

Scientific Benefit of Enlarging Gravitational Wave Detector Networks

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Abstract. Localising the sources of gravitational waves (GWs) in the sky is crucial to observing the electromagnetic counterparts of GW sources. The localisation capability is poor by a single GW detector yet can be improved by adding more detectors to the detector network. In this paper we review recent studies on scientific benefits of global detector networks and focus on their localisation capability. We employ Wen-Chen's formula to compare this merit of current and future detector networks for localising gravitational wave bursts. We find that the addition of a new detector located in Japan, or India, or Australia will increase angular resolution 3~5 fold with respect to current LIGO-Virgo network, and that the angular resolution improvement by adding a single detector in Australia is comparable to that achieved by adding detectors in both India and Japan. A six-site network achieves a 11-fold improvement in angular resolution compared with the existing three-site network.

1. Introduction

The first detection of gravitational waves (GW) is likely to take place within the coming decade. It will start a new era for GW astronomy. The Laser Interferometer Gravitational Wave Observatory (LIGO)[1] in the USA has achieved its initial sensitivity goal and the upgraded configuration, advanced LIGO (advLIGO) is expected to be operational in the next 4~5 years with a 10-fold improvement in sensitivity. During the same timescale, advanced Virgo[2], an upgrade of the current Virgo detector in Italy, is scheduled to achieve a similar sensitivity. Another advanced detector is under construction in Japan—KAGRA GRAvitational wave detector (KAGRA)[3]. Although LIGO-Australia (AIGO)[4] was not approved in 2011, LIGO-India (IndIGO)[5] has now been proposed, and as an alternative international collaboration for AIGO is now under discussion.

There are many well-known sources that can generate GWs, such as compact binaries system, stellar core collapse and supernovae, or pulsars. The advanced detectors are expected to detect at least several compact binary coalescence sources per year. The coalescence of neutron star/neutron star (NS/NS) is widely considered to be the progenitor of short hard gamma ray bursts (SHBs) and observation by GW detectors could tell us whether the events are indeed the cause of SHBs. It is critical to localise the event source accurately to allow the observation of electromagnetic counterparts. In this paper we will first review recent studies on the scientific

benefits of global detector networks, then narrow down our discussion on localisation capability of detector networks.

2. Recent studies on scientific benefits of detector networks

A global detector network excels a single detector mainly in the aspects that it can detect the signal more robustly and measure the source location more accurately. A LIGO report[6] gave an extensive review by comparing two possible detector networks: a 4 detector network using two detectors at Hanford (USA), one at Livingston (USA), and one at Cascina (Italy) and a new network obtained by moving one Hanford detector to the proposed AIGO site. The networks are denoted HHLV and AHLV (see Table 1 for detector labels). Comparisons were based on considerations of source localisation, source parameter estimation capacity, network sensitivity and false alarm rate, and how local environmental perturbations would affect the network. Schutz[7], assuming that coherent network analysis is the default method for detection, summarised the properties of detector networks into three criteria: a) triple detection rate, b) sky coverage, and c) angular precision. Schutz considered the possibility that individual detectors might be off-duty and characterised the detection rate of a given network to be the detection volumes of all sub-networks weighted by their on-duty probability. The sky coverage is often computed as sky reach above a certain threshold of a network property. The LIGO report used the network sensitivity for this property while Schutz used antenna power patterns.

The work of measuring how accurately an array of detectors can estimate source location has long been appreciated. So far two strategies have been developed. One is to perform Monte-Carlo simulations on posterior parameter sampling and calculate the variations with true values[8, 9, 10]. The other is to derive an analytical expression of localisation accuracy (or angular resolution, expressed as an angular area) directly. The difficulty in such a direct derivation will be illustrated in detail in section 3. Based on triangulation for source localisation, Fairhurst[11] presumed that source localisation is primarily related to timing information when the signal arrives at different detectors. Therefore he used the Fisher matrix to obtain time accuracies by which he derived a form of localisation accuracy. Wen and Chen[12] give a more generalized form of localisation accuracy appropriate for any unbiased localisation method. This deviation considered a full parameter space and marginalised the unnecessary parameters except source location parameters. Vitale and Zanolin[13] employed the asymptotic expansions to give a frequentist assessment of parameter estimation errors. The results of the above three strategies are comparable, and agree well with Monte-Carlo simulations given that signal-to-noise ratio (SNR) is relatively high[14]. A thorough study of the Fisher matrix for assessment of GW parameter estimation has been done by Vallisneri. This gives an SNR threshold above which the Fisher matrix is reliable[15]. Schutz[7] gave a figure of merit for angular resolution from a simplification of Wen-Chen's work. In this paper, we compare the performance of different detector networks with the aid of the Wen-Chen approach.

3. Parameter measurement

3.1. Detection and parameter estimation

In practice the detection and parameter estimation of GW signals are treated separately, as detecting a signal and solving all parameters of the signal simultaneously is not sensible due to immense computational cost. The purpose of detection is to recognise GW signals from noisy data efficiently and reliably. Much effort has been paid to reduce the complete form of the signal to make templates for detection. The most popular detection algorithm - matched filtering, uses templates defined on a reduced parameter space (the intrinsic parameter space) of the signal and time-frequency analysis techniques map the multi-dimensional parameter space to the 2-dimensional space.

For the matched filtering detection method, the estimation of intrinsic parameters are performed at the time of detection to maximise the SNR criterion. The remaining extrinsic parameters can be estimated using the maximum likelihood (ML) estimator. Advanced estimators like the maximum a posteriori estimator (MAP) or the Bayesian estimator are more reliable but are more difficult to compute.

We first take a quick look at the background on estimators. Let X be a random variable with probability function $f(x; \theta)$ which depends on an unknown parameter θ . The joint probability of a random sample from X is given by $\prod_1^n f(x; \theta)$, which is also known as the likelihood function for θ . An estimator of θ is any statistics denoted by $\Theta = s(x_1, \dots, x_n)$ and is itself a random variable. When $E(\Theta) = \tilde{\theta}$ (E is referred to expected value, $\tilde{\theta}$ is the true value), this estimator is said to be unbiased. A maximum likelihood estimator which maximise the likelihood function is an unbiased estimator.

One can understand that although detection and estimation can be treated separately, parameter estimation depends on the presence of the signal. In other words, the error of estimation is related to the confidence level of the presence. The SNR of an observation can be a criterion to reflect the presence of a signal. Higher SNR indicates the presence of signal is more likely and therefore the estimated parameter values are very likely to fall within the parameter space where signal is present. The statistical error is then well approximated by the inverse of the Fisher information which will be shown below.

3.2. Fisher information matrix and Cramér-Rao bound

The Fisher information matrix (or Fisher matrix) of θ , where θ has more than one parameter, is defined as:

$$\Gamma_{ij} = E \left[\left(\frac{\partial}{\partial \theta_i} \log f(x; \theta) \right) \left(\frac{\partial}{\partial \theta_j} \log f(x; \theta) \right) \right] \quad (1)$$

According to the Cramér-Rao bound [16], the lower bound of the variance of any unbiased estimator is determined by the inverse of the Fisher matrix: $Var(\Theta) \geq \Gamma^{-1}$. Using this fact, we can obtain the best precision of an unbiased estimator of any parameter. Here the focus of our paper is on angular position θ, ϕ . Wen and Chen (2010) have derived geometrical expressions for angular resolution in several practical situations. In the case of detecting short signals (in which time the motion of the detector network is negligible), the general expression of angular resolution is

$$\Delta\Omega = \frac{4\sqrt{2}\pi c^2}{\sqrt{\sum_{J,K,L,M} \Delta_{JK} \Delta_{LM} |(\mathbf{r}_{KJ} \times \mathbf{r}_{ML}) \cdot \mathbf{n}|^2}} \quad (2)$$

In this formula, J, K, L, M are detector indices; $\Delta\Omega$ is the angular resolution; $\Delta_{IJ} = \langle \Omega d_I | \Omega d_J \rangle \delta_{IJ} - \langle \Omega d_I | \partial_{\lambda_k} d_J \rangle \mathbf{B}_{\mathbf{kl}}^{-1} \langle \partial_{\lambda_l} d_J | \Omega d_J \rangle$, where d_I is the Fourier transform of noise weighted waveform at detector I , $\langle | \rangle$ is complex vector inner product, δ_{IJ} is the Kronecker delta, and $B_{kl} = \langle \partial_{\lambda_k} \mathbf{d} | \partial_{\lambda_l} \mathbf{d} \rangle$; \mathbf{r} is positional vector of connecting detectors; \mathbf{n} is the source direction vector.

Two special cases of short signals have been considered in [12] and also in this paper. For one case is the waveform and the arrival time of the signal are both unknown. This we refer to as the worst case. The best case is the one when the waveform is known and the arrival time is unknown. In this scenario, Δ_{IJ} can be simplified as $\Delta_{IJ} = -\frac{\xi_I \xi_J}{\sum_I \xi_I}$ for $I \neq J$, where $\xi_I \equiv \langle \Omega d_I | \Omega d_I \rangle$. We will use angular resolution for the best case and the worst case to compare the localisation capability of a range of detectors.

4. Results and discussion

The GW detector networks that we consider comprises combinations of three established interferometers: LIGO-Virgo (LHV), one partially funded: KAGRA (J), and two possible ones:

Table 1: Properties of detectors. Longitude is positive toward east and latitude positive toward north. The locations for L,H,V,J can be found on corresponding detector websites. The location of AIGO is set in Gingin, Western Australia. The location of IndIGO is found from[7]. There are two interferometers in Hanford, USA and the symbol "HH" is used when both detectors are considered.

Detector	Label	Longitude	Latitude	Sensitivity
LIGO Livingston, USA	L	-90.77°	30.56°	advL ^[1]
LIGO Hanford, USA	H	-119.41°	46.45°	advL
Virgo, Italy	V	10.5°	43.63°	advV ^[2]
KAGRA, Japan	J	137.18°	36.25°	advL
IndIGO, India	I	74.05°	19.09°	advL
AIGO, Australia	A	115.87°	-31.95°	advL

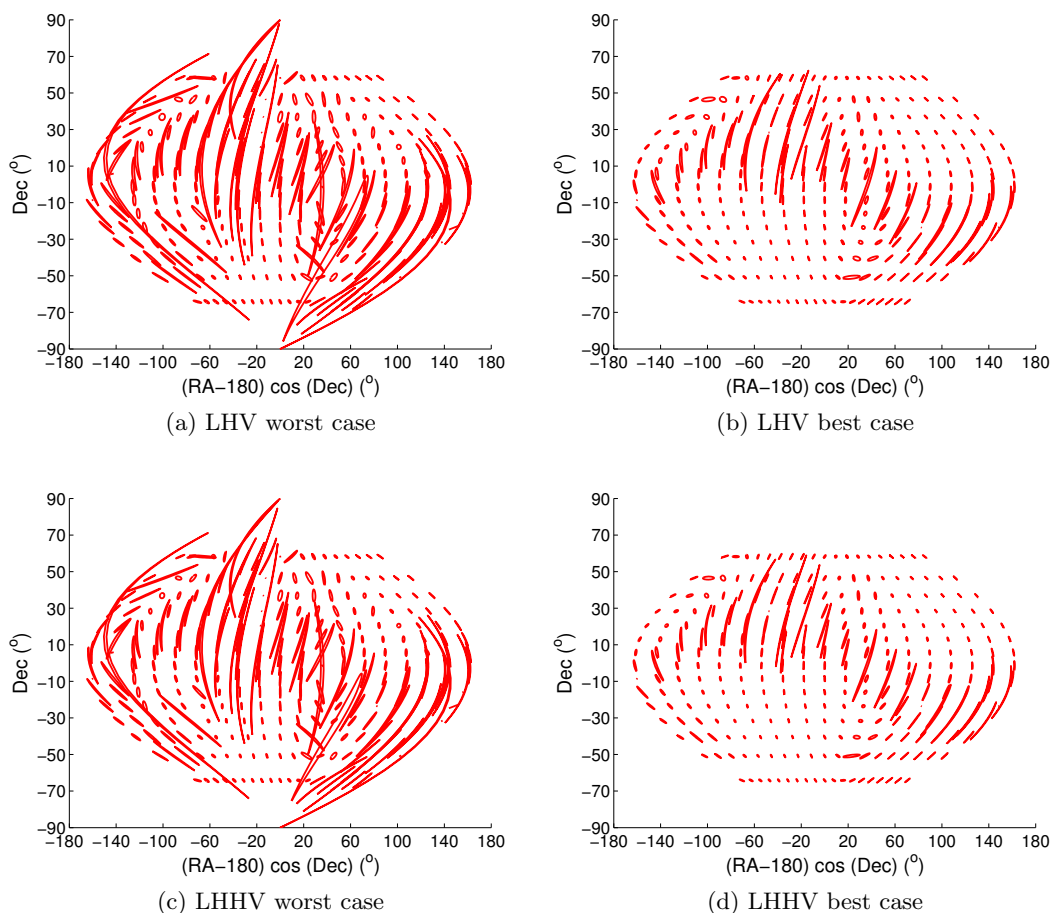


Figure 1

IndIGO (I) or AIGO (A). Other first generation GW interferometers detectors such as TAMA in Japan and GEO600 in Germany will not be discussed in this paper.

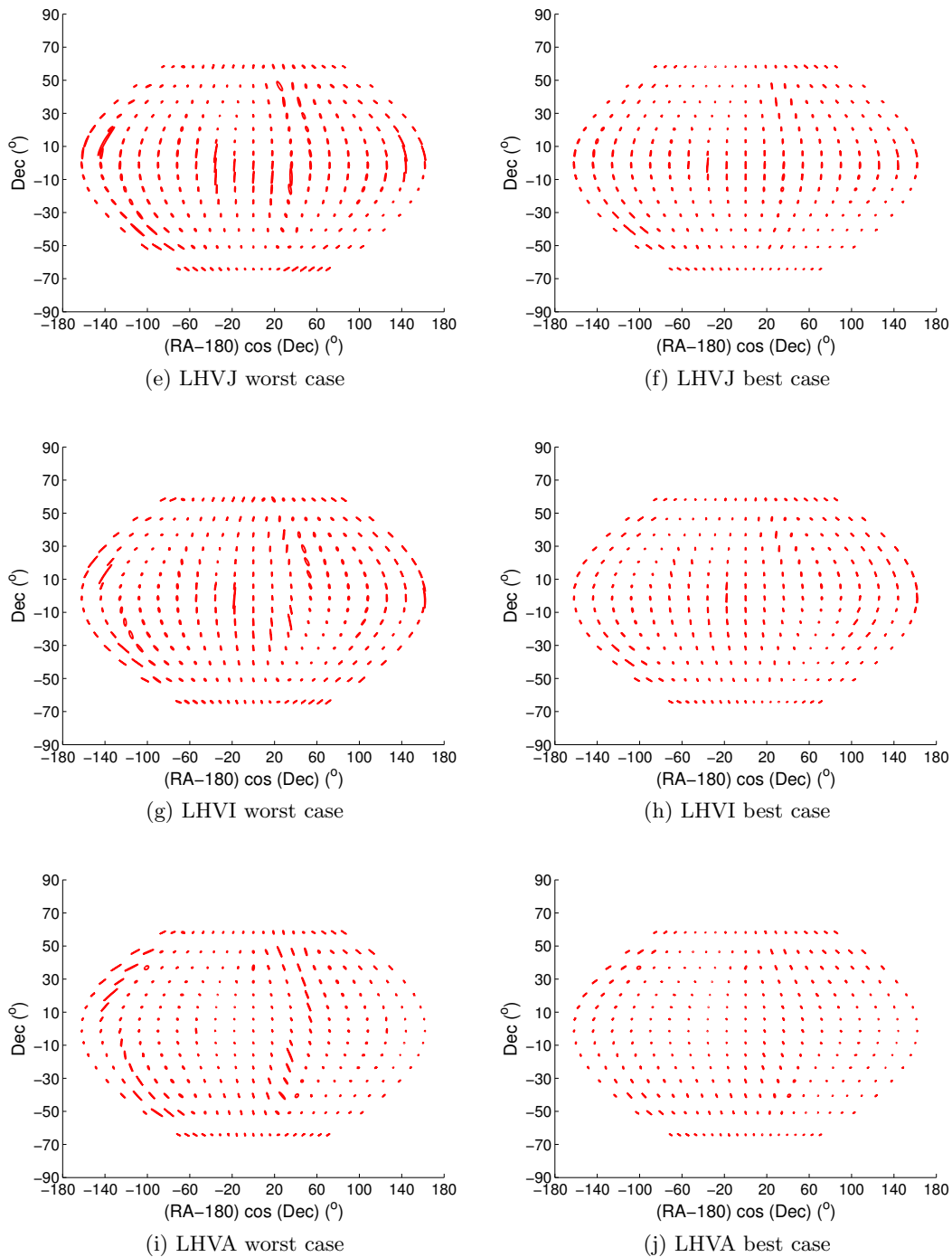


Figure 1

The GW signal for experiments here is the GW burst from supernova(U11 model)[17] with central frequency 600Hz. We adjust the distance of the signal so that the optimal network SNR of LHV is 10. Note that Eq.(2) obtained from the Fisher matrix is the lower bound of angular resolution. Therefore what we discuss in this paper is the best localisation capability of the detector networks.

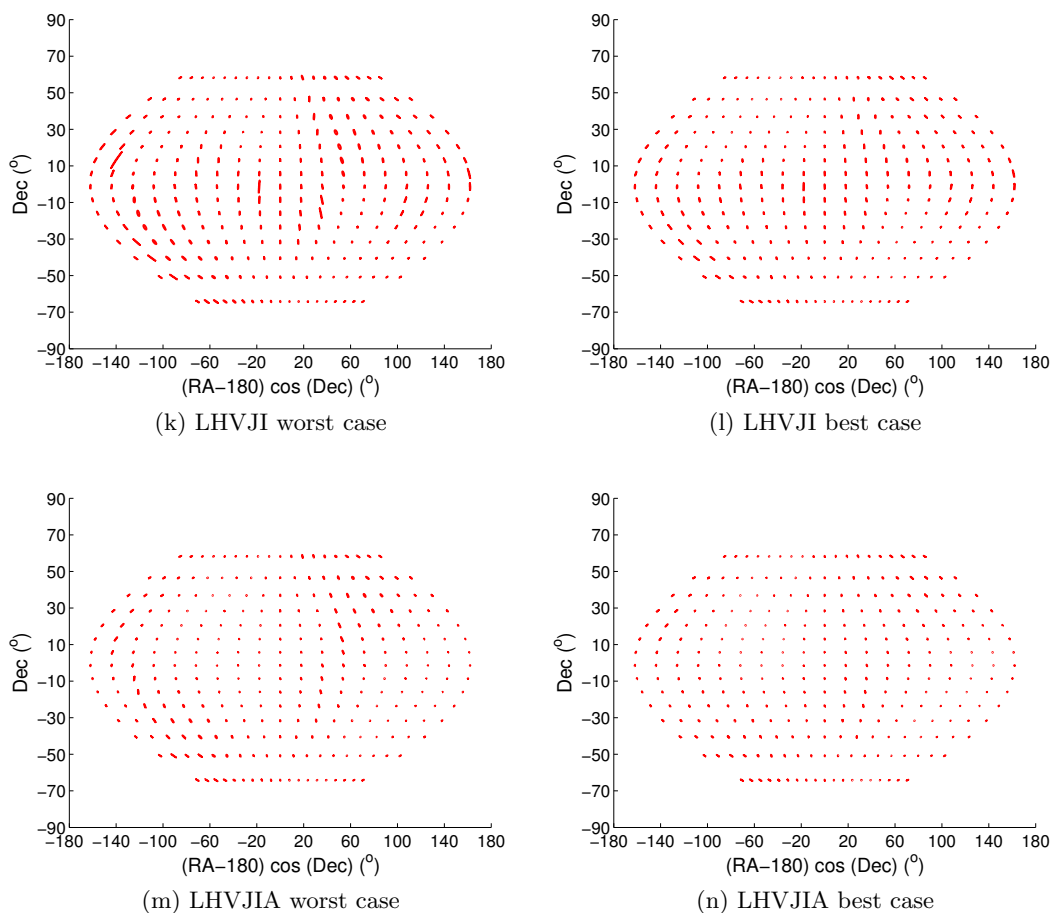


Figure 1: All sky-map error ellipses of 7 detector networks. They are LHV, LHHV, LHVJ, LHVI, LHVA, LHVJI, and LHVJIA. All angular areas are in 95% confidence interval.

We present our results in the term of all sky maps which show the distribution of error ellipses for point source located uniformly over the sky. For each detector network, we apply both two cases (explanations of best case and worst case can be found in Section 3).

Observation of the various all-sky maps gives a qualitative estimate of network performance in the best and worst cases as shown in Fig. 1. To obtain quantitative characteristics we perform accumulative distributions for the angular area as shown in Fig. 2. From Fig.1 and Fig.2, we can see the following:

- (i) If a signal is formed along the plane formed by detector network LHV, then it is hard to locate the signal accurately by this network.
- (ii) The improvement of angular resolution if an extra detector is added at the same place is negligible. The angular resolution of LHVJ and LHVI are comparable and so is LHVA and LHVJI. The reason why AIGO can improve source localisation significantly lies in that it can form the largest baselines with existing detectors and is roughly in antipodal position with LIGO.

Our results are consistent with Schutz[7] who used a simplified version of Wen-Chen formula, and with Vitale and Zanolin[13] who used a frequentist strategy, and with Fairhurst[11] who took advantage of timing information, and with Nissanke *et.al*[9] who employed Monte-Carlo

sampling. The results in Fig. 2 show the great improvement obtained by extending the network but also emphasize on outstanding benefit of the AIGO detector, as discussed further below.

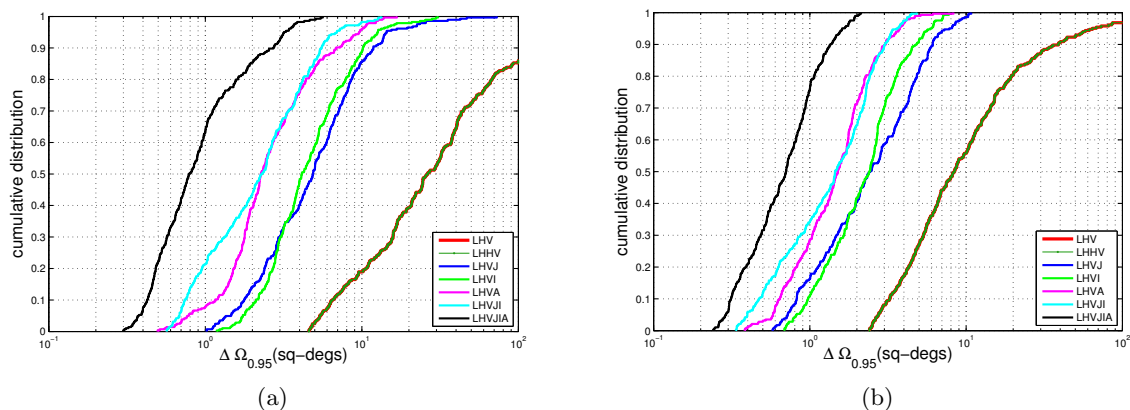


Figure 2: Cumulative distribution of error ellipses at 95% confidence in worst case (a) and in best case (b) of 7 detector networks. The best case is most likely to occur in reality which the signal waveform is unknown but the arrival time is unknown. In the best case scenario, 50% of the area of error ellipses will be below 8 deg^2 for LHV/LHHV networks, and goes down to around 2.5 deg^2 for LHVJ/LHVI networks, further down to less than 1.5 deg^2 for LHVJ/LHVJI networks, achieving its best at around 0.7 deg^2 for the LHVJIA network. The worst-case figure depicts more significant improvement of angular resolution when extending detector networks.

5. Conclusion

We have reviewed recent work on the scientific benefits of extending global detector networks. In particular we discussed the angular resolution of future networks. Our results show that the addition of new sites in Japan (KAGRA) or in India (IndIGO) will improve the angular resolution about 3-fold compared to the current LHV network. However, adding a new detector in Australia (AIGO) alone has similar angular resolution performance to KAGRA and IndIGO combined. AIGO gives a 5-fold better angular resolution compared with the LHV network. The best network is LHVJIA which includes LIGO Livingston, LIGO Hanford, Virgo, KAGRA, IndIGO, and AIGO. It has a 11-fold improvement compared with LHV, achieving angular resolution of 0.7 deg^2 for 50% of sources, and 0.3 deg^2 for 10% of sources. In future work we will repeat this analysis for compact binary coalescence sources and consider other performance measures for the networks.

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