

## Research Article

# The Energy Conservation in Our Universe and the Pressureless Dark Energy

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Recent observations confirm that a certain amount of unknown dark energy exists in our universe so that the current expansion of our universe is accelerating. It is commonly believed that the pressure of the dark energy is negative and the density of the dark energy is almost a constant throughout the universe expansion. In this paper, we show that the law of energy conservation in our universe has to be modified because more vacuum energy is gained due to the universe expansion. As a result, the pressure of dark energy would be zero if the total energy of our universe is increasing. This pressureless dark energy model basically agrees with the current observational results.

## 1. Introduction

In the past decades, the data from supernovae confirmed the accelerating expansion of our universe [1, 2]. This acceleration can be explained by assuming the existence of a cosmological constant  $\Lambda$  in the Einstein field equation. Usually, this constant is regarded as a kind of energy called “dark energy” that exists in our universe. The  $\Lambda$ CDM model, which is the most robust scenario nowadays to describe the evolution of our universe, suggests that the dark energy density  $\rho_\Lambda$  is a constant throughout the evolution of our universe. This model provides good fits for the data on the large-scale structure of the universe and the Cosmic Microwave Background [3, 4]. Besides this major model, there are some other dark energy models which can also satisfy the current observational constraints [5–9].

In fact, quantum physics shows that vacuum is not really nothing but contains energy. The discovery of the Casimir effect indicates that some nonzero energy exists in vacuum, which is called the vacuum energy [10]. Therefore, many cosmologists believe that dark energy is indeed the vacuum energy [11–13]. However, theoretical calculations show that the predicted value of vacuum energy is nearly 120 orders of magnitude larger than the observed value in our universe [11]. Although there are some theoretical suggestions which can alleviate the problem, no satisfactory explanation is obtained

[14–17]. Moreover, the idea of vacuum energy suffered from the “coincidence problem” [13]. It states that the ratio of dark energy density to matter energy density is extremely very small at the very beginning of universe expansion while the current value of dark energy density is so close to the present matter density [18]. Therefore, some suggest invoking a time-dependent dark energy, which is now known as the quintessence model, to solve the “coincidence problem” [19–21].

Although the idea of vacuum energy has two major problems, recent observations show that the dark energy density is really very close to a constant value [22–24]. If  $\rho_\Lambda$  is a constant, the parameter of the dark energy equation of state should be  $w = -1$ , where the pressure of dark energy is given by  $P_\Lambda = w\rho_\Lambda c^2$ . The most recent observational constraint is  $w = -0.99 \pm 0.06$  [25]. Therefore, it seems that it may not be necessary to invoke the idea of quintessence, which suggests that  $w$  is not exactly equal to  $-1$ . Furthermore, most of the quintessence models involve some free parameters and arbitrary scalar potential functions which make the model much more complicated than the vacuum energy model. Therefore, based on the observations and the simplicity of model, vacuum energy model is still a better one that can explain the required dark energy in our universe.

Since positive pressure and energy would produce attractive gravitational effect on our universe expansion, the negative pressure in dark energy is usually interpreted as

the effect of “antigravity”. It is very strange because we do not know anything that is positive in energy but produces negative pressure. However, the result of the negative pressure of dark energy is solely based on the assumption of energy conservation. What would be the equation of state of the dark energy if the total energy of our universe is increasing? In this paper, we show that dark energy can be pressureless if we assume that the total energy of our universe is increasing when the universe is expanding. We first review the essential equations that govern the evolution of our universe in the standard picture. Then we discuss the effect of the equations after allowing that our universe’s total energy is increasing.

## 2. The Evolution of Our Universe in the Standard Picture

The original Friedmann equation with dark energy term in a flat universe is given by

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2, \quad (1)$$

where  $a$  is the cosmic scale factor,  $G$  is the universal constant for gravitation, and  $\rho = \rho_r + \rho_m + \rho_\Lambda$  with  $\rho_m$  and  $\rho_r$  being the energy density of matter and radiation, respectively. On the other hand, if the total energy of our universe remains constant, the law of energy conservation gives

$$\frac{d}{dt} (\rho a^3) = -\frac{P}{c^2} \frac{d}{dt} (a^3), \quad (2)$$

where  $P = P_m + P_r + P_\Lambda$  is the total pressure with  $P_m$  and  $P_r$  being the matter and radiation pressure, respectively, and  $c$  is the speed of light. By differentiating (1), we get

$$\frac{\ddot{a}}{a} - \frac{4\pi G \dot{\rho}}{3} \frac{a}{\dot{a}} - \frac{8\pi G}{3} \rho = 0. \quad (3)$$

From (2), we get

$$\dot{\rho} = \left( -3\rho - \frac{3P}{c^2} \right) \frac{\dot{a}}{a}. \quad (4)$$

By substituting (4) to (3) and  $P_\Lambda = w\rho_\Lambda c^2$ , we get

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3} \left[ (-3w-1) \rho_\Lambda - \rho_m - \frac{3P_m}{c^2} - \frac{3P_r}{c^2} \right]. \quad (5)$$

The above equation is the standard equation to describe our universe expansion. Since the effects of  $P_m$  and  $P_r$  are negligible in the matter and dark energy dominated universe, the accelerating universe expansion requires  $w \leq -1/3$ ; that is, dark energy should have negative pressure. For the standard  $\Lambda$ CDM model,  $\rho_\Lambda$  is a constant and  $w = -1$ . Our universe is accelerating now since  $\rho_\Lambda > \rho_m$ .

## 3. The Increasing Energy in Our Universe

However, the above calculations are based on the assumption of the constant total energy (2). If dark energy is really the

vacuum energy, the total energy of our universe is increasing when the universe is expanding. The more the space is involved in our universe, the more the energy would be contained in our universe [26]. In other words, some extra energy is “flowing” into our universe. If this is the case, (2) should be rewritten as

$$\frac{d}{dt} (\rho a^3) = -\frac{P}{c^2} \frac{d}{dt} (a^3) + \dot{E}, \quad (6)$$

where  $\dot{E}$  is the rate of energy “flowing” into our universe. Since our universe is expanding, the actual “volume” of our universe is also increasing. Therefore the amount of dark energy (vacuum energy) in our universe is increasing due to the increasing vacuum “volume.” Since the density of vacuum energy is a constant and the cube of the scale factor  $a^3$  is directly proportional to the total “vacuum volume,” we should have

$$\dot{E} = \rho_\Lambda \frac{d}{dt} (a^3). \quad (7)$$

Therefore, by using (6) and (7), we get

$$\dot{\rho} = \left( 3\rho_\Lambda - 3\rho - \frac{3P}{c^2} \right) \frac{\dot{a}}{a}. \quad (8)$$

By putting (8) into (3), the additional term  $\dot{E}$  finally rewrites the acceleration equation as

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3} \left[ (2-3w) \rho_\Lambda - \rho_m - \frac{3P_m}{c^2} - \frac{3P_r}{c^2} \right]. \quad (9)$$

Obviously, the expansion of our universe can be accelerating even if  $w = 0$ . From the traditional model (our universe as a closed system), observations indicate that  $w \approx -1$  [25]. Surprisingly, by comparing (5) with (9), this is equivalent to  $w \approx 0$  in the new model (the total energy of our universe is increasing). This shows that dark energy is pressureless if we rewrite the energy conservation law.

## 4. Discussion

The traditional cosmological model considers the total energy of our universe being constant so that dark energy must provide negative pressure in our universe. However, if we assume that dark energy is indeed the vacuum energy, more energy would be contained in our universe as the universe is expanding. In other words, the total energy in our universe should be increasing. If we include the increasing energy term in the fluid equation, we need not require the dark energy to have negative pressure. Recent observations suggest that  $w \approx -1$  in the traditional model is actually equivalent to  $w \approx 0$  in our model. This indicates that dark energy is pressureless. Therefore, our model is consistent and compatible with the observational evidence about a negative  $w$ .

Basically, the idea of negative pressure in dark energy serves as the “antigravity” to balance the gravity effect from matter. If dark energy is pressureless, how can it balance the

attractive gravitational force? The above result tells us that the Einstein field equation can intrinsically give acceleration of universe expansion if the total energy of universe is increasing. In the traditional model, the increased vacuum energy due to universe expansion is used to do negative work so that the total energy remains unchanged. However, we have no clear idea why the dark energy has to do negative work to dissipate the energy gained by itself due to expansion. If dark energy is pressureless, no work has to be done by the dark energy. All the energy gained during universe expansion is vacuum energy. Since the term  $\rho_\Lambda$  in (6) would be cancelled by (7), the scale dependence of matter and radiation would not be affected by the amount of vacuum energy if  $w = 0$ .

In fact, assuming that the total energy of our universe is increasing and the dark energy is pressureless is mathematically equivalent to the view that the total energy of our universe remains constant with negative pressure dark energy. However, the physical interpretations are different from each other. The former view is a better interpretation because we need not assume some new form of energy which has negative pressure. In fact, the Casimir effect just shows us that vacuum energy exists, but not the existence of negative pressure. Also, no work has to be done by the dark energy when the universe is expanding. The only drawback is that we need to assume that the total energy of our universe is not always constant, which has never been proven to be a strict law in astrophysics. Nevertheless, law of energy conservation can still be applied in general situations as the effect of vacuum energy is negligible. This would not be true only if we consider our entire universe as an object.

In our model, we are not saying that the universe is expanding in a greater space with increasing its size and volume in a background space and proceeding in the environment. Based on general relativity, the “new volume” in our universe is created by the expansion of the universe. According to quantum mechanics, this simultaneously creates the “new vacuum energy” because the vacuum energy (dark energy) is associated with the volume. Therefore, the increasing energy in our universe is a direct consequence of the theories of general relativity and quantum mechanics.

Although our model can give a new interpretation of the nature of dark energy, it cannot offer a satisfactory explanation to the cosmological constant problem. Some new mechanisms are required to cancel out the large amount of vacuum energy. Also, this model cannot account for the scalar field that is responsible for inflation.

## 5. Conclusion

If the total energy of our universe is increasing during the universe expansion, the dark energy could be regarded as pressureless. This interpretation does not violate any observational results in cosmology.

## Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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