41}$$

From Eqs. (2.34) and (2.35) we have at the Planck scale:

$$\begin{aligned} \frac{3\tilde{g}_{U(1)}^2}{\pi} &\approx 27.7, \\ \frac{3\tilde{g}_{SU(2)/Z_2}^2}{\pi} &\approx 196, \\ \frac{3\tilde{g}_{SU(3)/Z_3}^2}{\pi} &\approx 212.0. \end{aligned} \tag{2.42}$$

But the correction of the running of finesructure constants from AntiGUT group (2.33) in the interval $\Delta t = \sqrt{40}$ gives:

$$\alpha^{-1}_{2,exp}(\mu_{Pl}) \approx 53.3; \qquad \alpha^{-1}_{3,exp}(\mu_{Pl}) \approx 59.4. \tag{2.43}$$

The corresponding Abelian values of $\frac{3\tilde{g}^2}{\pi}$ are:
for U(1):

$$\frac{3\tilde{g}^2_{U(1)}}{\pi} \approx 27.7, \tag{2.44}$$

for N=2:

$$\frac{3\tilde{g}^2_{U(1)}}{\pi} \approx 106.6, \tag{2.45}$$

for N=3:

$$\frac{3\tilde{g}^2_{U(1)}}{\pi} \approx 59.4 \cdot \sqrt{\frac{65}{27}} \approx 184.32. \tag{2.46}$$

2.4 The d_N-parameter.

As we suggested in the introduction we shall consider a correspondance between the nonAbelian and Abelian Chan Tsou monopoles and in this connection have a correspondance of couplings so that the ratio of corresponding monopole couplings is like that of the critical couplings for the same two theories; i.e., as in equation (2.32). Since we want to avoid having to try to use our bad knowledge of the ratio of the monopole mass scale at which the phase transition couplings are rather easily estimated and the Planck or fundamental scale, we are interested in beta-functions. So the most important feature of the gauge group for the purpose here is how the monopole coupling will run as a function of the scale. For this purpose we use $\beta_{\tilde{g}^2}$. Now if we consider - as is the simplest - an Abelian monopole we strictly speaking would have no group dependence of the beta function $\beta_{\tilde{g}^2}$, but now we imagine there there actually *is* an effect of the selfcouplings of the Yang Mills fields in the "electric" sector and include that. We shall do that by the assumption that there will be a term corresponding to it in such a way as to make the Dirac relation (or its replacement for a non Abelian monopole) (2.37) be valid for the running couplings at all scales. In (2.32) we have the relation between the Chan-Tsou $\widetilde{SU(N)}$ coupling $\tilde{g}^2_{SU(N)}$ for the critical coupling, which we here by definition of the relation between corresponding theories extend also to non-critical couplings:

$$\tilde{g}^2_{SU(N)} = C\tilde{g}^2_{U(1)\ \ corresp\ \ SU(N)}, \quad \text{where} \quad C = \sqrt{\frac{6}{(5+A)B}}. \tag{2.47}$$

In the Chan-Tsou formalism the replacement for the Dirac relation is that

$$\tilde{g}g = 4\pi n, \quad n \in \mathbf{Z}. \tag{2.48}$$

Combining (2.47) and (2.48) and taking n to be unity we get for the Abelian magnetic charge corresponding to that of a nonAbelian $SU(N)$

$$\tilde{g}^2_{U(1)\ \ corr\ \ SU(N)} = \frac{(16\pi^2)}{Cg^2_{SU(N)}}. \tag{2.49}$$

With the postulate - but that is really true - that the running of the couplings shall be consistent with the Dirac relation we can take the scale dependence of this equation (2.49) on both sides to obtain

$$\beta_{\tilde{g}^2_{U(1)\ \ corresp\ \ SU(N)}} = -(16\pi^2/C)\cdot\beta_{g^2_{SU(N)}}/g^4_{SU(N)}. \tag{2.50}$$

Now there is the group dependent contribution to the well known beta function

$$\beta_{g^2_{SU(N)}}|_{Y.M.contribution} = -g^4_{SU(N)}\cdot\frac{11N}{48\pi^2} \tag{2.51}$$

which to keep the Dirac relation valid at all scales must be tranfered to also exist in the beta function for the square of the monopole charge

$$\beta_{\tilde{g}^2}|_{Y.M.contribution} = (16\pi^2/C)\cdot\frac{11N}{48\pi^2} = \frac{11N}{3C} \stackrel{\wedge}{=} d_N \tag{2.52}$$

2.5 Our monopole coupling versus d curve and successful agreement

Let us make it quite clear what we mean by our *intersection point* and our so called *fourth point*.

Our intersection point is the point at which two functions that live in the space spanned by the variables $(d_N, \tilde{g}^2_{U(1)})$ intersect one another. One of these functions is our in d_N extrapolated curve of "experimental" values of function $\tilde{g}^2_{U(1)\ \ corresp\ \ SU(N)}$. The other function consists of values $(d_N, \tilde{g}^2_{U(1)\ \ corresp\ \ SU(N)})$ that satisfy the condition

$$\beta_{\tilde{g}^2_{U(1)\,corresp,\,SU(N)}} = \beta_{\tilde{g}^2_{U(1)}} + d_N = \frac{\tilde{g}^4}{48\pi^2} + \frac{\tilde{g}^6}{(16\pi)^2} + d_N = 0. \tag{2.53}$$

The value of $\tilde{g}^2_{U(1)\ \ corresp\ \ SU(N)}$ at the intersection point remains the same under scale changes of course since its β-function vanishes.

Our fourth point - we could denote it as $(d_{N_{4th}}, \tilde{g}^2_{U(1)\ \ crit\ \ max})$ -lives in the space of variables $(d_N, \tilde{g}^2)$. By definition the second coordinate is the maximum value value of $\tilde{g}^2_{U(1)\ \ crit}$ on the phase transition bounday which lives in the space of the variables $(\lambda, \tilde{g}^2_{U(1)})$. We have above denoted this maximum alias top point by the symbol $\tilde{g}^2_{U(1)\ \ crit\ \ max}$. The first coordinate of the fourth point - i.e., $d_{N_{4th}}$ - is the value of d_N obtained when $\tilde{g}^2_{U(1)\ \ crit\ \ max}$ is substituted into Eqn (2.53) above. We get for the fourth point

$$(d_N, \tilde{g}^2_{U(1)\ \ crit\ \ max}) = (-0.62, 15.11). \tag{2.54}$$

The success that we have in this paper is that the intersection point coincides with the fourth point to very high accuracy.

Maybe it's instructive to think of choosing a bunch of $\tilde{g}^2_{U(1)}$ values satisfying Eqn (2.53) (corresponding of course to a bunch of different d_N values). With this bunch of $\tilde{g}^2_{U(1)}$ values we know how to RG run them back and forth between Planck scale and monopole mass scale - also in the space spanned by $(\lambda, \tilde{g}^2)$ where the phase transition curve is located with its top point

$$(\lambda_{crit}, \tilde{g}^2_{U(1)\,crit\,max}) = (-7.13, 15.11).$$

This bunch of $\tilde{g}^2_{U(1)}$ values don't RG run at all by definition so they must be parallel to the λ axis in the space in which the phase transition boundary lives. What happens to these parallel to λ RG tracks of the bunch of $\tilde{g}^2_{U(1)}$ values depends only on the height (i.e., $\tilde{g}^2_{U(1)\ crit\ max}$) and not on the details (including the unknown scale) of the phase transition curve. Our intersection point follows the one horizontal RG track that can become a tangent to the phase transition boundary and the point of tangency is necessarily the top point. RG tracks below the one corresponding to our (fortuitous for MPP) intersection point would hit the phase boundary below the top point and therefore corresspond to having only a monopole condensate phase. And being within the condensate phase and removed from the phase transition boundary so that the Coulomb phase is energetically inaccessible would violate MPP. The (horizontal) RG tracks of $\tilde{g}^2_{U(1)}$ values above the top point would miss hitting the phase transition boundary and hence correspond to being in the Coulomb phase more or less energetically prohibited from being in the monopole condensate phase depending on how far above the top point, that the RG track is. This is also in violation of MPP. The only RG trajectory allowed by MPP is the one that goes through the fourth point. And our intersection point singles out just this RG trajectory.

Actually, MPP is put to a very stringent test here. Had our intersection point picked out any other RG trajectory then the one that hits the fourth point our MPP would have been falsified.

2.6 Conclusion and outlook

In the figures we see the plot of the values of $3\tilde{g}^2/\pi$ – for the "corresponding " $U(1)$ monopole coupling – versus our group characterising quantity d_N. The extrapolation to the point where it crosses the curve of $(d_N, 3\tilde{g}^2/\pi)$ combinations for which the β-function for $\tilde{g}$ is zero (the crossing point is drawn both for the one loop and two loop calculation and lies of course exactly on the extrapolated curve but the intersecting curve $\beta_{\tilde{g}^2} = 0$ is not drawn) it is remarkably close to being critical in the sense that its ordinate is very close to the critical value $3\tilde{g}^2/\pi = 3 \cdot 15.11/\pi = 14.46$ (the two loop critical value for the case of zero beta function of $\tilde{g}^2$ is 15.11 corresponding to the top point of the phase transition curve). We have drawn on the plot the point with this ordinate calculated using the relation $\beta_{\tilde{g}^2} = 0$.

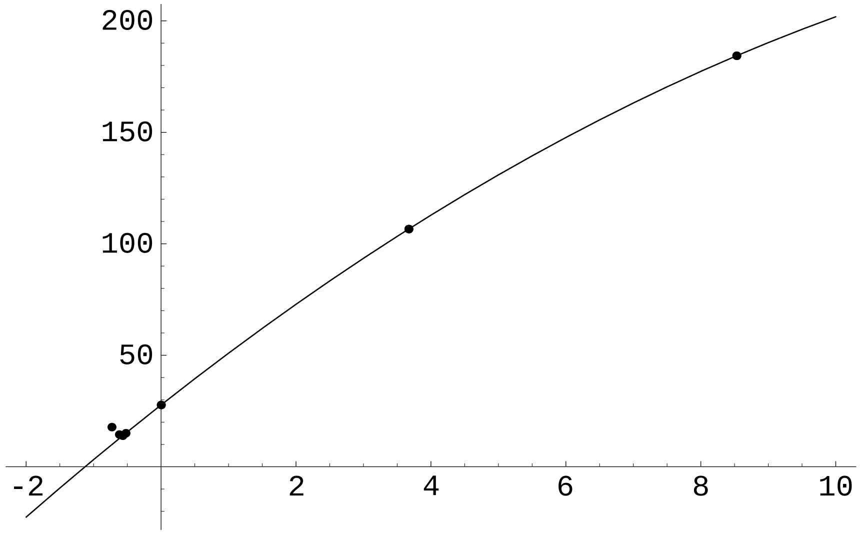

Fig. 2.1. Magnetic coupling $\frac{3\tilde{g}^2}{\pi}$ (ordinate) as a function of d_N (abcissa). The curve is determined to be the parabola that passes through the values of $\frac{3\tilde{g}^2}{\pi}$ corresponding to at "experimental values of gauge couplings (at positive d_N-values). The points at negative value of d_N lying off the parabolic fit (solid line) correspond to maximum values of $3\tilde{g}^2/\pi$ on phase transition curves calculated to one and two loops (see closeup in Fig. 2.2).

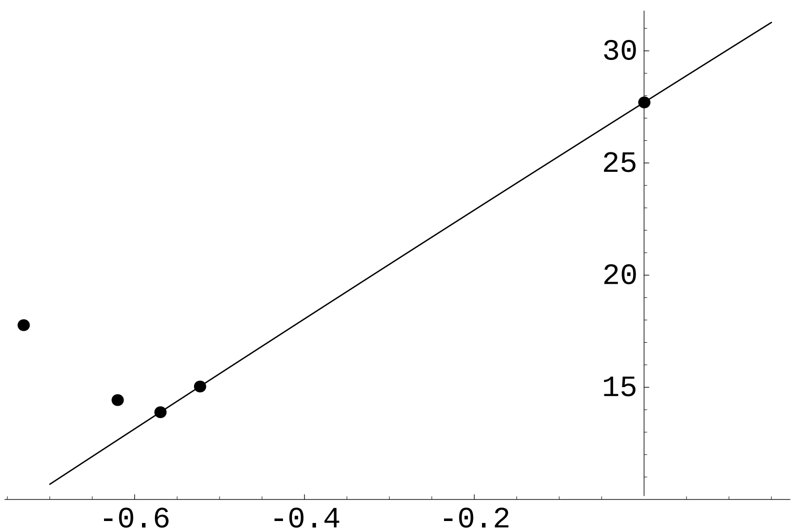

Fig. 2.2. Closeup of $\frac{3\tilde{g}^2}{\pi}$ vs. d_N for slightly negative values of d_N. The two points (at negative d) lying on the parabolic fit (solid line) correspond to to the intersection of the parabolic fit with the curves $\beta_{g^2} = 0$ calculated for one- and two loops the latter corresponding to the point at $d_N \approx -0.57$. The two points (also at negative values of d_N) not on the parabolic fit correspond to the maximum values of $3\tilde{g}^2/\pi$ on the phase transition curves calculated to one and two loop. The latter corresponds to the point at $d_N =\approx -0.62$. The points for highest order (i.e., at $d_N \approx -0.57$ on curve and at $d_N \approx -0.62$ off the curve) are seen to converge towards one another relative to the remaining two points calculated to one loop.

It is this coincidence which is either accidental or because our theory of Anti-GUT conbined with MPP is working so well that it only deviates in say the inverse $U(1)$ fine structure constant at the Planck scale by about 2·1.5 = 3 units.

If the Planck scale had not been taken to be $\mu_{Pl} = 1.2 \cdot 10^{19}$ GeV, as was the case for the figure but rather to the value obtained if we had used $8\pi G$ instead of G as the quantity from which to determine by dimensional arguments the Planck scale, the latter would have been a factor $\sqrt{8\pi}$ smaller. This would correspond to subtracting 1.61 from the $\ln \mu_{Pl}$ or 3.22 from the $\ln \mu_{Pl}^2$. With this lower value for the fundamental energy scale the $U(1)$ running coupling would be shifted up in value from the 55.4 of equation (23) by 1.84 to 57.2 which in turn would shift the 55.4/2=27.7 value plotted on the curve up by 0.92. This is seen by eye to shift the curve even closer to going through the fourth point than was the case for the Planck scale calculated simply using G. Have in mind that the $SU(2)$ point with almost compensating β-function contributions from the Yang Mills and the fermion contributions would only be moved very little by a slight change in the Planck scale so that this point would be almost unchanged by the replacement of G with $8\pi G$, while the $SU(3)$ point would go the opposite way, meaning down in $3\tilde{g}^2/\pi$-value, so that the curve would essentially be tilted so as to rise more slowly with increasing d_N.

Assuming that the correct fundamental scale should be obtained from $8\pi G$ rather than from G, the deviation would in terms of the inverse $U(1)$ fine structure constant be only about one out of 27.7 meaning about 1 in 55.4, which is only about 2%. We can not meaningfully expect better coincidence unless we were to calculate a three loop critical value since going from one to two loop in the critical monopole coupling squared gave a 20% correction so that 4% would be expected from three loops But 4% in the critical monopole coupling square would mean about 2% in the $U(1)$ finestructure constant (inverted) at the Planck scale.

We may also be concerned that we have not yet made two loop calculations of our correction factors C giving the correspondance between the nonAbelian couplings and the corresponding $U(1)$ coupling. But it is our experience numerically that the remarkable result of the intersection point having critical coupling is rather insensitive to the exact C-system used. We therefore believe that even with the C's only computed to one loop the accuracy of the calculation is already effectively a two loop calculation.

The question of the $8\pi G$ versus the G in determining the Planck scale or fundamental scale must be considered basically an uncertainty at least until after a very detailed discussion of this point.

Our result brings to mind the proverbial story of trying to find a needle in a large haystack. Actually we could even tell a better story. Start by standing at a distance from the haystack with an electric torch with a very narrow beam. Initially the torch is turned off. Now imagine that our task is to aim the torch in the dark it before we turn the torch on - we are not allowed to move the torch after turning it on. Now ask about the likelyhood of capturing the needle in our fixed narrow beam of light. We could improve the story by claiming that we stand with our torch at the Planck scale and must aim it at a haystack at the scale of the monopole mass before turning it on. There is even the added complication that we a priori don't know exatly what we're looking for. But we miraculously find the needle in our narrow beam of light. But maybe with the guiding light of our model this is not such a miracle after all. Nature may be telling us that we're on the right track with our MPP/AntiGUT model.

In this work we have only concerned ourselves with two of the three intersting phases for monopoles, namely the Coulomb phase and the phase with a monopole condensate. These phases have been treatable using the Higgs monopole description of monopoles as particles. We would also like to have the monopole confining phase at our multiple point. For every new phase we bring to the multiple point there is one more intensive parameter that becomes finetunned by MPP. This could be ratio of the monopole mass scale to that of the Planck mass if we had the confining phase. However taking the confining phase into account would require appending a string scenario to our approach here and doing this with the the high accuracy we otherwise have in this work is not yet possible.

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