

Physical properties of the double-Kerr solution

Miguel S Costa, Carlos A R Herdeiro and Carmen Rebelo*

Departamento de Física e Astronomia e Centro de Física do Porto, Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre, 687, 4169-007 Porto, Portugal

E-mail: miguelc@fc.up.pt, crherdei@fc.up.pt, mrebelo@fc.up.pt

Abstract. We analysed two special cases of the double-Kerr solution describing a system of two stationary co-axial Kerr black holes, with equal mass and either the same or opposite angular momentum. These cases are asymptotically flat and obey the axis condition, but there is always a strut (i.e. a conical singularity) between the black holes, preventing the collapse of the system. Studying the force associated to the strut, we show that the force between the two black holes is more attractive in the counter-rotating than in the co-rotating case. We also show that, for both the counter-rotating and co-rotating cases, 1) the angular velocity of the two black holes decreases as they approach one another, for fixed mass and angular momentum; 2) the extremal limit J/M^2 varies with the distance between the black holes and may even be larger than one. These results are interpreted in terms of rotational dragging effects.

1. Introduction

The main purpose of this communication is to summarise some curious features of the double-Kerr solution. This solution, which describes the nonlinear superposition of two Kerr black holes, was originally derived by Kramer and Neugebauer [1]. Subsequent analyses [2, 3, 4, 5] revealed that, for two under-extreme objects, the solution cannot be made free of singularities. The metric has always a conical singularity along the axis between the black holes. This conical singularity, that can be seen as a massless strut, balances the attraction force, avoiding the collapse of the system. This may be interpreted as follows: while in the the known Majumdar-Papetrou solution, describing a system of extremal Reissner-Nordström black-holes in equilibrium, the repulsive Coulomb force can balance the attractive gravitational force [6], in the double-Kerr solution the spin-spin repulsion is not strong enough to overcome gravitational attraction and hence two aligned Kerr black holes cannot be in stationary equilibrium.

Probably due to the presence of the strut, which implies that the solution cannot be seen as a realistic physical situation, little more was known about this particular solution. Recent studies [7, 8], focused on two special limits of the general double-Kerr solution where black holes have equal mass and either the same or opposite angular momentum, revealed that:

- the force between the two black holes is more attractive in the counter-rotating than in the co-rotating case;
- whenever the black holes are co- or counter-rotating, the angular velocity always decreases as the black holes approach one another and its maximum is always smaller than the angular velocity of an extremal isolated black hole with the same mass.

This peculiar behaviour of the angular velocity can be interpreted in terms of rotational dragging effects and allows the black holes to have a $J > M^2$.

2. Angular velocity

The co- and counter-rotating limits of the double-Kerr solution are characterised by three parameters: the mass M , the angular momentum J and the physical (i.e. proper) distance d between the black holes. As can be seen in figure 1, in both the counter-rotating and the co-rotating cases, the angular velocity Ω of the horizon, for each black hole, decreases when the two black holes get closer, keeping fixed their mass and the angular momentum, computed as Komar integrals.

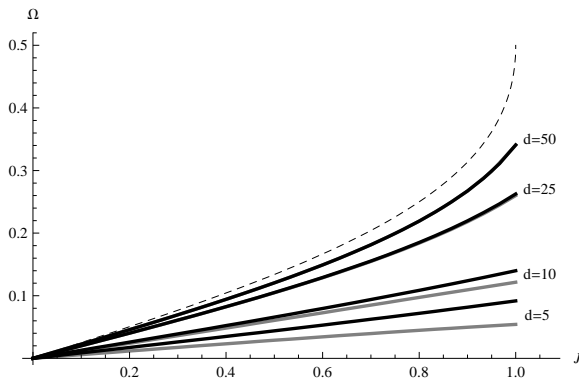


Figure 1. Horizon angular velocity as function of the angular momentum, for fixed mass $M = 1$, in the double Kerr system. The dashed line corresponds to the infinite distance limit (i.e. the usual Kerr solution). The black (grey) lines correspond to the co-rotating (counter-rotating) case for different values of the physical distance d . Note that in the co-rotating (counter-rotating) case, $\Omega = \Omega_1 = \Omega_2$ ($\Omega = \Omega_1 = -\Omega_2$). This figure was first shown in [9].

Moreover, it shows that the angular velocity is smaller, at the same physical distance, in the counter-rotating than in the co-rotating case. In the counter-rotating case, since black holes are rotating in opposite directions they are being counter-dragged and therefore they slow down. Using the same line of reasoning, one expects that the mutual rotational dragging, in the co-rotating case, speeds up the horizon angular velocity. Indeed one verifies that it is larger than in the counter-rotating case. However there must be a dominant effect responsible for the general slow down of the angular velocity. In [8, 9] it was suggested that each black hole is simultaneously dragging and being dragged; and dragging the other black hole always slows down the black hole which is dragging. This novel effect can only be seen in exact multi-black hole solutions and it is not an artifact of the strut. In fact, we can also observe this general slow down of the angular velocity in the balanced black Saturn [9], which is a regular (on and outside an event horizon) solution of the vacuum Einstein's equations in 5 dimensions. Considering a fat ring ¹, with fixed mass and angular momentum, when the mass of the central Myers-Perry black hole is increased, the angular velocity of the ring horizon decreases. This happens because the black ring has to drag a heavy black hole.

3. Extremal Limit

The decrease in the angular velocity, due to frame dragging effects, allows the black holes in binary solutions to carry a higher angular momentum than isolated ones. This can be easily seen analysing Smarr's relation $M = TA_H/2 + 2\Omega J$, where T and A_H denote respectively the temperature and horizon area of each black hole. For the counter-rotating case it is possible to find the explicit expression to the extremal limit in terms of M and of the coordinate distance ζ ² (in Weyl canonical coordinates), more specifically

$$\frac{|J_{ext}|}{M^2} = \sqrt{\frac{\zeta + 2M}{\zeta - 2M}}$$

¹ Just like Kerr black holes, fat rings have a positive moment of inertia, i.e. a higher J leads to a higher Ω .

² Note that ζ is a good measure of the physical distance, in the sense that d grows monotonically with ζ . In Weyl canonical coordinates $d = \int_{-\zeta/2}^{\zeta/2} dz \sqrt{g_{zz}}|_{\rho=0}$.

which gets larger as the counter-rotating black holes are approached.

The ratio J_{ext}/M^2 in the co-rotating case is displayed in figure 2, where it varies from one when the black holes are far away to two at the merging limit, corresponding to an extremal Kerr black hole with mass $2M$ and angular momentum $2J$, that is formed after merging.

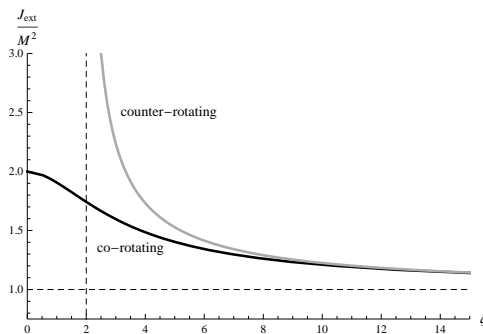


Figure 2. J_{ext}/M^2 in terms of the coordinate distance, for the co-rotating case and the counter-rotating case. We fixed $M = 1$ which determines the merging limit in the counter-rotating case to be at $\zeta = 2$. This figure was first shown in [8].

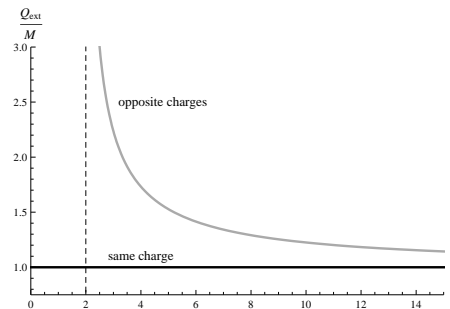


Figure 3. Q_{ext}/M in terms of the coordinate distance, for the cases where black holes have equal mass and either the same or the opposite charge. We fixed $M = 1$ which determines the merging limit in the dihole case to be at $\zeta = 2$.

It should be pointed out that this property concerning the upper bound for the angular momentum has a counterpart for charged solutions. Considering a system of two charged black holes with equal mass M and either the same [6] or the opposite [10] charge Q , one can see that in the case of opposite charge case one finds

$$\frac{|Q_{ext}|}{M} = \sqrt{\frac{\zeta + 2M}{\zeta - 2M}},$$

just like the ratio J_{ext}/M^2 in the counter-rotating case. Moreover, just like it happens with the horizon angular velocity in the double Kerr system, in the charged dihole solution [10] the electrostatic potential at the horizon decreases as black holes approach one another and therefore each black hole supports more charge without reaching the extremal limit. When the black holes have the same charge [6] the electrostatic potential at the horizons is independent of the distance between the black holes and, as can be seen in figure 3, the ratio Q_{ext}/M is always equal to one; this is the expected Q_{ext}/M value to the Reissner-Nordström black hole with mass $2M$ and charge $2Q$, that is formed after the merging and, as previously pointed out, whenever $Q = M$ the black holes are in equilibrium.

4. Interaction Force

The force associated to the strut reflects the interaction force between the black holes. With the aim of studying the spin-spin interaction, we computed the force between the black holes in terms of J , for fixed mass and physical distance. Figure 4 shows that in both co- and counter-rotating cases, the force increases, in modulus, as J increases, meaning that we are in the presence of an attractive force.

However, it must be taken into account that when we vary J , for fixed d and M , we are also varying the horizon shape and therefore higher gravitational multiple moments should contribute to the force at the same order as the spin-spin interaction. This may explain why even in co-

rotating case, where the spin-spin force is expected to be repulsive, the total force becomes more attractive when we increase J .

It is, still, reassuring to note that the force is greater, in modulus, in the counter-rotating case than in the co-rotating one. This should be expected from the properties of the spin-spin force which also explains why more energy can be extracted in a head on collision of Kerr black holes, if the spins are anti-aligned (counter-rotation), rather than aligned (co-rotation), with the direction of separation [11].

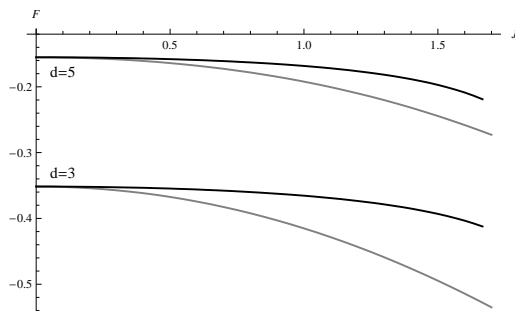


Figure 4. Force between the two black holes for two physical distances $d = 3$ and $d = 5$ as a function of J for $M = 1$, for the counter-rotating (grey lines) and co-rotating case (black lines). This figure was first shown in [8].

5. Conclusion

We have shown that due to frame dragging effects the angular velocity of two aligned black holes always decreases when the black holes are approached, even if they are co-rotating. Also due to this slow down in the angular velocity, black holes in binaries can carry more angular momentum than isolated black holes with the same mass. Thus, extremal black holes have $J > M^2$ in the double-Kerr system. Regarding the interaction force between the black holes we have observed that it is more attractive in the counter-rotating than in the co-rotating case. This makes clear why, in particular, it is always impossible to reach equilibrium.

Acknowledgments

C.R. is funded by FCT through grant SFRH/BD/18502/2004. C.R. would like to thank the organization of the “Spanish Relativity Meeting 2009” the grant to attend this meeting.

References

- [1] Kramer D and Neugebauer G 1980 *Phys. Lett.* **75A** 259
- [2] Dietz W and Hoenselaers C 1985 *Ann. Phys.* **165** 319
- [3] Manko V S, Ruiz E and Sanabria-Gómez J D 2000 *Class. Quantum Grav.* **17** 3881
- [4] Manko V S and Ruiz E 2001 *Class. Quantum Grav.* **18** L11
- [5] Neugebauer G and Hennig J 2009 *Gen. Rel. Grav.* **41** 2113
- [6] Azuma T and Koikawa T 1994 *Prog. Theor. Phys.* **92** 1095
- [7] Herdeiro C A R and Rebelo C 2008 *J. High Energy Phys.* JHEP10(2008)017
- [8] Costa M S, Herdeiro C A R and Rebelo C 2009 *Phys. Rev. D* **79** 123508
- [9] Herdeiro C A R, Rebelo C and Warnick C M 2009 *Phys. Rev. D* **80** 084037
- [10] Emparan R and Teo E 2001 *Nucl. Phys. B* **610** 190
- [11] Hawking S W 1971 *Phys. Rev. Lett.* **26** 1344