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Phase-synchronicity conditions from pulse-front tilted laser beams on one-dimensional periodic structures and proposed laser-driven deflection

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This article explores general particle-to-field phase synchronicity conditions in one-dimensional periodic phase modulation structures powered from a pulse front tilted laser beam. The analysis applies to speed-of-light particles. It is found that the synchronicity condition for the accelerating force is straightforward to accomplish, whereas synchronicity from a deflection force in these structures is only possible to attain with certain geometry conditions. When these conditions are met a synchronous deflection force that acts on the particle over a distance much greater than the laser wavelength is introduced. This opens the possibility for an effective laser-driven deflection microstructure.

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I. INTRODUCTION

One general condition that all practical particle accelerators satisfy is their ability to provide extended phase synchronicity between the relativistic particle bunch and the driving electromagnetic field, which allows for the application of a continuous force on the particle over a distance much greater than the wavelength. Many different methods have been developed, and common particle accelerator configurations rely on the electromagnetic wave energy co propagating with the particle and satisfy phase synchronicity either through the control of the particle's trajectory or more commonly through the control of the electromagnetic wave phase velocity with the aid of a medium or a waveguide structure.

Other particle accelerator architectures that do not rely on guiding of the electromagnetic field along the particle channel have been analyzed in the past [1,2,3]. In transverse-pumped accelerator structures such as [4] the EM energy flows at right angles to the particle beam, and the phase synchronicity is attained by the introduction of a periodic phase modulation of the field with a period equal to the wavelength of the electromagnetic wave. These structures have been analyzed by evaluating the average gradient in one structure period [4], and it has been suggested that a pulse-front tilted laser beam could deliver extended phase synchronicity ranging over many structure periods. However to the author's knowledge no rigorous analysis on this statement has been made.

Here, a general analytical evaluation of the accelerating and deflecting forces in periodic one-dimensional structures is presented and a simple criterion for the possibility of extended phase synchronicity in such systems is derived. First, the question of the

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extended phase synchronicity of the accelerating force in a periodic structure as proposed in [4] is addressed. In particular, phase-synchronicity from a pulse-front tilted laser beam is analyzed. Next, it is shown that in this structure the deflection force from the electromagnetic wave cannot maintain phase synchronicity. Finally, it is shown that tilting of the periodic structure with respect to the electron beam introduces a phase-synchronous deflection force.

II. THE GENERAL STRUCTURE GEOMETRY

Figure 1 shows six examples of conceptual one-dimensional periodic structures. Some of these, such as c) have been proposed for particle acceleration. All these structures posses a vacuum channel that runs along the *y*-axis and extrudes to infinity in the *z*-direction and have a repeating shape with a period λ_p where $\lambda_p \leq \lambda$. These structures can be viewed as multi-layer gratings of various shapes that are powered by an electromagnetic plane traveling in the *x*-direction.

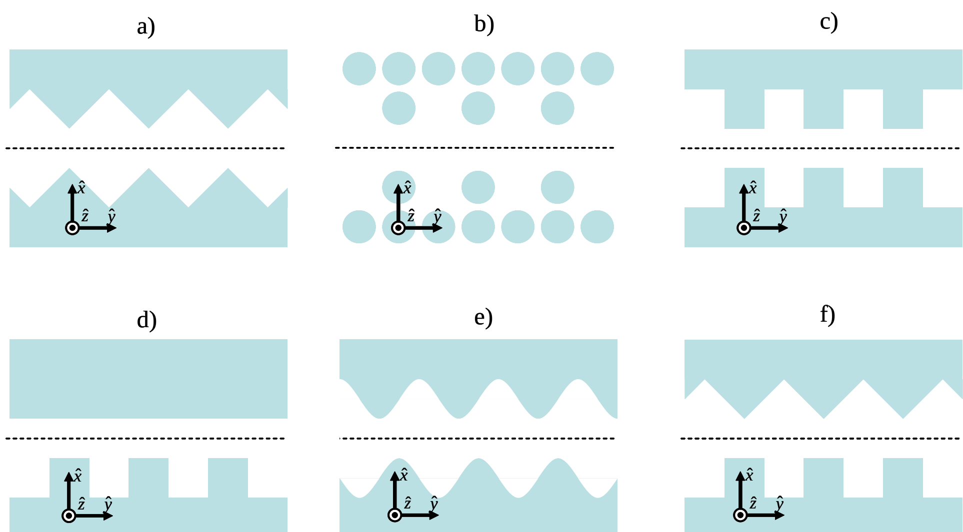

FIG. 1. Examples of possible symmetric (a,b,c) and non-symmetric (d,e,f) one-dimensional accelerator structures. The dashed line indicates the particle trajectory through the vacuum channel of the structure.

The examples a,b,c in Figure 1 posses mirror symmetry in the *yz*-plane while the examples d,e,f do not. The synchronicity considerations that follow are valid for all the periodic structures shown in Figure 1. First, a speed-of-light test particle traveling down the *y*-axis, such that $\vec{v}(t) = c\hat{y}$ and $\vec{r}(t) = y_0 + ct\hat{y}$ is considered. In the last section the case of a speed-of-light particle with a nonzero velocity component in the *z*-direction is analyzed.

III. PHASE SYNCHRONICITY OF THE ACCELERATING FORCE

Assume first a monochromatic plane wave of angular frequency ω that is incident on one of the structures in Figure 1. Any one of the electric or magnetic field components in the vacuum channel can be described by a function of the form

$$F(x,y,t) = A(x,y)e^{i\omega t} \tag{1}$$

$A(x,y)$ is the spatial field component and $e^{i\omega t}$ is the harmonic time dependence. Since the structure is infinite in the *z*-direction the spatial functions depend only on the *x*- and the *y*-coordinates. Because of the periodicity of the structure and the normal incidence condition of the wave the spatial field components have to satisfy

$$A(x,y) = A(x, y + n\lambda_p) \tag{2}$$

In the analysis presented here the period of the structure, denoted by λ_p, does not have to equal the wavelength of the electromagnetic wave λ in vacuum. As described in [4], for this two-dimensional problem there are two independent field solutions, each containing three of the six field components that are labeled as transverse electric TE and transverse magnetic TM solutions. Particle acceleration requires a nonzero electric field component in the *y*-axis, which is produced by the polarization associated with the TM solution. The acceleration field component is described as a periodic function that can be expanded as a Fourier series of the form

$$E_y(0,y,t) = \sum_{n=-\infty}^{+\infty} V_n e^{ik_p ny} e^{ikct} \tag{3}$$

where $k_p = 2\pi/\lambda_p$ and $k = 2\pi/\lambda$. The particle position in the vacuum channel is $y(t) = y_0 + ct$. Therefore the time variable can be expressed as $t(y) = y/c - y_0/c$ and hence the force from the accelerating electric field acting on the particle is

$$F_y(y, t(y)) = q\,\mathrm{Re}\big(E_y(0, y, y/c)\big) = q\,\mathrm{Re}\left(\sum_{n=-\infty}^{+\infty} V_n e^{ik_p ny} e^{iky} e^{-iky_0}\right) \tag{4}$$

The average acceleration gradient from this TM wave $\langle G_y \rangle_{\mathrm{TM}}$ is defined as

$$\langle G_y \rangle_{\mathrm{TM}} = \langle F_y(y, t(y))/q \rangle = \mathrm{Re}\left(\lim_{L\to\infty} \frac{1}{L} \int_0^L \sum_{n=-\infty}^{+\infty} V_n e^{ik_p ny} e^{iky} e^{-iky_0} dy\right) \tag{5}$$

The term e^{-iky_0} is a constant that represents the optical phase of the particle with respect to the field and can be taken out of the path integral. For $\langle G_y \rangle$ to be nonzero there has to be a component in the sum of equation 5 that possesses a non-oscillatory term, that is, $nk_p + k = 0$. Since *n* is integer, to satisfy phase synchronicity for one of the terms in the sum the structure period has to be an integer multiple of the wavelength of the driving electromagnetic wave ; that is, $\lambda_p = -n\lambda$. For periodic structures such as those shown in

Figure 1 the lowest order coefficients, V_{+1} or V_{-1}, are the largest and hence the most desirable to phase-synchronize to. The phase synchronicity condition for the lowest order term in equation 5 occurs when the structure period is equal to the electromagnetic field wavelength, $\lambda_p = \lambda$, and hence the average acceleration gradient is

$$\langle G_y \rangle_{\mathrm{TM}} = \mathrm{Re}\left(e^{-iky_0} V_{-1}\right) \tag{6}$$

The fact that in equation 6 the acceleration gradient depends on V_{-1} instead of V_{+1} is only a consequence of the specific phasor notation that was adopted in equation 3. As with all particle accelerators the term e^{-iky_0} describes the phase between the particle and the field, which can be accelerating or decelerating.

Equation 6 describes the phase synchronicity condition for a CW plane wave at normal incidence to the structure. On the other hand, a pulse-front tilted laser beam corresponds to a sum of CW plane waves with different directions of propagation that depend on the frequency of the particular plane wave component [5]. Hence to verify the possibility of extended phase-synchronicity from a pulse-front tilted laser beam the synchronicity of each plane wave component has to be found. Assuming an oblique incidence angle $\Delta\varphi$ of a plane wave the field in equation 1 acquires a pseudo-periodicity of the form [6,7]

$$F(x, y, t) = A(x, y) e^{i\omega t} e^{-ik\Delta\varphi y} \tag{7}$$

where $A(x, y)$ is still the same spatial periodic field function that can be expressed as a discrete Fourier series. The additional obliquity term modifies the accelerating gradient expression shown in equation 5 to

$$\langle G_y \rangle_{\mathrm{TM}} = \langle F_y(y, t(y))/q \rangle = \mathrm{Re}\left(e^{-iky_0} \lim_{L\to\infty} \frac{1}{L} \int_0^L \sum_{n=-\infty}^{+\infty} V_n e^{ik_p n y} e^{iky} e^{-ik_\omega \Delta\varphi y} dy \right) \tag{8}$$

The obliquity angle of the plane wave introduces a new phase synchronicity condition that relates $\Delta\varphi$ to the frequency of the wave. From equation 8 it can be observed that the synchronicity condition is

$$nk_p + k - k\Delta\varphi = 0 \tag{9}$$

Again, utilizing the lowest order nonzero term $n = -1$, and defining the change in the *k*-vector of the electromagnetic wave as $\Delta k = k - k_p$ equation 9 becomes $\Delta k - k\Delta\varphi = 0$ which can be rewritten as

$$1 = k \frac{\Delta\varphi}{\Delta k} \tag{10}$$

Hence, for a laser pulse with multiple wavelengths the general phase synchronicity for each wavelength has to satisfy equation 10, such that each frequency of the electromagnetic wave is oriented a different obliquity angle. The pulse front tilt angle γ of a laser beam with center wavelength corresponding to $k = 2\pi/\lambda$ is given by [5]

$$\tan\psi = k\frac{d\varphi}{dk} \tag{11}$$

From equations 10 and 11 it can be observed that the phase-synchronicity condition for a laser pulse on a one-dimensional periodic structure is the same as the expression of a pulse-front tilted wave with a tilt angle satisfying $\tan\psi = 1$. Not surprisingly, this is equal to a 45° pulse front tilted wave. The center frequency component has a wavelength $\lambda = \lambda_p$ and is at normal incidence to the structure while the other frequency components are oriented at an angle proportional to their frequency offset.

IV. THE CANCELATION OF SYNCHRONOUS DEFLECTION FORCES

In [4] it was predicted that for a plane wave powering a periodic one-dimensional structure there exists an optical phase where the speed-of-light particle experiences a nonzero deflection force. Here it is shown that the residual deflection force of the geometry calculated by FDTD methods in a single structure period in [4] does not maintain extended phase synchronicity over a larger number of structure periods and hence is not usable for an effective particle deflection. To analyze this problem analytically the same argument as in the previous section can be applied. The deflection force is composed of the lateral components of the total Lorentz force acting on the speed-of-light particle. For the TM solution

$$\vec{F} = q\,\mathrm{Re}\begin{pmatrix} E_x \\ E_y \\ 0 \end{pmatrix} + q\begin{pmatrix} 0 \\ c \\ 0 \end{pmatrix} \times \mathrm{Re}\begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix} = q\,\mathrm{Re}\begin{pmatrix} E_x + cB_z \\ E_y \\ 0 \end{pmatrix} \tag{12}$$

The deflection force is $F_x = \mathrm{Re}(E_x + cB_z)$. As before, for a normal-incidence plane wave E_x and B_z are periodic in the *y*-direction, and for a tilted plane wave they acquire the same phase slippage term $e^{-ik\Delta\varphi y}$. Hence E_x and B_z have the same form as that of E_y described in equation 3.

$$\begin{aligned} E_x(0,y,t) &= \sum_{n=-\infty}^{+\infty} U_n e^{ik_p ny} e^{ikct} e^{-ik\Delta\varphi y} \\ B_z(0,y,t) &= \sum_{n=-\infty}^{+\infty} W_n e^{ik_p ny} e^{ikct} e^{-ik\Delta\varphi y} \end{aligned} \tag{13}$$

In the vacuum the time harmonic fields E_x and B_z can be shown to be related by

$$cd_y B_z = ikE_x \tag{14}$$

This establishes the relation between the Fourier coefficients in equation 13 that satisfy

$$cW_n = \frac{k}{nk_p - k\Delta\varphi} U_n \tag{15}$$

Hence the average deflection gradient can be described in terms of the Fourier coefficients

$$\langle G_x \rangle_{\mathrm{TM}} = \mathrm{Re}\left(e^{-iky_0} \sum_{n=-\infty}^{+\infty} \left[\left\{ 1 + \frac{k}{nk_p - k\Delta\varphi} \right\} U_n \lim_{L\to\infty} \left(\frac{1}{L} \int_0^L e^{i(k_p n + k - k\Delta\varphi)y} dy \right) \right] \right) \tag{16}$$

Once again, the extended phase synchronicity is satisfied for the coefficient for which the oscillatory term in the path-integral is constant, that is, when $nk_p + k - k\Delta\varphi = 0$, which not surprisingly is the same condition as in the previous section, shown in equation 9. However, when this condition is satisfied the factor in the curly brackets in equation 16 also becomes zero. Hence for any k, n or $\Delta\varphi$ the average extended deflection gradient from this structure experienced by speed-of-light particle is zero; $\langle G_x \rangle_{\mathrm{TM}} = 0$. The same calculation can be carried out for the average defection gradient from the TE solution; $\langle G_z \rangle_{\mathrm{TE}} = \langle \mathrm{Re}(E_z - cB_x) \rangle$, and it is found that no extended phase synchronicity can exist for this deflection component either.

V. GEOMETRY FOR A NONZERO SYNCHRONOUS DEFLECTION FORCE

As a means to introduce a nonzero deflection force the particle is allowed to acquire a velocity component in the *z*-direction, such that the particle's velocity vector is described by $\vec{v}(t) = c(\hat{y}\cos\alpha + \hat{z}\sin\alpha)$, where α represents a tilt angle between the particle trajectory and the extruded structure dimension. A perspective view of the new geometry is shown in Figure 2.

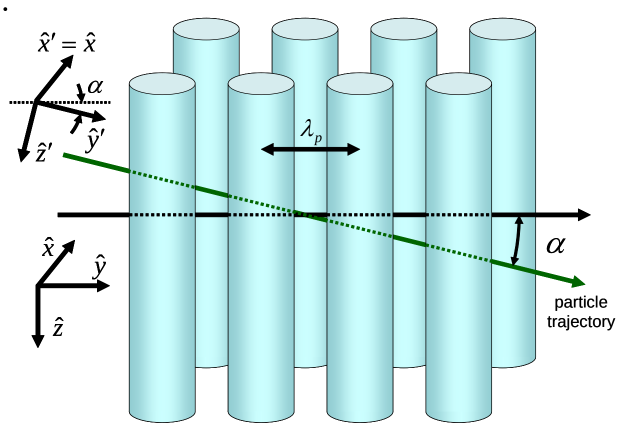

FIG. 2. Perspective view of an oblique particle trajectory with angle α in the one-dimensional periodic structure.

The Lorentz force from the TM wave acting on such a particle is

$$\vec{F} = q\,\mathrm{Re}\begin{pmatrix} E_x + cB_z\cos\alpha \\ E_y \\ 0 \end{pmatrix} \tag{17}$$

The $\cos\alpha$ term, which can be made to be different from unity, allows for an imbalance between the opposing electric and magnetic force components whose average produces a residual net deflection. The deflection gradients are given by $\langle G_x\rangle_{\mathrm{TM}} = \langle F_x/q\rangle$, $\langle G_y\rangle_{\mathrm{TM}} = \langle F_y/q\rangle$, and $\langle G_z\rangle_{\mathrm{TM}} = \langle F_z/q\rangle$ evaluated along the particle path $y' = y/\cos\alpha$. In terms of the Fourier expansions of the field components of equations 3 and 13 the average gradients are

$$\begin{aligned}
\langle G_x\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(e^{-iky_0}\sum_{n=-\infty}^{+\infty}\left(1+\frac{k\cos\alpha}{nk_p - k\Delta\varphi}\right)U_n \lim_{L\to\infty}\left(\frac{1}{L}\int_0^L e^{i\left(k_p n - k\Delta\varphi + k/\cos\alpha\right)y(y')}dy'\right)\right)\\
\langle G_y\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(e^{-iky_0}\sum_{n=-\infty}^{+\infty}V_n \lim_{L\to\infty}\left(\frac{1}{L}\int_0^L e^{i\left(k_p n - k\Delta\varphi + k/\cos\alpha\right)y(y')}dy'\right)\right)\\
\langle G_z\rangle_{\mathrm{TM}} &= 0
\end{aligned} \tag{18}$$

Equation 18 shows that the phase synchronicity condition is $k_p n + k/\cos\alpha - k\Delta\varphi = 0$, and for the lowest order component

$$-k_p + k\cos\alpha = k\Delta\varphi \tag{19}$$

when synchronicity of the lowest order coefficient is satisfied the average gradients of equation 18 read

$$\begin{aligned}
\langle G_x\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(e^{-iky_0}U_{-1}\right)\sin^2\alpha\\
\langle G_y\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(e^{-iky_0}V_{-1}\right)\\
\langle G_z\rangle_{\mathrm{TM}} &= 0
\end{aligned} \tag{20}$$

These are the average gradient components in the *xyz* coordinate system that is aligned to the structure shown in Figure 2. In the particle's *x'y'z'* coordinate system the gradients from the TM wave read

$$
\begin{aligned}
\left\langle G_{\perp,x'} \right\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(e^{-iky_0} U_{-1}\right)\sin^2\alpha \\
\left\langle G_{\|,y'} \right\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(+ e^{-iky_0} V_{-1}\right)\cos\alpha \\
\left\langle G_{\perp,z'} \right\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(- e^{-iky_0} V_{-1}\right)\sin\alpha
\end{aligned} \tag{21}
$$

For the TE wave, which includes the B_x, B_y and E_z components, a similar analysis can be performed. The Lorentz force from a TE wave in the *xyz* coordinate system is

$$
\vec{F} = q\,\mathrm{Re}\begin{pmatrix} -cB_y \sin\alpha \\ +cB_x \sin\alpha \\ E_z - cB_x \cos\alpha \end{pmatrix} \tag{22}
$$

Proceeding with the same type of analysis as that for the TM wave B_x, B_y and E_z are expressed as a Fourier expansions of the form

$$
\begin{aligned}
B_x(0,y,t) &= \sum_{n=-\infty}^{+\infty} X_n e^{ik_p ny} e^{ikct} e^{-ik\Delta\varphi y} \\
B_y(0,y,t) &= \sum_{n=-\infty}^{+\infty} Y_n e^{ik_p ny} e^{ikct} e^{-ik\Delta\varphi y} \\
E_z(0,y,t) &= \sum_{n=-\infty}^{+\infty} Z_n e^{ik_p ny} e^{ikct} e^{-ik\Delta\varphi y}
\end{aligned} \tag{23}
$$

The gradients are the average values of the force components of equation 22. Inspection of the terms in equation 23 reveals that the same phase synchronicity as that for the TM wave given in equation 19 applies, and hence the average gradients are

$$
\begin{aligned}
\left\langle G_x \right\rangle_{\mathrm{TE}} &= \mathrm{Re}\left(- e^{-iky_0} cY_{-1}\right)\sin\alpha \\
\left\langle G_y \right\rangle_{\mathrm{TE}} &= \mathrm{Re}\left(e^{-iky_0} cX_{-1}\right)\sin\alpha \\
\left\langle G_z \right\rangle_{\mathrm{TE}} &= 0
\end{aligned} \tag{24}
$$

In the *x'y'z'* coordinate system these gradient components from the TE wave are

$$
\begin{aligned}
\left\langle G_{\perp,x'} \right\rangle_{\mathrm{TE}} &= \mathrm{Re}\left(- e^{-iky_0} cY_{-1}\right)\sin\alpha \\
\left\langle G_{\|,y'} \right\rangle_{\mathrm{TE}} &= \mathrm{Re}\left(+ e^{-iky_0} cX_{-1}\right)\sin\alpha\cos\alpha \\
\left\langle G_{\perp,z'} \right\rangle_{\mathrm{TE}} &= \mathrm{Re}\left(- e^{-iky_0} cX_{-1}\right)\sin^2\alpha
\end{aligned} \tag{25}
$$

The synchronicity condition in equation 19 describes a different pulse-front tilt angle than the 45° found for the simple accelerating structure described earlier. Using the relation

$\Delta k = k - k_p$ equation 19 becomes $-(k-\Delta k)/\cos\alpha + k/\cos\alpha = k\Delta\varphi$, which can be rewritten as

$$\frac{1}{\cos\alpha} = k\frac{\Delta\varphi}{\Delta k} \tag{26}$$

This is still a pulse front tilt condition for an electromagnetic wave where the new pulse front tilt angle ψ is steeper and is given by $\tan\psi = 1/\cos\alpha$. The steeper pulse front tilt angle condition is a consequence of the oblique trajectory of the speed-of-light particle with respect to the structure and field coordinates.

VI. EVALUATION OF THE AVERAGE GRADIENTS AT SYNCHRONICITY

If the input electromagnetic wave satisfies the phase synchronicity condition of equation 20 it is possible to evaluate the average deflection from the path integral of the field components within a single structure period. As seen for the specific gradient components in equation 18 for either the TE or the TM solution the average gradient components have a general form

$$\langle G_j \rangle = \mathrm{Re}\left(\lim_{L\to\infty}\left(\frac{1}{L}\int_0^L \sum_{n=-\infty}^{+\infty} P_n(k,\alpha,\Delta\varphi) e^{-iky_0} e^{i(k_p nk - \Delta\varphi + k/\cos\alpha)y(y')} dy' \right)\right) \tag{27}$$

where $P_n(k,\alpha,\Delta\varphi)$ is polynomial that depends on the field components and on the particular geometry. At phase synchronicity for the lowest order equation 19 is satisfied. Hence the complex exponent $k_p n - k\Delta\varphi + k/\cos\alpha$ becomes $k_p(n+1)$ and $\langle G_j \rangle$ simplifies to

$$\langle G_j \rangle = \mathrm{Re}\left(\lim_{L\to\infty}\left(\frac{1}{L}\int_0^L \sum_{n=-\infty}^{+\infty} P_n(k,\alpha,\Delta\varphi) e^{-iky_0} e^{ik_p(n+1)y(y')} dy' \right)\right) \tag{28}$$

Since n is integer and k_p enforces a periodicity of λ_p or an integer fraction of it all the terms of the sum in the path integral $(0,L)$ be expressed in terms of a path integral within a single structure period which corresponds to $(0,\lambda)$.

$$\langle G_j \rangle = \mathrm{Re}\left(e^{-iky_0} \frac{1}{\lambda}\int_0^\lambda \sum_{n=-\infty}^{+\infty} P_n(k,\alpha,\Delta\varphi) e^{ik_p(n+1)y(y')} dy' \right) \tag{29}$$

Since $y' = y/\cos\alpha$ equation 29 can be expressed as

$$\left\langle G_j \right\rangle = \mathrm{Re}\left(\frac{1}{\lambda_p} \int_0^{\lambda_p} \sum_{n=-\infty}^{+\infty} P_n(k,\alpha,\Delta\varphi) e^{-iky_0} e^{ik_p(n+1)y} dy \right) \tag{30}$$

The expression $\sum_{n=-\infty}^{+\infty} P_n(k,\alpha,\Delta\varphi) e^{-iky_0} e^{ik_p(n+1)y}$ corresponds to the instantaneous gradient component $G_j(y) = F_j(y)/q$ at location y when phase synchronicity for $n=-1$ is satisfied. Therefore when this condition is met the average gradient is

$$\left\langle G_j \right\rangle = \mathrm{Re} = \frac{1}{\lambda_p} \int_0^{\lambda_p} F_j(0, y, t(y))/q\, dy \tag{31}$$

Applying equation 31 for the gradient components from a TM wave

$$\begin{aligned}
\left\langle G_{\perp,x'} \right\rangle_{\mathrm{TM}} &= \frac{1}{\lambda_p} \int_0^{\lambda_p} \mathrm{Re}(E_x + cB_z \cos\alpha) dy \\
\left\langle G_{\perp,y'} \right\rangle_{\mathrm{TM}} &= \frac{1}{\lambda_p} \int_0^{\lambda_p} \mathrm{Re}(E_y \cos\alpha) dy \\
\left\langle G_{\perp,z'} \right\rangle_{\mathrm{TM}} &= \frac{1}{\lambda_p} \int_0^{\lambda_p} \mathrm{Re}(-E_y \sin\alpha) dy
\end{aligned} \tag{32}$$

Similarly, the gradient components from the TE wave are

$$\begin{aligned}
\left\langle G_{\perp,x'} \right\rangle_{\mathrm{TE}} &= \frac{1}{\lambda_p} \int_0^{\lambda_p} \mathrm{Re}(-cB_y \sin\alpha) dy \\
\left\langle G_{\perp,y'} \right\rangle_{\mathrm{TE}} &= \frac{1}{\lambda_p} \int_0^{\lambda_p} \mathrm{Re}(cB_x \sin\alpha \cos\alpha) dy \\
\left\langle G_{\perp,z'} \right\rangle_{\mathrm{TE}} &= \frac{1}{\lambda_p} \int_0^{\lambda_p} \mathrm{Re}(-cB_x \sin^2\alpha) dy
\end{aligned} \tag{33}$$

For a linear superposition of TE and TM polarizations the total transverse deflection gradient is the vector sum of the total x and z components

$$\begin{aligned}
\left\langle G_{\parallel,\mathrm{total}} \right\rangle &= \left\langle G_{\parallel,y'} \right\rangle_{\mathrm{TE}} + \left\langle G_{\parallel,y'} \right\rangle_{\mathrm{TM}} \\
\left| \left\langle G_{\perp,\mathrm{total}} \right\rangle \right| &= \sqrt{\left(\left\langle G_{\perp,x'} \right\rangle_{\mathrm{TE}} + \left\langle G_{\perp,x'} \right\rangle_{\mathrm{TM}} \right)^2 + \left(\left\langle G_{\perp,z'} \right\rangle_{\mathrm{TE}} + \left\langle G_{\perp,z'} \right\rangle_{\mathrm{TM}} \right)^2}
\end{aligned} \tag{34}$$

The total deflection gradient has an orientation that may not align with the coordinate axes

$$\tan\theta = \left(\left\langle G_{\perp,x'}\right\rangle_{\mathrm{TE}} + \left\langle G_{\perp,x'}\right\rangle_{\mathrm{TM}}\right) \Big/ \left(\left\langle G_{\perp,z'}\right\rangle_{\mathrm{TE}} + \left\langle G_{\perp,z'}\right\rangle_{\mathrm{TM}}\right) \tag{35}$$

Equations 34 and 35 indicate that a given input electromagnetic wave may produce an acceleration and a deflection gradient simultaneously. The value of the individual components depends on the polarization state, on the optical phase and on the structure geometry, and their evaluation can be performed through direct numerical integration with equations 32 and 33. It is also possible to gain a qualitative insight on the phase relation between the individual gradient components by realizing, for example, that the Fourier coefficients U_n, V_n and W_n of the TM wave are not independent. Since the particle was assumed to travel at $x = 0$ these were treated as constants. However, it can be shown that in the vacuum channel the fields satisfy Helmholtz equation [6] and since $\lambda_p \le \lambda$ and $n \ne 0$ the propagation constant is complex. Hence U_n, V_n and W_n are amplitudes of an evanescent order and have a dependence on the *x*-axis of the form that is a linear superposition of exponential decay factors [8].

$$\begin{aligned}
U_n(x) &= u_{n,+}e^{+\Gamma_n x} + u_{n,-}e^{-\Gamma_n x} \\
V_n(x) &= v_{n,+}e^{+\Gamma_n x} + v_{n,-}e^{-\Gamma_n x} \\
W_n(x) &= w_{n,+}e^{+\Gamma_n x} + w_{n,-}e^{-\Gamma_n x}
\end{aligned} \tag{36}$$

With equation 14 and the additional condition $cd_x B_z = -ikE_y$ the coefficients $u_{n,\pm}, v_{n,\pm}$ and $w_{n,\pm}$ are related by

$$cw_{n,\pm} = \frac{k}{nk_p - k\Delta\varphi} u_{n,\pm} \quad , \quad v_{n,\pm} = \frac{\pm i\Gamma_n}{nk_p - k\Delta\varphi} u_{n,\pm} \tag{37}$$

Therefore in terms of $u_{n,\pm}$ the gradient components for the TM wave at $x = 0$ are

$$\begin{aligned}
\left\langle G_{\perp,x'}\right\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(e^{-iky_0}\left(u_{-1,+} + u_{-1,-}\right)\right)\sin^2\alpha \\
\left\langle G_{\parallel,y'}\right\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(+ie^{-iky_0}\left(u_{-1,-} - u_{-1,+}\right)\Gamma_{-1} \big/ \left(k_p + k\Delta\varphi\right)\right)\cos\alpha \\
\left\langle G_{\perp,z'}\right\rangle_{\mathrm{TM}} &= \mathrm{Re}\left(-ie^{-iky_0}\left(u_{-1,-} - u_{-1,+}\right)\Gamma_{-1} \big/ \left(k_p + k\Delta\varphi\right)\right)\sin\alpha
\end{aligned} \tag{38}$$

Equation 38 reveals that there is an optical phase shift between the maximum of $\left\langle G_{\perp,x'}\right\rangle_{\mathrm{TM}}$ and the maximum of $\left\langle G_{\parallel,y'}\right\rangle_{\mathrm{TM}}$ and $\left\langle G_{\perp,z'}\right\rangle_{\mathrm{TM}}$ that depends on the coefficients $u_{-1,+}$ and $u_{-1,-}$. For the TE mode a similar set of relations for the gradient coefficients is found

$$
\begin{aligned}
\langle G_{\perp,x'} \rangle_{\mathrm{TE}} &= \mathrm{Re}\left(i e^{-iky_0} \left(x_{-1,+} - x_{-1,-} \right) \Gamma_{-1} / \left(k_p + k\Delta\varphi \right) \right) c \sin\alpha \\
\langle G_{\parallel,y'} \rangle_{\mathrm{TE}} &= \mathrm{Re}\left(e^{-iky_0} \left(x_{-1,-} + x_{-1,+} \right) \right) c \sin\alpha \cos\alpha \\
\langle G_{\perp,z'} \rangle_{\mathrm{TE}} &= \mathrm{Re}\left(- e^{-iky_0} \left(x_{-1,-} + x_{-1,+} \right) \right) c \sin^2\alpha
\end{aligned}
\tag{39}
$$

Figure 3a shows the cross-section of a specific structure example where the particle trajectory is assumed to lie at $\alpha = 20°$ in the yz plane. Figure 3b shows a diagram of the directions of the gradient components relative to the particle trajectory. Figures 3c and 3d show the magnitude of the gradient components with respect to the optical phase of a TE and a TM polarized electromagnetic wave. The gradients were evaluated by numerical path integration of the fields as indicated by equations 32 and 33. The incoming laser wave is assumed to have an electric field amplitude of $|E_{laser}| = 1$.

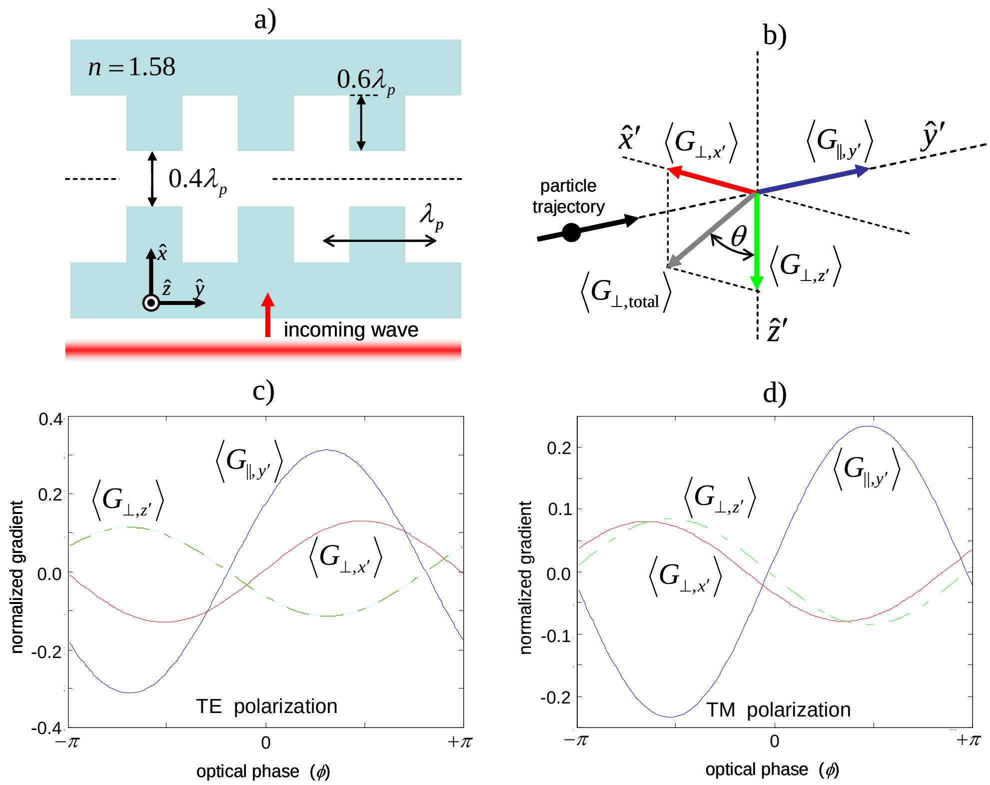

FIG. 3. Example of a periodic grating structure where the grating lines have a tilt angle of $\alpha = 20°$ to the particle trajectory. (a) Top view of the structure example. (b) Diagram of the gradient components. (c) The gradient components from the TE mode as a function of the optical phase. (d) The gradient components of the TM incoming wave as a function of the optical phase.

As predicted by equations 39 and 40 $\langle G_{\parallel,y'}\rangle$ and $\langle G_{\perp,z'}\rangle$ have opposite sign and have an optical phase offset to $\langle G_{\perp,x'}\rangle$. Figure 3 shows that for both the TE and the TM wave the acceleration gradient is larger than either deflection gradient.

VII. A PROPOSED LASER-DEFLECTION STRUCTURE

As seen in Figure 3 the magnitude of the average deflection components $\langle G_{\perp,z'}\rangle$ and $\langle G_{\perp,x'}\rangle$ represents a significant fraction of the incoming laser plane wave amplitude. This is a motivation to explore the possibility of employing the synchronous deflection force to steer the particle beam with a laser. However as shown in Figure 3 the all three deflection force components occur from the TE and the TM wave and for a practical deflection unit it will be desirable to have the ability to apply the desired deflections in the *x* and *z* directions independently. Linear superposition of the incoming waves can be applied to accomplish this functionality. For example, applying two electromagnetic waves from the opposite sides of the structure can provide for a means to cancel $\langle G_{\parallel,y'}\rangle$. Assuming the structure is symmetric about the *y*-axis a TM plane wave with the same amplitude as in equation 36 but incident from the opposite side of the structure has coefficients

$$
\begin{aligned}
U'_n &= u_{n,+}e^{-\Gamma_n x} + u_{n,-}e^{+\Gamma_n x} \\
V'_n &= v_{n,+}e^{-\Gamma_n x} + v_{n,-}e^{+\Gamma_n x} \\
W'_n &= w_{n,+}e^{-\Gamma_n x} + w_{n,-}e^{+\Gamma_n x}
\end{aligned}
\tag{40}
$$

For this wave $v_{n,\pm}$ and $cw_{n,\pm}$ are related to $u_{n,\pm}$ by

$$
cw_{n,\pm} = \frac{k}{nk_p - k\Delta\varphi}u_{n,\pm} \quad , \quad v_{n,\pm} = \frac{\mp i\Gamma_n}{nk_p - k\Delta\varphi}u_{n,\pm}
\tag{41}
$$

And hence at $x = 0$ the gradient components are

$$
\begin{aligned}
\langle G_{\perp,x'}\rangle'_{\mathrm{TM}} &= \mathrm{Re}\left(e^{-iky_0+\phi}\left(u_{-1,+} + u_{-1,-}\right)\right)\sin^2\alpha \\
\langle G_{\parallel,y'}\rangle'_{\mathrm{TM}} &= \mathrm{Re}\left(-ie^{-iky_0+\phi}\left(u_{-1,-} - u_{-1,+}\right)\Gamma_{-1}/\left(k_p + k\Delta\varphi\right)\right)\cos\alpha \\
\langle G_{\perp,z'}\rangle'_{\mathrm{TM}} &= \mathrm{Re}\left(+ie^{-iky_0+\phi}\left(u_{-1,-} - u_{-1,+}\right)\Gamma_{-1}/\left(k_p + k\Delta\varphi\right)\right)\sin\alpha
\end{aligned}
\tag{42}
$$

If the two incident TM waves have the same optical phase $\phi = 0$ the total gradient components are

$$
\begin{aligned}
\left\langle G_{\perp,x'} \right\rangle_{\mathrm{TM}} + \left\langle G_{\perp,x'} \right\rangle'_{\mathrm{TM}} &= 2\,\mathrm{Re}\left(e^{-iky_0'} \left(u_{-1,+} + u_{-1,-} \right) \right) \sin^2 \alpha \\
\left\langle G_{\|,y'} \right\rangle_{\mathrm{TM}} + \left\langle G_{\|,y'} \right\rangle'_{\mathrm{TM}} &= 0 \\
\left\langle G_{\perp,z'} \right\rangle_{\mathrm{TM}} + \left\langle G_{\perp,z'} \right\rangle'_{\mathrm{TM}} &= 0
\end{aligned}
\tag{43}
$$

And when the incident TM waves have opposite optical phase, such that $\phi = \pm\pi$

$$
\begin{aligned}
\left\langle G_{\perp,x'} \right\rangle_{\mathrm{TM}} + \left\langle G_{\perp,x'} \right\rangle'_{\mathrm{TM}} &= 0 \\
\left\langle G_{\|,y'} \right\rangle_{\mathrm{TM}} + \left\langle G_{\|,y'} \right\rangle'_{\mathrm{TM}} &= 2\,\mathrm{Re}\left(+ ie^{-iky_0} \left(u_{-1,-} - u_{-1,+} \right) \Gamma_{-1} \big/ \left(k_p + k\Delta\varphi \right) \right) \cos \alpha \\
\left\langle G_{\perp,z'} \right\rangle_{\mathrm{TM}} + \left\langle G_{\perp,z'} \right\rangle'_{\mathrm{TM}} &= 2\,\mathrm{Re}\left(- ie^{-iky_0} \left(u_{-1,-} - u_{-1,+} \right) \Gamma_{-1} \big/ \left(k_p + k\Delta\varphi \right) \right) \sin \alpha
\end{aligned}
\tag{44}
$$

The same analysis can be performed for the TE waves, and with a similar result. When the optical phase between the two TE waves is opposite only the transverse deflection gradient in the *x*-direction is nonzero and when their optical phase is equal the other two components are nonzero. As can be appreciated from Figure 3 the TE wave provides the larger deflection oriented in the *x*-direction and has a maximum of $\left\langle G_{\perp,x'} \right\rangle_{\max} \sim 0.13 E_{laser}$.

Notice that for both the TE and TM polarizations a net deflection in the *z'* direction is always accompanied by a net acceleration or deceleration component. This is shown in equation 44 for the TM wave. However as shown in Figure 1 for the structures in question the critical dimension where steering is important is the *x*-direction. Since these planar structures are two-dimensional the precise beam position in the z' axis is less critical and therefore steering in this direction is likely to be required less frequently than steering in the critical *x*-direction.

If only a single laser beam is available to power the deflection structure a possible approach to provide a net deflection with no net acceleration is to employ linear superposition of a TE and a TM wave incident from the same side of the structure. For the example shown in Figure 3, choice of the amplitude of the TM wave to be 1.34 times the amplitude of the TE wave and lagging by an optical phase of 0.16π produces a cancellation of $\left\langle G_{\|,y'} \right\rangle$ and $\left\langle G_{\perp,z'} \right\rangle$. This combination of TE and TM modes corresponds to an elliptically polarized input wave with a Jones vector of the form

$$
J = \frac{1}{\sqrt{a_{\mathrm{TE}}{}^2 + a_{\mathrm{TM}}{}^2}} \begin{pmatrix} a_{\mathrm{TE}} \\ a_{\mathrm{TM}} \end{pmatrix} \sim \frac{1}{1.67} \begin{pmatrix} 1 \\ 1.34 \times e^{i\phi} \end{pmatrix} \quad ; \quad \phi \sim 0.16\pi \tag{45}
$$

Figure 4a shows the resulting deflection gradient in the *x*-direction having a maximum value of $\left\langle G_{\perp,x'} \right\rangle_{\max} \sim 0.13 E_{laser}$. The maximum deflection gradient is the same as that resulting from the application of two opposite TE waves considered earlier. Both schemes for cancelling the acceleration component present advantages and difficulties. While the first method relies on linearly polarized light it requires the superposition of two separate laser beams which typically increases the complexity of the optical system that powers the structure. The second approach, while only employing one laser beam, requires a set of polarization components to produce the elliptically polarized wave.

If residual acceleration or deceleration in the deflection structure has no significant impact on the electron beam no superposition of TE or TM modes is required, and as shown in Figure 4b a TE wave produces a total maximum deflection gradient $\left\langle G_{\perp,\text{total}} \right\rangle_{\max} \sim 0.16 E_{laser}$. At the optical phase of maximum deflection the orientation of the deflection in the *x'z'* plane, indicated by the angle θ, is approximately 40° and the acceleration gradient $\left|\left\langle G_{\|,y'} \right\rangle\right| \sim 0.3 E_{laser}$.

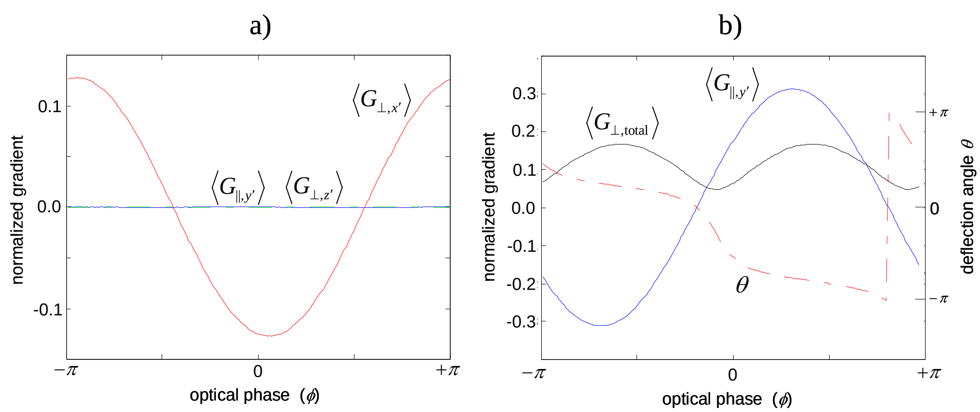

FIG. 4. (a) Use of elliptically polarized light to provide a net deflection with no acceleration. (b) The total deflection from a single TE wave. The dashed line θ is the orientation angle of the deflection in the *x'z'* plane (see Figure 3b)

VIII. CONCLUSIONS

Pulse-front tilted waves can maintain extended phase synchronicity with a speed-of-light particle traveling inside a periodic structure. An oblique orientation between the periodic structure and the particle trajectory introduces a nonzero synchronous deflection force, and the choice of polarization and optical phase provide control for the direction of the deflection force. This allows for the possibility of laser-driven steering elements for relativistic charged particles. The means to provide a laser-driven deflection was briefly analyzed here. Other interesting applications such as the possibility of laser-driven undulators will be explored in further depth in an upcoming article.

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