Relative Entropy Estimates in Statistical Mechanics and Field Theory

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To Professor Jan Łopuszański
for his 75^{th} birthday

Abstract. We review numerous applications of relative entropy estimates in Statistical Mechanics and Field Theory.

The Particle Structure Implies Relative Entropy Bounds

It is well known that one can represent the physical Hilbert space $\mathcal{H}$ of the free scalar massive field theory as $\mathbb{L}_2(\mu_G)$ defined with a mean zero Gaussian measure of covariance $G = (-\Delta + m^2)^{-\frac{1}{2}}$. This Hilbert space has a natural Fock space structure

$$\mathcal{H} = \bigoplus_{n=0}^{\infty} \mathcal{H}_n$$

that is it can be represented as a direct sum of orthogonal n-particle subspaces $\mathcal{H}_n$ which are preserved by the semigroup $P_t \equiv e^{-tH}$, $t \geq 0$, where H denotes the physical Hamiltonian of the free field. At the end of sixties it has been discovered that this Particle Structure implies the following very special property of the semigroup P_t

$$||P_t f||_{\mathbb{L}_q} \leq ||f||_{\mathbb{L}_2} \qquad (\mathbb{H})$$

with $q \equiv q(t) = 1 + e^{2t/c}$ for some positive constant c. That means the semigroup is not only contractive in the physical Hilbert space, (which follows from the fact that physical Hamiltonian has non-negative spectrum), but maps this space into a strictly smaller subspaces consisting of more smooth vectors. Since then this property is called the Hypercontractivity. For the references to the related publications including those of J. Glimm, E. Nelson, B. Simon, R. Hoegh-Krohn and others, see e.g., (Simon 1974) and (Glimm and Jaffe 1987).

A. Borowiec et al. (Eds.): Proceedings 1998, LNP 539, pp. 149−160, 2000.

In (Federbush 1969), P. Federbush studied perturbation of free Hamiltonian by $\lambda : \varphi^4 :$ interaction. He has shown that the Hypercontractivity property implies the following infinitesimal condition, called Logarithmic Sobolev inequality,

$$\mu_G \left(f^2 \log f^2\right) \leq 2c\, \mu_G \left(fHf\right) \;, \tag{$\mathbb{LS}$}$$

where f belongs to the quadratic form domain of the Hamiltonian and is normalised by $\mu_G f^2 = 1$. For later purposes we note that the quadratic form on the right hand side of ($\mathbb{LS}$) can be regarded as a Dirichlet form, that is it can be represented as an expectation of a square of (infinite dimensional) gradient, (Araki 1960), (Herbst 1976), (Albeverio and Hoegh-Krohn 1977). Later L. Gross showed, (Gross 1976), that actually this Relative Entropy bound is equivalent to the Hypercontractivity property.
We remark that, because of our normalisation condition, f^2 in the above inequality can be regarded as a probability density with respect to the measure μ_G and so the quantity on the left hand side of ($\mathbb{LS}$) can be interpreted as the relative entropy of the corresponding measures. Thus we see that in the Free Field Theory the Particle Structure implies the Relative Entropy bound.

1 The Relative Entropy Bounds for Gibbs Measures

One of the main properties of ($\mathbb{LS}$) is the fact that whenever it holds for any two measures, it is also true for their product. Thus such entropy bounds are naturally suitable for description of large or even infinite physical systems. At the time when ($\mathbb{LS}$) was introduced the only known examples of measures satisfying it were given by the Gaussian or some product measures. This situation persisted till the mid of eighties when Bakry and Emery introduced a very efficient criterion for the case when the underlying configuration space was given as $\Omega = \mathbb{M}^\Gamma$ with $\mathbb{M}$ being a Riemannian manifold with strictly positive Ricci curvature and Γ a countable set, (Bakry and Emery 1984). It has been applied in (Carlen and Stroock 1986) to show that ($\mathbb{LS}$) hold for infinite volume measures describing some continuous spin systems on a lattice at very high temperatures. A new idea which allowed to extend this result came at the end of eighties from the Statistical Mechanics (where some other relative entropy bounds proved to be a useful tool in the study of infinite systems). Studying a uniqueness problem for disordered spin systems, the author realised that one can use the Gibbs structure related to the spin systems to prove ($\mathbb{LS}$). It allowed him to show this relative entropy estimate not only for continuous spins when the single spin space do not satisfy $Ricc > 0$, (as for example in planar rotators), (Zegarliński 1990), but also for discrete spin systems, (Zegarliński 1990), (Zegarliński 1992). To describe the related idea and some results, we need to recall the basic notion of the Gibbsian description. We begin from introducing *the finite volume Gibbs measures*

$$\mu_\Lambda^\omega(f) \equiv \delta_\omega\left(\frac{\mu_0^\Lambda(e^{-U_\Lambda}f)}{\mu_0^\Lambda(e^{-U_\Lambda})}\right),$$

where δ_ω is the Dirac measure fixing the external configuration $\omega \in \Omega$ of the system outside a finite subset Λ of the lattice $\mathbb{Z}^d$, (denoted later on by $\Lambda \subset\subset \mathbb{Z}^d$); μ_0^Λ denotes a free product measure in Λ and the interaction energy is given by

$$U_\Lambda \equiv \sum_{X\cap\Lambda\neq\emptyset} \Phi_X(\sigma_X).$$

The potential $\Phi \equiv \{\Phi_X : X \subset\subset \mathbb{Z}^d, |X| < \infty\}$, for simplicity of the exposition, is assumed to be of finite range, that is $\Phi_X \equiv 0$ if $diam(X) > R$ for some fixed $R > 0$.
The infinite system is described by a Gibbs measure μ which by definition is a solution of the celebrated Dobrushin - Lanford - Ruelle equation

$$\mu\left(\mu_\Lambda^{\cdot}(f)\right) = \mu(f). \qquad (\mathbb{DLR})$$

Using this equation one can represent the relative entropy as follows

$$\begin{aligned}\mu\left(f^2\log f^2/\mu f^2\right) &= \mu\left(\mu_\Lambda^{\cdot}(f^2\log f^2/\mu f^2)\right)\\ &= \mu\left(\mu_\Lambda^{\cdot}(f^2)\log\frac{f^2}{\mu_\Lambda^{\cdot}(f^2)}\right) + \mu\left(\mu_\Lambda^{\cdot}(f^2)\log\left(\mu_\Lambda^{\cdot}(f^2)/\mu(\mu_\Lambda^{\cdot}(f^2))\right)\right).\end{aligned}$$

In this way we split the estimate into two parts. The first one involves the local relative entropy estimate with the measure $\mu_\Lambda^{\cdot}$ and as a finite dimensional problem is usually easy. On the other hand the second term has a similar structure but involves a new density $\mu_\Lambda^{\cdot}(f^2)$ which is in some way smoother. Choosing another finite set, (given as a translation of Λ), we can apply the same idea to that second term. It is an interesting fact that under some mixing condition such procedure can be iterated and leads to convergent expansion which results with the desired relative entropy estimate.

2 Equivalence of Equilibrium and Non-Equilibrium Descriptions

An interesting outcome of the research on relative entropy estimates is contained in the following result.
Theorem: *The following conditions are equivalent*
(I) *Strong Mixing :* $\exists M > 0 \forall \Lambda \subset\subset \mathbb{Z}^d, \omega \in \Omega$

$$|\mu_\Lambda^\omega\left((f - \mu_\Lambda^\omega(f))(g - \mu_\Lambda^\omega(g))\right)| \leq C(f,g)e^{-M\cdot dist(\Lambda_f,\Lambda_g)}$$

for any local observable f and g localised in a bounded set Λ_f and Λ_g, respectively.

(II) *Spectral Gap :* $\exists m > 0 \forall \Lambda \subset\subset \mathbb{Z}^d, \omega \in \Omega$

$$m\,\mu_\Lambda^\omega\,(f-\mu_\Lambda^\omega(f))^2 \;\le\; \mu_\Lambda^\omega(\sum_i |\nabla_i f|^2)$$

for any f in the domain of the Dirichlet form.

(III) *Logarithmic Sobolev Inequality :* $\exists c > 0 \forall \Lambda \subset\subset \mathbb{Z}^d, \omega \in \Omega$

$$\mu_\Lambda^\omega\left(f^2 \log f^2/\mu_\Lambda^\omega(f^2)\right) \;\le\; 2c\,\mu_\Lambda^\omega(\sum_i |\nabla_i f|^2)$$

for any f in the domain of the Dirichlet form.

(IV) *Asymptotic Sobolev Inequality :*
$\exists C > 0 \forall \Lambda \subset\subset \mathbb{Z}^d, \omega \in \Omega,\ \forall p \in [2, 2+\frac{1}{|\Lambda|}]$

$$||f||^2_{\mathbb{L}_p(\mu_\Lambda^\omega)} \;\le\; ||f||^2_{\mathbb{L}_2(\mu_\Lambda^\omega)} \;+\; (p-2)C\mu_\Lambda^\omega(\sum_i |\nabla_i f|^2)$$

for any f in the domain of the Dirichlet form.

The equivalence of the (I)-(III) has been proven in (Stroock and Zegarliński 1992). The last point has been added only recently in (Zegarliński 1998).

The first condition is a statement from statistical mechanics which says that in the given systems one has a fast uniform decrease of correlations. It means, (Dobrushin and Shlosman 1985, 1987), that in such system one has no phase transitions in the strongest possible sense of analytic dependence of expectations on the potential.

The second has an interpretation within the spectral theory of the selfadjoint Markov generator described by the Dirichlet form on the right hand side of the inequality. It means that one has a gap at the bottom of the spectrum of this generator. Thus it carries an information about the ergodicity of the corresponding semigroup in the $\mathbb{L}_2$ sense.

The third statement is our relative entropy estimate. It is well known that ($\mathbb{LS}$) implies the spectral gap. On the other hand one can show examples when the spectral gap inequality is true, but ($\mathbb{LS}$) does not hold. The important point here is that the spectral gap is uniform with respect to the volume Λ and external conditions ω.

Finally the last property involves a classical Sobolev inequality which is one of the cornerstones of the twentieth century analysis. It tells us that given the $\mathbb{L}_2$ information about the gradient of a function we can estimate its $2+\delta$ moment with strictly positive delta δ. This improvement of square by a small root is much stronger than the logarithmic one in ($\mathbb{LS}$), but is relaxed in the thermodynamic limit. The equivalence of (III) and (IV) follows from the very special behaviour of the coefficient at the Dirichlet form.

The condition (I) is a condition of the equilibrium statistical mechanics. If we would think of (II) - (IV) as some features of dissipative dynamics with generator described by the corresponding Dirichlet form, we could say that the above Theorem establishes an equivalence between equilibrium and non-equilibrium description of a physical system.

3 Strong Decay to Equilibrium

One of interesting consequences of the hypercontractivity of the dissipative dynamics is the fact that the corresponding systems decays exponentially fast to the unique equilibrium.

Theorem: ((Stroock and Zegarliński 1995))
If a Gibbs measure μ satisfies

$$\mu\left(f^2 \log \frac{f^2}{\mu f^2}\right) \leq 2c\,\mu(fHf)$$

then

$$||e^{-tH}f - \mu f||_\infty \leq e^{-mt} C_V |||f|||$$

with a positive constant C_V dependent only on a finite set V in which an observable f is localised, and with an arbitrary $m \in (0, gap_2 H)$, provided some suitable seminorm $|||f|||$ of f is finite.

It is interesting to remark that the region where ($\mathbb{LS}$) remains true, in many systems, extends to the critical point. (It includes for example the Ising ferromagnet with nearest neighbours interactions.)
Results about the decay to equilibrium, besides their theoretical and esthetical value, play an important role in numerical analysis. One should recall that the only practical way of making actual computations of equilibrium expectations of large system is via running a stochastic process on a computer. To illustrate the computational difficulty, consider the Ising model on ten by ten square of the integer lattice $\mathbb{Z}^2$. This certainly can not be regarded as a large system if we compared it with a small macroscopic piece of ferromagnet which is known to contain 10^{23} elements. Yet the corresponding configuration space contains $2^{100} \geq 10^{30}$ different configurations of the ± 1 spins. That is we would have to compute more than 10^{30} terms $\exp\{-U_\Lambda(\sigma)\}$ to get a value of expectation with a Gibbs measure. Computing 10^9 terms per second, (which is a very good speed !), one would need 10^{21} seconds. That is about 10^{14} years. Compare that with the age of the Universe which when estimated using the Big Bang theory is equal approximately 10^{10} years !
For related development see also (Martinelli and Olivieri 1994), (Lu and Yau 1993), (Zegarliński 1990)–(Zegarliński 1998) and references there in.

4 Strong Decay to Equilibrium in Disordered Systems

It is well known that the systems with random interactions can exhibit an interesting behaviour. A simplest example of such a system is given by the Edwards - Anderson model described by the following interaction energy

$$U = \sum_{|i-j|=1} J_{ij}\sigma_i\sigma_j \,,$$

where the couplings J_{ij} are random i.i.d. variable. If the couplings can take on arbitrary large values, the corresponding system exhibits a non-analytic behaviour even at the high temperature region. In view of the previous discussion it is natural to expect that one should have also different non-equilibrium behaviour. The first numerical evidence that at the high temperatures one should have a stretched exponential decay has been published in the mid of eighties by Ogielski, (Ogielski 1985). For a mathematical results one needed to wait for a long time. By adapting the strategy based on the hypercontractivity the following result has been proven for Glauber dynamics of two dimensional models.

Theorem ((Guionnet and Zegarliński 1996, 1997))
Almost surely

$$||e^{t\mathcal{L}_J} f - \mu_J f||_\infty \leq e^{-t^\alpha} C(J) |||f|||$$

with some $\alpha \in (0,1)$ *and a random variable* $C(J)$*, where* J *denotes the random configuration of the couplings.*

See also (Cesi et al. 1997) for further development on that subject.

5 The Relative Entropy Estimates in Quantum Systems

In the description of quantum spin systems we describe observables as elements of a C^* algebra $\mathcal{A} = \overline{\cup_{\Lambda\subset\subset \mathbb{Z}^d}\mathcal{A}_\Lambda}$, where $\mathcal{A}_\Lambda$ is isomorphic to $\mathbb{M}^\Lambda$ the algebra of all complex $n\times n$ matrices. The free state is given by a normalised trace Tr, satisfying the usual properties

$$Tr\mathbf{1} = 1, \qquad Tr(a^*a) > 0 \text{ for } a \neq 0 \text{ and } \qquad Tr(ab) = Tr(ba)\,.$$

With the trace Tr we can associate a family of partial traces Tr_X, $X \subset\subset \mathbb{Z}^d$ possessing all basic properties of conditional expectations. A Gibbs state ω on the algebra $\mathcal{A}$ is given by

$$\omega_{|\mathcal{A}_\Lambda}(f) = Tr(\rho_\Lambda f)\,,$$

where ρ_Λ is a density matrix. In this setting a dissipative dynamics is described as a Markov semigroup, that is a semigroup satisfying the following properties

$$P_t\mathbf{1} = \mathbf{1}, \qquad P_t f^* f \geq 0,$$

and possibly also the following *Feller Property*

$$P_t(\mathcal{A}) \subset \mathcal{A}\,,$$

Frequently we want to distinguish a'priori a family of invariant states that is the states satisfying

$$\omega(P_t f) = \omega(f)\,.$$

One convenient way of doing that is by assuming the following detailed balance condition with respect a scalar product associated to the state ω

$$< P_t f, g >_{\mathcal{H}_\omega} = < f, P_t g >_{\mathcal{H}_\omega}\ .$$

A construction of a dissipative dynamics preserving positivity in the algebra and simultaneously satisfying this symmetry condition constitutes one of the toughest problems of mathematical physics; for some progress in that direction see (Majewski et al. 1998) and references therein.
We recall that, unlike as in the classical case, in the non-commutative theory one can consider many scalar products associated to a given state $\omega = Tr(\rho\,\cdot)$. Some examples are given by

$$< f, g >_{\omega,s} \equiv Tr(\rho^{s/2} f \rho^{(1-s)/2})^* (\rho^{s/2} f \rho^{(1-s)/2})\,.$$

In particular if we set $s = 0$ one gets the usual scalar product used in the GNS construction. An integral over $s \in [0,1]$ gives a scalar product relevant to the linear response theory.
Additionally one can associate to ω an interpolating family of $\mathbb{L}_p(\omega, s)$ spaces defined by the following norms

$$||f||^p_{\mathbb{L}_p(\omega,s)} \equiv Tr|\rho^{s/p} f \rho^{(1-s)/p}|^p\,.$$

Note that for $1 \leq p \leq q \leq \infty$ we have

$$\mathbb{L}_p \supset \mathbb{L}_q \supset \mathcal{A}\,.$$

We note that a symmetric in $\mathbb{L}_2(\omega, s)$ Feller - Markov semigroup can be extended to a contractive semigroup in all $\mathbb{L}_p(\omega, s)$

$$||P_t f||_{\mathbb{L}_p(\omega,s)} \equiv ||f||_{\mathbb{L}_p(\omega,s)}$$

in a full analogy to the classical theory.

6 Hypercontractivity in Noncommutative $\mathbb{L}_p$ Spaces

Given a family of noncommutative $\mathbb{L}_p(\omega, s)$ spaces, in a natural way we can define a Hypercontractive semigroup by the following condition

$$||P_t f||_{\mathbb{L}_p(\omega,s)} \leq ||f||_{\mathbb{L}_2(\omega,s)}\,,$$

where $p = 1 + e^{\frac{2}{c}t}$, with some $c \in (0, \infty)$. Later on $s = \frac{1}{2}$.
Theorem: ((Olkiewicz and Zegarliński 1999))
Hypercontractivity in $\mathbb{L}_q(\omega, \frac{1}{2})$ spaces implies the following Quantum Relative Entropy bound

$$QE_p(f) \equiv Tr|\rho^{1/2p} f \rho^{1/2p}|^p \left(\log|\rho^{1/2p} f \rho^{1/2p}| - 1/p \log \rho \right)$$
$$- ||f||^p_{\mathbb{L}_p(\omega,\frac{1}{2})} \log ||f||_{\mathbb{L}_p(\omega,\frac{1}{2})}$$
$$\leq c(p)\, \mathcal{E}_p(f)\,,$$

where $c(p) = \frac{cp}{2(p-1)}$ *and*

$$\mathcal{E}_p(f) =< I_{p,q}(f), \mathcal{L}_p f >_{\mathbb{L}_2(\omega,\frac{1}{2})}$$

with isometry $I_{p,q} : \mathbb{L}_p \to \mathbb{L}_q$.
Moreover if

$$\mathcal{E}_2(I_{2,p} f) \leq \frac{q^2}{4(q-1)} \mathcal{E}_q(f)$$

then

$$QE_2(f) \leq c\, \mathcal{E}_2(f)$$

implies hypercontractivity
The statement simply says that basically the quantum relative entropy estimate is equivalent to hypercontractivity in this more general noncommutative setting. Note that the theorem introduces a new kind of quantum relative entropy not considered before in the literature. In the particular case when the observable f is nonnegative and commutes with the density matrix ρ, the renormalization of the logarithm gives us the classical formula for the relative entropy.

7 Spectral Theory of Hypercontractive Semigroups

Suppose $P_t = e^{-t\mathcal{L}}$ is a symmetric Markov semigroup in $\mathbb{L}_2(\mu)$. If its generator would have a discrete spectrum, (as it happens for example in case of the Laplace - Beltrami operator on a compact Riemannian manifold), we would have the following representation

$$P_t f = \sum_n e^{-t\lambda_n} (\Psi_n, f) \Psi_n$$

with Ψ_n being a normalised eigenfunction corresponding to an eigenvalue λ_n. Using this representation an equivalent condition of the hypercontractivity property

$$\exists T \in (0,\infty) \forall t > T\,, \qquad ||P_t f||^4_{\mathbb{L}_4} \leq ||f||^4_{\mathbb{L}_2}$$

can be written as follows

$$\sum_{n_1,..,n_4} e^{-t(\lambda_{n_1}+..+\lambda_{n_4})} \mu(\Psi_{n_1}..\Psi_{n_4}) \prod_{l=1,..,4} (\Psi_{n_l}, f) \leq \sum_{n_1,n_2} (\Psi_{n_1}, f)^2 (\Psi_{n_2}, f)^2$$

for all $t > T$. This means that for hypercontractivity to be true we need very special properties of the spectrum and overlapping property of the eigenfuctions, (that is the random variables Ψ_n have to be in some sense weakly dependent and behave similarly to the random variables with Gaussian distribution). As we have mentioned at the beginning of this lecture, in case of free scalar massive field one can derive hypercontractivity from the particle structure of the theory. More precisely one uses the following properties.
(I) *Existence of Invariant Subspaces* $\forall n \in \mathbb{Z}^+ \exists \mathcal{H}_n \subset \mathbb{L}_2(\mu)$

$$P_t\mathcal{H}_n \subset \mathcal{H}_n, \qquad \mathcal{H}_n \perp \mathcal{H}_{n'}, \qquad \text{and } \cup_n \mathcal{H}_n = \mathbb{L}_2(\mu)\,.$$

(II) *Particle Structure of the Spectrum*

$$\exists \varepsilon \in (0,\infty) \forall n \in \mathbb{N} \qquad \inf \sigma(H|\mathcal{H}_n) \geq n\varepsilon\,.$$

(III) *Gaussian Bounds*

$$\exists C \in (0,\infty) \forall n \in \mathbb{N} \quad \forall f \in \mathcal{H}_n \qquad ||f||_{\mathbb{L}_4(\mu)} \leq C^n ||f||_{\mathbb{L}_2(\mu)}\,.$$

We mention that recently the following further examples of such structure has been exhibited, (Bodineau and Zegarliński 1998).
Example A: The Glauber dynamics in $D = 1$ Ising model

– $\mathcal{H}_n = \overline{Span\{\sigma_X, |X| = n\}}$

– $\sigma(H|\mathcal{H}_n) \subset [\eta_- n, \eta_+ n]$, *with some constants* $0 < \eta_- < \eta_+ < \infty$, *(Minlos and Trishch 1994)*

– *Gaussian Bounds*

$$\exists C \in (0,\infty) \forall n \in \mathbb{N} \quad \forall f \in \overline{Span\{\sigma_X, |X| = n\}} \qquad ||f||_{\mathbb{L}_4(\mu)} \leq C^n ||f||_{\mathbb{L}_2(\mu)}\,,$$

where μ is the infinite volume Gibbs measure of the model.
This method is a bit simpler than the one used in (Zegarliński 1990), but also offers more precise estimates on the Logarithmic Sobolev coefficient c.
Example B: The Free Dynamics for Quantum Spin Systems

Let $\omega \equiv \otimes_i \omega_{\Lambda_i}$, where $\Lambda_i \cap \Lambda_j = \emptyset$ for $i \neq j$,ω_Λ is a state on $\mathbb{M}^\Lambda$. Then the Logarithmic Sobolev inequality holds with generator

$$Hf = \sum_i (f - \omega_{\Lambda_i} f)\,.$$

8 A Problem

The problem of the particle structure of a physical theory is one of the important problems which still remain weakly understood. Some partial results, (see references in (Glimm and Jaffe 1987)), show that in the two dimensional models of scalar fields with polynomial interactions one has the particle structure up to a level N provided the coupling constant $\lambda = \lambda(N) > 0$ is sufficiently small. Similar structure has been proven to exist for generators of Glauber dynamics in classical spin systems on the lattice in an interesting paper (Minlos 1996).
Taking into account the progress made in the last decade in understanding the relative entropy estimates and our discussion presented in this lecture, it would be very interesting to prove that under some reasonable general conditions present in the physical models, one has the following implication

$$\textbf{Relative Entropy Bound} \qquad \Longrightarrow \qquad \textbf{Particle Structure}$$

Acknowledgment

The author would like to thank the organisers for this interesting and memorable meeting and is pleased to acknowledge the support of British Council. The research described in this paper has been supported by EPSRC Grants GR/K95635, GR/L58873 GR/K76801 and EU Stochastic Analysis Network.

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