O'NEILL: I realise that the present state of this work is very theoretical and that Veksler would not want to predict the parameters of some final machine that would certainly work.

VEKSLER: At the present moment no, because to make certain machines it is essential at first to analyse, at least in a first approximation, the problem of stability.

I would like to add to this that we also have started practically from the same concept as Ohkawa was talking about here the day before yesterday. Nevertheless we soon were convinced that we could not expect a bunch of such a form to be stable during acceleration. We carried out an extensive theoretical work, of which this is but a short summary.

Any further attempt on my part to give numbers that might conceivably come out from these investigations at a later date would, at the present time, be pure guess work.

WANIEK: At the very high temperatures radio-frequency containment of plasma looks rather unfeasible as it has been shown during the past two years by Weibel at S.T.L. and by a group at Argonne. Radio-frequency acceleration, of course, makes good use of the shallow skin depth in the plasma body even at low temperatures. Nevertheless the field interpenetration time has to be compatible with the acceleration time. This still requires moderately good conductivity in the plasma.

I am just wondering whether the fields and the electric energy density which is required in such a case, namely for containment and for the additional acceleration, look reasonable at all.

VEKSLER: The first proposal for plasma containment by radio-frequency was made, I suppose, some years ago by my collaborators and myself when we first discussed the question of coherent acceleration of plasma bunches. Our present investigations show that the electric power involved in containment and acceleration (containment is not an additional process) is of a reasonable size.

The currents in such a kind of acceleration will be rather high: but I would like to state that I do not think that this method of acceleration will be applicable directly to obtain very high energies. I would say that even though the number of particles per pulse may be very large, the mean currents will not be high.

THE CAPTURE OF ELECTRONS INJECTED INTO THE BETATRON

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1. INTRODUCTION

Since the first successful operation of the betatron by Kerst [1], the injection process has not been satisfactorily explained until now. The published theories [2-5] are not in accordance with experimental facts. From the experimental material presented in this preliminary report, it is possible to form a consistent picture of the mechanism which enables the capture of injected electrons. The described mechanism is directly applicable to continuous injection into fixed-field alternating gradient accelerators which seems to have remained unexplained [6].

The electrons starting from the injector have to remain in the vacuum chamber without colliding with the injector structure. Using the well-known Poincaré's recurrence theorem, it can easily be shown that the injected electron can be captured only when it forms a non-conservative system.

There are two ways of making the system non-conservative. Either the external fields are made rapidly varying with time or there exists an energy exchange between the injected electrons. Both methods have been used, but the majority of betatrons makes use of the well-known scheme originally proposed by Kerst [1], where the time variation of the external magnetic field during injection is so small that only electron interactions can cause a nonconservative motion.

Whereas the capture of electrons injected into a time-dependent field is quite clear, the case of injection into a static or slowly varying magnetic field would remain unexplained. The questions to be answered are: what kind of electron interaction takes place, and what mechanism insures the capture of electrons? The present report tries to answer these questions in a descriptive manner.

2. THE APPARATUS

The experiments were performed on a 15 MeV betatron. The mean diameter of the vacuum chamber was 110 mm, the radial aperture 40 mm, the axial aperture 30 mm. The field index was $n = 0.6$. The injector used was of Kerst-type, giving either a rectangular or a cylindrical beam. The injection voltage, variable between 3 to 15 kV, had a repetition frequency of 50 c/s and a pulse length of 0.5 to 3 μs. In all experiments a static magnetic field was used, which enabled the introduction of unusual measuring methods.

It was experimentally shown that the number of captured electrons is the same, whether a static magnetic field or an alternating magnetic field (of frequency 50 c/s) is used. The injection process was observed with the aid of miniature probes intruding into the interior of the vacuum chamber[7]. The toroidal beam of captured electrons was investigated by displacing the beam at an arbitrary time after injection radially on a target or probe by the aid of a rapid decrease of the magnetic field[8].

3. ELECTRON INTERACTION

The injected electrons form a beam which winds itself several times around the axis of the magnetic field until it hits the chamber walls and the injector. The form of this beam is discussed by the author[8]. Under favourable conditions an interpenetration of the different windings of the beam occurs. The electrons travel with approximately the same velocity in the azimuthal direction. Where the beam crosses itself the electrons move radially against each other with a velocity corresponding to the energy of betatron oscillations. Thus favourable conditions for amplification of small fluctuations in space-charge density distribution are realized. The mechanism of space-charge wave amplification was first described by Haeff[9] and is used in double-stream amplifiers for amplification of microwave signals[10].

The amplified space-charge density fluctuations are carried along the beam and, after one or more revolutions, they again return to the intercrossing of the beam. Thus a regenerative loop is formed giving rise to electron oscillations.

These oscillations were found experimentally. In fact, it is so easy to detect them that it is strange why they have not been found earlier. The detection method consisted simply in connecting a wavemeter to a probe protruding into the vacuum chamber. The wavemeter crystal detector was connected to a video amplifier and oscillograph.

For every injection voltage there was found a discrete set of frequencies which stood in harmonic ratio to the electron revolution frequency. Let us denote the oscillations modes by the number N of oscillations occurring during one revolution. Varying the injection voltage from 4 to 15 kV, frequencies from 30 to 1600 Mc/s were found corresponding to oscillation modes $N = 1/3$ to $N = 18$. As a rule, more modes were simultaneously present although their amplitudes were quite different. The frequency of every oscillation mode increased with the square root of injection voltage, as is shown in Fig. 1. This convincingly proves that the regenerative loop is formed by the electron beam itself. It deserves notice that the lines in Fig. 1, corresponding to different modes, were drawn independently of measured values only with the presumption that the electron orbit radius equals 110 mm. The measured frequencies are in good agreement with the predicted lines.

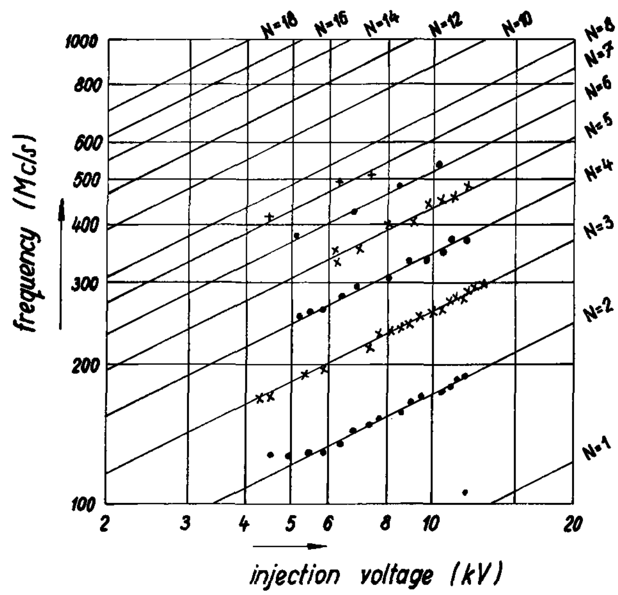

Fig. 1 Theoretical dependence of the frequency of electron oscillations compared with the measured values.

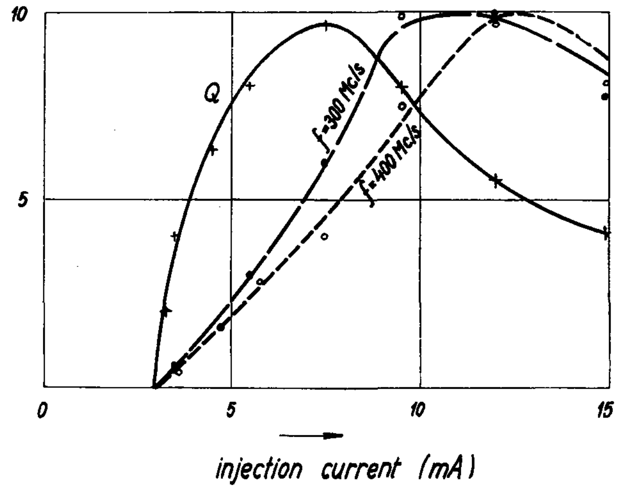

Fig. 2 Full curve (Q) represents the dependence of the captured charge on injection current. Dashed curves represent the dependence of the square of oscillation amplitude for 2 oscillation modes ($f = 300$ Mc/s, $f = 400$ Mc/s) on injection current.

Figure 2 shows the dependence of the square of oscillation amplitude on the injection current at an injection voltage of 8 kV. Amplitudes of two oscillation modes are plotted. The oscillations start at a definite threshold current, and their amplitude rises with increasing current until it reaches saturation. In the same figure the captured charge is also plotted. It shows the well-known dependence. The capture of electrons begins at a definite threshold current, and the captured charge rapidly rises with increasing current until it reaches a maximum value from which it slowly falls. It is remarkable that the two threshold currents coincide. The coincidence of capture and oscillation was established quite generally. Every time when capture occurred oscillation was also present. On the other hand conditions could be realised, when strong oscillations existed with small capture.

It can be deduced from these facts that the described electron oscillations are the primary cause of electron capture. Some oscillation modes were still present after the end of the injection pulse. Obviously they were produced by captured electrons.

4. ELECTRON CAPTURE

The motion of the injected electrons can be decomposed, in a well-known way, into a rotarory motion along the equilibrium circle and into a superposed motion representing betatron oscillations. The energy of the betatron oscillations, which usually is about 2 per cent of the over-all injection energy, will be called incremental energy. The simplest way for capturing the injected electrons consists in decreasing the incremental energy. This is accomplished by the aid of electron oscillations described in the previous chapter.

The energy necessary for the build-up of electron oscillations is drawn from the relative motion of the electrons relative to each other, that is from the incremental energy. Every electron leaving the injector has approximately the same incremental energy. During the build-up of electron oscillations a re-arranging of incremental energy distribution is accomplished, some electrons getting an increased incremental energy and some a decreased incremental energy. The electrons with a high incremental energy strike the injector or vacuum chamber walls and are lost. They carry away a substantial part of the incremental energy belonging to the whole group of injected electrons. The mean incremental energy of the remaining electrons is thus reduced. In this way a part of the injected electrons may be captured on account of the loss of the others. It can be shown mathematically that the described mechanism leads to electron capture. Kovrizhnykh and Lebedev obtained a corresponding solution of the Boltzmann's equation [11] and the author used a simple statistical approximation [7].

The capturing process was experimentally investigated. The loss of electrons on the chamber walls, due to build-up of electron oscillations, can be visually observed by spraying the inner walls of the vacuum chamber with a fluorescent powder. The amplitude of electron oscillations was observed with the aid of the co-axial probe. It had not the same value for every injection pulse. Whenever it reached a high value a flash on the chamber walls appeared, caused by electrons striking the walls.

Fig. 3 represents the measured distribution of charge density across the cross-section of the vacuum chamber in an azimuthal distance of 150° from the injector. The injection pulse was 2.2 μs long, the injection voltage 5 kV, and the injection pulse current 10 mA. The charge density was measured with the aid of a small moving co-axial probe. The upper three figures represent curves of constant charge density in a time t_1 after the beginning of the injection pulse.

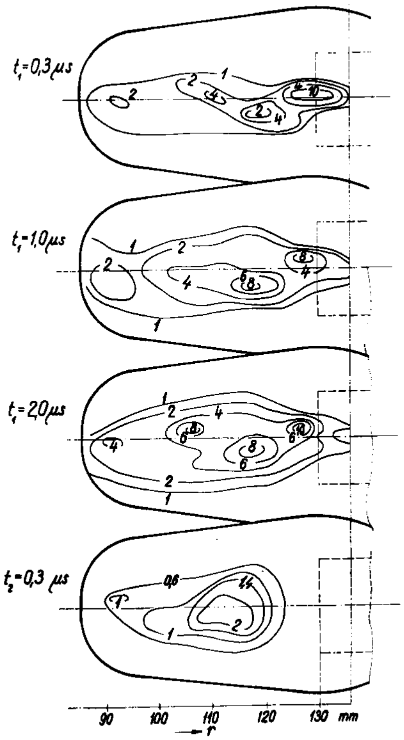

Fig. 3 Distribution of charge density across the cross-section of the vacuum chamber at different times t_1 after the beginning of the injection pulse and at a time $t_2 = 0.3$ µs after the end of the injection pulse.

The bottom figure represents the captured charge density distribution at 0.3 µs after the end of the injection. The numbers at the curves indicate the measured charge density in per cent of the maximum possible charge density (in MKS system)[8)]

$$\sigma_{\max} = \frac{2\varepsilon_0 V}{r_0^2},$$

where V is the injection voltage, and r_0 the radius of equilibrium orbit. The figures clearly show a gradual widening of the space occupied by electrons during injection. This is due to electron scattering, caused by electron oscillations.

The gradual increase of captured charge density during injection is directly seen on oscillograms represented in Fig. 4. The time dependence of charge density in the centre of the vacuum chamber was registered with the aid of a co-axial probe at four different injection currents. The injection pulse length was again 2.2 µs. In the first case (Fig. 4a) the injection current was smaller than the threshold current. No capture occured, which is manifested by constant pulse height. (The overshoot at the beginning is due to the injection pulse edge). In the second case (Fig. 4b) the injection current was a little higher than the threshold value. Capturing occurred, but the captured charge fluctuated considerably from one injection pulse to the other. A gradual increase of charge density with increasing time during the injection pulse is noticed. The third case (Fig. 4c) corresponds to optimal injection current. (In this and the next figure, the sensitivity of the measuring apparatus was 10 times reduced.) Here the density of the captured charge rises rapidly and reaches saturation. The exponential decay of the pulse is caused by gradual absorption of the captured charge. The fourth figure (Fig. 4d) corresponds to a higher injection current than the optimal. Saturation sets in earlier. These pictures clearly show that all theories which ascribe a significance to injection pulse edges are false.

Not all of the captured electrons are preserved in the vacuum chamber, even in a very good vacuum. As was pointed out in the previous chapter, electron oscillations exist for several microseconds after the end of the injection. The mechanism of their generation and their consequences are the same as described for the injection. It was shown experimentally by the author[7)], that a continuous decrease of the number of captured electrons and of their mean incremental energy occurs for a time of about 20 µs

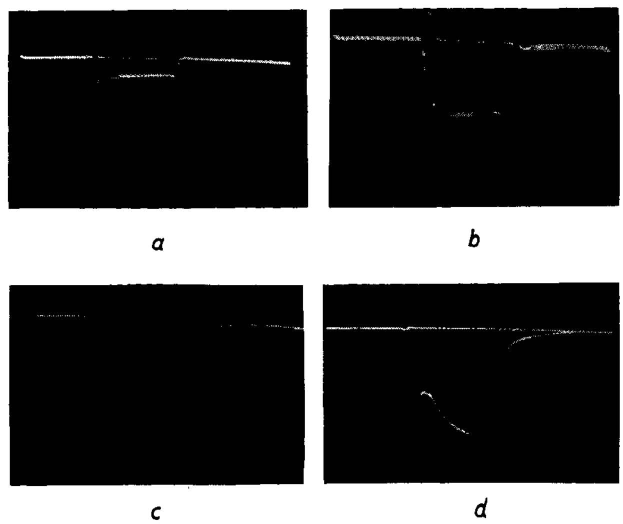

Fig. 4 Time dependence of charge density in the centre of vacuum chamber during injection for four different injection currents.

after injection. The decrease of the mean incremental energy is manifested by a reduction of the cross-section of the toroidal beam formed by the captured electrons. The charge density distribution in the toroidal beam follows a Gaussian curve very precisely, indicating a Maxwellian energy distribution of captured electrons[7, 12]. As long as the collective electron oscillations persist, the loss of captured electrons continues. Finally, the density of electrons in the toroidal beam decreases to such a low value that no electron oscillations can be generated. The electrons existing in the beam at this moment are definitely captured.

In this way the collective electron oscillations are, on the one hand, the primary cause of the capture of injected electrons and, on the other hand, they represent a fundamental limitation in the magnitude of capturable charge.

Acknowledgment

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ADDITIONAL PAPERS

The following papers, related to the subject of this session, were not read but are included in the Appendix.