Massless Particles in de Sitter Space

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1 Periodic time and periodic motion in de Sitter space

De Sitter space here means the vacuum solution of Einsteins field equation with negative cosmological term and maximal symmetry. Locally it can be embedded by

$$\eta^{\alpha\beta}\overline{u}_\alpha\overline{u}_\beta \equiv \overline{u}_1^2 + \overline{u}_2^2 + \overline{u}_3^2 - \overline{u}_4^2 - \overline{u}_6^2 = -r^2 \tag{1}$$

in a 5-dimensional Lorentzian space; its group of motions is the de Sitter group $S\tilde{O}_0(3,2)$. Time, which rotates the $\overline{u}_4\overline{u}_6$-plane, is chosen as $t = r\arctan(\overline{u}_4/\overline{u}_6)$. Clearly it lies in the range $0 \leq t < 2\pi r$ for the embedding 1. By extending it to $-\infty < t < +\infty$ we get global coordinates for de Sitter space. It is conformal to half of Einstein spacetime $S^3 \times R$. In one possible mapping, the equator S^3 becomes spatial infinity of de Sitter space. Fig. 1 also shows a timelike geodesic, i.e. the trajectory of a massive free point particle. As can be seen, the motion is periodic with period $\Delta t = 2\pi r$. Also, after $\Delta t = \pi r$ the motion is reflected in phase space. In local coordinates this corresponds to a mapping $\Re : \overline{u}_\alpha \to -\overline{u}_\alpha$. Interacting point particles move on periodic trajectories between collisions only. For them the world does not repeat after $\Delta t = 2\pi r$.

2 Scalar particles in de Sitter space

The unitary irreducible representations of positive energy of the universal covering group of $S\tilde{O}_0(3,2)$ can be labelled $D(E_0, s)$, where E_0 is the energy and s the spin of their lowest weight multiplet. Angular momenta and energy are the quantum numbers of $S\tilde{O}(3) \times S\tilde{O}(2) \subset S\tilde{O}_0(3,2)$. For the possible values of E_0 and s see Fig. 2.

We use projective coordinates

$$u^2 < 0,\ u_\alpha \equiv \lambda u_\alpha,\ \lambda > 0,\ \overline{u}_\alpha = u_\alpha/\sqrt{ru^2}. \tag{2}$$

The behavior of fields along the half-rays is fixed by choosing a degree of homogeneity, e.g. $u\cdot\partial\Phi = -\Phi$.

The Lagrangean for scalar particles with $E_0 \geq 1$ is

$$L = \partial_\alpha\Phi^*\partial^\alpha\Phi + M^2\Phi^*\Phi,\ M^2 = (E_0-1)(E_0-2)/y^2; \tag{3}$$

the Klein-Gordon scalar product is

$$-i\int d^3y\,\Phi^*\overleftrightarrow{\partial_t}\,\Phi. \tag{4}$$

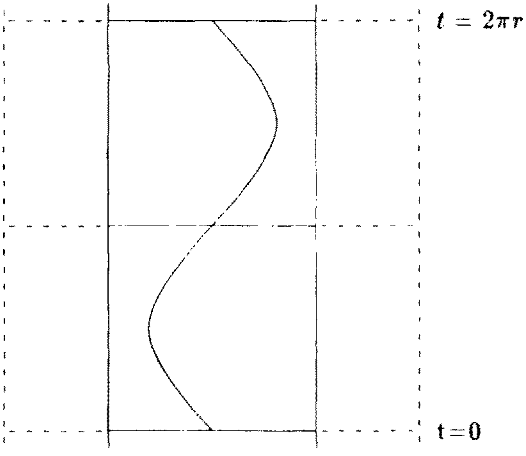

Figure 1: De Sitter space is conformal to half of Einstein space $S^3 \times R$. The horizontal lines represent S^3, the solid vertical lines represent spatial infinity of de Sitter space. Also shown is the world line of a point particle with speed 0.8c.

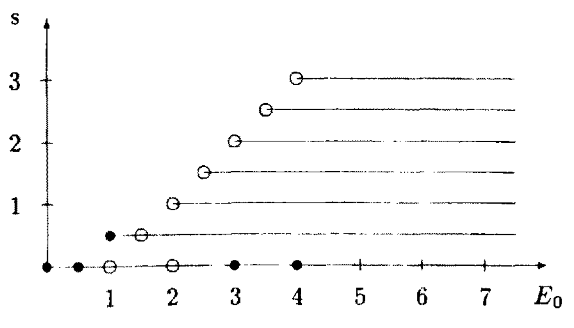

Figure 2: The lowest weight multiplets of unitary irreducible representations of $S\tilde{O}_0(3,2)$. Circles denote massless representations, dots are representations mentioned in the text.

For mass 0 ($E_0 = 1, 2$), the field equation is

$$\partial^2 \Phi = 0. \tag{5}$$

It has two lowest weight solutions,

$$\Phi_1(u) = (u_4 + iu_6)^{-1},\ \Phi_2(u) = \sqrt{-u^2}\,(u_4 + iu_6)^{-2}. \tag{6}$$

Φ_1 is an odd function under $\mathfrak{R}$, Φ_2 an even one:

$$\Phi_{1,2}(-u) = \pm\Phi_{1,2}(u). \tag{7}$$

They are lowest weights of a D(1,0) resp. D(2,0). In general, E_0 of $D(E_0, 0)$ fixes the $\mathfrak{R}$-transformation of the scalar field to [1]

$$\Phi_{E_0}(-u) = (-1)^{E_0}\Phi_{E_0}(u). \tag{8}$$

We have the same situation as for spherical harmonics, where for opposite points on the S^2 holds

$$Y_{lm}(-(\theta, \Phi)) = (-1)^l Y_{lm}(\theta, \Phi).$$

The two states (6) are not orthogonal when integrating in the scalar product (4) over one spacelike hypersurface, say at $t = 0$. But due to their different behavior under $\mathfrak{R}$, they are orthogonal when integrating over two surfaces, say at $t = 0$ and $t = \pi r$. So we can describe massless scalars either with a disconnected initial value surface and one field with arbitrary $\mathfrak{R}$-behavior, or with a connected initial value surface and fields with fixed $\mathfrak{R}$-behavior [2].

3 Helicity in de Sitter space

Massless representations of the Poincaré group with helicity λ can be mapped one to one on conformal massless representations. When restricting these to the de Sitter group $S\tilde{O}_0(3,2)$, the scalar representation gives the direct sum $D(1,0) \oplus D(2,0)$, while the helicity λ and $-\lambda$ representations give the same $D(1+ \mid \lambda \mid, \mid \lambda \mid)$. We want to show for low spins, that this degeneracy can be resolved in de Sitter field theory.

Photons in de Sitter space can be described by a field potential $A_\alpha(u)$ with degree of homogeneity (-1) and the field equation

$$\partial^2 A_\alpha = 0. \tag{9}$$

The tensor products of the massless scalar representations and the five-dimensional one give us two positive energy solution spaces

$$D(1,0) \otimes D_5 = [D(1,1) \to \{D(2,1) \oplus D(0,0)\} \to D(1,1)] \oplus D(1,0), \tag{10}$$

$$D(2,0) \otimes D_5 = [D(3,0) \to D(2,1) \to D(3,0)] \oplus D(2,0). \tag{11}$$

The arrows denote leaks in the indecomposable representations. The photons D(2,1) have gauge freedom carrying a D(1,1) resp. a D(3,0) [3].

We can decompose the vector potential in symmmetric and antisymmetric functions under $\Re$:

$$A_\alpha^\pm(-u) = \pm A_\alpha^\pm(u). \tag{12}$$

From the symmetry of the scalar fields (7) we conclude that A^+ describes the (3,0)-gauge theory and A^- describes the (1,1)-gauge theory. Explicit calculation shows that the ground states of de Sitter electrodynamics are linear combinations of the ground states of conformal electrodynamics [4] with fixed helicities:

$$A_0^\pm = A_0^{\lambda=+1} \pm A_0^{\lambda=-1}. \tag{13}$$

For massless spin 2 fields in de Sitter space there are two possible potentials, a symmetric 2-tensor and a 3-tensor with mixed symmetry. In each case, there are two possible gauge theories, in which the pure gauge fields carry a D(0,2) resp. a D(4,1). The first gauge field is symmetric under $\Re$, the second one is antisymmetric [5].

The spin 1/2 representations $D(E_0, 1/2)$ with $E_0 \geq 3/2$ can be realized by 4-spinor fields with degree of homogeity (-2) and field equations [6,7]

$$(\beta\cdot u)(\gamma\cdot\partial)\Psi_{1,2} = \pm(E_0 - 3/2)\Psi_{1,2}. \tag{14}$$

The 4×4 matrices β_α, γ_α satisfy

$$\beta_\alpha\gamma_\beta + \beta_\beta\gamma_\alpha = 2\eta_{\alpha\beta}.$$

The two fields are related by

$$\psi_2 = \sqrt{-u^2}^{\,-1}(\beta\cdot u)\gamma_5\psi_1. \tag{15}$$

The situation is as in flat spacetime, where the Dirac equation $(\gamma\cdot\partial + m)\psi = 0$ is equivalent to $(\gamma\cdot\partial - m)\gamma_5\psi = 0$.

For the massless case $E_0 = 3/2$ there is only one field equation and we have chiral invariance

$$\psi \to \exp(ia\sqrt{-u^2}^{\,-1}(\beta\cdot u)\gamma_5)\Psi. \tag{16}$$

Comparing ground states $\Psi_0^{\lambda=\pm 1/2}$ of conformal spinors with helicity $\pm 1/2$ and ground states $\Psi_0^\pm$ of $\Re$-eigenstates gives

$$\Psi_0^\pm = \Psi_0^{\lambda=+1/2} \pm \Psi_0^{\lambda=-1/2}. \tag{17}$$

Locally each of the fields $\Psi^\pm$, $A^\pm$ has enough states to describe both local "helicities", e.g. left- and right-polarized light. But if only one type of photons would be excited, the background radiation would have only half the intensity, as the density of states would be halfed as compared to the conformal electrodynamics [8].

4 Minimal coupling

Minimal coupling of the massive spinor field gives a current

$$J_\alpha^- = -\frac{ie}{2}\overline{\Psi}[\beta_\alpha(\gamma\cdot u) - (\beta\cdot u)\gamma_\alpha]\Psi, \tag{18}$$

which is antisymmetric as the (1,1)-electrodynamics is. For massless spinors there is due to chiral invariance also a conserved current

$$J_\alpha^+ = -\frac{ie}{2}\{\overline{\Psi}^+[\beta_\alpha(\gamma\cdot u) - (\beta\cdot u)\gamma_\alpha]\Psi^- + \overline{\Psi}^-[\beta_\alpha(\gamma\cdot u) - (\beta\cdot u)\gamma_\alpha]\Psi^+\}, \tag{19}$$

which is symmetric under $\mathfrak{R}$ like the (3,0)-electrodynamics. In conformal electrodynamics both currents $J^\pm$ appear, but in *massive* de Sitter electrodynamics J^+ is *not* conserved.

For interacting theories we should not expect $\mathfrak{R}$-symmetry of the fields. So it is possible to contemplate couplings which are not $\mathfrak{R}$-invariant. But their free limit would be non-continuous.

In perturbation theory the (in)- and (out)-states are described by free fields with $\mathfrak{R}$-symmetry. As we use the universal covering space, the Feynman propagators should be globally causal, i.e.

$$D_F = \Theta(t-t')D^+ + \Theta(t'-t)D^-, \tag{20}$$

where $D^\pm$ is the sum over normalized positive resp. negative energy eigenstates [9]. Although physical states must have integer E_0 for integer spin and half-integer E_0 for half-integer spin [10], virtual states can have all energies $E_0 \in R$; they do not have to be continuous on the hyperbola (1).

The appearance of the trivial spurion state with gauge freedom, $D(0,0) \to D(1,1)$ in the Gupta-Bleuler-triplet (10) may justify some remarks. It does not satisfy the Lorentz-condition, has negative norm and can be written as a gradient

$$A_\alpha = S\partial_\alpha \ln\left(\frac{u_4 + iu_6}{\sqrt{-u^2}}\right). \tag{21}$$

Minimal coupling of a classical spurion field to matter is equivalent to the gauge transformation

$$\Psi \to \left(\frac{u_4 + iu_6}{\sqrt{-u^2}}\right)^{eS}\Psi; \tag{22}$$

it shifts all energies by $\Delta E = eS/r$. So it fixes the origin of the energy scale and shouldn't cause any infrared problems.

5 The massless Higgs model in de Sitter space

In flat space the Higgs potential [11]

$$V(\Phi) = m^2\Phi^2 + \lambda\Phi^4 \tag{23}$$

requires an imaginary mass to allow a non-zero minimum. The second order Casimir operator of the de Sitter group has for scalar representations the eigenvalues

$$C_2 = E_0(E_0 - 3), \tag{24}$$

which are negative for the conformal massless case $E_0 = 1, 2$. This suggests that, due to the negative curvature, spontaneous symmetry breaking may be possible for massless scalar fields with self-coupling.

To test this idea we consider the classical massless Goldstone model of two real massless fields with self-coupling,

$$\partial^2\Phi_{1,2} - \frac{2}{u^2}\Phi_{1,2} + \frac{f^2}{2u^2}(\Phi_1^2 + \Phi_2^2)\Phi_{1,2} = 0. \tag{25}$$

We have chosen the degree $(u\cdot\partial)\Phi = 0$ here; there is a constant solution, around which we can expand:

$$\Phi_1 = 0 + \Phi,\ \Phi_2 = 2/f + \chi. \tag{26}$$

In linear approximation we obtain

$$\partial^2\Phi = 0, \tag{27}$$

$$\partial^2\chi + 4\chi = 0. \tag{28}$$

So the "Goldstone-particle" Φ carries a unitary D(3,0), while the "Higgs-particle" χ carries a unitary D(4,0).

When coupling minimally to electrodynamics,

$$\partial_\alpha\Phi_1 \rightarrow \partial_\alpha\Phi_1 - eA_\alpha\Phi_2,$$
$$\partial_\alpha\Phi_2 \rightarrow \partial_\alpha\Phi_2 + eA_\alpha\Phi_1.$$

we get the massless Higgs model. Expanding again around the constant solution we obtain after the gauge transformation

$$B_\alpha = A_\alpha - \frac{f}{2e}\partial_\alpha\Phi_1$$

in linear approximation a massive vector field

$$\partial^2 B_\alpha = \frac{4e^2}{f^2u^2}B_\alpha \tag{29}$$

whith mass $2e/(fr)$, and - as above - a Higgs-particle with mass 4/r.

Experimentally this Higgs mass would be zero; so the massless Higgs-model cannot be used for symmetry breaking in the standard models. Yet we want to stress that both, the kinetic and the interaction term are conformally invariant. The scale in the effective theory is entirely due to the curvature of de Sitter space.

Acknowledgements

I have the pleasure to thank Prof. H.D.Doebner for encouragement and advice.

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