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Causal Sets Dynamics: Review & Outlook

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Abstract.

Causal sets is an approach to quantum gravity, where spacetime is replaced by a causal set. It is fundamentally discrete, and the causal relations between spacetime elements is the only structure that remains. A complete theory should have (i) kinematics (ii) dynamics and (iii) phenomenology. In this contribution we will explore the dynamical part of the theory, focusing on recent developments. We will analyse (a) classical dynamics *of* the causal set, (b) quantum dynamics of matter and fields on a classical causal set and finally (c) quantum dynamics of the causal set.

1. Motivation

Constructing a quantum theory of gravity, consistent, widely accepted and confirmed by experiments is probably the most important open problem in current theoretical physics. The mathematical and conceptual difficulties that arise, suggest we may need to abandon some of the a-priori assumptions we make about nature. This is essentially done in all approaches to quantum gravity. In this contribution, we examine the causal sets approach (first appeared in [1] while in [2] some reviews can be found). It is the nature of spacetime as a differential manifold with a Lorentzian metric on it that is questioned in this approach. It is replaced by a causal set that is (i) a discrete entity with (ii) only remaining structure the causal relations between spacetime points. Both assumptions, discreteness and causality are based on indications from current physics and naturality arguments.

Approaches to quantum gravity, that start with a continuous spacetime, observe a possibly effective, discreteness. Examples of the above, are the discreteness of the volume operator in Loop Quantum Gravity and the dualities between small and large scales in string theory. Discreteness of spacetime, could also remove the infinities we face in quantum field theory, which are problematic even though we have "learned living" with them. Furthermore, singularities in general relativity could be avoided. Indications for discreteness of spacetime, exist in black hole thermodynamics, where the finiteness of the entropy that is proportional with the area of the horizon can be understood easier if that corresponded to the number of fundamental

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area units that cover the horizon. Finally, certain modifications of gravity that account for the smallness of the cosmological constant also point towards discreteness of spacetime. All these arguments suggest, that a natural starting point for a quantum gravity theory, would be a discrete spacetime.

The second feature that is used in causal sets in order to replace the standard view of spacetime, is the *causality*. The concept that two spacetime points are causally related, has the direct explanation, that objects/fields at the one (the one being "earlier") can affect objects/fields at the other. It is a natural and intuitive concept. Even though we are used to the standard view of spacetime, arguably the topology and differential structure and the Lorentzian signature metric, are far more abstract concepts than the simple notion of causality. Moreover, as we will see below, the causal relation can encode most of the information of the Lorentzian signature metric on a manifold.

In section 2 we will define mathematically a causal set and explain in which sense it can replace spacetime, give some basic definitions that will be required later on and divide the analysis of a theory on 3 parts, the kinematics, dynamics and phenomenology. Section 3 is the main part, dealing with the dynamics of causal sets. In section 3.1 we examine the classical dynamics of a causal set, in section 3.2 the developments that have occurred in the quantum dynamics of matter and fields on a classical/fixed causal set and in section 3.3 the quantum dynamics of the causal sets themselves. Finally in section 4 we will summarise and conclude.

2. Definition and basics

Causal set is the mathematical entity that replaces spacetime in this approach to quantum gravity. Mathematically is a set C with the following features.

(i) A partial order relation $\prec$ which is (a) irreflexive ($x \not\prec x$) and (b) transitive ($x \prec y \prec z \Rightarrow x \prec z$)

(ii) The partial order corresponds to the causal relation between elements of C, so if $x \prec y$ it means that x is in the past of y.

(iii) Locally finite: $[x, y] \equiv |\{y \text{ such that } x \prec y \prec z\}| < \infty \ \forall \quad x, z \in C$. And $|A|$ indicates cardinality of the set A. It is this condition that imposes the discreteness of spacetime, since it requires that between every pair of elements of the causal set, to be only a finite number of other elements.

Here we should point out that the standard spacetime, if we use the metric to define causal relations, satisfies the first two conditions we required above and it is only the locally finiteness condition that is not satisfied.

Following the definition of a causal set, an explanation is needed on why could such an entity replace the continuum spacetime. It is based on works of Hawking [3] and later of Malament [4] and in particular on this theorem by Malament:

> The metric of a globally hyperbolic spacetime can be reconstructed uniquely from its causal relations up to a conformal factor.

This theorem states that most of the information of the metric is encoded in the causal relations of the spacetime points. Having the conformal metric, means that the metric is completely fixed, once we determine the spacetime volume of each neighborhood. However, the discreteness of spacetime provides a very natural way to fix the volume. Since each spacetime interval, by definition, has a finite number of elements, by assuming that each element corresponds to some fundamental discrete volume (Plank volume), the volume of each spacetime interval is uniquely

determined. In other words we make the correspondence "Volume = number of elements × Plank volume". In a schematic way we have that "Order + Number = Geometry ". This leads to the central conjecture of causal sets (also called the Hauptvermutung):

> **Central Conjecture**: Two distinct, non-isometric spacetimes cannot arise from a single causal set.

The conjecture is still not proven, however it has very solid theoretical and numerical evidence for its validity[1]. Having a causal set as the real structure existing in nature, we may ask under what circumstances a spacetime (Lorentzian manifold) can be said to approximate a given causal set. To make the above precise we define the concept of a *faithful embedding*:

> A faithful embedding is a map ϕ from a causal set $\mathcal{P}$ to a spacetime $\mathcal{M}$ that:
> (i) preserves the causal relation ($i.e., x \prec y \iff \phi(x) \prec \phi(y)$)
> (ii) is "volume preserving", meaning that the number of elements mapped to every spacetime region is Poisson distributed, with mean the volume of the spacetime region in fundamental units, and
> (iii) $\mathcal{M}$ does not possess curvature at scales smaller than that defined by the "intermolecular spacing" of the embedding (discreteness scale).

The central conjecture can be restated as "a causal set cannot be faithfully embedded in two non isometric spacetimes".

While physically, it is the spacetime approximating the causal set, we can ask the inverse question and in particular to ask to find a causal set that approximates a given spacetime. This will happen, when the number of elements of the causal set is large and we speak of continuum approximation, which is the analogue of a continuum limit[2].

An interesting thing to point out, is that following the definition of a faithful embedding, we can see that a regular lattice does not correspond to Minkowski spacetime since there is no faithful embedding between them. The condition (ii) states that *every* spacetime interval should have a number of elements of the causal set mapped that is proportional to the volume (on average). Considering a boosted frame one can easily notice that there are large volumes with no elements at all. It is well known that there is a tension between discreteness and Lorentz invariance. However, we *can* have a causal set such that it faithfully embeds in Minkowksi spacetime, by considering a random lattice. One can generate such a lattice, by sprinkling elements in Minkowski spacetime, randomly with probability $P(n) = \frac{(\rho V)^n \exp^{-\rho V}}{n!}$ and variance $\sqrt{V}$ which is called a Poisson sprinkling.

Causal sets constructed in this manner, respect Lorentz invariance at the kinematic level, and evade the above mentioned tension between discreteness and Lorentz invariance. Other discrete approaches, do not satisfy this condition on kinematic level and expect Lorentz-invariance to emerge at a later stage (e.g. through superposition of Lorentz-violating objects). Further analysis of the above can be found in [11].

We should note that according to the definition, a given causal set is not necessarily faithfully embedded in a manifold. In other words, a random causal set does not always correspond to some continuous spacetime. Moreover, if we take a typical causal set consisting of a large number of elements n, it does not look like a manifold, but rather as a Kleitman-Rothschild

[1] Most of the work on the kinematics of causal sets, regarding the dimension (e.g. [5, 6]), topology ([7]), timelike and spacelike distances ([8, 9]), provide support to the conjecture.

[2] It is analogue, since the actual limit of taking some scale going to exactly zero, is never taken as for example in CDT [10].

(KR) order [12]. A typical causal set is selected, by pure counting, if we assume that any possible order (causal set) has equal weight. The KR orders have only three layers (i.e. longest chain is a 3-element chain) and 1/4 of elements are in the first layer, 1/2 in the second and the remaining 1/4 in the third. These causal sets are clearly not manifold-like (they have only "three moments of time"). It remains a task for the dynamics of the theory, to select a causal set that can be faithfully embedded on a continuous spacetime rather than one that is of the KR type.

To this end, we will give some definitions for causal sets that will be needed later in the text. We call *links* a pair of elements $x \prec y$ such that $\nexists\ z$ such that $x \prec z \prec y$. In other words two elements are linked, if there is no element "between" them. We call *chain* C, a collection of elements that $\forall \quad x, y \in C$, either $x \prec y$ or $y \prec x$. In other words, all the elements in the cain are ordered, and it is the causal set analogue of a timelike curve. Closely related is the definition of a *path* which is a chain with the extra condition that each pair of consecutive elements are links. An *anti-chain* is a collection of elements A, that $\forall \quad x, y \in A$ neither $x \prec y$ nor $y \prec x$. I.e. it is a collection of unrelated elements. To consider a *maximal* anti-chain, means that it is an anti-chain such that all the remaining elements of the causal set are to the past or to the future of at least one element of the anti-chain, i.e. we cannot extend the anti-chain and still remain anti-chain. The maximal anti-chain is the causal set analogue of spacelike surface.

When constructing a theory, there are three elements that one needs to address. First is the kinematics of the theory, and for causal sets, one can see how the concepts of dimension [5, 6], topology [7], distances (temporal and spatial) [8, 9], and geometry can arise. Second is the dynamics, that is the topic of this contribution. Finally, we have the phenomenology. It deals with the observable consequences that the novel elements of the theory lead us. For causal sets people have explored the cosmological constant problem that attains a potential solution [13, 14], entropy bounds and black hole entropy and thermodynamics [15, 16] and deviations/diffusion of motion due to the fundamental discreteness [17].

3. Dynamics

While from the kinematics of a theory one can gain some insight in the understanding of nature, the basic physical content of a theory is encoded in the dynamics. We will divide our analysis into three parts. Firstly we will examine the possible classical dynamics that a causal set can have. This should be done in a way that is intrinsic to the causal set itself and make use solely of the causal relations. The analogue in continuum physics, would be the theory of general relativity which is the classical theory for spacetime. Secondly, we will explore, the dynamics of quantum matter and fields on a given "classical" causal set. This is the analogue of quantum field theory on a fixed curved spacetime. Finally we will analyse, the possible quantum dynamics of the causal set itself, which is the final aim in order to construct a quantum theory for gravity.

There are two issues that one needs to consider when, in the attempt to quantise gravity, takes as starting point an alternative (possibly discrete) structure for spacetime. The first issue is the so-called "entropy" problem. It is common in all discrete approaches. By pure counting, a typical discrete structure (in our case a causal set) does not correspond in the appropriate limit to a continuous spacetime. As we have seen the typical causal set is a KR order. In constructing the appropriate dynamics, one should overcome this difficulty and obtain a dynamical way of selecting causal sets that do correspond to continuous spacetimes.

The second issue, is which direction one would take for the (classical or quantum) dynamics. On can have a *bottom-up* point of view, where we start from the fundamental relations of causal

sets and by imposing conditions that are natural to the order itself we obtain the dynamics. This is the most ambitious but also principled way. Alternatively, we can have a *top-down* approach, where one gets motivation from continuous spacetime, rephrase the concepts in terms of the causal sets relations and uses them in order to obtain the dynamics. While, this is not conceptually preferable, if assume that nature *is* a causal set in reality, technically is easier to proceed and it may lead to useful conclusions.

Finally, having formally quantise causal sets is not the end of the story. To make contact with experiments, one needs a proper way to interpret the theory. There are two aspects that need consideration. First one needs to formally construct questions that are invariant from the arbitrary choices that were made during the quantisation process and do not correspond to physical degrees of freedom. This is guaranteed by dealing with re-labelling invariant questions, which is the discrete analogue of diffeomorphisms invariance. The second aspect has to do with the interpretation of quantum theory for the case that we have a single closed no-repeatable system. The problem is twofold, first we need an interpretation that does not require an external observer and secondly that gives predictions for single systems rather than ensembles of identically prepared systems. These thoughts, along with the inability of having a canonical approach due to the fundamental spacetime nature of causal sets, leads to interpretations based on histories, such as the co-event formulation [18] or the decoherent histories approach [19].

3.1. Classical Dynamics for Causal Sets

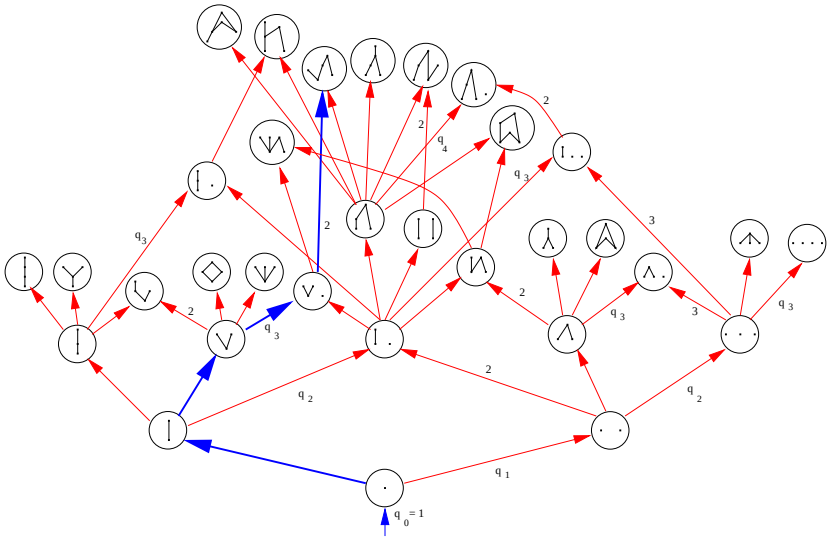
The classical dynamics of causal sets that have been analysed so far, belong to the "bottom-up" approach. We start with the basic concepts of causal sets, and by imposing certain physical conditions we generate the most general dynamics that can be obtained. The dynamics that we consider, have certain free-parameters that along with assumptions on the initial condition, determine the type of "evolution" of a causal set. In this general set up, the dynamics are *not* deterministic, but rather stochastic. For a more detailed description the reader is directed to the original references [20].

The arena that the dynamics of causal sets take place, is the set of all causal sets. This set has a natural partial order on it, which is that two elements (i.e. causal sets), are related, if the one is subset of the other, when viewed as partial orders. This partial order of partial orders, is called *poscau* (see Fig. 3.1). It is the analogue of superspace for geometrodynamics, or more precisely in our case, the space of all spacetimes.

The dynamics we will consider, generate the causal set in the following manner. We start with one element, and then grow the causal set by adding one more element at every step/level. In each level, the newborn element has to be determined whether it is at the past or future or unrelated to every other, already existing element of the causal set. This corresponds to a move from one point of poscau to another, since the old causal set is obviously subset of the new. The most general dynamics, would allow with some probability any of those transitions. This process of growing the causal set with adding a new element, with some probability of where to add it, is called *Classical Growth Dynamics* (CGD). This process has an "internal/parameter time" which is the number of born elements, which however, should not be confused with the time observed by observers in the causal set itself. These stochastic dynamics are actually generalisations of random walks.

However we are not totally free to choose any parameter for the probabilities of the newborn elements. There are the following physical conditions that our probabilities are required to satisfy, that turn out to be very restrictive.

(i) **Internal temporality**. Each new element is born to the future or spacelike to all existing



The poset of finite causal sets

elements (never at the past). A natural requirement, that guarantees that the "present time" causal set (in the internal time defined above), is not affected by adding later more elements.

(ii) **Discrete general covariance**. The probability of arriving at a particular causal set does not depend on the order the elements are born. In other words the probability does not depend on the path chosen on the Figure, but only on the starting and ending point on the poscau. This condition guarantees label-invariance of the theory and it is the discrete analogue of diffeomorphism invariance at general relativity.

(iii) **"Bell's causality"**. The probability for the newborn element depends only on the elements of the existing causal set that belong to the past of the new element and *not* from the total causal set. This condition guarantees some sense of locality for the evolution of the causal set. Quantum paradoxes, such as the Bell inequalities, that arise due to non-locality gave the name for this condition. It is therefore believed, that for quantum dynamics of the causal set this condition might need to either be dropped or relaxed to accommodate those paradoxes. However, non-local effects on a causal set may arise even if the dynamics of the causal set itself are local in the way defined above and therefore altering this condition is not theoretically necessary.

(iv) **The Markov Sum rule**. Starting from a given causal set with n elements (i.e. a point at the poscau) and adding a single element, give rise to many possible different causal sets with $n+1$ elements. The sum of the probabilities for all these $n+1$ elements causal sets, need to sum up to one.

The above 4 conditions, are very well physically motivated and seem very general. However, they restrict the possible parameters and there is only *one* free parameter for each level of the growth process. By each level, we mean, the total number of elements of the causal set (see details in [20]). The relation these parameters have between them give rise to different dynamics. For example, setting them equal, give rise to the well known and explored example of transitive percolation (see references in [20]). The choices for dynamics are still many (remember that a typical realistic causal set has huge number of elements), and the resulting causal sets differ. Since we have started with concepts intrinsic to causal sets we have followed

a strictly bottom-up approach, and this is how far we can reach without resorting to concepts arising from continuous spacetimes. Despite the fact that certain choices of the parameters give rise to causal sets with notable similarities with cosmological models [21] (and certainly nothing like the KR typical causal sets), it is believed that the resulting causal sets are not in general manifold-like. It is thus believed that in order to explain the continuum-like behaviour, we may need to resort to quantum dynamics, as we will see in section 3.3.

3.2. Quantum Matter on a fixed Causal Set

The second type of dynamics we consider, is the dynamics of matter and fields on a single fixed causal set. As already stated, this is the analogue of fixing the background to a curved, in general, background. One needs to first recover the standard results from continuous spacetimes, and then explore the possibility for deviations from them due to the underlying fundamental discreteness. The effects are typically small and may depend on free parameters and possibly on the full quantum dynamics. In any case, this analysis may give us hints and directions both for developing quantum dynamics and for testing (with experiments eventually) the theory.

Matter on a causal set may arise in two ways. Either from the relations between elements of the causal set or by adding matter or fields on a given causal set. The first one is more intrinsic to the causal set but will not be explored here, since not much progress has been made. Matter degrees of freedom could appear e.g. in a Kaluza-Klein way, from variations of the causal structure at different parts of the total causal set. The second one will be explored in the following.

First we will explore the case of a causal set that faithfully embeds into Minkowski spacetime. The way this causal set arises is the issue of quantum dynamics of the causal set and indications for that will be reported in the next subsection. Secondly, using certain results from the flat case, we will consider causal sets that faithfully embed into curved spacetimes.

One thing to do is to consider point particles (massive for now), moving along a chain of the causal set that faithfully embeds into Minkowski spacetime. Several models were considered [17, 22] that shared the common feature, that in the continuum approximation the particle followed roughly a timelike geodesic, slightly deviating (swerving) from this motion. The resulting effect, is that of a drift motion, and the evolution equation turns out to be a diffusion equation that depends on a single parameter the "diffusion strength" k. We will present here briefly the first model as in [22]. The particle follows trajectories that are chains of the causal set $\{e_1, e_2, \cdots\}$. There is a time t_f, below which the causal set may behave non-locally. It is called "forgetting time" and above this it behaves normally. If the particle has position e_n with four momentum p_n the next element is chosen such that:

- e_{n+1} is at the causal future of e_n and at proper time t_f.
- The momentum change $|p_n - p_{n+1}|$ is minimized. Note, that p_{n+1} is on the mass shell and proportional to the vector between e_n and e_{n+1}.

The effect of this, is a motion which is as straight as possible, with some deviations, due to random fluctuations that the particle's momentum has. Taking the continuum approximation leads to the following diffusion equation (see [22]).

$$\frac{\partial \rho}{\partial \tau} = k\nabla^2 \rho - \frac{1}{m} p^\mu \partial_\mu \rho \tag{1}$$

The next stage is to examine the consequences of discreteness for massless particles. Technically, there is an extra difficulty, which arises due to the fact that the trajectories are

no longer chains, since chains correspond by definition to timelike curves. We are lead to an equation with two parameters (diffusion and drift) k_1, k_2. The reader is referred to [17].

The next thing to consider are quantum fields, and in order to do so, we have to look on how to obtain the d'Alembertian and the propagators. In [23], the d'Alembertian was introduced. Here we follow [24], to obtain the retarded propagator for a particle on a causal set.

To get the propagator $K(x, y)$ from an element x to y in its future, one needs to sum over all the chains (or paths) on a causal set from element x to element y, weighted with a particular amplitude. This is in direct analogy with quantum mechanics where the quantum amplitudes to go from one point to another are obtained by summing over all the possible trajectories. It is required to choose what is the analogue for the causal set, of the trajectories that we will sum over. It can be either the chains or (more restricted) the paths. The choices made should lead to the desired behaviour when considering causal sets faithfully embedded in Minkowski spacetime, i.e. give rise to the propagator of the Klein-Gordon equation (with the choices made in [24] we will get the retarded propagator). The amplitude depends on two parameters a which is the probability that the particle "hops" once along the trajectory from one element to another, and b which is the probability that the particle stops at an element of the trajectory. For a chain of length n (where we have n hopes and $n-1$ intermediate stops) the amplitude would be $a^n b^{n-1}$. For causal set in 1+1 Minkowski and 3+1 Minkowski we can choose the values of a and b in such a way to recover the retarded Klein-Gordon propagator. The values depend only on the mass of the particle and the volume corrseponding to each causal set element, i.e. the density. For 1+1 dimension these values are:

$$a = \frac{1}{2}, \quad b = -\frac{m^2}{\rho} \tag{2}$$

while for 3+1 dimensions the values are:

$$a = \frac{\sqrt{\rho}}{2\pi\sqrt{6}}, \quad b = -\frac{m^2}{\rho}. \tag{3}$$

Further details can be found in [24]. Continuing in the same line of research, in [25] scalar quantum field on a causal set was considered, and the Feynman propagator was computed. Of crucial importance for that work, was the use of Pauli-Jordan function [26] $\Delta(x) := G_R(x) - G_A(x)$ and its analogue for a causal set. Further generalisations involving curved spacetimes could be possible.

A different direction was taken in [23] and [27]. A slowing varying (at some frame) field $\phi(x)$ was considered and they used:

$$\Box_d \phi(u, v) = \frac{1}{a^2}(\phi(u, v) - \phi(u - a, v) - \phi(u, v - a) + \phi(u - a, v - a)) \tag{4}$$

in order to define the following B operator:

$$\begin{aligned} B\phi(x) &= \frac{4}{a^2}(-\frac{1}{2}\phi(x) + \sum_{y \in N_1(x)} \phi(y) \\ &\quad -2 \sum_{y \in N_2(x)} \phi(y) + \sum_{y \in N_3(x)} \phi(y)) \end{aligned} \tag{5}$$

$N_i(x)$ is the set of elements y, such that the number of elements z that satisfy $y \prec z \prec x$ is exactly i (i.e. i inclusive intervals). It has be shown (and confirmed by simulations) that this

operator, on average, gives the value of the d'Alembertian for flat space in the suitable limit. Further care is needed to guarantee the variations are also controlled [23].

This work is the starting point on extending the work for causal sets that faithfully embed in curved spacetime. Using expressions for the Ricci scalar in terms of volume and proper distance of small causal intervals [6, 28] the expression for the operator B (d'Alembertian) in flat space changes by a term:

$$B\phi(x) = (\Box - \frac{1}{2}R)\phi(x) \tag{6}$$

However, if we apply this expression to a constant field $\phi(x) =$ constant it gives an expression of the Ricci scalar for the causal set. This leads to an important step for causal sets, which is the derivation of an analogue of the Einstein-Hilbert action in curved spacetime called the *Benincasa-Dowker* (BD) Action (see [27]). The expressions for 2 and 4 dimensions are the following:

$$\frac{1}{\hbar}S^{(2)} = N - 2N_1 + 4N_2 - 2N_3 \tag{7}$$

$$\frac{1}{\hbar}S^{(4)} = N - N_1 + 9N_2 - 16N_3 + 8N_4 \tag{8}$$

N_i are the i-inclusive intervals as defined above. Generalisation for arbitrary dimensions also exist. The importance of this development, is that one can use this BD action, for constructing quantum dynamics.

Finally, recently an alternative expression for the Ricci scalar was derived in [29], having used the concept of neighborhood. Rather than using intervals as in the BD action, the authors used chains (see eq. 47 of [29]). While the expressions for arbitrary dimensions were clearer and could be used as an improved dimension estimator, this action, cannot be used directly for quantum dynamics, since it is local and requires the concept of a neighborhood. It is difficult to define such concept from the causal relations themselves without resorting to the background structure.

3.3. Quantum Causal Sets

The first important observation, is that causal sets have a spacetime nature, from their definition. A spacelike surface is simply a collection of unrelated elements, and other candidates for this concept, such as "thickened" anti-chains (see [7]) are not intrinsic to the causal set. It follows that a canonical quantisation of causal sets, even if possible, would contradict the spirit of the approach. This leads, to path integral quantisations and in general in "histories" formulations.

The mathematical aim is the assignment of a quantum amplitude to each causal set. Alternatively, one can define a quantum measure (see [30]) on the space of all causal sets (poscau). Interpreting the quantum measure (or the amplitudes) is complicated as already mentioned at the end of section 2 and a histories approach ([18, 19]) should be preferred.

Technically, there are two ways to proceed. The first one, is to attempt to generate the quantum measure, through a growth process, similarly with the CGD and is a bottom-up approach. It is a generalisation of quantum random walks. The second is to directly assign a weight on causal sets and this is usually done in a manner justified from continuum physics and is thus a top-down approach.

Quantum Growth Dynamics: There are two notable differences, in the quantum case. First of all, instead of a classical measure, we have either a quantum measure, or quantum amplitude.

In those cases, it is technically more involved to extend the measure to the full histories space. However, progress in this direction has been made for measurable sets of histories (see [31]). The second issue, is that the requirement of Bell's causality, may not be a suitable choice for quantum causal sets. It is well known in quantum theory, that the Bell's inequalities are violated and thus we require a novel locality condition that allows for these violations. For the moment, there is no widely accepted suitable generalisation of this condition.

Path Integral Approach: A first and very interesting result appeared in [32], from consideration of the so-called 2-dimensional orders. To this end, we should stress what is meant by a 2-dimensional order. It is a technical term for partial orders, and in general has nothing to do with spacetime dimension. The definition is the following:

Assume that we have a set $\mathcal{P} = \{e_1, e_2, \cdots\}$ along with two linear orderings of this set. Then we take the intersection of this linear orderings, which means that an element $e_1 \prec \epsilon_2$ iff e_1 is before e_2 in both linear orderings. The resulting structure is a partial order that is called 2-dimensional. Conversely,if we have a partial order that can be generated from the intersection of 2 linear orderings it is called 2-dim order. By analogy we can define n-dimensional partial order if it is the intersection of n linear orderings. For the special case of 2-dimensional partial orders, it turns out that there is a correspondence with 2-dimensional spacetimes. However, this analogy breaks down for higher dimensions.

Brightwell et al in [32] considered 2-dim orders and attempted to find what the typical 2-dim order would look like. Note, that as we have already mentioned a typical causal set is not manifold-like but a KR order, which is the source of the "entropy" problem. In Causal Dynamical Triangulations approach to quantum gravity, the crucial observation they made, was that instead of summing over all possible triangulations, they restricted attention only to the triangulations that obeyed their "causality" condition. In similar spirit, in [32] they restricted their potential causal sets, to 2-dim partial orders. The surprising result, was that within the 2-dim orders the major contribution comes from causal sets that are manifold-like and moreover that are faithfully embeddable in 2-dim Minkowski. We can view this as an example of how manifold-like behaviour (and more precisely flat) emerges from general considerations on causal sets.

Finally, one can use expressions for the causal set analogue of the Einstein-Hilbert action, to assign quantum amplitudes and examine the consequences. The more suitable choice, is the BD action eqs. (7, 8). However, other attempts may also be worth exploring, such as the expressions for the Ricci scalar found in [29] or in [33].

4. Summary and Conclusion

We introduced the causal sets which is a discrete replacement for standard continuous spacetime. We then focused on the dynamics of the theory, which divides to three parts. (1) The classical dynamics of causal sets, (2) the quantum dynamics of matter and fields on a classical causal set and finally (3) the quantum dynamics of the causal set. The following major issues concerning all three parts, were stressed. The "entropy" problem, which is that a typical causal set is not manifold-like. The direction chosen for the dynamics i.e. whether we take a bottom-up approach starting from concepts intrinsic to causal sets or the top-down approach where motivation for our choices comes from standard continuous physics. The interpretation of quantum causal sets, has the following two aspects. Constructing label-invariant questions, and constructing an interpretation of quantum theory suitable for quantum causal sets. The issues taken into consideration are that (a) we need a path integral approach (rather than canonical), (b) an observer independent formulation (since nothing is outside the total causal set) and (c) an interpretation that is valid for single, non repeatable system. The suggested

interpretations were the co-event formulation and decoherent histories approach. Applications of those formulations to quantum causal sets, is definitely something that needs to be pursued.

The classical dynamics presented, were bottom-up and in general stochastic [20]. Despite the fact that we constructed the most general dynamics, four physical conditions, restricted vastly the possible dynamics. The typical causal sets for any given choice of the parameters differs, and while it avoids the KR causal sets it appears that does not in general give manifold-like causal sets.

Particles moving on a given fixed causal set, move as expected with some small deviation that appears as a drift. With reasonable choice of parameters, however, this deviation is not observable. There are several attempts to define the d'Alembertian and using this to obtain the propagator. This was done initially on a causal set that faithfully embeds in flat spacetime. In [24, 25] the retarded Klein-Gordon propagator and eventually the Feynman propagator were computed. The greatest development is, however, the Benincasa-Dowker [27] action that applies to general causal sets that faithfully embed in curved spacetimes.

Finally, we explored the developments on the quantum dynamics of causal sets. There are two directions, the one being the generalisation of the CGD, and the other is the direct assignment of amplitudes to causal sets. The difficulties for the first approach were both technical (how to extend the quantum measure) and conceptual (what replaces the Bell's causality condition). For the second approach, two major developments have been made. The first was in [32] where for some sub-class of causal sets, namely the 2-dim orders, the typical causal set not only faithfully embeds in a manifold, but it is 2-dim Minkowski. This is very interesting, since it shows how quantum dynamics may evade the "entropy" problem. The second development, is the existence of an analogue of the Einstein-Hilbert action in arbitrary dimensions. One can use this, namely the BD action, to attribute weights to different causal sets and analyse the consequences. This direction is currently the priority, and accomplishing this, even for some special class of causal sets, would give a well defined quantum causal sets theory.

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