Tsallis-Pareto like distributions in hadron-hadron collisions

G. G. Barnaföldi¹, K. Ürmössy^{1,2}, T. S. Biró¹

 $^1\rm MTA$ KFKI RMKI, Konkoly-Thege Miklós út 29-33, H-1121 Budapest, Hungary $^2\rm E\"otv\"os$ University, Pázmány Péter sétány 1/A, H-1117 Budapest, Hungary

E-mail: bgergely@rmki.kfki.hu

Abstract. Non-extensive thermodynamics is a novel approach in high energy physics. In highenergy heavy-ion, and especially in proton-proton collisions we are far from a canonical thermal state, described by the Boltzmann–Gibbs statistic. In these reactions low and intermediate transverse momentum spectra are extremely well reproduced by the Tsallis-Pareto distribution, but the physical origin of Tsallis parameters is still an unsettled question.

Here, we analyze whether Tsallis-Pareto energy distribution do overlap with hadron spectra at high- p_T . We fitted data, measured in proton-proton (proton-antiproton) collisions in wide center of mass energy range from 200 GeV RHIC up to 7 TeV LHC energies. Furthermore, our test is extended to an investigation of a possible \sqrt{s} -dependence of the power in the Tsallis-Pareto distribution, motivated by QCD evolution equations. We found that Tsallis-Pareto distributions fit well high- p_T data, in the wide center of mass energy range. Deviance from the fits appears at $p_T > 20-30 \text{ GeV/c}$, especially on CDF data [7]. Introducing a p_T -scaling ansatz, the fits at low and intermediate transverse momenta still remain good, and the deviations tend to disappear at the highest- p_T data.

1. Introduction

Heavy ion physics is one possible testbed for non-extensive thermodynamics, due to the fact that high-energy nuclear reactions are few-body and non-thermalized processes. Furthermore, strong correlations between particles, especially at the final state or during the hadronization, rule out the Boltzmann–Gibbs and rather lead to non-extensive statistics [1].

Non-extensive thermodynamics is a novel approach to high-energy heavy ion collisions, it is particularly successful for describing the intermediate and high transverse momentum regimes. Based on the idea in Refs. [2, 3] we found that the hadron production in AuAu collisions at RHIC energies can be well fitted by the Tsallis–Pareto distribution [4].

Recent experimental data measured at the LHC in proton-proton collisions [5, 6] have also shown Tsallis – Pareto-like distribution, but the fitted parameters are not satisfactorily interpreted. Furthermore, fits for earlier Tevatron data [7] result in a deviance from the parameterization at the highest transverse momentum region, as it was pointed out in Refs. [8, 9].

In this short contribution we report on various measured spectra from RHIC to LHC energies. We test a particular, scale evolution *ansatz* for the power in the Tsallis–Pareto distribution, suitable to fit the highest momentum data. We compare our results to several existing experimental data, the parameters are estimated up to the highest LHC energies.

2. Tsallis – Pareto fit on various experimental data

We investigated the neutral pion (π^0) and charged hadron (h^{\pm}) production spectra in protonproton (pp) and proton-antiproton $(p\bar{p})$ collisions in a wide center of mass energy (\sqrt{s}) range from 200 GeV (RHIC,UA1) up to 7 TeV (LHC). In all cases we selected data with similar experimental acceptance, taken at around midrapidity $(y, \eta \approx 0)$ in symmetric (pseudo)rapidity regions. We plotted the data as a function of logarithmic transverse momenta, on the *left panel* of Fig. 1. Points are: PHENIX data [10] blue full triangles, CERN UA1 data [11, 12] blue open triangles, CDF data [7, 13] red full squares, ATLAS data [14] magenta full stars, earlier CMS data [5] green full circles, while the latest CMS (upscaled) data [6] green full triangles. The p_T bins and errors are also indicated on the data points with bars on all panels of Fig. 1.

We assumed that the spectra measured in pp or $p\bar{p}$ collision are Tsallis–Pareto-like. They are determined by two essential parameters: the effective temperature, T refers to the average slope and the parameter q determines the tail of the distribution. For the Boltzmann–Gibbsdistribution, when $q \rightarrow 1$, 1/T gives the logarithmic slope of the curve, while for non-unity qvalues this effective temperature is p_T -dependent, especially at higher momenta. This behavior modifies the shape of the whole distribution-curve to a power-law tailed one as p_T increases. Since fits to existing proton-(anti)proton data well agree with $q \neq 1$, one concludes that these spectra are Tsallis–Pareto-distributions.



Figure 1. Tsallis fits on experimental data plotted on a logarithmic plot on the *left panel*. 'Data/Tsallis' best fit for the Tsallis–Pareto distribution on the *middle panel*. On the *right panel* 'Data/Scaling Tsallis' points are shown. (Color online.)

The data above were fitted by using the least-square method. We used the standard parameterization of the Tsallis – Pareto distribution given by:

$$E\frac{\mathrm{d}N}{\mathrm{d}p_T} = A \cdot m_T \left(1 + \frac{1}{n \cdot T}m_T\right)^{-n},\tag{1}$$

where $m_T = \sqrt{m^2 + p_T^2}$ is the *transverse mass* of the hadrons and the power is n = 1/(q-1). Values of T and q are obtained via the best fit. The results of the fits are drawn by *solid curves* on Fig. 1. (We note, some of the distributions were upscaled to avoid multiple overlaps.)

These Tsallis fits are in a remarkably good agreement with the experimental data, except CDF at 1.96 TeV. It shows up to 3-4 orders of magnitude deviations beyond $p_T \approx 20-30$ GeV/c. For a better comparison we plot the ratio of the experimental data to the fitted Tsallis distribution on the *middle panel* of Fig. 1. Here the 3-4 orders of magnitude deviations are clearly seen (all points above 30 GeV/c are off scale).

Motivated by the QCD scale evolution idea we investigate a similar distribution ('Scaling Tsallis') as given by Eq. (1), but with a p_T -dependent power:

$$n \longrightarrow n(p_T) = \frac{1}{q-1} - 2 \cdot \log \log \left(1 + \kappa \cdot p_T\right) \quad ,$$
 (2)

with a parameter $\kappa = 0.013$ (GeV/c)⁻¹. The obtained 'Data/Scaling Tsallis' points are plotted on the right panel of Fig. 1. Here, all experimental data are in close agreement with the 'Scaling Tsallis' formula, except the latest CMS result. It has a factor of 2-4 deviance for low ($p_T < 1$ GeV/c) and at the highest ($p_T > 15$ GeV/c) transverse momenta. Based on this latter panel, introduction of a scaling may help on the earlier data especially for Tevatron. On the other hand latest CMS results explicitly contradict to this behavior at high- p_T .

3. Analyzing Tsallis parameters

We investigated the center of mass energy dependence of the parameters fitted to the Tsallis – Pareto-distributions in Fig. 1. We summarize all of these properties on Fig. 2.

On the *left panel* of Fig. 2 we plot the obtained $q(\sqrt{s})$ relation with *blue* signs for the standard 'Tsallis' and with *red* ones for the scaling Tsallis – Pareto-distribution. For both cases, the values of the Tsallis-q parameter are slightly increasing then saturating with \sqrt{s} , within the values 1.07 < q < 1.18. Generally the Tsallis-q parameters display the same function of the \sqrt{s} for both fits.



Figure 2. The \sqrt{s} dependence and the parameter space of fitted Tsallis parameters T and q.

We indicate the $T(\sqrt{s})$ dependence on the *middle panel* of Fig. 2. Here the two parameterizations deviate from each other. The value of the effective temperature in case of 'Tsallis' is around $\approx 0.1 \text{ GeV}$ – almost independently of \sqrt{s} (see *blue points*). Meanwhile, after introducing scaling by Eq. (2) into Eq. (1), an increase can be seen on the *T* value starting from ≈ 0.2 GeV, and may saturate around 0.3 GeV between 2.5 - 3.5 GeV for the highest energies, $\sqrt{s} > 1$ TeV (see *red points*). These latter results are closer to the physical picture of high-energy collisions stating that a higher energy density reflects hotter matter. To make an estimate for the values of the effective temperature at higher \sqrt{s} values we use the following two parameterizations: (i) a power-like: $T(\sqrt{s})[\text{GeV}] = (0.24 \pm 0.006) \cdot (\sqrt{s})^{0.12\pm0.02}$ and (ii) a logarithmic one: $T(\sqrt{s})[\text{GeV}] = (0.029 \pm 0.005) \cdot \log(\sqrt{s}) + (0.242 \pm 0.006)$.

The parameter space for the standard 'Tsallis' and for the 'Scaling Tsallis' are summarized on the right panel of Fig. 2. Since both parameters have explicit dependence on the \sqrt{s} , the evolution of the parameters can be inspected clearly on the plot. As already noted, the Tsallis-q values are similar for both cases, but due to the different effective temperature finally two deviating trajectories appear. They cover well separated regions in the parameter space. However, due to the inclusion of the above mentioned low- p_T CMS data with huge errors, a given degree of uncertainty remains. Based on the scattering of the fitted values around $q \leq 1.22$, we surmise that the Tsallis-q distribution is not yet reached within the present statistic. This may correspond to a strong correlation of partons or to the limited number of steps during hadronization, as applied in Monte Carlo generators Ref. [15].

4. Conclusions and outlook

Based on various experimental data from RHIC to LHC energies, we fitted Tsallis–Pareto distributions to transverse momentum spectra measured in pp and $p\bar{p}$ collisions. Two sets of Tsallis parameterization were applied: 'standard' and 'scaling' – this latter with p_T -dependent power ansatz. Parameters T and q were obtained by using the least square method.

We investigated the \sqrt{s} dependence on the parameter values for the original and for a modified, scaling Tsallis–Pareto distribution. Tsallis-q parameters were found to be almost \sqrt{s} independent distributed around $q = 1.12 \pm 0.6$. This value seems to be constant (or slowly increasing) beyond recent LHC energies, which supports the idea that these experiments are just getting to reach Tsallis-q distribution with q = 1.22. It may then be reached at the highest energies of the LHC.

The effective temperature, T behaves differently in the cases of two parameterizations. Standard Tsallis has an almost constant value $T = 0.8 \pm 0.2$ GeV as a function of \sqrt{s} . With this parameterization no further changes are expected as increasing the energy. By using the 'scaling Tsallis–Pareto' ansatz, defined by Eqs. (2) and (1), values of T increase with \sqrt{s} . We speculate that this results is physically reliable, since increasing the \sqrt{s} may result in a higher energy density and higher temperature. Assuming both a power like and a logarithmic function, we estimated a common value of $T \approx 0.32 - 0.33$ for pp collisons at 14 TeV LHC energy.

Based on the experimental data, the Tsallis–Pareto distribution seems to be a solid and invariant assumption for high-energy hadron spectra both in proton-(anti)proton and heavy-ion collisions. The possible scaling property of the power, however, needs to be investigated in more detail in the light of forthcoming data.

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