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Spectroscopy of quantum-corrected Schwarzschild black hole

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Abstract

The quantization of black hole parameters is a long-standing topic in physics. Adiabatic invariance, periodicity of outgoing waves, quantum tunneling, and quasi-normal modes are the typical tools via which the area and the entropy of a black hole are quantized. In this paper, the quantum spectra of area and entropy of quantum-corrected Schwarzschild black hole are investigated. The deformation in the space–time of Schwarzschild black hole was perused by Kazakov and Solodukhin. Here the deformed Schwarzschild metric is taken into account, and the effect of the space–time modification on the minimal area and entropy increment for the Schwarzschild black hole is scrutinized, utilizing two different procedure: Jiang-Han's method of the adiabatic invariant integral and Zeng et al.'s approach of the periodic property of outgoing waves. The analyses of this paper draw the conclusion that the quantum correction to the space–time does not alter the quantum characteristics of the Schwarzschild black hole.
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1. Introduction

At the early eighties of XX century, quantum aspects of black hole became an intense focusing issue of the physicists, when Bekenstein posited that the entropy of a black hole is proportional to its horizon area, and conjectured that the minimal increment in the horizon area is $\Delta\mathcal{A} = 8\pi\hbar$ [1,2]. He further argued, considering the Ehrenfest principle, that the horizon area of a non-extremal black hole is an adiabatic invariant quantity [3–6], that is, the area of

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black holes changes very slowly in comparison with the variation of external perturbations. Bekenstein considered the neutral test particle of size in the order of Compton wavelength to calibrate the spectrum of Kerr black hole. At the same time, Hod evaluated two different area quanta–taking into account the absorption of charged particles by Reissner–Nordström black hole, $\Delta\mathcal{A} = 4\hbar$ [7]; and considering the Bohr correspondence principle along with studying the asymptotic behavior of the real part of highly damped quasi-normal frequencies, $\Delta\mathcal{A} = 4\hbar\ln 3$ [8]. Later, Kunstatter merged the suggestion of Bekenstein that the horizon area of a black hole is an adiabatic invariant with the method of quasi-normal mode frequencies of a perturbed black hole employed by Hod along the way of quantizing the horizon area, and replaced the adiabatic quantity $\mathcal{I} = \int \mathrm{d}E/\omega_\mathrm{c}(E)$ by $\mathcal{I} = \int \mathrm{d}M/\omega_\mathrm{R}(E)$, where $E,\ \omega_\mathrm{c}(E),\ M$, and $\omega_\mathrm{R}(E)$ are the system-energy, the classical vibrational frequency, the mass of the oscillating black hole, and the real part of quasi-normal frequency, respectively [9]. Applying the Bohr–Sommerfeld quantization rule, Kunstatter found exactly the same quantum previously derived by Hod as $4\hbar\ln 3$. But this outcome in future faced some deficiencies [10–12]. Maggoire, taking the proper frequency as $\omega_\mathrm{c} = \left[|\omega_\mathrm{R}|^2 + |\omega_\mathrm{I}|^2\right]^{1/2}$, ω_I being the imaginary part of the quasi-normal frequency, and contemplating solely on the shift in two successive quantum levels, recovered the same Bekenstein spectrum [13]. After these pioneering contributions, many black holes in a variety of space–times have been quantized till today. Some of the recent works can be found in Refs. [14–24].

Along with the quasi-normal mode frequencies, different forms of adiabatic invariance have been proposed so far [25–31]. Particularly in Refs. [26–28], Barvinsky et al. put forward a quantization technique independently of the quasi-normal modes, using some basic properties of Euclidean black hole thermodynamics. They treated the mass and the charge of a black hole as canonical conjugates, and the time as Euclideanized. Also in Ref. [31], Majhi and Vagenas proposed a fascinating but straightforward form of the adiabatic invariant integral inspired by the outgoing positive-energy particle emanated from the Hawking radiation passing through the event horizon to infinity, and causing an oscillation. Majhi–Vagenas adiabatic expression can be expressed as

$$\mathcal{I} = \int p_i \mathrm{d}q_i = \int\int_0^{p_i} \mathrm{d}p_i' \mathrm{d}q_i = \int\int_0^{p_0} \mathrm{d}p_0' \mathrm{d}q_0 + \int\int_0^{p_r} \mathrm{d}p_r' \mathrm{d}q_r,$$

where p_i is the momentum conjugate to q_i, $q_0 = \tau$ which is the Euclidean time, and $q_r = r$ is the radial coordinate. In the meantime, a similar expression but with a closed integral was proposed by Jiang and Han as $\mathcal{I} = \oint p_i \mathrm{d}q_i$ [32], because the integral $\int p_r \mathrm{d}r$ was found to violate the invariancy under canonical transformation, while instead the contour integral $\oint p_r \mathrm{d}r$ obeys the property [33–35]. On the other hand, Zeng et al. adopted a different method, namely the periodicity of outgoing wave to derive the area spectrum of Schwarzschild, Kerr, and BTZ black holes [36,37]. Outside the horizon of an excited black hole, the outgoing wave performs periodic motion. In Kruskal coordinates, the gravity system is periodic with respect to Euclidean time, and the motion of a particle in this periodic gravity system possesses a period given by the inverse of the Hawking temperature. Therefore, they deduced that the frequency of the outgoing wave is given by the inverse of the Hawking temperature.

In this paper, the quantum spectra of horizon area and entropy of the quantum-corrected Schwarzschild black hole are derived, using the adiabatic invariance integral from Jiang-Han's method [32], and the periodicity of the outgoing waves from Zeng et al.'s approach [36]. The next section contains a short note on the quantum deformation of the Schwarzschild metric taken from the literature, Sect. 3 contains the spectroscopy measurements of the quantum-corrected

Schwarzschild black hole via the adiabatic integral technique, and Sect. 4, the periodicity of the outgoing wave technique. Finally, Sect. 5 is devoted to summarize this work.

2. A brief review on quantum correction to Schwarzschild metric

The quantum-deformed Schwarzschild metric has been derived by Kazakov and Solodukhin in Ref. [38]. In accordance with the Hawking–Penrose results [39–41], the curvature of space–time increases without bound in the proximity of singularities, and the classical theory of gravitation breaks down. A successful quantized theory of gravity is essential to avoid the predictions of geodesically incomplete space–time manifolds [42]. Due to quantum excitation of the metric and the matter field, Kazakov and Solodukhin assumed that the general line element $g_{\mu\nu}$ can be written as $g_{\mu\nu} = g^{\text{sph}}_{\mu\nu} + h_{\mu\nu}$, where $g^{\text{sph}}_{\mu\nu}$ is the spherically symmetric part, and $h_{\mu\nu}$ is the non-spherically symmetric deviation term. For the Schwarzschild black hole space–time under correction, they derived the metric coefficient as $f(r) = \frac{1}{r}\left[-2M + \int^r U(\rho)\mathrm{d}\rho\right]$, where M is the black hole mass, and $U(\rho)$ is the renormalizable potential. Wontae and Yongwan considered the vacuum quantum fluctuation, and found that $U(\rho) = \mathrm{e}^{-\rho}\left[\mathrm{e}^{-2\rho} - \frac{4}{\pi}G_{\mathrm{R}}\right]^{-1/2}$ [43], where $G_{\mathrm{R}} = G_{\mathrm{N}}\ln(\mu/\mu_0)$, G_{N} is the Newton's gravitational constant, and μ is the scale parameter. Performing the integral of the renormalizable potential, the metric coefficient for the quantum-deformed Schwarzschild black hole can be derived as $f(r) = \frac{1}{r}\left[-2M + \left(r^2 - a^2\right)^{1/2}\right]$, where the correction term $a = 2\left(G_{\mathrm{R}}/\pi\right)^{1/2}$, and $r > a$. The event horizon of this black hole is at the radius $r_{\mathrm{H}} = \left[4M^2 + a^2\right]^{1/2}$. Recently the quantum-deformed Schwarzschild black hole has been investigated through different perspectives, like the phase transition by Wontae and Yongwan [43], the quasi-normal mode frequencies for gravitational and Dirac perturbations [44] and the energy and thermodynamics [45] by Saleh et al., the quasi-normal modes for electromagnetic perturbations by Wang et al. [46], and the phase transition in rainbow gravity by Shahjalal [47].

3. Area and entropy spectra via adiabatic invariance

In this section, the proposal of Majhi and Vagenas regarding the adiabaticity of black hole [31] is followed, where their line integral is substituted by the closed integral recommended by Jiang and Han [32]. The contour of the integral almost coincides with the horizon of the black hole. The metric defining the quantum-corrected Schwarzschild black hole, in the Lorentzian time coordinate t has the form

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2,$$

where

$$f(r) = \frac{1}{r}\left[-2M + \sqrt{r^2 - a^2}\right], \tag{1}$$

$\mathrm{d}\Omega_2^2$ is the unit 2-sphere line element, M is the mass of the quantum-corrected Schwarzschild black hole, and $c = G = 1$. By applying the Wick rotation in the Lorentzian time coordinate t via the transformation $t \to -\mathrm{i}\tau$, the Euclideanized form of the metric is

$$\mathrm{d}s^2 = f(r)\mathrm{d}\tau^2 + f^{-1}(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2.$$

As the only dynamic degree of freedom is the radial coordinate r in the adiabatic invariance, the integral quantity can be expressed as

$$\mathcal{I} = \oint p_r \mathrm{d}q_r = \oint \int_0^{p_r} \mathrm{d}p_r' \mathrm{d}r, \tag{2}$$

where momentum p_r is to be replaced by the energy. For a massless point particle, the equation of motion is the radial null geodesic $\dot{r} = \mathrm{d}r/\mathrm{d}\tau$ [48], and for a massive particle, the null geodesic is replaced by the phase velocity $\dot{r} = v_p$ [49]. Because the adiabatic invariance scheme put forward by Majhi and Vagenas has the attention only on the outgoing path, the particle being massless or massive is inconsequential [50]. The Hamiltonian relationships for the present system are

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}\tau} = \frac{\mathrm{d}\mathcal{H}'}{\mathrm{d}p_r'}, \qquad \mathcal{H}' = M'. \tag{3}$$

Only the $(\tau - r)$ section determines the radial path $\dot{r}$, with a vanishing unit 2-sphere line element. $M' = M - \omega'$ is the mass of the Euclideanized quantum-corrected Schwarzschild black hole after the particle with energy $E' = \omega'$ tunnels through the event horizon. When the relationships in (3) are placed in (2), the expression of the adiabatic invariant takes the form

$$\mathcal{I} = \oint \int_0^H \frac{\mathrm{d}\mathcal{H}'}{\dot{r}} \mathrm{d}r = \oint \int_0^M \mathrm{d}M' \mathrm{d}\tau. \tag{4}$$

Now, the period T of the Euclidean time along the integrating orbit about the event horizon is [51]

$$T = \oint \mathrm{d}\tau = 2\pi/\kappa_{r_\mathrm{H}}, \tag{5}$$

where κ_{r_H} is the surface gravity of event horizon for the quantum-corrected Schwarzschild black hole, which can be calculated explicitly, using equation (1) as

$$\kappa_{r_\mathrm{H}} = \frac{1}{2} \left| \frac{\mathrm{d}f(r)}{\mathrm{d}r} \right|_{r=r_\mathrm{H}} = \frac{1}{4M}.$$

Note that the deformation in the space–time does not affect the surface gravity for the Schwarzschild black hole. In other words, the surface gravity of the conventional and the quantum-corrected Schwarzschild black hole are the same, and consequently, the Hawking temperature of the two black holes are alike as

$$T_\mathrm{BH} = \frac{\hbar \kappa_{r_\mathrm{H}}}{2\pi} = \frac{\hbar}{8\pi M}. \tag{6}$$

Expression in (5) leads the right-most term of (4) to

$$\mathcal{I} = 2\pi \int_0^M \mathrm{d}M'/\kappa_{r_\mathrm{H}}'. \tag{7}$$

The periodic range of τ is $0 \le \tau \le 2\pi/\kappa$. The end form of the adiabatic contour integral, implementing the correspondence between the surface gravity and the Hawking temperature stated in equation (6) in (7) is

$$\mathcal{I} = \hbar \int_0^M \mathrm{d}M'/T_\mathrm{BH}' = 4\pi M^2.$$

On the other hand, the area of the event horizon is $\mathcal{A} = 4\pi r_{\mathrm{H}}^2 = 4\pi a^2 + 4\mathcal{I}$. Then, according to the Bohr–Sommerefeld quantization rule, $\mathcal{I} = 2\pi n\hbar$, where $n = 1, 2, \ldots$, the eigenvalues of the event horizon are $\mathcal{A}_n = 4\pi a^2 + 8\pi n\hbar$, and the area spectrum of the quantum-corrected Schwarzschild black hole is

$$\Delta\mathcal{A} = \mathcal{A}_n - \mathcal{A}_{n-1} = 8\pi\hbar.$$

At last, the Bekenstein–Hawking area-entropy relationship [52] gives the entropy spectrum as

$$\Delta\mathcal{S} = \Delta\mathcal{A}/4\hbar = 2\pi.$$

4. Area and entropy spectra via gravitational wave periodicity

The minimum increment in the area and the entropy of the quantum-corrected Schwarzschild black hole using the periodic property of the outgoing gravitational wave proposed by Zeng et al. [36] are derived in this section. There are two ways of constructing the wave function, the Klein–Gordon equation

$$g^{\mu\nu}\partial_\mu\partial_\nu\Phi - \frac{m^2}{\hbar^2}\Phi = 0,$$

or the Hamilton-Jacobi equation

$$g^{\mu\nu}\partial_\mu S\partial_\nu S + m^2 = 0,$$

where the wave ansatz Φ and the action S have the relation $\Phi = \exp\left[\frac{\mathrm{i}}{\hbar}S(t, r, \theta, \varphi)\right]$. In Klein–Gordon equation, the wave ansatz $\Phi = \frac{1}{4\pi\omega^{1/2}}\frac{1}{r}R_\omega(r, t)Y_{l,m}(\theta, \varphi)$ [53], and the space–time metric have to be replaced, and then the wave function can be derived. The action S can be decomposed for the static spherically symmetric black hole metric as [54,55]

$$S(t, r, \theta, \varphi) = -Et + W(r) + J(\theta, \varphi),$$

where E stands for the energy of the emitted particle observed at infinity. Near the horizon, $J = 0$, and $W(r) = \mathrm{i}\pi E/f'(r_{\mathrm{H}})$. The wave function near the outside of the horizon is $\Phi = \exp\left[-\frac{\mathrm{i}}{\hbar}Et\right]\Psi(r_{\mathrm{H}})$, where $\Psi(r_{\mathrm{H}}) = \exp\left[-\pi E/\left(\hbar f'(r_{\mathrm{H}})\right)\right]$. It is obvious that Φ is periodic with the period

$$T = 2\pi\hbar/E = 2\pi/\omega,$$

where $E = \hbar\omega$. The gravity system is periodic in Kruskal coordinates with respect to the Euclidean time. Hence a particle moving in this gravity system naturally owns the same period of the system [56]

$$T = \frac{2\pi}{\kappa_{r_{\mathrm{H}}}} = \frac{\hbar}{T_{\mathrm{BH}}}. \tag{8}$$

The area of the quantum-corrected Schwarzschild black hole is $\mathcal{A} = 4\pi r_{\mathrm{H}}^2 = 4\pi\left(4M^2 + a^2\right)$, and

$$\Delta\mathcal{A} = 32\pi M\mathrm{d}M = 32\pi M\hbar\omega = 32\pi M\hbar\frac{2\pi}{T}.$$

Using the relationships (8), and replacing the value of the Hawking temperature, one finds the area spectrum $\Delta\mathcal{A} = 8\pi\hbar$, and therefore the entropy spectrum, $\Delta\mathcal{S} = 2\pi$.

5. Outlook

This paper aims to find out any discrepancy in the minimal area and entropy increment in the Schwarzschild black hole when a quantum deformation is applied to the black hole space–time. To derive the area and the entropy eigenvalues, the property of the black hole adiabaticity and the periodicity of the outgoing waves near the horizon have been considered. As one exploits the periodicity of the Euclidean time coordinate, one no longer needs to keep the quasi-normal modes in the formulation. The outcomes in both of the strategies show that the quantum correction term a, which has the length dimensionality, makes no impression on the area or the entropy increment. It clearly denotes that the external perturbations leave no footprint to the quantization of the black hole parameters. Thence the eigenvalues for the classical and the quantum-corrected Schwarzschild black hole are identical for a fixed quantum level.

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