# Small scale ring laser gyroscopes as environmental monitors

**Filippo Bosi, Angela D. V. Di Virgilio, Umberto Giacomelli, Andrea Simonelli, Giuseppe Terreni**
INFN Sez. di Pisa, Largo B. Pontecorvo 3, Pisa, Italy

**Andrea Basti, Nicolò Beverini, Giorgio Carelli, Donatella Ciampini, Francesco Fuso, Enrico Maccioni, Paolo Marsili, Fabio Stefani**
Dept. of Physics of the Univ. of Pisa, Largo B. Pontecorvo 3, Pisa, italy

**Abstract.** Inertial sensors are of interest for many different applications, and at present ring laser gyros with area larger than one square meter are the most sensitive ones; a few ring lasers with area between 15 and 72 square meters are operative with sensitivity of the order of tens of prad/s in 1 second measurement and high duty cycle. Smaller scale rings are severely limited by the back-scattering induced coupling between the counter-propagating beams, that in presence of just the angular rotation velocity vector of the Earth, eventually lock together the two frequencies making the device blind. The problem is usually overcome by increasing the frequency distance between the two modes, i.e. increasing the size of the ring. Even when the RLG is large enough and the two modes are not locked, back-scattering affects the signal. However, this is not a fundamental limit, and we have recently shown that it can be analytically solved. In this paper the sensitivity of a ring laser with a square side of 1 m will be discussed.

The Sagnac effect has been demonstrated by Georges Sagnac more than 100 years ago. It states that the difference of time of flight of two light beams counter-propagating inside a closed path is proportional to the angular rotation rate of the frame. Then, recombining after one round trip the two beams, the phase shift $\Delta\phi$ is:

$$\Delta\phi = \frac{8\pi A\Omega}{\lambda c}\cos\theta \tag{1}$$

Due to the Sagnac effect, the two counter-propagating beams in a ring laser (called Ring Laser Gyro, RLG) oscillate at a slightly different frequencies. The beat frequency $f_s$ between the two beams (Sagnac frequency) is related to the the angular velocity $\Omega$ by

$$f_s = S\Omega\cos\theta \ ; \quad S = 4\frac{A}{\lambda L} \tag{2}$$

where $L$ is the perimeter length, $A$ the enclosed area, $\theta$ the angle between the area vector of the ring and the rotational axis. $S$ is the geometrical scale factor. Note that this instrument has in principle several advantages: it does not require any external laser source and it is based on a frequency measurement, which is by itself advantageous, since it is based on the time standards,

which can be very accurate, and guarantees a very large dynamic range of the measurement. Up to now the measured beat note $f_m$ of the two counter-propagating beams was considered equal to $f_s$, but this must be considered an idealisation, since neglects the fact that the two laser beams are coupled together by the laser active medium[1]. As described by Aronowitz (1), this produces a pulling of the two counter-propagating modes and the beat note $f_m$ is not equal to the expected value of $f_s$ but this is not a fundamental problem. It has been recently shown that the true Sagnac frequency $f_s$ can be recovered analytically from the available signals of the RLG, i.e. the measured beat note $f_m$ and the records of the mono beam signals (2). However it is necessary to keep the coupling between the two modes low enough in order to avoid complete locking. In short it is necessary to design the RLG such that the expected Sagnac frequency is higher than a frequency threshold $f_{lock}$, which depends on the quality of the mirrors, on the size of the ring cavity and on the mean value of the rotation rate acting on the device. Considering a RLG attached to the Earth, the bias induced by the Earth motion is a function of the orientation of the ring. We note that the RLG cannot have area vector orthogonal to the Earth rotation axis. Assuming the Earth bias $0.7 \times \Omega_\oplus$ (i.e. 45° between the RLG area vector and the Earth rotation axis), a rough estimation of the locking frequency for a square cavity gives $f_{lock} = \frac{c\mu_s\lambda}{\pi dL}$, where $c$ is the velocity of light, $d$ the diameter of the electromagnetic field of the beam, and $\mu_s$ the total scattered fractional amplitude at each reflection. Considering the conservative values: $d = 0.5$mm and $\mu_s = 1.4 \times 10^{-3}$, it is straightforward to find that $\frac{f_{lock}}{f_s} < 1$ for RLG square cavity with the side larger than 80cm.
The very high sensitivity and dynamic range, together with their bandwidth extending from the DC up to tens Hz, could open important perspectives for transportable RLGs in scientific and industrial environmental applications. Typical application should be the control of soil subsidence in volcanic region or as a consequence of anthropic activities, the risk evaluation of many geological phenomena or the structural characterization of large engineering structures or monuments. Underground laboratories exhibit very high thermal stability, and low seismic noise, ideal conditions for top sensitivity experiments. For example, the Japanese antenna KAGRA is underground located, and the third generation gravitational waves detectors will be underground located. Small scale RLG would be important as environmental monitor, for Newtonian noise subtraction and to improve the performances of the mirror suspensions at low frequency.(3; 4)
The potential sensitivity of a small scale RLG is shown in Fig. 1, indicating that an instrument with 1 m of side could achieve a sensitivity near to $10^{-11}$ rad/s in 1 hour of integration time. We note that Sagnac effect is sensitive to the rotation relative to an inertial frame and therefore is not affected by Newtonian noise, while, on the contrary, balance tilt-meter are sensitive to rotation relative to the local $\vec{g}$ direction. This kind of properties is rather timely in future generation gravitational waves research and, in general, for seismological study.
However, when the ring laser approaches the threshold region, its effective scale factor $S$ becomes mono more and more sensitive to the highly non-linear laser dynamics and the consequent random noise becomes more and more significant, cutting the sensitivity in low frequency band. In order to provide a reliable signal $f_s$ must be evaluated taking into account the laser dynamic, which is feasible utilising the measured $f_m$ and making use of diagnostic signals of the laser ( i.e. the two mono beam intensity and the active region discharge fluorescence). Starting from the stationary solution of the ring laser equations and taking into account the diagnostic signals, it is possible to find an analytical relation and evaluate $f_s$. Note that, to simplify the notation, in the following we will indicate the Sagnac frequency as pulse frequency $\omega_s = 2\pi f_s$ in rad/s. Following our analysis(2; 5), $\omega_s$ can be evaluated as the linear sum of several terms:

$$\omega_s \approx \omega_{s0} + \omega_{ns1} + \omega_{ns2} + \omega_{K1} + \omega_{K2} + \omega_{nsK} \tag{3}$$

[1] Small scale RLG developed for inertial navigation utilises special technique, as dithering of the mirrors, to avoid the locking, this allows the functionality of the gyro, limiting the sensitivity of the apparatus.

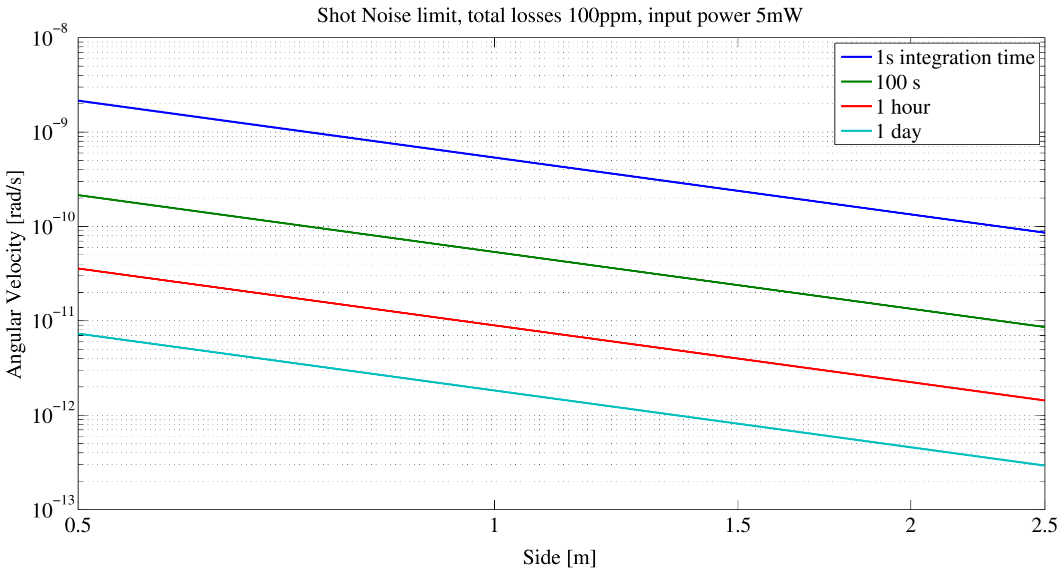


**Figure 1.** Sensitivity shot noise limit of a square RLG as a function of its side length.It is assumed total cavity losses of 100 ppm (conservative estimation compared to present best mirrors quality), intra-cavity power of 5 mW.

which is a linear expansion up to the second order in the parameter $\delta_s$ and $K$ of the analytical steady state solution of ring laser (Aronowitz model). The six terms can be evaluated independently by inserting in the elaboration program the data collected by the diagnostics channels (2; 5). We have validated this approach by applying the procedure to the data acquired on GP2, a square RLG with a perimeter of 6.40 m Figure 2 shows the histogram of the Sagnac frequency data of GP2 in a 24 hours run before and after the application of the algorithm to the data. The standard analysis leads to a broader and highly irregular distribution, typical of non linear systems, with an estimation of the Earth rotation rate systematically larger than the expected value. These non linearities are highly reduced applying the reconstruction method for $f_s$, the final Gaussian-like distribution makes us confident that the non linearities are pretty much reduced. Also the estimated value of the rotation rate provided by $f_s$ is now in perfect agreement with the expected one, with a relative systematic error of 1 part in $10^4$, equivalent to an improvement in accuracy of a factor 60 against what obtained with the standard analysis.
GP2 is not the optimal apparatus. It is located in a quite noisy environment and at the time of the above measurement its mirrors were not top quality ones. Despite all above said, the best observed sensitivity was 2 nrad/s, obtained on 30 s integration time. This test shows that a middle size RLG can work continuously in a standard laboratory with a sensitivity of the order of a few nrad/s in the range 0.01-10 Hz, a range that can be extended at lower frequency by a smarter design of RLG experimental set up. This result also demonstrates that relative small instruments, with an area of the order of 1 m$^2$, have a potential utility as very sensitive sensors of ground tilting. In conclusion it is now worthwhile to apply all we have learned on the large area RLGs to develop smaller size ones, let say square rings with about 1 m side, with the aim of reaching the sensitivity close, or even below, 1 nrad/s in 1 second measurement.

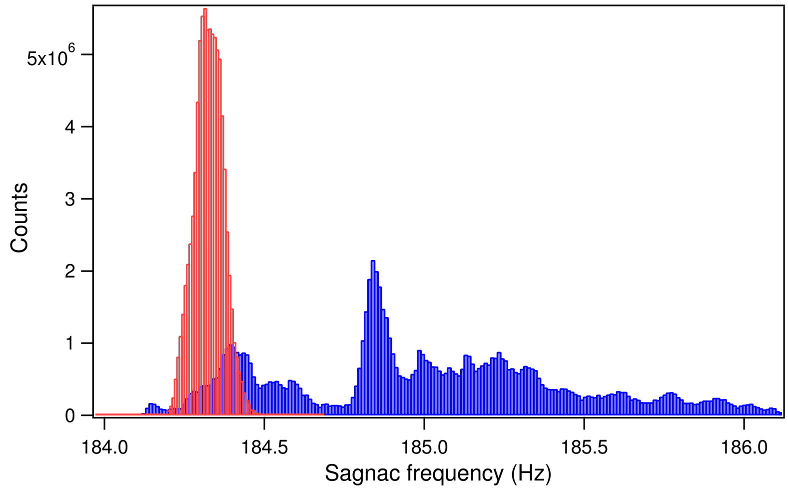


**Figure 2.** Comparison of the histograms of the Sagnac frequency estimated before (blue) and after (red) the application of the new procedure. Clearly the new method leads to a narrower and more Gaussian-like distribution, with mean value 184.29 Hz, in agreement with the expected one *(adapted from ref.(2))* .

## References

[1] Aronowitz F and Collins R J 1970 *Journal of Applied Physics* **41** 130–141 ISSN 0021-8979 URL `http://aip.scitation.org/doi/10.1063/1.1658312`

[2] Di Virgilio A D V, Beverini N, Carelli G, Ciampini D, Fuso F and Maccioni E 2019 *The European Physical Journal C* **79** 573 ISSN 1434-6044 URL `http://dx.doi.org/10.1140/epjc/s10052-019-7089-5`

[3] Harms J 2019 *Living Reviews in Relativity* **22** 6 ISSN 1433-8351 URL `https://doi.org/10.1007/s41114-019-0022-2`

[4] Di Virgilio A, Beverini N, Bradaschia C, Carelli G, Ferrante I, Gennai A, Maccioni E, Passuello D, Porzio A, Schreiber U and others 2007 *Virgo note VIR-0019E-07*

[5] Di Virgilio A D V, Giacomelli U, Beverini N, Carelli G, Ciampini D, Fuso F, Maccioni E and Ortolan A 2019 *Subbmitted to EPJC* URL `http://arxiv.org/abs/1906.11338`