Resonant States and Strong Interactions

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1. Introduction

Over the past year, there has been a substantial increase in the number of established resonance states, and in our knowledge of their production and decay characteristics. The large number of these resonance states testifies eloquently to the great strength appropriate to the "Strong Interactions". At the present stage, a discussion of the systematics of such a number of states really calls for the consideration of particular dynamical models for them. In this review lecture, we shall carry out this discussion in terms of the simplest possible model. This model supposes that the mesons and baryons are composite objects made up of the *quarks* proposed by Gell-Mann[1] and by Zweig[2], the simplest scheme for the triplet objects which could give rise to the SU(3) symmetry observed for the strong interactions.

We denote the triplet of quarks by Q_α, for $\alpha = 1, 2, 3$. The quarks (Q_1, Q_2) form an isodoublet with charges (q, q-1), baryon number b, and hypercharge y. The quark Q_3 is then an isosinglet with charge (q-1), baryon number b, and hypercharge (y-1). The antiquarks are denoted by $Q^\alpha = \overline{Q}_\alpha$. To form an octet state from the quarks, the two simplest possibilities are

$$M^\alpha_\beta = (Q^\alpha Q_\beta - \tfrac{1}{3}Q^\gamma Q_\gamma \delta^\alpha_\beta), \tag{1.1a}$$

$$B^\alpha_\beta = \epsilon^{\alpha\lambda\mu}Q_\lambda(1)Q_\mu(2)Q_\beta(3) - \tfrac{1}{3}\delta^\alpha_\beta(\epsilon^{\sigma\lambda\mu}Q_\lambda(1)Q_\mu(2)Q_\sigma(3)). \tag{1.1b}$$

The first possibility corresponds to a state with B = 0, hence to meson states. The second possibility corresponds to a state with baryon number 3b. There is also the state conjugate to (1.1b),

$$\overline{B}^\alpha_\beta = \epsilon_{\beta\lambda\mu}Q^\lambda(1)Q^\mu(2)Q^\alpha(3) - \tfrac{1}{3}\delta^\alpha_\beta(\epsilon_{\sigma\lambda\mu}Q^\lambda(1)Q^\mu(2)Q^\sigma(3)), \tag{1.1c}$$

with baryon number (-3b). All octet states can be formed from tensor combinations of products of these three octets, allowing the formation of baryon states with B = 3nb, for n integral, positive or negative. The simplest possibility for forming baryon states B = 1 corresponds to the choice $b = \frac{1}{3}$. With this choice, the observed properties of the baryons require the choices $q = \frac{2}{3}$, $y = \frac{1}{3}$.

As pointed out by Gell-Mann[1], at least one of these quarks must be a stable particle. Yet no quarks have yet been found in nature. The simplest interpretation is that they are massive objects, formed rarely in cosmic ray collisions and therefore elusive to detect. The strongest limit on their mass M_Q is that provided by Dorfan et al.[3], who found $M_Q > 4.5$ GeV.

2. The Mesons

In this model, the mesons are envisaged as bound states of quark and antiquark, $\overline{Q}$-Q. These states are necessarily limited to *octet* states (described by (1.1a)) and *singlet* states (described by $S = Q^\alpha Q_\alpha$). To date, essentially all of the meson states established are consistent with singlet or octet multiplets. These meson states are characterized by total spin S (with S = 0 or 1), and orbital angular momentum L, giving both unitary octet and singlet configurations ^SL_J, for which the charge-configuration parity is $C = (-1)^{L+S}$. Excited states of more complicated structure could also be considered, for example the configuration $\overline{Q}\overline{Q}QQ$. We suppose that such configurations lie high in excitation energy; their consideration will become necessary if there become established any meson multiplets belonging to the {27} representation, or to the $\{10+\overline{10}\}$ representation.

An attractive hypothesis due to Fujii[4a] is that the $\bar{Q}$-Q binding is due to a neutral vector field V coupled with the baryon current. Such an interaction would have the following consequences for the $\bar{Q}$-Q system:

(i) the $\bar{Q}Q$ interaction would be F-independent (where F denotes the unitary spin),

(ii) the $\bar{Q}Q$ interaction would have only weak spin-dependence,

(iii) V-exchange gives rise to attraction between Q and $\bar{Q}$ (since they have opposite baryon number),

(iv) V-exchange gives rise to a repulsive spin-orbit interaction between Q and $\bar{Q}$ (i.e. repulsive in the $\bar{Q}Q$ state of highest J, for given L).

Hence the octet and singlet states of the $\bar{Q}Q$ system are expected to lie close in mass value, and therefore to undergo strong mixing in consequence of the SU(3) symmetry-breaking interactions, the resulting set of nine states being referred to as a *nonet*. The weak F- and spin-dependence of the $\bar{Q}Q$ forces gives rise to a situation closely related with that described by SU(6) symmetry, but not identical with it. For L = 0, there are 36 meson states, corresponding to the $^1S_0$ and the $^3S_1$ nonets. With the SU(6) symmetry alone, these separate into the ($\underline{35}$+$\underline{1}$) representations; there are more relationships between these 36 meson states than those of SU(6) symmetry. For L = 1, there will be four nonets (split in mass by the spin-orbit coupling) corresponding to the states $^3P_2$, $^3P_1$, $^3P_0$ with C = +1, and $^1P_1$ with C = -1; this situation does not correspond to SU(6) symmetry at all.

A frequent objection to the $\bar{Q}$-Q model is that such a system may be highly relativistic, since the binding energy is comparable with the total quark mass, and correspondingly very complicated in structure. Although this could very well be the case, it has been pointed out by Morpurgo[4b] that this need not necessarily be the situation. For $\bar{Q}$-Q force range R, the typical quark momenta in the state will be $\hbar/R$, as follows from the uncertainty principle, to be compared with the momentum $M_Q c$. If R is sufficiently large, $R \gg \hbar/M_Q c$, then the internal motions are non-relativistic. This condition can readily be met; for example, $R \approx \hbar/m_V c$ for exchange of the neutral vector meson V, and we then require only $m_V/M_Q \ll 1$. It is true, on the other hand, that the mass of the state, m = m(U), must be a singular function of the potential strength U at the critical value U* for which m = 0, but this appears to be an independent question.

We consider only the simplest possible SU(3)-breaking mechanism, the hypothesis that the quark Q_3 is heavier than (Q_1, Q_2) by mass Δ. This difference leads to mass splitting for the meson and baryon states; in turn, these mass-splittings may generate further mass-splitting potentials between the quarks and antiquarks. We should add that we do not imagine the mass difference Δ to be the primary cause for SU(3)-symmetry breaking, but we shall not enquire further into the origin of the mass difference Δ here.

A nonet is characterized by an octet mass m_8, and a singlet mass m_1, the masses appropriate to pure SU(3) symmetry in the limit $\Delta \to 0$. When the mass splitting term Δ is introduced, the nonet masses become perturbed to the following values:

(i) the I = 1, Y = 0 state remains at mass $M_1 = m_8$,

(ii) the I = $\frac{1}{2}$, Y = ±1 states have mass $M^* = m_8 + \Delta$,

(iii) the I = Y = 0 states become mixed, their mass matrix being

$$\begin{pmatrix} m_8 + \frac{4}{3}\Delta & -\frac{2\sqrt{2}}{3} I\Delta \\ -\frac{2\sqrt{2}}{3} I\Delta & m_1 + \frac{2}{3}\Delta \end{pmatrix} \tag{2.1}$$

In the off-diagonal terms of (2.1), a factor I has been included to represent the overlap integral between the octet and singlet radial wavefunctions. The I = Y = 0 masses M_0', M_0'' are obtained by diagonalizing the matrix (2.1); they satisfy the relation

$$\begin{aligned}(M_0' - M_1)(M_0'' - M_1) - \frac{4}{3}(M^* - M_1)(M_0' + M_0'' - 2M^*) \\ = \frac{8}{9}(M^* - M_1)^2(1 - I^2) \geq 0. \end{aligned} \tag{2.2}$$

With I = 1, as may be expected to hold when $m_1 = m_8$, the right-hand side of Eq. (2.2) vanishes and we then have the Schwinger mass relation[5]. The mixing angle θ between the physical I = Y = 0 states, defined by

$$\varphi(M_0') = \cos\theta\, \varphi_1 + \sin\theta\, \varphi_8 ,$$

$$\varphi(M_0'') = -\sin\theta\, \varphi_1 + \cos\theta\, \varphi_8 , \qquad (2.3)$$

is then given by the expression

$$\tan\theta = \frac{2\sqrt{2}}{3} I\Delta / \left(\frac{m_8 - m_1}{2} + \frac{\Delta}{3} + \sqrt{\left\{\left(\frac{m_8 - m_1}{2} + \frac{\Delta}{3}\right)^2 + \frac{8}{9} I^2\Delta^2\right\}}\right). \qquad (2.4)$$

For the case $m_8 = m_1$, and $I = 1$, the mixing angle is given by

$$\tan\theta_0 = \frac{1}{\sqrt{2}} , \qquad (2.5)$$

corresponding to $\theta_0 = 35.3°$, to which we shall refer as the ideal mixing angle. In this ideal situation, the state M_0' would be described by the configuration $(Q^1Q_1 + Q^2Q_2)/\sqrt{2}$, the state M_0'' by the configuration $-Q^3Q_3$.

The vector mesons observed fit well with this nonet structure. Their mass values satisfy the Schwinger relation. The nonet parameters are m_1 = 810 MeV, m_8 = 770 MeV, with Δ = 122 MeV and overlap integral $I \approx 1$. The mixing angle is $\theta_V = +40°$ when M_0' is taken to correspond to the ω meson. There is no clear check on this mixing angle yet. Two pieces of data are relevant, however:

(i) the decay mode $\varphi \rightarrow \rho + \pi$ is expected to be forbidden for the ideal mixing angle θ_0, since the φ state is then composed of Q_3 and $\bar{Q}_3$, whereas the ρ and π are composed of Q_1, Q_2 and $\bar{Q}_1$, $\bar{Q}_2$. Glashow and Socolow[6] have pointed out that the decay rate $\Gamma(\varphi \rightarrow \rho + \pi)$ is therefore sensitive to the deviation of θ_V from θ_0, especially as $\omega \rightarrow \rho\pi$ is the dominant ω decay mode; they give the relationship

$$\Gamma(\varphi \rightarrow \rho\pi) = 17 \tan^2(\theta_V - \theta_0)\Gamma(\omega \rightarrow \rho\pi). \qquad (2.6)$$

From the observed branching ratio[7], $\Gamma(\varphi \rightarrow \rho\pi)/\Gamma(\varphi) = 32 \pm 8\%$, Glashow and Socolow obtain one solution $\theta_V = 39 \pm 1°$, in good agreement with the above expectation.

(ii) the decay mode $\omega \rightarrow e^+ + e^-$ has been observed by Binnie et al.[8] On the basis of three events, the observed rate is $2 \times 10^{-4}\Gamma(\omega)$. With the value $g^2_{\omega NN}/4\pi = g^2_{\rho NN}/4\pi \approx 2$ appropriate to SU(6) symmetry, the expected rate[9] is related with the contribution of the ω meson to the neutron charge form factor,

$$\Gamma(\omega \rightarrow e^+e^-)/\Gamma(\omega) = 2.9 \times 10^{-4}\, C^2_{\omega}/(g^2_{\omega NN}/4\pi) = 2 \times 10^{-4}, \qquad (2.7)$$

using the value $C_{\omega} = 1.21$ quoted by Pipkin[10], so that the observed rate is in good qualitative accord with theoretical expectation. This comparison does not yield any estimate of θ_V, of course, owing to the inherent uncertainties in the interpretation of the electromagnetic form factors observed for the nucleons; a determination of θ_V free from these uncertainties requires also a knowledge of $\Gamma(\varphi \rightarrow e^+e^-)$. However, the comparison does show that the $\omega \rightarrow \gamma$ interaction is allowed, so that the ω-meson is not a pure unitary singlet state.

For the pseudoscalar mesons, it appears that $(\text{mass})^2$ should be used in all mass relations. This has been argued by Gursey et al.[11] on the basis of the small mass values observed for the pseudoscalar octet, with the hypothesis that these masses should tend to zero in the limit of exact SU(3) symmetry; their arguments are well supported for the case of small mass values by an explicit model calculation. Certainly, a satisfactory interpretation of the pseudoscalar mass values requires the use of $(\text{mass})^2$ in the mass relations. It is possible that $(\text{mass})^2$ should be used generally for all boson multiplets, but this question is far from settled at present.

The η' meson at 959 MeV lies relatively far from the pseudoscalar octet states, so that m_1 and m_8 must be well separated. It is not surprising therefore that the pseudoscalar mesons do not satisfy the Schwinger $(\text{mass})^2$ relation; the $(\text{mass})^2$ analogue to Eq. (2.2) requires the value $I = 0.65$ for the overlap integral, not an unreasonable value. The parameters for this nonet are m_1 = 830 MeV, m_8 = 140 MeV. The equality of the mass difference Δ for the pseudoscalar and vector nonets is expressed by the relation

$$K^2 - \pi^2 = K^{*2} - \rho^2 , \qquad (2.8)$$

which is satisfied to better than 10% accuracy. The mixing angle is $\theta_P = -10°$ when M_0' is taken to correspond to the η' meson. A method has been proposed[12] for the determination of this mixing angle from the rates $\Gamma(\pi^0 \rightarrow \gamma\gamma)$, $\Gamma(\eta \rightarrow \gamma\gamma)$, and $\Gamma(\eta' \rightarrow \gamma\gamma)$, but no empirical estimate for θ_P is possible yet.

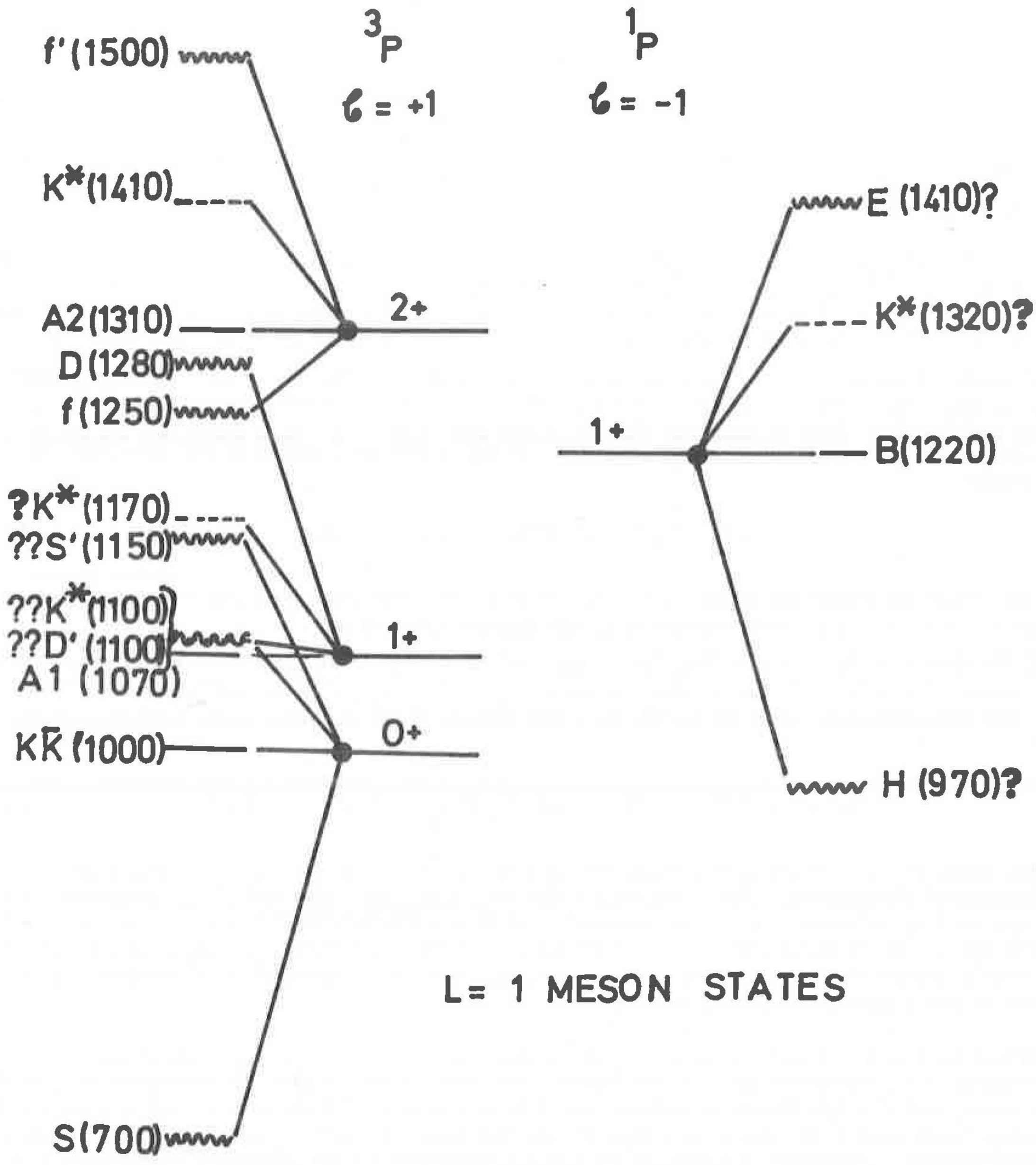

Figure 1

The mesonic resonance states with positive parity, related to their parent L = 1 configurations according to the quark-antiquark model. The states marked (?) are either not yet established or do not definitely have the quantum numbers assigned to them. The states marked (??) are purely speculative and have been predicted on the basis of the nonet model, assuming the Schwinger mass relation to hold.

The lowest excited meson nonets* are expected to correspond to L = 1. We therefore expect a series of positive parity states, corresponding to the $\bar{Q}$-Q configurations $^3P_2$, $^3P_1$, $^3P_0$ which have C = +1 and spin-parity (2+), (1+) and (0+), respectively, and the configuration $^1P_1$, which has C = -1 and spin-parity (1+). Generally, there is the possibility of mixing between the $^3P_1$ and $^1P_1$ configurations. Here, mixing between these configurations is forbidden for the neutral Y = 0 states, since they have opposite C; mixing between the I = 1, Y = 0 states is forbidden (except for electromagnetic effects) since they have opposite G-parity. Mixing is allowed only between Y = +1 (or -1) states, and then only by SU(3)-breaking interactions which link S = 0 and S = 1 states. Here, such interactions must be proportional to $(\underline{\sigma}_1 - \underline{\sigma}_2).\underline{L}$ and will not be considered further; mass-splitting by the quark mass-difference Δ does not lead to this mixing effect.

A (2+) nonet now appears well established, consisting of A2(1320), K†(1410), f(1250) and f'(1500). Isospin I = 0 is not yet established for f'(1500); it has a non-zero J and decays to K_1K_1, so that the spin-parity assignment (2+) is the most plausible. The masses fit the Schwinger relation and correspond to the parameter values m_1 = 1230 MeV, m_8 = 1320 MeV, Δ = 90 ±15 MeV and I ≈ 1. We note that this estimate for Δ is in reasonable accord with the values obtained from L = 0 nonets. The mixing angle obtained is θ_T = 30°, when M_0' is identified with the f(1250) meson. Glashow and Socolow[6] have shown that the branching ratios observed for their decay modes are in satisfactory accord with their interpretation as a nonet. The most striking features are the large f → ππ width (100 MeV) compared with the small width $\Gamma(A2 \to K\bar{K}) \approx$ 5 MeV, and the absence of evidence for f' → ππ. For these decay processes there are two amplitudes appropriate, say M for the transitions $T_8 \to P_8+P_8$, and N for the transitions $T_1 \to P_8+P_8$. With these, the decay amplitudes are

$$M(f \to \pi\pi) = \frac{1}{2}\sqrt{\frac{3}{2}}N\cos\theta_T - \sqrt{\frac{3}{5}}M\sin\theta_T , \qquad (2.9a)$$

$$M(A2 \to K\bar{K}) = \sqrt{\frac{3}{5}}M , \qquad (2.9b)$$

$$M(f' \to \pi\pi) = \frac{2}{2}\sqrt{\frac{3}{2}}N\sin\theta_T + \sqrt{\frac{3}{5}}M\cos\theta_T \qquad (2.9c)$$

The large ratio $\Gamma(f \to \pi\pi)/\Gamma(A2 \to K\bar{K})$ is due in part to the phase-space ratio, including centrifugal barrier penetration factor (about 5.8 over-all, assuming (momentum)^5 dependence), but also requires constructive interference between N and M, with ratio N/M ≈ -2. In this case, there will be strongly destructive interference between N and M in the f' → ππ amplitude (2.9c); despite the large phase space ratio in favour of f' → ππ decay, the predicted width is rather small, Γ(f' → ππ) ≈ 1 MeV, the precise value being rather sensitive to the input data and assumed barrier penetration factors.

With repulsive spin-orbit coupling, the L = 1 nonets will be split to an equal spacing pattern†, as shown in Figure 1, in the sequence $^3P_2$, $^1P_1$, $^3P_1$ and $^3P_0$ in order of decreasing mass value. For each nonet, the m_1 and m_8 values may differ a little as a result of some unitary-spin dependence of the $\bar{Q}$-Q interaction. Next, the introduction of the mass difference Δ splits each nonet into the states corresponding to masses M_1, M*, M_0' and M_0''.

The m_8 values for these nonets may be compared by considering the masses of the I = 1, Y = 0 states, A2(1310), B(1220), A1(1070) and $K\bar{K}$(1000) appropriate to the sequence (2+), $(1+)^-$, $(1+)^+$ and (0+). Here we have identified the (0+) state with the I = 1 $K\bar{K}$ enhancement reported[15] by the CERN group from their studies of the reactions $\bar{p}p \to \bar{K}K\pi$ and $\bar{K}K\pi\pi$; it may be that this $K\bar{K}$ interaction is simply

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*The inclusion of orbital angular momentum L = 1 in the classification of meson states was first discussed by Borchi and Gatto[13] within the framework of SU(6) symmetry. Their scheme was equivalent to the introduction of spin-orbit coupling and led to C = +1 nonets for (0+), (1+) and (2+), together with a C = -1, (1+) octet.

†Gatto et al.[14] have recently proposed a $\tilde{U}(12)$ scheme for the excited meson states, which runs parallel with the picture provided by a quark-antiquark model with L = 1, and with SU(6) symmetry. The symmetry-breaking which they assume allows unitary-spin dependence and SU(3)-breaking effects for the spin-orbit interaction, but still predicts equal spacing for the (2+), $(1+)^-$, $(1+)^+$, and (0+) states with I = 1, Y = 0. However, there are included additional terms for the singlet state and for the singlet-octet mixing amplitude, just for the (0+) system, as well as terms which mix the $(1+)^-$ and $(1+)^+$ K* states.

an s-wave threshold scattering-length effect*, and that the $\eta\pi$ peak pointed out by Alitti et al.[17] at 1040 MeV from their compilation of all $\eta\pi$ data is a real effect and does represent a (0+) resonant state. This $\eta\pi$ state would then have the right quantum numbers (G = -1, C = +1) to be identified with the I = 1, (0+) state predicted from this $\bar{Q}$-Q model. Although the expectation of equal spacing is not particularly well satisfied, these states do appear in the sequence predicted. The mean spacing due to the spin-orbit interaction is about 80 MeV, comparable in magnitude with the SU(3)-breaking mass shifts, typified by $\Delta \approx$ 100 MeV.

The experimental situation concerning K* mesons is obscure. If all of the data on $K\pi\pi$ final states in π^-p collisions near 3 GeV/c are taken together[18-20], the K*(1170) peak first reported by Wangler et al.[18] still stands out. Its decay mode $K^*\pi$ is compatible with expectation for (1+) and its mass value agrees with expectation for the K* state corresponding to A1(1070). The K*(1320) proposed by Almeida et al.[21] from the reaction $K^+p \rightarrow K^+\pi^+\pi^-p$ at 5 GeV/c would similarly agree with expectation for the K* state corresponding to B(1220). However, although there is a broad hump in the region 1200-1400 MeV, neither of these states appear distinctly in the $K\pi\pi$ mass distribution reported by Ferro-Luzzi et al.[22] in their more extensive study of the reactions $K^+p \rightarrow KN\pi\pi\pi$ (including the reaction process and momentum value of Almeida et al.). No $K\pi$ resonance is known in the mass region (about 1100 MeV) anticipated for the (0+) K* state. The only $K\pi$ resonance which has been proposed to date and is appropriate to the assignment (0+) is K* (725), whose situation is now rather obscure (cf. ref. 23); its mass value is far from the expected value. Quite apart from the model discussed here, such K* states are expected to exist simply on the basis of SU(3) symmetry, as counterparts to other octet states already established.

The known positive-parity states have been fitted into the scheme of four L = 1 nonets in Figure 1. Spin parity (1+) and C = +1 have been established for D(1280); C = +1 holds for A1(1070) and spin-parity (1+) is the most probable assignment. K*(1170) has been assigned to this nonet; on this basis, the Schwinger relation requires the ninth $(1+)^+$ meson D' to lie at about 1100 MeV. † With this mass, the D' meson lies below the threshold for any allowed decay mode V+P, in fact below the threshold $K\bar{K}\pi$. The simplest decay processes allowed by the strong interactions are $D' \rightarrow \eta\pi\pi$ and $D' \rightarrow 4\pi$; the simplest electromagnetic decays are the electric dipole transitions $D' \rightarrow \rho + \gamma$ and $\omega + \gamma$. With these mass values, m_1 and m_8 lie rather close. with values m_1 = 1110 MeV, m_8 = 1070 MeV.

The $(1+)^-$ nonet is rather speculative. The existence of the B-meson appears quite certain, and the simplest interpretation of its decay correlations corresponds to spin-parity (1+), in which case its quantum numbers are those appropriate to this nonet. As discussed above, the existence of K*(1320) is quite uncertain at present. The H(975) meson was reported by Bartsch et al.[24] from the π^+p reaction at 4 GeV/c,

$$\pi^+ + p \rightarrow N^*(1238)^{++} + H(970) , \qquad (2.10)$$

followed by the decay $H \rightarrow \pi^+\pi^-\pi^0$. A similar but less pronounced enhancement has been observed at this mass value by Goldhaber[23] in a study of the same reaction at 3.65 GeV/c, but there has not yet been any clear-cut confirmation of this meson. The decay characteristics reported by Bartsch et al. required abnormal parity for the H-meson so that spin-parity $(1+)^-$ is the only possibility available for it in the scheme of Fig. 1. The Schwinger mass relation would then require the H' state to lie at about 1380 MeV; the H' meson would be expected to have the decay modes $\pi\rho$, $\bar{K}K^*$ and $\bar{K}^*K$. It is not impossible that the H' meson may be identical with the E(1410) meson; the E(1410) decay characteristics are consistent with spin-parity (1+) and the direct evidence on C is consistent with either C = ±1. This assignment is not consistent with the decay mode $E \rightarrow (K\bar{K})+\pi$ where $(K\bar{K})$ denotes an I = 1 s-wave $K\bar{K}$ system, the interpretation adopted here by Rosenfeld[15].

The (0+) nonet is also very speculative. For the I = 1 state, we have adopted the identification $K\bar{K}$(1000), although the $\eta\pi$(1040) peak reported by Alitti et al.[17] might well prove to be the proper identification. The K*(725) meson is the only $K\pi$ resonance known which is consistent with s-wave decay, but it is far displaced from the expected location of about 1100 MeV. The S(700) meson, known from the decay mode $S \rightarrow 2\pi^0$, has the quantum numbers appropriate to this nonet; the S'(1150) state given in Figure 1 is based only on the Schwinger mass relation and the expected K*(1100) state, so that this entry is completely speculative.

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*We may note also that Kienzle et al.[16] have reported at this Conference the observation of a narrow peak at m_X = 962±5 MeV in the reaction $\pi^- + p \rightarrow X^- + p$ where only the recoil proton was observed. It is possible that this peak could correspond to an $\eta\pi$ resonance (the $\eta\pi^0$ decay of the corresponding X^0 state produced in a reaction such as $K^- + p \rightarrow \Lambda + X^0$ would be difficult to identify, and the closeness of the observed η' (959) meson mass could be coincidental) and that this resonance might be related with the I = 1 $K\bar{K}$ threshold effect mentioned above. In the approximation of a constant scattering length, the observed mass m_X and width $\Gamma_X \leq 5$ MeV would correspond to an I = 1 $K\bar{K}$ scattering length a+ib = (1.6+i0.1)F. This mass value would be in good accord with expectation from the L = 1 $\bar{Q}$-Q model with spin-orbit coupling.

†This estimate for the D' mass value is sensitive to the K* mass assumed. If K* is increased to 1190 MeV, for example, the estimate for D' falls to 975 MeV.

The next highest nonets are then expected to correspond to L = 2, and to have spin parities (3-), (2-), and (1-) with C = -1, and (2-) with C = +1, in the order (1-), $(2-)^-$, $(2-)^+$, (3-) for increasing mass values. Some evidence for higher mesonic resonances, in the mass region of 1600 MeV, has been presented at this conference, but quantum numbers have not yet been established for any of these states.

Confirmatory evidence[22] has been presented at this Conference for an $I = \frac{3}{2}$ $K\pi\pi$ resonance at 1270 MeV. There is no evidence for a $K\pi$ decay mode for this state, which strongly suggests that it must have abnormal parity, (1+), (2-), ... This state could be a member of a $\{10+\overline{10}\}$ representation, or of a {27} representation. In either case, the existence of this state would require the existence of corresponding Y = ±2 states (decaying to KK* and $\overline{K}\overline{K}$*, respectively), which should be identified in order to establish the nature of the unitary multiplet; I = 0 for the Y = ±2 states would correspond to a $\{10+\overline{10}\}$ representation, I = 1 for the Y = ±2 states would correspond to a {27} representation. There has also been evidence presented for an I = 1 KK resonance at 1280 MeV.[25] Such a resonance must belong to a {27} representation, in which case the most striking companion states would be those giving rise to corresponding resonances in the $I = \frac{3}{2}$ $K^+\pi^+$ and $K^-\pi^-$ systems, and in the I = 2 $\pi^+\pi^+$ and $\pi^-\pi^-$ systems. No evidence has been found for these companion resonances despite considerable search over the relevant mass region. Here we emphasize that the $\{10+\overline{10}\}$ representation or the {27} representation occur first in excited quark-antiquark configurations of the type $\overline{Q}\overline{Q}QQ$; if such configurations lie so low in mass value, then such configurations will also contribute to the singlet and octet states and this will mean that there is little possibility for such a simple description of the mesonic states as the $\overline{Q}Q$ model discussed here. It is of the greatest importance to establish whether or not the $K^*_{3/2}$ and KK peaks which have been reported do correspond to resonant states, and to identify the complete unitary multiplets to which they belong.

3. The Baryonic Resonances

In this model, the baryon states are systems composed of three quarks. Only *singlet, octet,* and *decuplet* states can be formed in this way, and all of the established baryonic resonances are consistent with these unitary multiplets. The baryon wavefunction will generally consist of sums of products of a unitary-spin wavefunction g, a spin wavefunction χ, and a space wavefunction φ. The permutation symmetries appropriate to these wavefunctions are tabulated in Table I.

Spin S	{α}	Unitary-spin Wavefunction g	Spin Wave-function χ	Space Wave-function φ (i) Fermi Statistics	(ii) Para-Statistics
$\frac{3}{2}$	{10}	S	S	A	S
$\frac{3}{2}$	{ 8}	M	S	M	M
$\frac{3}{2}$	{ 1}	A	S	S	A
$\frac{1}{2}$	{10}	S	M	M	M
$\frac{1}{2}$	{ 8}	M	M	S, M or A	S, M or A
$\frac{1}{2}$	{ 1}	A	M	M	M

Table I. The permutation symmetries appropriate to three-quark wave-functions, according as quarks satisfy (i) Fermi statistics, or (ii) parafermi statistics with p = 3. S denotes complete symmetry, A complete antisymmetry, and M denotes mixed (or [21]) symmetry.

Greenberg[26] has proposed that quarks should satisfy parafermi statistics with p = 3. This is equivalent to the hypothessis that there are three kinds of quark, denoted by Q^i_α for i = 1, 2, 3, such that Q^i_α, Q^j_β commute unless $i \neq j$, and anticommute for i = j, but such that no physical interaction distinguishes between the three types of quark. In this situation, the three-quark wavefunction is symmetrical under permutations, and the space wavefunction is required to have the symmetry listed in the last column of Table I. This hypothesis is attractive since it allows the space wavefunction to be symmetrical for the lowest three-quark states observed, for the $S = \frac{3}{2}$, L = 0 decuplet state and the $S = \frac{1}{2}$, L = 0 octet state, as one would normally expect for attractive, non-exchange, potential interactions between the quarks. Another attractive feature is that the n-quark system then saturates for n = 3. As a result, a repulsion would be expected for close approach between the two baryons, when their three-quark wavefunctions overlap, just as there occurs a strong repulsion between two alpha-particles at close approach. It is conceivable that this effect could account for the short-range repulsion known to occur between two nucleons, and would lead to a universal character for this baryon-baryon repulsion.

With the more conventional view (which we shall assume in our further remarks) that quarks should satisfy Fermi statistics, the $S = \frac{3}{2}$, L = 0 decuplet state requires a totally antisymmetric space wavefunction. According to Table I, the other three-quark state giving a totally antisymmetric space wavefunction is the $S = \frac{1}{2}$, L = 0 octet. In the limit that QQ forces are spin and unitary-spin independent, these two states would necessarily have the same energy; in this limit of SU(6) symmetry, these two configurations together form the $\underline{56}$ representation. It is not clear what features are required for the QQ interaction to lead to an antisymmetric ground-state space wavefunction. Apparently they must have a strong space-exchange component; as shown by Kuo and Radicati[27], a repulsive space-exchange QQ potential gives attraction in a totally anti-symmetric state φ, no attraction in a state of mixed symmetry, and repulsion in a symmetrical state. With the vector model V mentioned above, the QQ interaction would be repulsive. Kuo and Radicati[27] have therefore suggested that there may exist three-body QQQ interactions with a strong space-exchange component; this would have the advantage that no bound states would be expected to occur for the QQ systems, but it is not clear what mechanism could give rise to strong three-body forces of this character.

Becchi and Morpurgo[28] have shown that the magnetic moment predictions known for the $(\frac{1}{2}+)$ octet and $(\frac{3}{2}+)$ decuplet from SU(6) symmetry are readily obtained from this model, using the quark magnetic moment operator,

$$\underline{M}(Q) = \mu \left\{\frac{2}{3}\underline{\sigma}(Q_1) - \frac{1}{3}\underline{\sigma}(Q_2) - \frac{1}{3}\underline{\sigma}(Q_3)\right\} \tag{3.1}$$

The form of this operator is dictated by SU(3) symmetry; the scale moment μ is regarded as a free parameter. For the nucleon magnetic moments, the results

$$\mu_p = \mu, \qquad \mu_n = -\frac{2}{3}\mu \tag{3.2}$$

are obtained. These follow from the symmetry imposed on the product $g\chi$ of the spin and unitary-spin wavefunctions by the assumption that the space wavefunction has A symmetry for the $(\frac{1}{2}+)$ octet (an assumption dictated by the belief that the $(\frac{1}{2}+)$ octet and $(\frac{3}{2}+)$ decuplet states are closed related, as would be the case for spin and unitary-spin independent interactions). We note that the scale moment μ is large relative to the quark magneton,

$$\mu/(e\hbar/2M_Q c) = (2.79\frac{M_Q}{M_p}) > 15 , \tag{3.3}$$

implying a very large anomalous moment for the quarks, for which there is no ready explanation.

For the electromagnetic transition $\gamma + B_8 \rightarrow B^*_{10}$, Becchi and Morpurgo[29] have pointed out that the E2 amplitudes vanish in the quark model, since the transition is L = 0 → L = 0. This is in good accord with the small values $E_{23}/M_{13} = 0.0 \pm 0.04$ obtained by McDonald et al.[30] from the π^0 angular distribution in the reaction $\gamma p \rightarrow p\pi^0$, and 0.04 ± 0.04 obtained by Drickey and Mozley[31] from their study of π^0 production by polarized photons. The M1 amplitude is calculated to give the value

$$(p, \tfrac{1}{2}|\underline{M}|N^*_{\frac{3}{2}}, \tfrac{1}{2}) = \frac{2\sqrt{2}}{3}\mu_p , \tag{3.4}$$

as found by Beg et al.[32] using SU(6) symmetry. The empirical value[33] is somewhat larger than this, being $(1.25 \pm 0.02)\times\frac{2\sqrt{2}}{3}\mu_p$. This is a substantial discrepancy in this model, since the corrections expected (admixture of M and S space wavefunctions for the $(\frac{1}{2}+)$ octet, or incomplete overlap between the A space wavefunction φ for the octet and decuplet states) would tend to depress the theoretical estimate.

Thirring[34], Becchi and Morpurgo[28], and Anisovitch et al.[35] have pointed out that the amplitude for the M1 transition $\omega \rightarrow \pi^0\gamma$ is given by the scale moment $\mu = \mu_p$, according to the quark model. With this amplitude, the decay width $\Gamma(\omega \rightarrow \pi^0\gamma)$ is predicted to be 1.18 MeV, in excellent agreement with the empirical estimate $\Gamma(\omega \rightarrow \pi^0\gamma) = 1.27 \pm 0.2$ MeV obtained from $\Gamma(\omega)$ and the $(\pi^0\gamma)$ branching ratio for ω-decay[15].

The nucleon beta-decay interaction can be calculated on the basis of a quark beta-interaction $Q_2 \to Q_1 + e^- + \bar{\nu}_e$. The ratio G_A/G_V for the nucleon is then found to be (5/3) $(G_A/G_V)_Q$, as given by Beg and Pais[36]. For the axial vector interaction, the quark model also leads directly to the Beg and Pais result,

$$F/(D+F) = 2/5, \tag{3.5}$$

in excellent agreement with the value 0.37 found by Willis et al.[37] in their analysis of the leptonic decay rates for the baryons on the basis of the unitary symmetry hypothesis of Cabibbo.

Mass splitting within the octet and decuplet may be introduced through the quark mass difference Δ. As pointed out by Zweig[2], this leads to equal spacing for the decuplet, as observed, with the estimate Δ = 147 MeV, rather larger than the estimates obtained from the mesonic states. For the baryons, the Ξ-N mass difference is then expected to be 2Δ, which leads to the estimate Δ = 190 MeV, much larger still. Also, this model for SU(3) symmetry-breaking requires equality for the Λ and Σ states, whereas they are split by about 80 MeV. We have to conclude that, for the QQ system, there are appreciable symmetry-breaking potential terms of an exchange character, which are able to generate the Λ-Σ mass difference.

The octet and decuplet are separated by some spin or unitary-spin dependence in the QQ interaction, but the mass splitting coefficients are expected to be the same for the two multiplets, insofar as they have identical space wavefunctions, leading to the mass formula

$$M = M_0 + aY + b(I(I+1) - \tfrac{1}{4}Y^2) + cS(S+1) + dF^2 \tag{3.6}$$

where the last two terms are equivalent since there are only two unitary multiplets involved, with different values for both S and F^2. For the $(\frac{1}{2}+)$ octet, the coefficients a, b are

$$a = (N - \Xi)/2 = -189 \text{ MeV}, \tag{3.7a}$$
$$b = (\Sigma - \Lambda)/2 = 39 \text{ MeV}, \tag{3.7b}$$

For the $(\frac{3}{2}+)$ decuplet, $I = \frac{1}{2}Y + 1$, and the expression (3.6) becomes linear in Y, leading to equal spacing with $\Delta M = a + \frac{3}{2}b$. With (3.7), the expected spacing is 130.5 MeV, about 10% smaller than the observed spacing of 147 MeV; the discrepancy may be attributed to higher order terms* in the symmetry-breaking interaction or to differences between the space wavefunctions φ appropriate to the octet and decuplet states. The octet-decuplet mass splitting is measured by

$$Y_1^* - \Sigma = (3c + 18d) = 190 \text{ MeV}, \tag{3.8}$$

since F^2 = 18 for an octet state, and 36 for a decuplet state.

Many baryonic resonances with negative parity are known over the mass range 1400-1800 MeV. It appears natural to expect such resonances to correspond to the L = 1 excited configurations of the three-quark system. These resonant states include unitary singlets (such as $Y_0^*(1405)$, with spin-parity $(\frac{1}{2}-)$), octets (with spin-parity $(\frac{3}{2}-)$, such as $N_{1/2}^*(1510)$, and with spin-parity $(\frac{5}{2}-)$, such as $N_{1/2}^*(1680)$ and $Y_1^*(1750)$) and decuplets. With L = 1, the $(\frac{5}{2}-)$ octet requires the $S = \frac{3}{2}$, {8} configuration, for which Table I indicates a space wavefunction with permutation symmetry M. A space wavefunction of the same symmetry is also required by the $S = \frac{1}{2}$, {10} configuration, the $S = \frac{1}{2}$, {8} configuration, and the $S = \frac{1}{2}$, {1} configuration. With QQ interactions without spin or unitary-spin

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*That is, terms which are not of the tensor form T_3^3. Such terms certainly exist[38]; after including the electromagnetic self-masses, the baryon masses do not satisfy the Gell-Mann-Okubo mass formula within errors, the discrepancy being equivalent to the addition of a term fY^2 to expression (3.6) with f = 6.9 ± 0.3 MeV. This term is comparable in magnitude with the electromagnetic mass differences and cannot be understood as arising from SU(3)-breaking corrections to the electromagnetic masses.

dependence, these four configurations would have the same energy; without spin-orbit interactions, the states with given m_L would form the basis for a $\underline{70}$ representation of the SU(6) group.† With corrections due to the spin and unitary-spin dependence of the QQ interactions, and with spin-orbit interaction, the 210 states separate into nine unitary multiplets, as shown in Figure 2.

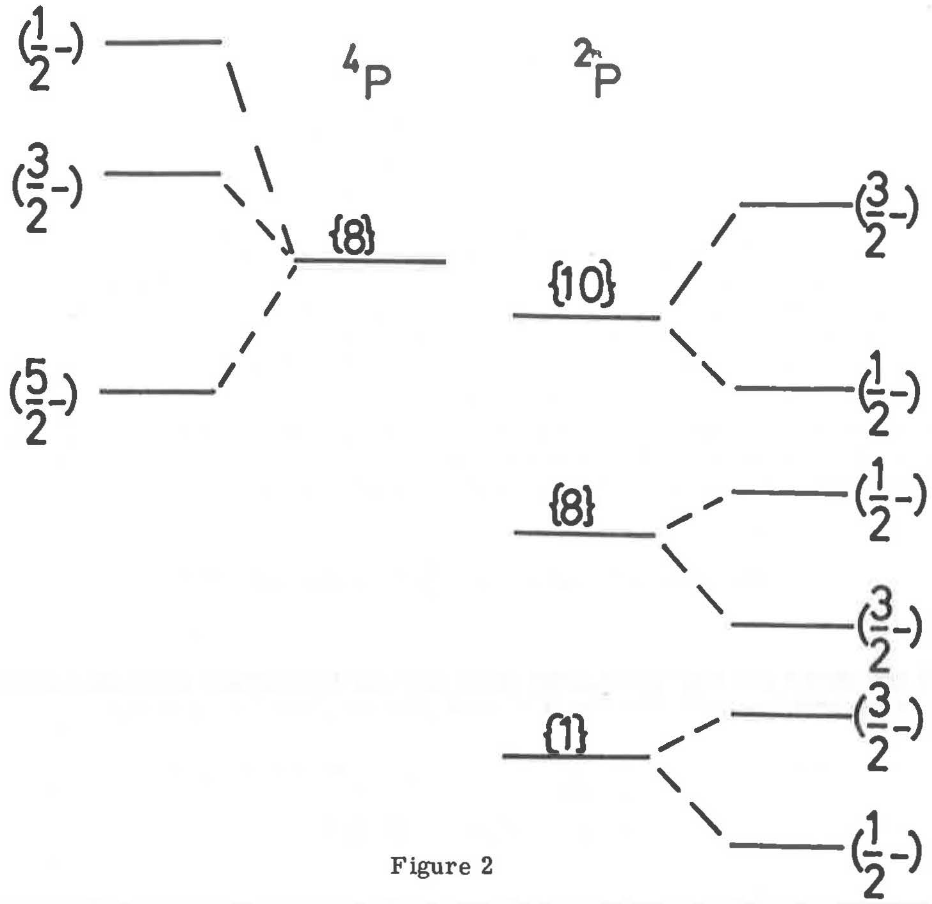

Figure 2

The baryonic resonance multiplets with negative parity expected to occur for an L = 1 space wavefunction with M symmetry. The sequence of unitary multiplets $\{\alpha\}$ has been chosen to put the known resonances in about the correct relative locations; for each unitary multiplet, the sign of the spin orbit coupling has been chosen such as to give the observed spin-parity to the lowest state known for each multiplet. As discussed in the text, resonance states belonging to these multiplets are known for all cases except the (^{4}P, $\{8\}$) and (^{2}P, $\{10\}$) multiplets with spin-parity ($\frac{3}{2}$-).

We shall now discuss briefly the negative-parity baryonic resonances in terms of the multiplets shown in Figure 2.

(1) $^2P_{1/2}$, $\{1\}$. $Y_0^*(1405)$ is now believed to have spin-parity ($\frac{1}{2}$-). This is based on the properties now established for the I = 0 $\bar{K}N$ scattering interaction at low energies by the recent studies of Sakitt et al.[40] and Kim[41] on the low-energy cross-sections for the K^-p scattering and reaction processes. Their analysis leads to the value $A_0 = (-1.67 \pm 0.04 + i(0.72 \pm 0.04))$F. quoted by Kim for the I = 0 $\bar{K}N$ scattering length; in the approximation of a constant scattering length, this value corresponds to the existence of a Y_0^* resonant state at 1411 MeV, with width 37 MeV. Since the observed Y_0^* state is at 1405 MeV, with width 35 ± 5 MeV, it is very plausible to identify the $Y_0^*(1405)$ with this s-wave $\bar{K}N$ virtual bound state and to assign spin-parity ($\frac{1}{2}$-) to $Y_0^*(1405)$. A direct determination of the spin and parity of $Y_0^*(1405)$ from the polarization properties of its decay mode $\Sigma^+\pi^-$ would still be exceedingly desirable.

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†Gyuk and Tuan[39] have proposed to assign the negative parity baryonic resonances directly to the $\underline{70}$ representation of SU(6) symmetry, assigning $Y_0^*(1405)$ to the ($\frac{1}{2}$-) singlet state, the $N\eta$ and $\Lambda\eta$ threshold effects (see below) to the ($\frac{1}{2}$-) octet, and the established $d_{3/2}$ resonances $N_{1/2}^*(1510)$, $Y_0^*(1520)$, ... to the ($\frac{3}{2}$-) octet. This proposal gives no explanation for the negative parity assigned to the $\underline{70}$ representation and does not allow place for the ($\frac{5}{2}$-) resonances now established in the same mass region.

(2) $^2P_{3/2}$, {1} and $^2P_{3/2}$, {8}. The $(\frac{3}{2}-)$ resonances which are established are $N^*_{1/2}(1518)$ and $Y_0^*(1520)$. Smith et al.[42] have shown that the data on $\Xi^*(1820)$ is consistent with spin-parity $(\frac{3}{2}-)$ (the data are also consistent with spin-parity $(\frac{5}{2}+)$, but the assignment $(\frac{3}{2}-)$ is more probable in terms of unitary multiplets and the location of the known $(\frac{5}{2}+)$ baryonic resonances). In terms of unitary multiplets, the most probable interpretation for $Y_1^*(1660)$ is spin-parity $(\frac{3}{2}-)$, although there are conflicting experimental data on this point.[43-46]

It appears quite possible that $Y_0^*(1520)$ is essentially a unitary singlet state. The case for this assignment has been argued in detail by Martin[47] and others. $Y_0^*(1520)$ lies very low in mass relative to the other members of the $(\frac{3}{2}-)$ octet; the Gell-Mann-Okubo mass formula would lead to the expectation of mass 1670 MeV for the $(\frac{3}{2}-)$ octet Y_0^*. Apart from $Y_0^*(1520)$, the octet states are approximately equally spaced, corresponding to the reasonable value $\Delta = 150$ MeV. Also, the branching ratio $(\bar{K}N)/(\Sigma\pi)$ for $Y_0^*(1520)$ is compatible with expectation for a singlet state and is not easy to reconcile with the PB branching ratios reported for the octet states.

Here we shall adopt this interpretation, that $Y_0^*(1520)$ is a $(\frac{3}{2}-)$ unitary singlet, and that the $(\frac{3}{2}-)$ octet is centred at mass value about 1660 MeV. This involves the hypothesis of a further $(\frac{3}{2}-)$ Y_0^* state at about 1670 MeV, for which there is no indication yet. Some mixing is expected to occur between these two Y_0^* states, due to the SU(3)-breaking interaction; since they both belong to the $^2P_{3/2}$ configuration, even the quark mass difference Δ will cause mixing between them.

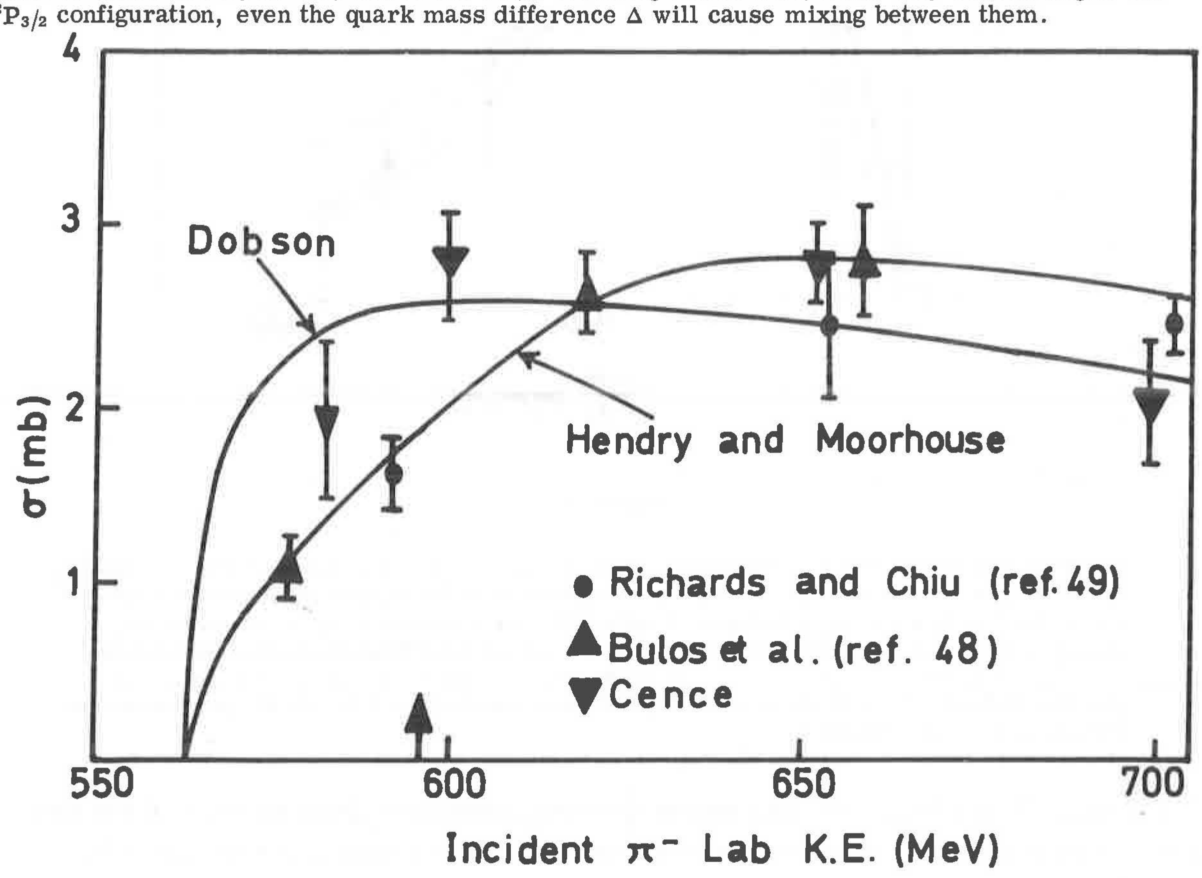

Figure 3

The total cross sections $\sigma(\pi^- p \rightarrow n\eta)$ available near the $n\eta$ threshold are plotted as function of the incident pion laboratory energy. The points due to Richards and Chiu[49] and Bulos et al.[48] correspond to direct measurements of this cross-section. The points marked Cence (Phys. Rev. Letters, to be published (1965)) give the total inelastic cross-sections deduced from his phase shift analysis of the πN scattering data in this region (these particular cross- sections are in good agreement with those given by other independent phase shift analyses). The theoretical curves show the constant K-matrix fit obtained by Dobson[56] (who actually obtained the K-matrix parameters from a best fit to the points of Cence), and the resonance fit obtained by Hendry and Moorhouse[55] (using an energy dependent K-matrix). The resonance energy found by Hendry and Moorhouse is shown by the upward arrow on the energy axis.

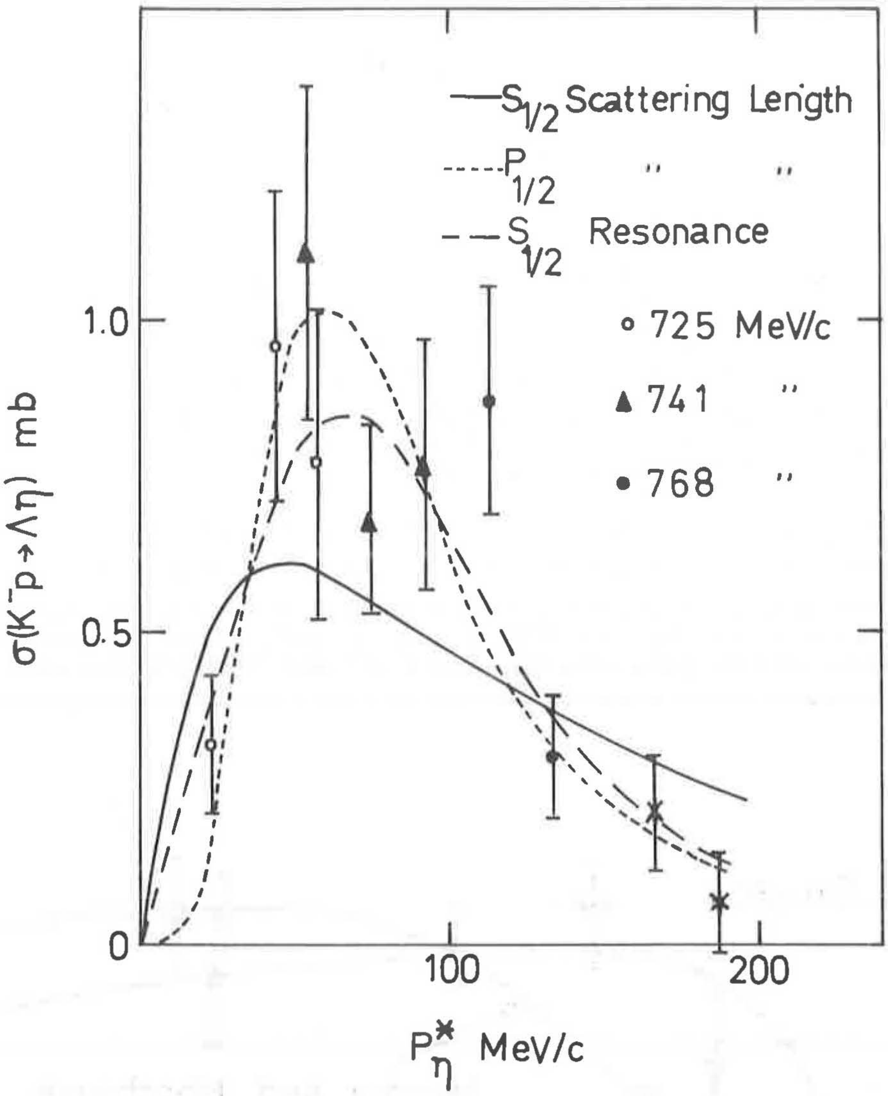

Figure 4

The cross-sections $\sigma(K^-p \to \Lambda\eta)$ obtained by Berley et al.[51] are shown plotted as function of the η c.m. momentum p^*_η. They are compared with the energy dependences expected for a constant s-wave $\Lambda\eta$ scattering length and for a constant p-wave $\Lambda\eta$ scattering length. The latter possibility is rejected since the angular distributions observed are isotropic and since the p-wave scattering length is exceedingly large. They are compared satisfactorily with the energy dependence expected for a Breit-Wigner resonance with an s-wave $\Lambda\eta$ channel.

(3) ${}^2P_{1/2}$, {8} and ${}^4P_{1/2}$, {8}. Two sets of ($\frac{1}{2}$-) octet resonances appear to occur. It has been known for some time that η production is very strong close to threshold in the reactions[48-51],

$$\pi^- + p \to n + \eta, \quad (3.9a)$$

$$K^- + p \to \Lambda + \eta \quad (3.9b)$$

and there are indications[52] that the same is true for the reaction $K^-p \to \Sigma\eta$. In phase shift analyses of πN scattering data[53,54], there appears a strong cusp in the S_{11} amplitude at the $n\eta$ threshold, corresponding to this strong threshold production.

For the πN situation, a number of authors[55-58] have made K-matrix analyses of the data on S_{11} elastic scattering and on the reaction (3.9a). The most complete discussion is that of Hendry and Moorhouse[55] who allow energy-dependence for the K-matrix (such that K^{-1} is a linear function of the total energy) and who allow for the possibility of other inelastic channels (such as $N\pi\pi$) in addition to the $N\eta$ channel. Hendry and Moorhouse find that the K-matrix must have a pole for a real energy not far above the $N\eta$ threshold, in other words that there must be a ($\frac{1}{2}$-) $N^*_{1/2}$ resonance at about

1510 MeV, in order to account for the energy dependence observed. Their fit for $\sigma(\pi^- p \to n\eta)$ is compared with the experimental data on Figure 3, and with a typical fit appropriate to the assumption of a constant K-matrix (which does not require the existence of a resonant state). The essential feature of the data which requires the existence of a resonant $N^*_{1/2}$ state is the low value observed for the cross-section close to the threshold, a feature which cannot be fitted unless the K-matrix is allowed to have energy-dependence (in which case it indicates the existence of a resonant state).

The data for the $\Lambda\eta$ situation[51] are shown in Figure 4. Here, the rise and fall of the cross-section occur over a much narrower energy range (from K^- laboratory momentum 724 MeV/c at threshold to 820 MeV/c for the uppermost datum point on Figure 4). Again, the assumption of a constant scattering length gives an exceedingly poor fit; with a real effective range term, the energy-dependence of the cross section takes the form

$$\sigma(K^- p \to \Lambda\eta) = Cp^*_\eta / \{ (\alpha + \frac{1}{2} r p^{*2}_\eta)^2 + (\beta + p^*_\eta)^2 \} , \qquad (3.10)$$

where p^*_η denotes the η c.m. momentum and $(\alpha - i\beta)^{-1}$ denotes the threshold $\Lambda\eta$ scattering length. This allows an excellent fit to the data if the term $(\alpha + \frac{1}{2} r p^{*2}_\eta)$ passes through zero in the physical region. In this case, however, expression (3.10) is essentially equivalent to the Breit-Wigner resonance formula, and an adequate fit to the data has been obtained by Willis et al.[51] with an s-wave resonance formula for resonance mass about 1675 MeV.

It appears natural to identify these threshold resonances with the $^2P_{1/2}$ octet expected from the pattern of Figure 2. Since this octet overlaps the $^2P_{3/2}$ octet quite strongly, we must conclude that the spin-orbit splitting is relatively weak for the ^2P octets, the mean separation between the octets being of order of 30 MeV, as compared with the splitting of 115 MeV (of opposite sign) observed for the ^2P singlet states.

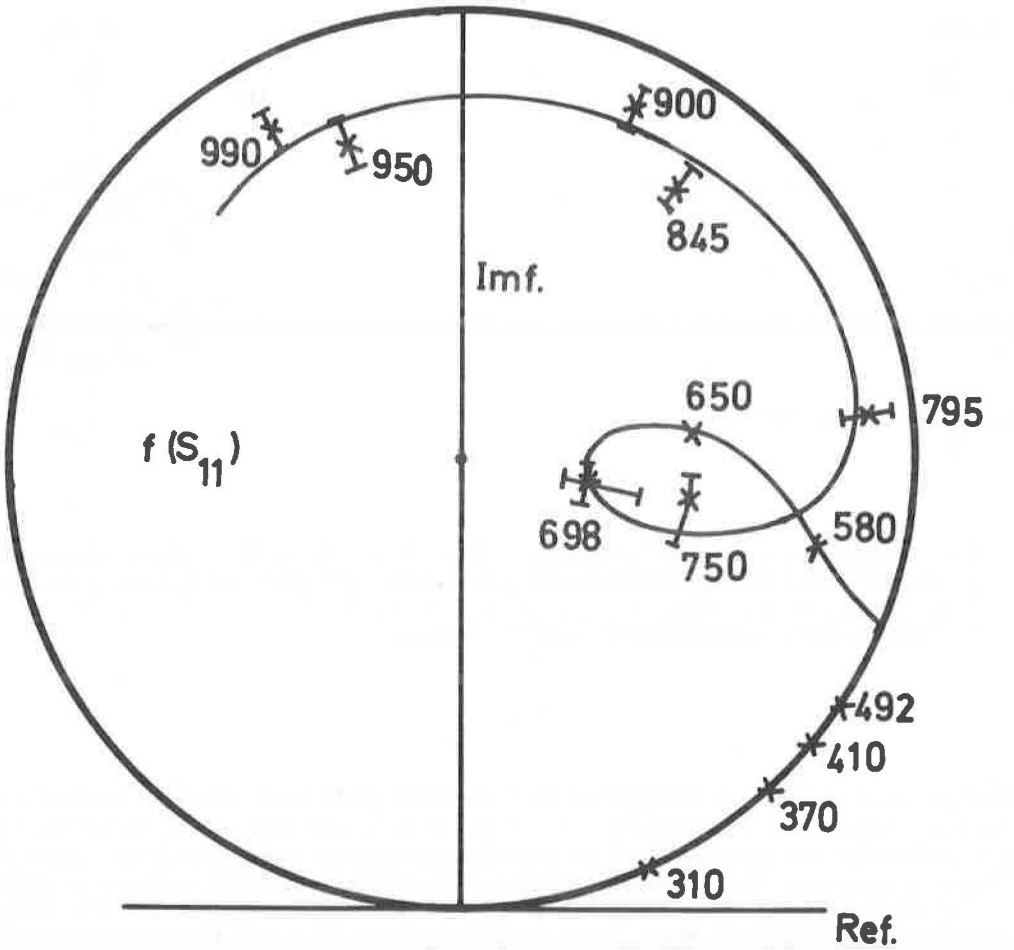

Figure 5

The scattering amplitude $f(S_{11})$ obtained by Bareyre et al.[54] in their phase shift analysis of the pion-nucleon elastic scattering and polarization data is plotted on an Argand diagram as function of pion laboratory energy (MeV). After a strong cusp at the $n\eta$ threshold, the amplitude first follows a looped path (where the analysis of Hendry and Moorhouse[55] indicates a resonance close to 600 MeV) and then rapidly traces out the upper part of a second circular loop (which is interpreted here to reflect the existence of a resonant state at about 900 MeV).

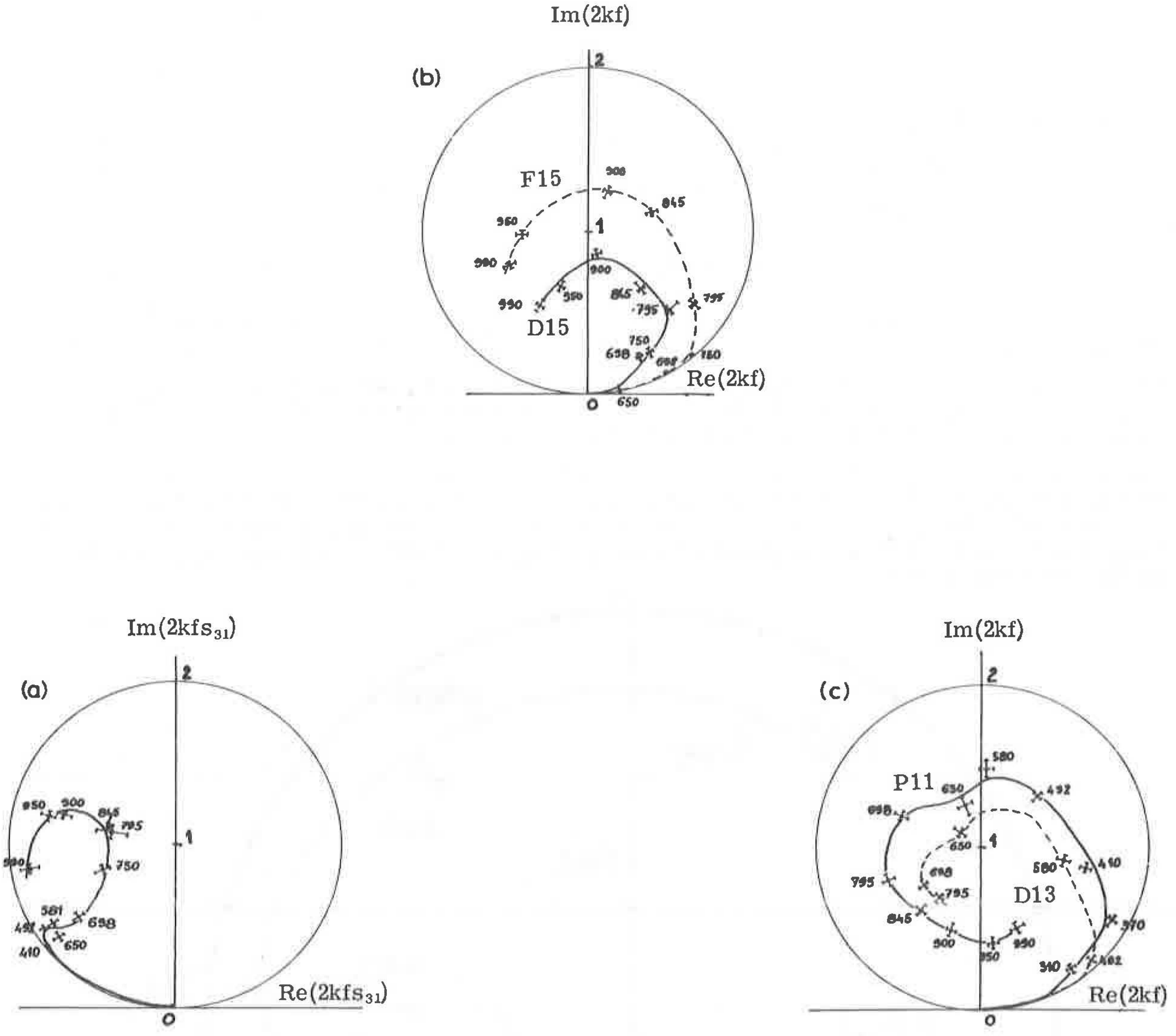

Figure 6 The scattering amplitudes f obtained by Bareyre et al.[54] in their phase shift analysis of the pion-nucleon elastic scattering and polarization data are plotted on an Argand diagram as function of the pion laboratory energy (MeV) for (a) the S_{31} state, (b) the D_{15} and F_{15} state, and (c) the P_{11} and D_{13} states.

The recent phase shift analysis of Bareyre et al.[54] of all the scattering, charge-exchange and polarization data available for pion-nucleon scattering below 1 GeV laboratory energy (including the extensive scattering and polarization data recently reported by Duke et al.[59]) has given the S_{11} phase up to higher energies. The S_{11} scattering amplitude obtained by their analysis is plotted in Figure 5 as function of energy on an Argand diagram. After a cusp behaviour (90° left-hand turn) at the $n\eta$ threshold, the amplitude describes a small loop associated with the resonance behaviour discussed above and then goes on to follow a larger circular orbit which it is natural to associate with a further S_{11} resonance in the neighbourhood of 1700 MeV. This $N^*_{1/2}(1700)$ will be a member of a further $(\frac{1}{2}-)$ octet. In the scheme of Figure 2, there is indeed a place available for this octet, as the $^4P_{1/2}$, {8} configuration.

(4) $^2P_{1/2}$, {10}. The phase shift analysis of Bareyre et al.[54] just mentioned shows a remarkable behaviour for the S_{31} amplitude. As shown in Figure 6(a), the amplitude corresponds to repulsion up to about 500 MeV, and then suddenly describes a small circular loop in an anti-clockwise direction in the complex plane, the latter being the behaviour characteristic for a strongly inelastic

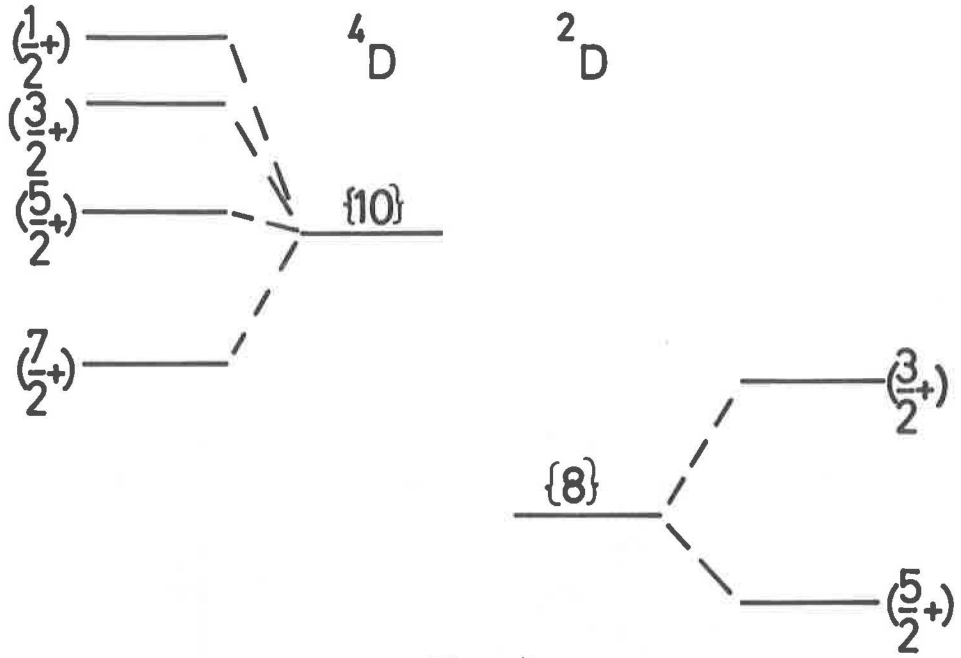

Figure 7

The baryonic resonance multiplets with positive parity, expected to occur for an L = 2 space wavefunction with A symmetry. The spin-orbit coupling has been chosen attractive, to place the highest spin state lowest in each unitary multiplet.

resonant state. A similar behaviour for the S_{31} amplitude has recently been reported also by Donnachie et al.[60] from a more sophisticated analysis (not necessarily more reliable) which fits the phenomenological S_{31} phase shifts by a dispersion-theoretic expression which takes into account the nearest left-hand branch cuts for the amplitude $f(S_{31})$. This resonance is the main component of the 800 MeV shoulder known in the π^+p total cross-section data. This $(\frac{1}{2}-)$ resonance must belong to a decuplet representation, and there is just one such state in the scheme of Figure 2, corresponding to the $^2P_{1/2}$, {10} configuration.

(5) $^4P_{5/2}$, {8}. The pion-nucleon scattering data of Duke et al.[59] has shown rather clear indications for the existence of an $I = \frac{1}{2}$ $D_{5/2}$ resonant state lying at almost the same mass value as the $I = \frac{1}{2}$, $F_{5/2}$ resonance at 1688 MeV. This $(\frac{5}{2}-)$ $N^*_{1/2}(1680)$ state appears rather clearly in the plot of of the D_{15} amplitude (see Figure 6) given by the phase-shift analysis of Bareyre et al.[54] Spin-parity $(\frac{5}{2}-)$ has also become established now for $Y_1^*(1760)$ from the work of Armenteros et al.[61] reported at this Conference, based on the angular distributions observed for $Y_0^*(1520)$ production and decay following $Y_1^*(1760)$ formation,

$$K^- + p \rightarrow Y_1^*(1760) \rightarrow Y_0^*(1520) + \pi^0 . \qquad (3.11)$$

These states appear to form part of a $(\frac{5}{2}-)$ octet, for which the natural interpretation is the $^4P_{5/2}$ configuration in the present model.

It is remarkable that all of the baryonic resonances known to have negative parity can be accomodated within this L = 1 configuration. However, there is still no evidence for members of two unitary multiplets predicted for this configuration, the $^2P_{3/2}$, {10} and $^4P_{3/2}$, {8} multiplets. As shown in Figure 2, these $(\frac{3}{2}-)$ decuplet states are expected to lie high in mass value and it is possible that they still lie above the mass range covered in the present phase-shift analyses. On the other hand, these $(\frac{3}{2}-)$ octet states are expected to lie between the $^4P_{5/2}$ and $^4P_{1/2}$ configurations, to which we have assigned the $(\frac{5}{2}-)$ $N^*_{1/2}(1680)$ state and the $(\frac{1}{2}-)$ state $N^*_{1/2}(1700)$. One possibility is that the assignment of the $N\eta$ and $\Lambda\eta$ threshold effects to resonant states may still be in doubt and that the $(\frac{1}{2}-)$ N*(1700) should be assigned to the $^2P_{1/2}$, {8} configuration; in this case the $^4P_{1/2}$, {8} configuration would be expected to lie high above the as-yet-unobserved $^4P_{3/2}$, {8} configuration, as shown in Figure 2. It is also possible that these states may be strongly absorptive (at least for the N* member of the multiplet, where the search has been most detailed) in which case they could only be detected through the phase-shift analysis of exceedingly accurate elastic data (such states could be detected more easily from the study of multiparticle final states, of course).

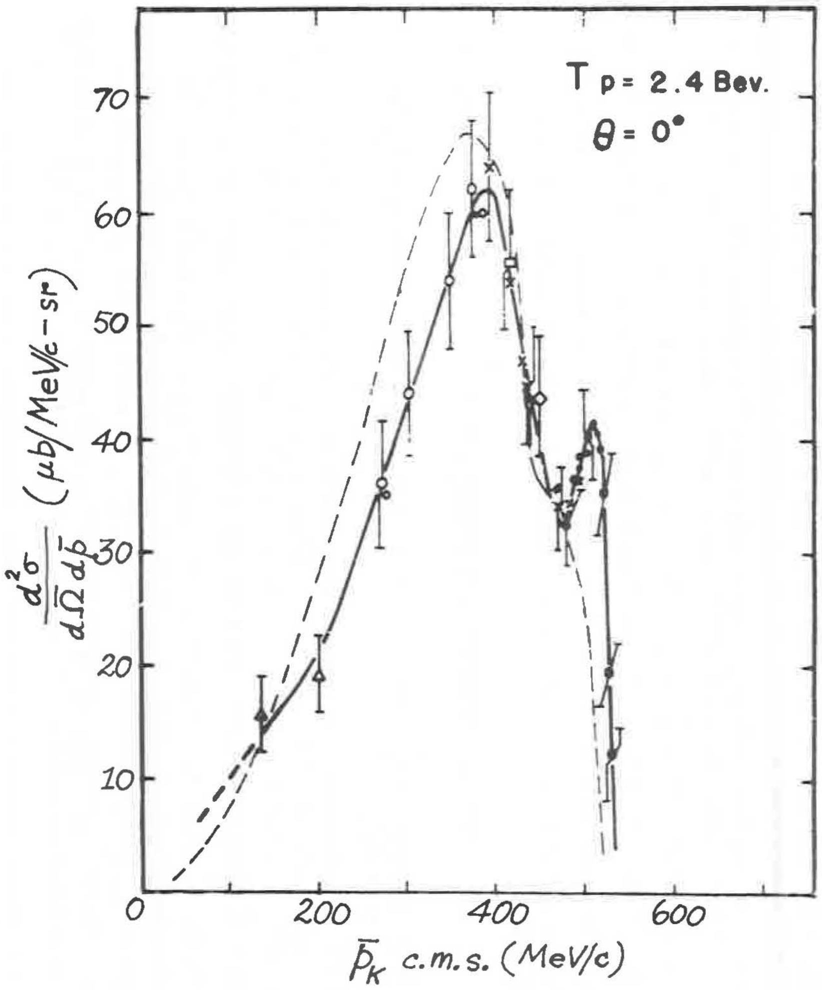

Figure 8a

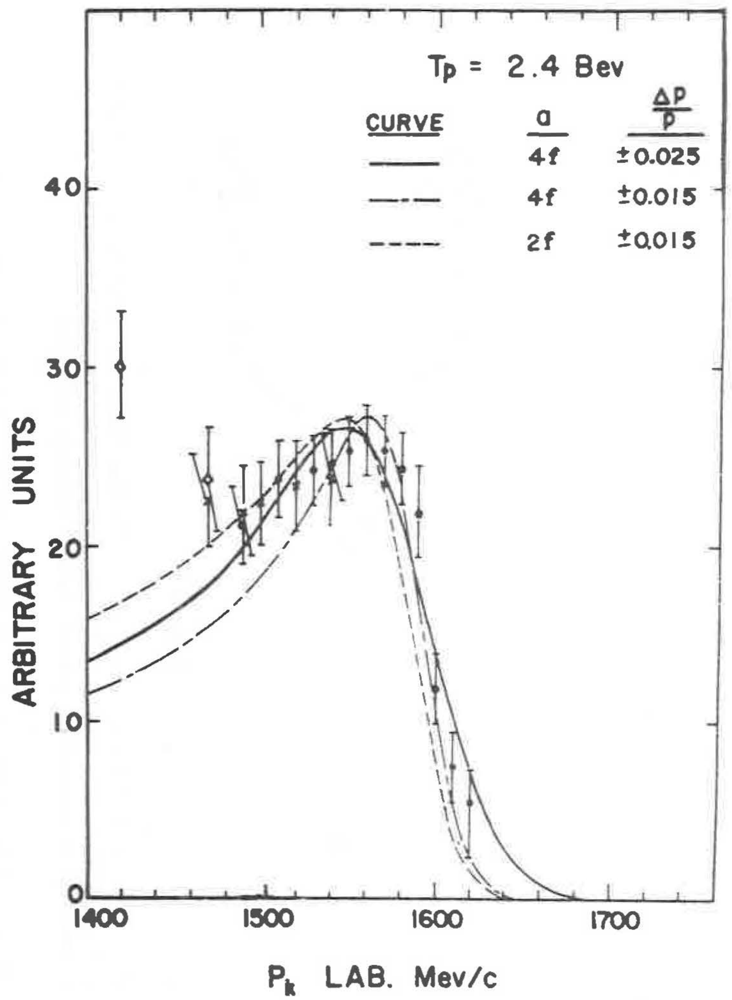

Figure 8b

The K^+ momentum spectrum observed by Melissinos et al.[68] at 0° for the reaction $pp \rightarrow YNK^+$ at 2.4 GeV proton laboratory energy is shown in Figure 8(a). The peak at the upper end of the spectrum is shown in detail in Figure 8(b). The fits shown to this peak correspond to phase space with the inclusion of an s-wave final-state Λp interaction with scattering length a and zero effective range, taking into account the experimental resolution.

We comment briefly on the pattern of states shown in Figure 2. The sequence of mass values observed for the configurations (^SL, $\{\alpha\}$) is in accord with an expression

$$M = M_0 + cS(S+1) + dF^2 \qquad (3.12)$$

with c and d positive; the value of d appears to be given (very roughly) by 18d $\approx$ 200 MeV, the value of c is not yet known and it may be quite small. The nature of the spin-orbit couplings is not easy to understand. As remarked earlier, the exchange of the vector meson V between two quarks would be expected to lead to attractive spin-orbit coupling, which would place the state of highest spin J lowest for the configurations ^SL with $S = \frac{3}{2}$, and highest for the configuration with $S = \frac{1}{2}$, but this does not correspond with the pattern observed. For the ^2P configurations, the highest spin J does lie highest in the $\{1\}$ and $\{10\}$ multiplets, whereas the reverse appears to hold in the $\{8\}$ multiplets; for the ^4P configuration, the spin-orbit coupling appears small (or strongly attractive, if the ($\frac{1}{2}$-) octet has been misidentified).

The next series of baryonic resonant states have positive parity. The established unitary multiplets are (($\frac{5}{2}$+), $\{8\}$) and (($\frac{7}{2}$+), $\{10\}$), the known members of the former being $N^*_{1/2}(1688)$ and $Y^*_0(1815)$, the known members of the latter being $N^*_{3/2}(1920)$ and probably $Y^*_1(2065)$[62]. It appears reasonable to expect these levels to correspond to L = 2 excitations of the QQQ system. If the wavefunction φ has the same permutation symmetry as the L = 0 ground configuration (this is possible since the spherical harmonic $Y_2^m(\mathbf{r}_{12})$ includes a symmetrical component, the configurations expected are (^4D, $\{10\}$) leading to decuplets with spin-parity $\frac{7}{2}$+, $\frac{5}{2}$+, $\frac{3}{2}$+, $\frac{1}{2}$+, and (^2D, $\{8\}$) leading to octets with spin-parity $\frac{5}{2}$+, $\frac{3}{2}$+. The absence of evidence for all but the multiplets of highest spin suggests that an attractive spin-orbit coupling is appropriate here. This group of states repeats the (S, $\{\alpha\}$) pattern of L = 0 states, the ($\frac{1}{2}$+) octet and the ($\frac{3}{2}$+) decuplet, but the level pattern expected has a different appearance because of the spin-orbit coupling.

There is also a low-lying baryonic resonance with positive parity, typified by the ($\frac{1}{2}$+) $N^*_{1/2}(1450)$, which provides quite a puzzle. The assignment ($\frac{1}{2}$+) with $I = \frac{1}{2}$ was first suggested by the fact that its excitation appears especially marked in the inelastic proton spectrum observed at small angles in proton-proton collisions, for example in the recent data of Bellitini et al.[63] This assignment has been confirmed from the πN scattering data by the phase-shift analysis of Bareyre et al.[54]; their P_{11} amplitude is shown in Figure 6 (earlier phase-shift analyses were in conflict about the energy-dependence of the P_{11} amplitude and the existence of a P_{11} resonance, but these suffered from lack of accurate data, especially polarization data, above the P_{11} resonance energy). We expect $N^*_{1/2}(1450)$ to be a member of a ($\frac{1}{2}$+) octet, but no other members of this octet are yet known. It is rather surprising to find the first excited ($\frac{1}{2}$+) octet less than 500 MeV above the baryon octet. It is hardly reasonable to suppose that this octet belongs to an L = 2 excitation, since that would imply the existence of a large number of positive-parity resonances (which are not seen below 1800 MeV); the L = 2 excitation just proposed above does not include a ($\frac{1}{2}$+) octet, and also lies much higher in mass. The most economical supposition is that this ($\frac{1}{2}$+) octet belongs to the configuration ($^2S_{1/2}$, $\{8\}$) with space symmetry S; Table I indicates that the other multiplet with the same space symmetry corresponds to the singlet configuration ($^4S_{3/2}$, $\{1\}$). Again, it is surprising that the L = 0 state with S symmetry should lie so low in mass relative to the L = 0 state with A symmetry. This interpretation would then require the existence of a ($\frac{3}{2}$+) Y^*_0 resonance in the low mass region, for which there is no indication at present.

4. Dibaryonic States

ΛN interaction.

The spin values of the light Λ-hypernuclei have been known for some time, ($\frac{1}{2}$+) for $_\Lambda H^3$, (0+) for $_\Lambda H^4$ and $_\Lambda He^4$, and ($\frac{1}{2}$+) for $_\Lambda He^5$. This leads to the conclusion that, for the ΛN system, the $^1S_0$ attraction is stronger than the $^3S_1$ attraction. Following this conclusion, the strength of the $^1S_0$ attraction is determined primarily by the binding energy of $_\Lambda H^3$, since its potential energy then has the form $(3V_s + V_t)/2$. The $^3S_1$ attraction contributes more strongly to the binding energy of $_\Lambda He^5$, for which the potential energy is $(V_s + 3V_t)$. Using hard core ΛN and NN potentials, rather reliable numerical calculations have now been made for $_\Lambda H^3$ by Smith and Downs[64] and for all the s-shell hypernuclei by Herndon et al.[65], giving consistent potentials V_s and V_t. In the extensive calculations of Herndon et al., the ΛN potentials were assumed to have hard core radius $r_c = 0.4$ F and intrinsic range b = 1.5 F (the intrinsic range found by DeSwart and Iddings[66] for their ΛN potentials calculated with a two-channel (ΛN and ΣN) formalism, for pion exchange processes generated through $\Lambda\Sigma\pi$ and $\Sigma\Sigma\pi$ couplings); the core nucleons were assumed to have the same spin structure as for the corresponding ground state nuclei. The ΛN potentials found in this way may be characterized by the scattering lengths $a_s = 2.9^{+0.6}_{-0.4}$ F, $a_t = 0.7 \pm 0.06$ F (since these are the parameters which are relatively insensitive to the details of the potential shapes, for example to whether or not there is a hard core repulsion) and the effective ranges $r_s = 1.9 \pm 0.1$ F, $r_t = 3.8 \pm 0.2$ F; the errors quoted here are only those arising from the uncertainties in the hypernuclear binding energies.

Table II

Curve (Fig. 9)	a_s(F)	r_s(F)	a_t(F)	r_t(F)	ΛN Interaction Form
-	$2.9^{+0.6}_{-0.4}$	1.9±0.1	0.7±0.06	3.8±0.2	Hard core potentials ($r_c = 0.4$ F, b = 1.5 F)[65]
A'	1.93	2.1	0.84	2.9	Yukawa (b = 1.48 F), with $^1P_1$ component for $_\Lambda H^3$.
B	1.93	2.1	1.4	2.3	Case A', with triplet potential arbitrarily increased.
C	1.37	1.1	1.0	1.2	Yukawa (b = 0.84 F), with $^1P_1$ component for $_\Lambda H^3$.
D'	2.3	2.1	0.77	3.6	Case A', with hard core repulsion included in potential.
F	2.3	2.1	1.4	2.8	Case D', with triplet potential arbitrarily increased.
G	3.0	3.0	1.0	5.5	Yukawa, with hard core repulsion and b = 2.07 F.

The singlet and triplet ΛN scattering parameters deduced from Λ-hypernuclei. The parameter sets A'-G have been taken from the paper of Bodmer[67]. Parameter set B is obtained from A' by increasing the strength (shape unchanged) of the triplet potential to give a fair fit to the data of Figure 9 on the Λp cross sections; parameter set F is similarly obtained from set D' by increasing the triplet potential strength. The intrinsic range for case C corresponds to a Yukawa potential with range parameter $\hbar/m_K c$; the intrinsic range for case G corresponds to a Yukawa potential with range parameter $\hbar/3m_\pi c$ outside a hard core repulsion $r_c = 0.42$ F.

Bodmer[64] has recently pointed out that such a strong ΛN spin dependence may be expected to give rise to spin flip transitions in $_\Lambda H^3$, leading to a $^1P_1$ component for the np system in $_\Lambda H^3$ (with which a p-orbital motion is required for the Λ-particle, by parity conservation). Since this $^1P_1$ component is energetically unfavorable, its inclusion in $_\Lambda H^3$ calculations naturally leads to the conclusion that the spin dependence in the ΛN interaction should be reduced; with the mean potential $(3V_t+V_s)/4$ fixed from $_\Lambda He^5$, this means a reduction in the $^1S_0$ scattering length, and an increase in the $^3S_1$ scattering length. The scattering lengths $a_s = 2.3$ F, $a_t = 0.77$ F thus obtained by Bodmer for hard core potentials are listed in Table II (case D'); they do not lie far outside the errors quoted for the scattering parameters obtained by Herndon et al.

Clear and direct experimental verification of the strength of the ΛN interactions has recently been obtained in two ways:

(i) Melissinos et al.[68] have examined the K^+ spectrum from the reaction

$$p + p \rightarrow \Lambda + p + K^+ , \qquad (4.1)$$

as shown in Figure 8. The feature of interest is the marked peak observed at the upper end of this spectrum, which corresponds to the low-energy threshold region for the Λp system. The natural interpretation of this peak is that it is due to a strong s-wave final-state interaction for the Λp system in this reaction (analogous to that known for the nn interaction in $\pi^- d \rightarrow nn\gamma$, and for the np interaction in $pp \rightarrow np\pi^+$); the best fit to the data corresponds to a scattering length a = 3 ± 1 F for the Λp state which gives rise to this peak.

(ii) the scattering cross section σ(Λp) has been measured between 120 MeV/c and 320 MeV/c laboratory momentum by Sechi-Zorn et al.[69] and Alexander et al.[70] from systematic searches for Λp scattering events following the Λ production reactions

$$K^- + p \rightarrow \pi^0 + \Lambda , \qquad (4.2a)$$

$$\rightarrow \pi^0 + \Sigma^0, \quad \Sigma^0 \rightarrow \Lambda + \gamma , \qquad (4.2b)$$

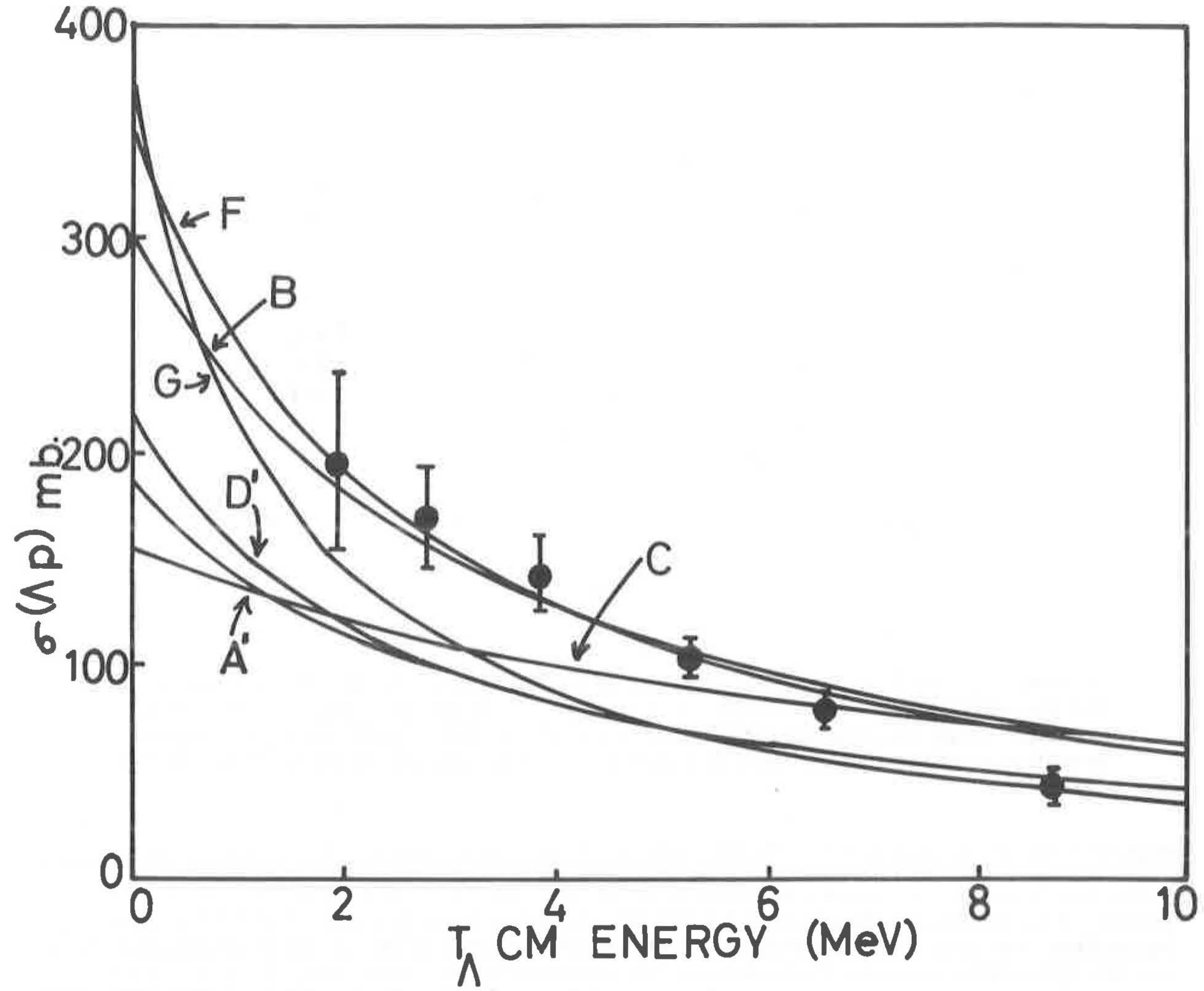

Figure 9

The total elastic cross-sections $\sigma(\Lambda p)$ observed by Sechi-Zorn et al.[69] and by Alexander et al.[70] are plotted as function of the Λ c.m. energy, and are compared with calculated curves corresponding to the expression (4.3) for the various sets of $^3S_1$ and $^1S_0$ ΛN scattering parameters, listed in Table II.

for K^- mesons coming to rest in hydrogen. Since their cross-section values are in excellent agreement, we have combined their results; the cross sections are shown on Figure 9, where they are compared with the cross-sections calculated for the singlet and triplet ΛN scattering parameters deduced from the analysis of the light hypernuclei, using the expression

$$\sigma(\Lambda p) = \frac{\pi}{(\frac{1}{a_s} + \frac{1}{2}r_s k^2)^2 + k^2} + \frac{3\pi}{(\frac{1}{a_t} + \frac{1}{2}r_t k^2)^2 + k^2} \qquad (4.3)$$

where k denotes the Λp c.m. momentum. We note that the calculated cross-sections are quite insensitive to the existence of a hard core repulsion, especially in the energy range of the cross-section data. For all the appropriate sets of scattering parameters (A', D', C and G), the experimental cross-sections are systematically and significantly larger (by about 30%, typically) than the calculated cross-sections.

The expression (4.3) for $\sigma(\Lambda p)$ is weighted in favour of the triplet scattering parameters. Consequently, the empirical cross-section values are readily reached by a moderate increase in the triplet potential, corresponding to an increase in a_t from 0.77 F to 1.4 F, as shown by curves B and F on Figure 9. We note that the steepness of the calculated cross section as function of the c.m. energy increases monotonically with the intrinsic range; the choice b = 1.5 F appears to be not much smaller than the optimum value.

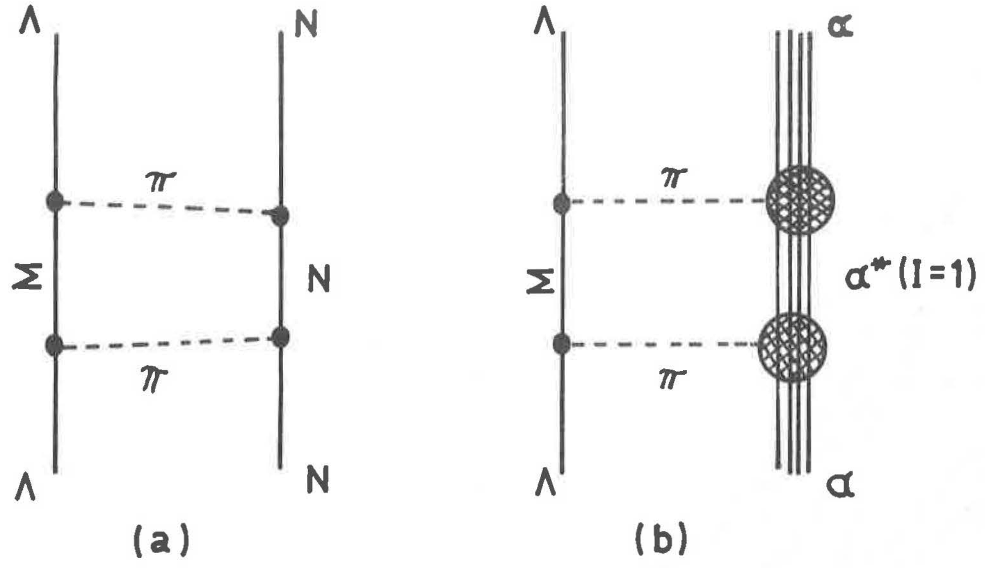

Figure 10

Diagram showing (a) a two-pion exchange graph contributing to the ΛN interaction, such that the intermediate state consists of ΣN only, and (b) the corresponding two-pion exchange graph contributing to the Λ-He^4 interaction. In the latter case, isospin conservation requires that the intermediate state without pions be of the form Σ -$He^{4*}(I = 1)$.

Bodmer[67] has suggested that the ΛN interaction may be suppressed in ${}_{\Lambda}He^5$, owing to an effect due to the requirements of isospin conservation in the intermediate states giving rise to this interactions. It is known that the graph of Figure 10(a) is a particularly strong contributor to the $^3S_1$ ΛN interaction; one-pion exchange generates a strong tensor force in the off-diagonal potential $V_{\Sigma\Lambda}$, and this contributes strongly to the s-wave ΛN scattering in the triplet state[66]. This graph is characterized by a ΣN intermediate state, whose threshold energy is only 80 MeV above that for the ΛN system. For the corresponding graph in the Λ-He^4 interaction, the intermediate Σ-He^4 state is forbidden by isospin conservation (and by parity conservation, which forbids $He^4 \rightarrow He^4 + \pi$). The lowest intermediate state of this type which is allowed is Σ-He^{4*}, where He^{4*} denotes the I = 1 excited state of He^4, at about 20 MeV excitation energy. This increase in the energy denominator for this graph corresponds to a suppression of the contribution of this process to the effective triplet potential V_t. Bodmer has made a rough estimate of this effect, based on the meson-theoretic potential calculations of De Swart and Iddings[66], simply by increasing the threshold energy for the ΣN channel in their coupled channel calculation by the excitation energy E*. For parameters corresponding to a_t = 1.4 F, the value E* = 30 MeV leads to a reduction to a_t = 1.2 F, E* = 80 MeV leads to 0.8 F, so that it appears plausible that this effect could be sufficient to reconcile the hypernuclear parameters with the Λp cross-section data.

Dibaryonic States

It is of interest to consider the dibaryonic systems in relation to broken SU(3) symmetry. The deuteron state is a member of the $\{\overline{10}\}$ representation; the corresponding potential $V(\overline{10})$ is therefore sufficiently strong to generate a bound state for the NN system. The $^1S_0$ NN system is a member of the {27} representation; the potential V(27) is almost strong enough (well depth parameter about 0.95) for binding of the NN system. For the potentials appropriate to the other substates of these representations, two symmetry-breaking effects are of particular importance:

(i) insofar as the BB potential is due to meson exchange, the large mass differences between the pseudoscalar mesons introduces a strong symmetry-breaking for these potentials. For example, one pion exchange generates a strong tensor potential in the $^3S_1$ NN system; the corresponding K-meson exchange in the ΛN potential generates a tensor potential whose range is about 3.5 times less and which is correspondingly ineffective in low-energy ΛN interactions.

(ii) the various BB channels which contribute to a given substate of the unitary multiplet $\{\alpha\}$ have thresholds which are often widely separated, owing to the large mass-splittings within the baryon octet. If these channels are s-wave, or if the potential interactions are of long range (so that the centrifugal barrier provides little shielding of the interior state), these threshold splittings can lead to very strong distortion from the pattern of resonance states expected for the representation $\{\alpha\}$, as pointed out by Oakes and Yang[71].

Deloff and Wyld[72] have carried out an instructive calculation on point (ii) for the Y = 1 BB systems. They neglect the possibility (i) and adopt unitary symmetric potentials $V(\overline{10})$ and V(27) which fit the $^3S_1$ and $^1S_0$ NN data, respectively. The Y = 1 $\{\overline{10}\}$ state has the form $(\Lambda N + \Sigma N)/\sqrt{2}$, and so the two-channel potential effective in this case (neglecting the potential contributions arising from other representations) has the form

$$\begin{pmatrix} \frac{1}{2}V(\overline{10}) & \frac{1}{2}V(\overline{10}) \\ \frac{1}{2}V(\overline{10}) & \frac{1}{2}V(\overline{10}) \end{pmatrix} \begin{matrix} \Lambda N \\ \Sigma N \end{matrix} \tag{4.4}$$

For channel threshold separation $\Delta = 0$, the Schrodinger equation diagonalizes, with eigenpotential $V(\overline{10})$; for large separation $\Delta \to \infty$, the potential is $\frac{1}{2}V(10)$ in the ΛN channel, too weak to give rise to a bound state. The physical situation (Δ = 80 MeV) is intermediate; Deloff and Wyld find that this potential (4.4) then gives no bound state, nor any resonant state (i.e. bound ΣN state which can decay through its coupling to the ΛN channel) for the Y = 1 $^3S_1$ system.

For the {27} representation, the Y = 1, $I = \frac{1}{2}$ state has the form $(3\Lambda N + \Sigma N)/\sqrt{10}$, for which the two-channel potential is

$$\begin{pmatrix} \frac{9}{10}V(27) & \frac{3}{10}V(27) \\ \frac{3}{10}V(27) & \frac{1}{10}V(27) \end{pmatrix} \begin{matrix} \Lambda N \\ \Sigma N \end{matrix} \tag{4.5}$$

Here the diagonal potential $V_{\Lambda\Lambda}$ is 0.9 times V(27); since the ΛN reduced mass is 9.5% larger than for the NN system, this state is quite close to binding, especially when the attraction arising from the off-diagonal potential $V_{\Lambda\Sigma}$ is included. As a result, even with this crude model, Deloff and Wyld found stronger ΛN scattering in the $^1S_0$ state than in the $^3S_1$ state, in accord with the present indications.

Other dibaryonic systems could well have bound states. For the $\{\overline{10}\}$ representation, the $I = \frac{3}{2}$, Y = -1 state has the form $\Xi\Sigma$, with potential $V(\overline{10})$. Since the reduced mass is 30% larger than for the NN system, this $\Xi\Sigma$ system would have quite strong binding (however, with $f \approx 0.4$ for the meson-baryon coupling, the one-pion-exchange tensor potential is very much less than for the NN system, since $G_{\Xi\Xi\pi}G_{\Sigma\Sigma\pi}$ is then only $-0.16\ G^2_{NN\pi}$, so that this bound system could well fail to exist as a result of the symmetry-breaking effect (i)). For the {27} representation, the $I = \frac{3}{2}$, $Y = \pm 1$ states, the I = 2, Y = 0 state and the I = 0, Y = -2 state each have potential V(27). In view of their large reduced mass, each of these systems could well be bound; again, the symmetry-breaking effects (i) need to be examined for these systems (although the exchange of one pseudo-scalar meson does not represent such a major contributor for the $^1S_0$ potential).

This situation illustrates explicitly the sensitivity which (as was pointed out by Oakes and Yang[71]) s-wave bound states and resonances may show with respect to the SU(3) symmetry-breaking produced by the separations introduced between the channel thresholds by the mass splittings which occur within the low-lying meson and baryon multiplets. It is clearly possible for some of the submultiplets of a given unitary multiplet simply not to occur, either as bound states or as resonances, if the binding of the unitary multiplet is due to the long range forces between the mesons and baryons. On the other hand, if the dibaryonic bound states were due to quark interactions, these states would only be shifted, but would not go away, when the physical channel thresholds were introduced, since the cause of their existence would then have little to do with these channels. In fact, these bound states and resonance states would appear only as CDD poles for these physical channels.

The discussions in the literature on dibaryonic (and multibaryonic) unitary multiplets tacitly assume that these states are due to very short range forces (for example due to the interactions between quarks), so that the splitting of the thresholds for the lowest sets of channels linked by unitary symmetry and the mass splittings among the lightest mesons are only of minor importance for the properties of the multiplet states. From what is known of the properties of the deuteron (and of other light nuclei) and their relation with the properties of the pion and the pion-nucleon interaction, there appears little reason to believe that this assumption is at all appropriate to the dibaryonic (and multibaryonic) systems. In fact, it appears quite an interesting possibility that the interactions in the six-quark system may actually correspond to the short range repulsion which occurs between two baryons.

5. Conclusions

The value of the quark model lies in the fact that it provides a qualitative basis for understanding the patterns observed for the mesonic and baryonic resonance states in physical terms, in terms of the properties of the interactions between quarks and antiquarks, which allows the possibility of co-ordinating the properties observed for different multiplets. Of course, it is entirely possible that whatever parallelism is found between the data and the simple quark model could simply be a reflection of the existence of general relationships which might hold in a more sophisticated and complicated theory of elementary particle stuff. To date, the success of the quark model in providing a simple interpretation for the data is rather spotty, yet not insignificant. It appears reasonable to conclude that the $\bar{Q}Q$ and QQ interactions are essentially independent of spin and unitary spin. The deviations from this σ- and F-independence are as follows:

(i) A weak unitary-spin (and perhaps spin) dependence of the QQ interaction is required for the baryonic states, to provide the term $(cS(S+1) + dF^2)$ for the mean masses of the unitary multiplets belonging to the QQQ configurations with a given permutation symmetry for the space wavefunction. The splittings observed are typically of order 200 MeV. For the mesonic states, the separation of the pseudoscalar octet from the pseudoscalar singlet and the vector meson states is strong (of order 600 MeV) and suggests that the $\bar{Q}Q$ interaction has a substantial spin dependence as well as unitary-spin dependence.

(ii) A spin-orbit coupling which is repulsive for the $\bar{Q}Q$ system. This leads to the sequence of spin-parity values observed for the even-parity mesonic states; the spin-orbit splitting is of order 100 MeV. For the baryonic states, the situation appears obscure; the spin-orbit splitting again appears to be typically of order 100 MeV (e.g. $Y_0^*(1520)$-$Y_0^*(1405)$ = 115 MeV) but its sign appears to differ from one unitary multiplet to another in the L = 1 configurations in a way which is not completely understood. For the L = 2 multiplet it is clear that the QQ spin-orbit coupling must be attractive.

(iii) An SU(3) symmetry-breaking due mainly to the quark mass difference Δ, for which a typical estimate is of order $\Delta \approx 100$ MeV. There may also be SU(3)-breaking potentials; in fact this appears definitely to be necessary for the QQ system. The dominance of the term Δ would provide a simple and natural explanation for the systematic nature of the mass-splittings among the different unitary multiplets. For the mesonic states, the mass M^* appears to lie about 100 MeV above M_1, at least for the (0-), (1-), (2+) and possibly $(1+)^\pm$ nonets (for the pseudoscalar mesons, see Eq. (2.8)). For the baryonic multiplets, the isospin substates always have mass increasing with decreasing hyper-charge. The estimates obtained for Δ are as follows[73]:

(i) for L = 0 multiplets. Δ = 147 MeV for the $(\frac{3}{2}+)$ decuplet and 190 MeV for the $(\frac{1}{2}+)$ octet,

(ii) for L = 1 multiplets. $\Delta \approx 85$ MeV for the $(\frac{5}{2}-)$ octet ($Y_1^*(1765)$-$N_{1/2}^*(1680)$). $\Delta \approx 150$ MeV for the $(\frac{3}{2}-)$ octet ($[\Xi_{1/2}^*(1816)$ - $N_{1/2}^*(1516)]$)/2 and ($Y_1^*(1660)$ - $N_{1/2}^*(1516)$),

(iii) for L = 2 multiplets. $\Delta \approx 125$ MeV for the $(\frac{5}{2}+)$ octet ($Y_0^*(1815)$-$N_{1/2}^*(1690)$). $\Delta \approx 145$ MeV for the $(\frac{7}{2}+)$ decuplet ($Y_1^*(2065)$-$N_{3/2}^*(1920)$).

At least all these estimates have the same sign and are of the same order of magnitude. The differences between them can probably be attributed to the SU(3)-breaking potential terms whose contributions will vary in structure (i.e. in I, Y dependence) according to the configuration considered.

It is remarkable that all these three interactions, those which separate the unitary multiplets belonging to a given supermultiplet and those breaking unitary symmetry, appear to be of comparable order of magnitude.

We now need accurate experiments with large statistics to search for and to identify all of the members of these unitary multiplets (as well as to determine the patterns of the higher supermultiplets), and to determine their decay properties. This data would provide many tests for the assignment of these supermultiplet configurations, and will provide the necessary information for us to develop some understanding of the nature of the spin and unitary-spin dependence of the QQ and $\bar{Q}Q$ interactions and of the SU(3)-breaking mechanisms.

It should be emphasized here that the states discussed are all considered to exist in consequence of the forces between quarks. This interpretation means that the long range forces usually considered in the dynamical discussions attempted in the literature for these resonant states are essentially irrelevant as concerns the existence of these resonance states. These long range forces would certainly be expected to affect the precise location of the resonant states but their effects are not then the cause of these resonant states. In this view, these resonant states would appear as CDD poles in such theoretical discussions of them in terms of the channels open at or near the resonance energy. This view receives some support so far from the fact that the complete multiplets identified to date satisfy the SU(3) patterns and relationships so well; if these states were primarily due to long range forces, it would appear reasonable to expect strong distortions from the SU(3) patterns to occur, since these would generally be significantly sensitive to the threshold separations between the various channels open near the resonance energy and related by unitary symmetry, and to the mass splittings within the unitary multiplets of particles whose exchanges generate these long range forces. On the other hand, it is probable that not all bound states and resonances are due only to the short range forces. For example, the existence of the deuteron is believed to result from the long range forces between two nucleons due to the exchange of pions between them.

From the former viewpoint (short range forces), the deuteron should be treated as a six-quark system and its existence would be understood as due to the strong, short-range QQ interactions. From the latter viewpoint (long-range forces), the deuteron is to be treated as a two-baryon system, the binding attraction being provided by interactions occurring when the the two baryons are well separated (as two spatially separate three-quark systems), their interaction being strongly repulsive at close approach (where the two quark-systems overlap). These two situations are, of course, not necessarily completely distinct. For the case of the deuteron, the difference is whether the interactions are strongly attractive for a system of six closely-packed quarks, or whether the interactions are repulsive in this situation, in which case the two three-quark systems prefer to keep spatially separate, in a physical situation such that there exist the long range attractions available and necessary to provide the binding of the system. In general, it may be possible for both short and long range interactions to contribute to the attraction needed to produce the resonant state.

The system of Be^8 provides an interesting (although incomplete) analogy to the two-baryon situation, as seen from the quark model. Although Be^8 consists of eight nucleons, it prefers to break up into the two alpha-particle system. The forces between the α-particles are strongly repulsive at short distances (due to the effect of the Pauli principle), attractive at intermediate distances, and repulsive again at large distances (due to Coulomb interaction); the α-α potential can be deduced to a considerable extent from the analysis of α-α scattering data. The Be^8 ground state (unstable, but long lived on the nuclear scale) might then be regarded as the result of the long-range α-α attraction, as the overlap between the two alpha-particles is unimportant in this ground state for a large fraction of the time. On the other hand, the Be^8 nucleus may be regarded more properly as a piece of nuclear matter, due to the strong attraction between the nucleons, which happens to be unstable with respect to the energy of two separated α-particles and whose break-up involves an intermediate stage in which the two α-particles remain relatively close for very many nuclear periods, owing to their low energy release and their difficulty to escape through the potential barrier arising from the Coulomb repulsion. We are accustomed to think of the deuteron in terms of the first of these pictures, yet it is not excluded à priori that the second picture could be the correct dynamical description. As emphasized by the dispersion theorists, the relationship between the asymptotic form of the deuteron and the one-pion exchange interaction is one which follows from rather general principles and which does not depend on the dynamical origin of the deuteron. However, the absence of any evidence for the existence of other members of the $^3S_1$ BB $\{\overline{10}\}$ multiplet (especially for the Y = 1, $I = \frac{1}{2}$ system) does argue against the view that these states are due to the short range interactions in the six-quark system.

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