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A MOMENTUM CALCULATION FOR CHARGED TRACKS WITH MINUTE CURVATURE

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ABSTRACT

ADJUST is a calculational method written in A.N.S.I. Fortran IV to correct the momenta of charged tracks with minute radius of curvature and large fractional momentum error [$K < 0.0014$ $(GeV/c)^{-1}$ and $\Delta p/p > 0.30$]. Single application of the method to straight tracks eliminates remeasurements and avoids creating additional biases against high multiplicity events ($N_{CH} > 8$ tracks). Although ADJUST originated from the analysis of bubble-chamber events, the method is not restricted to bubble-chamber data.

I. INTRODUCTION

A calculational tool is developed which reestimates the momentum of poorly measured tracks. Previously, momentum corrections have employed parabolic fits to short tracks that have failed the circle fit, linear fits to tracks with minute curvature, and range-momentum relations for particles which experience energy loss through ionization (pions, kaons, and protons deaccelerate in the chamber liquid). In all three categories the calculated momentum is underestimated and comparable to values computed from range-momentum tables.[1]

The text is organized into four sections: 1) a discussion of the geometrical reconstruction program, 2) the correction method, 3) results of an application to 6136 charged-current events, and 4) two appendices which contain Fortran code and derivations of formulae used in the correction algorithm.

Data used in this study were generated in a high energy νN experiment (E-546) at the 15-ft bubble chamber filled with an atomic mixture of 47% Ne-H. During an exposure of 326,000 pictures, we obtained 10,200 reconstructed charged-current events (8900 ν and 1300 $\bar{\nu}$ events) which had muon momentum greater than 4.0 GeV/c. Downstream of the chamber was the External Muon Identifier (EMI) distributed into two planes containing 21 + 18 multiwire-proportional chambers; altogether the apparatus represented 198 pion absorption lengths.

II. THE GEOMETRICAL RECONSTRUCTION

The main task facing the geometry program is to calculate the momentum **p** and error matrix $\langle \delta_{p_i} \delta_{p_j} \rangle$ for each charged track in an event. All variables used to specify the momentum are calculated from measurements of E-546 film and are determined with respect to the center of the track. These variables are the azimuthal angle ϕ, the dip angle θ, and the reciprocal of the absolute value of the momentum $1/p$ in units $(\mathrm{GeV}/c)^{-1}$.

$$\frac{1}{p} = \frac{10^3 \cos\theta}{0.3\, H_z \rho} \qquad H_z \text{ in kG; } \rho = \rho \text{ (cm)}$$

The geometrical reconstruction is divided into four steps. First, the program makes a fit to the space point coordinates found on the track by the method of corresponding points (in three camera views). Second, a circle is fit to surviving space points and tracks too straight for curvature (sagitta, $\delta < 0.01$ cm), are fit with a straight line. Results of the second step are used to fit a helix to the rays in all three views of the track. Then a mass-dependent correction is added to the fit, because pions, kaons, and protons deaccelerate in the chamber liquid. Helix fits are iterative, so values of the first fit become starting values for the iteration. Our experience is that the procedure converges after four iterations.

Errors in the measured track length propagate to calculated, kinematic quantities, e.g., momentum and energy. In order to improve the resolution of the calculated quantities, we have applied minimum cuts to the track length and momentum distributions (3.0 cm and 30.0 MeV/c). Equation (1) illustrates the effect of minimum length cuts on the momentum resolution of tracks; in addition to the two sources of error, multiple coulomb scattering and the setting error from measuring tables.[2]

$$\left(\frac{\Delta p}{p}\right)^2 = \Delta^2 \begin{Bmatrix} \text{multiple} \\ \text{coulomb} \\ \text{scattering} \end{Bmatrix} + \Delta^2 \{\text{setting error}\}$$
$$= \left(\frac{57.0}{\beta H \cos\theta \sqrt{LX}}\right)^2 + \left(\frac{26.0\ p\delta}{HL^2 \cos\theta}\right)^2, \qquad (1)$$

substituting E-546 values into Eq. (1), where

$\beta = v/c \sim 1$;

$\cos\theta \sim 1$, θ is the dip angle;

$H = H_z = 30$ kG, a vertical magnetic field with resolution $(\Delta H/\Delta R)$ equal to 0.1% at center chamber;

$X = 53$ cm, radiation length in chamber;

$\delta = 0.03$ cm, average measurement setting error;

$p = p$ (GeV/c);

$L = L$ (cm).

$$\left(\frac{\Delta p}{p}\right)^2 = \frac{0.068}{L} + \frac{0.001\ p^2}{L^4}. \qquad (2)$$

According to Eq. (2), the track momentum and length contribute to the fractional-momentum error. Short tracks ($L < 3.0$ cm) and (or) large momentum ($p > 74.0$ GeV/c) produce poor resolution. The length dependence of the momentum resolution is illustrated in Fig. 1, for 126,748 tracks.

III. THE METHOD

The momentum correction takes place in two subprograms, subroutine ADJUST (LPF, LTF, PTRK) and function CALCP (PTRK, TL, DL, DPOP1, DPOP2), the algorithm. Link words LPF and LTF direct the program to the point fit and track fit banks respectively, where measurements are unpacked from large common arrays of HYDRA formatted data.[3] Such measurements include the uncorrected momentum PTRK, track length TL, error in track length DL, and the fractional momentum error DPOP2 (with DPOP1 = 0.3). See Appendix A for a copy of the code.

Corrections to the initial momentum fall into four categories:

1) tracks with $0.30 < \Delta p/p < 1.0$ and no secondary interactions,
2) tracks with $0.30 < \Delta p/p < 1.0$ and secondary interactions,
3) tracks with $\Delta p/p > 1.0$ and no secondary interactions, and
4) tracks with $\Delta p/p > 1.0$ and secondary interactions.

Momenta from categories 1 and 2 are corrected by a multiplicative factor which scales the supplied error (DPOP2) by the minimum error required for correction (DPOP1).

$$p\ (\text{after}) = \frac{\left(\frac{\Delta p_0}{p_0}\right)_{\text{minimum}}}{\left(\frac{\Delta p}{p}\right)_{\text{supplied}}} * p\ (\text{before})$$

$$= \frac{\text{DPOP1}}{\text{DPOP2}} * p\ (\text{before}). \tag{3}$$

Equation (3) can be rewritten in terms of the curvature (1/p) **before** and **after** correction.

$$\frac{1}{K\ (\text{after})} = \frac{\text{DPOP1}}{\text{DPOP2}} * \frac{1}{K\ (\text{before})}$$

$$K\ (\text{after}) = \frac{\text{DPOP2}}{\text{DPOP1}} * K\ (\text{before}) \qquad (4)$$

Momenta from categories 3 and 4 are reestimated according to a method which parametrizes the radius of curvature in terms of track length and the experimental value for the minimum sagitta ($\bar{\delta} = 700\,\mu m$).

$$K = \sqrt{\frac{252.19}{L^{7/3}}}\ (\text{GeV}/c)^{-1} \qquad (5)$$

The standard deviation for the calculated curvature is

$$\sigma_K = \left\{ \left(\frac{\partial K}{\partial L}\ \Delta L \right)^2 \right\}^{1/2}_{L = \langle L \rangle},$$

where $\langle L \rangle = 61.0$ cm and ΔL is the error in track length.

$$\sigma_K = 0.002 \Delta L,$$

and

$$p\ (\text{new}) = \frac{1}{K + \sigma_K}$$

$$= \frac{1}{\frac{15.88}{L^{13/6}} + 0.002\ \Delta L} \qquad (6)$$

See Appendix B for derivations of Eqs. (5) and (6).

Momenta of tracks in categories 2 and 4 are corrected according to the following process. Each track at the secondary vertex has the momentum minimized (with CALCP); then vector summed. Results of all calculations in a particular category, are compared, and the maximum value (PTRK) is returned to the main program.[4]

IV. RESULTS

The total event sample contains 135,665 charged tracks in which 128,018 are labeled clean (tracks with momentum resolution $\Delta p/p < 0.30$) and 7647 labelled dirty (tracks with any combination of the following characteristics, poor resolution $\Delta p/p > 0.30$, minute radius of curvature, problems with end point reconstruction, or length less than 3.0 cm). The sample of clean tracks (CT) clearly outnumbers the dirty tracks (DT) by a factor of 17; however, results based upon 7647 CT are indistinguishable from results incorporating 128,018 CT.

ADJUST operates solely on the DT sample; therefore momentum distributions are generated before and after correction. Each spectrum is parametrized according to the quantities $\langle p \rangle$, σ_p, and p_{max}; so that differences in spectra have a quantitative basis for comparison. Results of the study appear in Table I. Observe the differences between corrected DT and uncorrected DT spectra. A ratio made of corresponding parameters (corrected/uncorrected) equals 1/2.

$$\frac{\langle p\rangle^{cor}}{\langle p\rangle^{uncor}} \approx \frac{\sigma_p^{cor}}{\sigma_p^{uncor}} \approx \frac{p_{max}^{cor}}{p_{max}^{uncor}} \approx \frac{1}{2}$$

It is possible to produce systematic biases in the DT momentum spectrum following the application of ADJUST, e.g., the correction could create an excess of high (low) momentum tracks which are absent in the CT spectrum. In later stages of analysis, the uncertainties would propagate to kinematic expressions and damage the resolution of the measurements.

The CT and corrected DT spectra agree (see Table I), and the ratio (CT/corrected DT) of corresponding parameters equals one. Notice the scatter plot of momenta before and after correction (Fig. 2). Nearly 90% of the corrected values are reduced relative to the input values ($p^{cor} < p^{uncor}$), while 10% of the tracks have ($p^{cor} > p^{uncor}$), an effect which results from summing the momenta of secondaries in routine CALCP.

ACKNOWLEDGMENTS

The author wishes to thank Dr. F. R. Huson for his continuous interest in his physics endeavors and the other members of Fermilab Experiment #546.

REFERENCES

[1] There are serious limitations to momentum correction when tracks are too straight to find curvature and (or) too short to fit with a circle. For example, consider fitting a parabola to a track with the following parameters: L = 10.0 cm, σ_{xy} = 0.03 cm (collaboration average for the x,y point correlation), and δ (sagitta) = 0.07 cm. The sagitta ($\delta = L^2/8\rho$) must be greater than a given constant ($3\sigma_{xy}$) before a parabolic fit applies. $\delta > 3\sigma_{xy}$, but $0.07 \text{ cm} \not> 0.09$ cm. Therefore the given track must be fit with a straight line.

[2] J. W. Burren and J. Sparrow, "The Geometrical Reconstruction of Bubble Chamber Tracks," Rutherford High Energy Laboratory NIRL/R/14, 1963.

[3] R. K. Böck and J. Zoll, Invitation to HYDRA, CERN/D.Ph II/PROG 74-4, 24.5.1974. The HYDRA applications library is a CERN system written in A.N.S.I. Fortran. HYDRA is capable of the geometrical and kinematic reconstruction of multiparticle final states in high energy scattering experiments. The system of coding resembles the mythological Hydra in that data are stored within tree-like structures. Link words are used to connect information banks throughout the network.

[4] P_A and P_B are calculated in each category.

$$P_A = \left\{ \begin{array}{ll} \dfrac{1}{K + \sigma_K} & \text{, for tracks with } \Delta p/p \geq 1.0 \\ \dfrac{DPOP1}{DPOP2} * P_{BEFORE} & \text{, for tracks with } 0.30 \leq \Delta p/p < 1.0 \end{array} \right\}$$

and

$$P_B = \begin{cases} 0, & \text{for tracks without secondary interactions} \\ \text{MINIMUM } \left|\sum_i \vec{P}\ (\text{after})\right|, & \text{i runs over all secondaries connected to the primary track} \end{cases}$$

PTRK = MAXIMUM (P_A, P_B)

PTRK is returned to main program.

The 135,665 tracks are classified into two groups 1) a sample of 56,188 tracks from the primary, interaction vertex and 2) 79,477 tracks from secondary interactions.

[5] P. H. Garbincius and L. G. Hyman, Phys. Rev. **A2**, 1834 (1970).

[6] F. R. Huson, Brookhaven National Laboratory Report 11368 (1968).

7 R. D. Evans, **The Atomic Nucleus** (McGraw-Hill, New York, 1967), p. 652.

TABLE I. Parameters for CT and DT Momentum Distributions.

Sample	N(p < 5 GeV/c) tracks	N(p > 5 GeV/c) (tracks)	<p (GeV/c)>	σ_p (GeV/c)	p_{max} (GeV/c)
Uncorrected					
(CT) $\Delta p/p < 0.30$	109,189	18,829	5.2	17.6	359.9
(DT) $\Delta p/p > 0.30$	5,572	2,075	12.7	44.8	982.4
Corrected					
(DT) $\Delta p/p > 0.30$	6,108	1,539	6.0	20.2	367.4

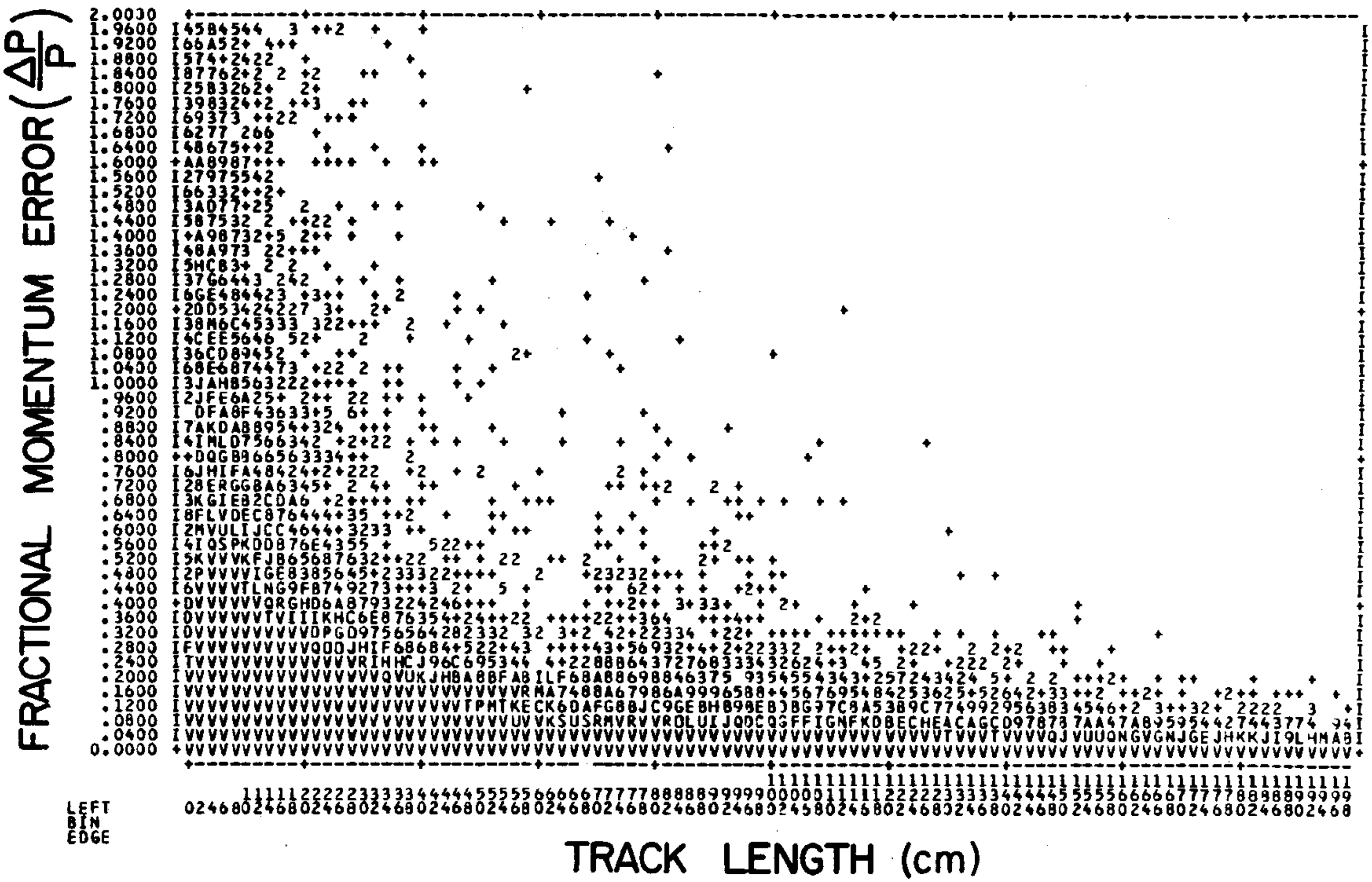


Fig. 1. Momentum resolution as a function of track length.

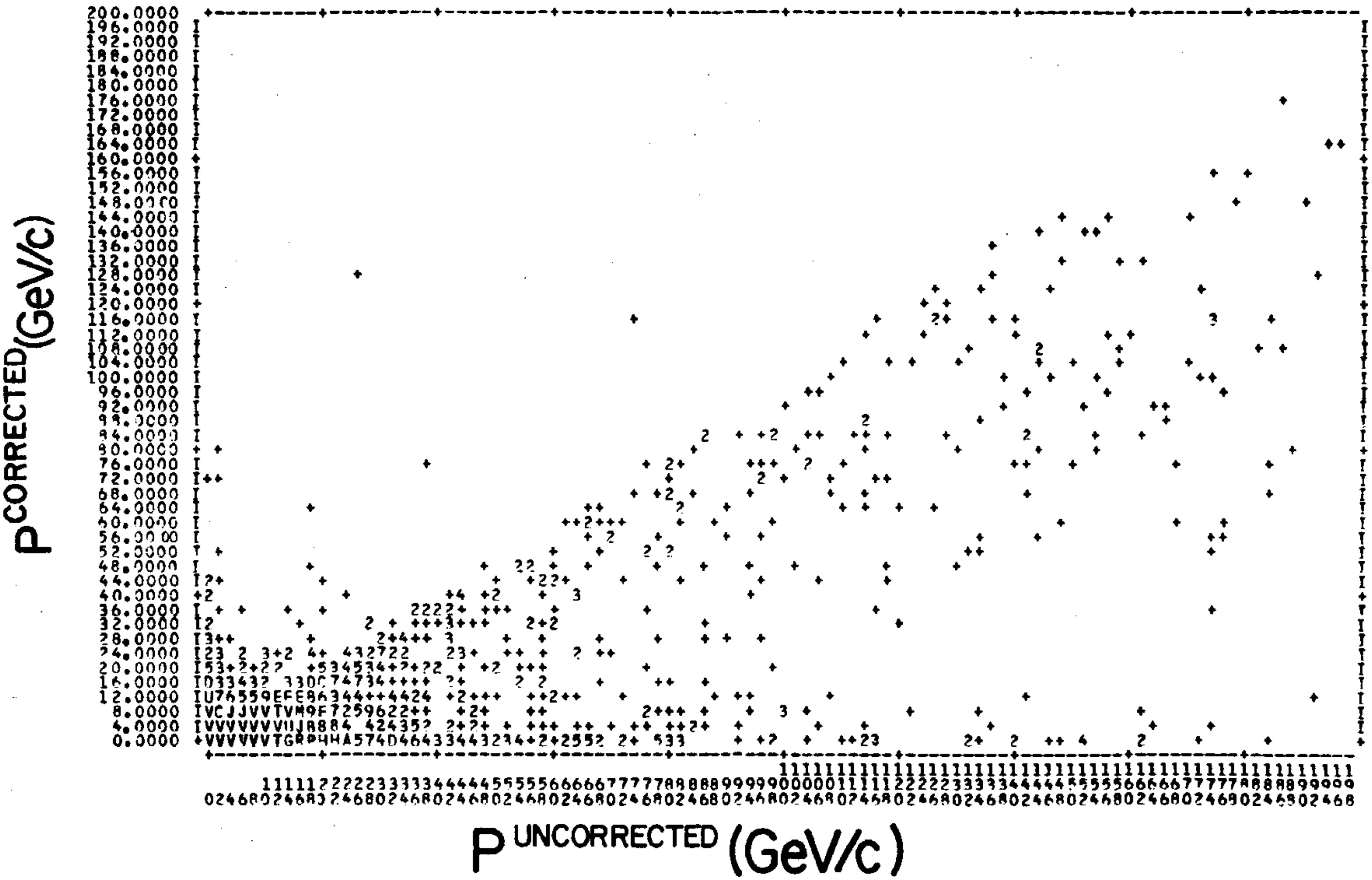


Fig. 2. Corrected versus uncorrected momentum.

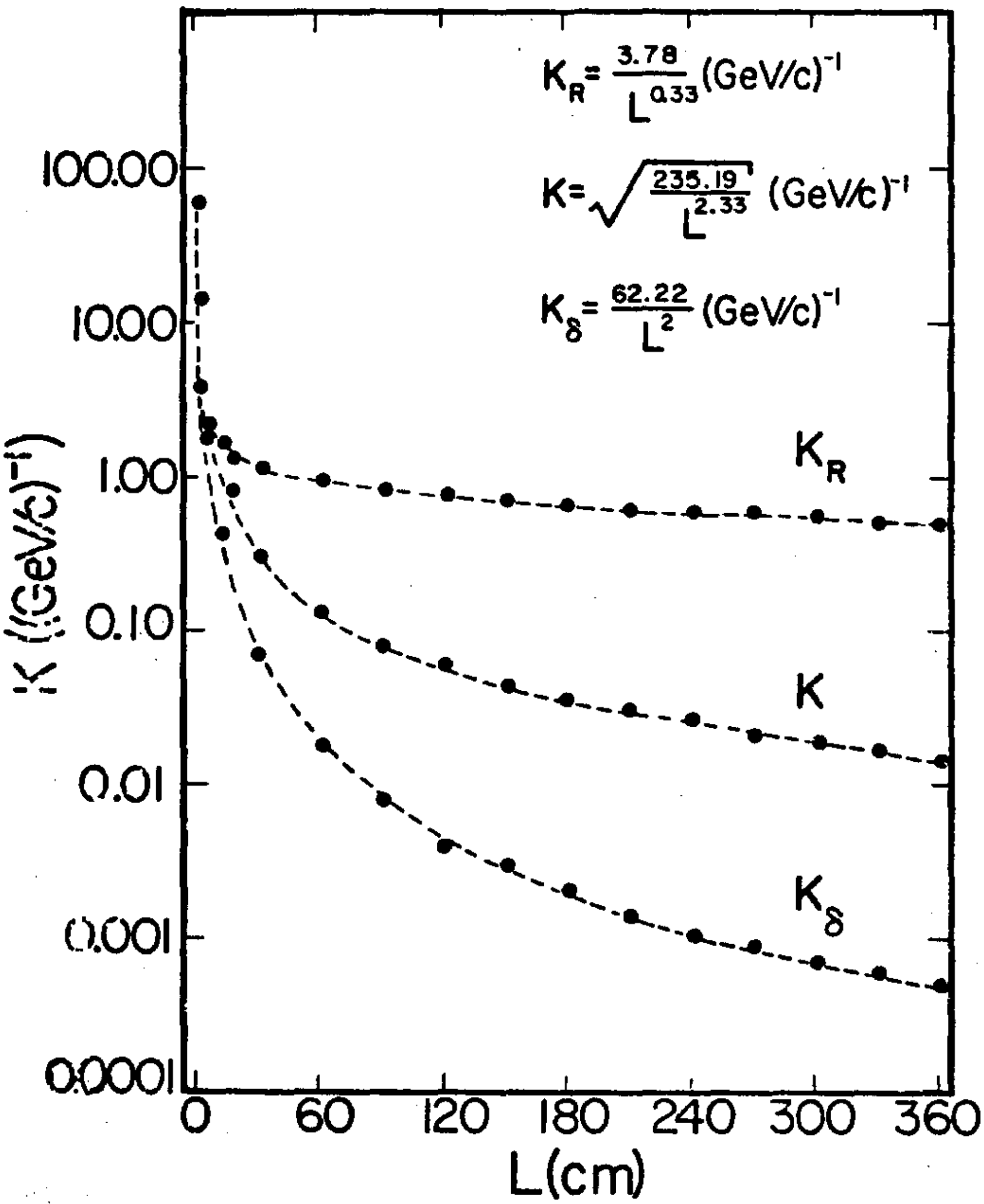


Fig. 3. Reciprocal momentum functions.

APPENDIX A

```
      SUBROUTINE ADJUST(LPF,LTF,PTRK)
      DIMENSION ID(25000)
      COMMON /DMAIN/ TWORDS,D(25000)
      COMMON /TERROR/ DPOPO,DPOP,SIGMK
      EQUIVALENCE (D(1),ID(1))
      PTRKA= 0.
      PTRKB= 0.
      PX= 0.
      PY= 0.
      PZ= 0.
      IF( LPF*LTF.LE.0 )RETURN
      TRKL= ABS( D(LTF+2) )
      DTL = D(LTF+3)
      TYPE= AMOD( D(LTF+13),100.  )
C USE ENDPOINT COORDS. OF EXTRAPOLATED BC TRACKS FOR MUON CANDIDATES.
      IF( TYPE.EQ.25..OR.TYPE.EQ.26.  )19,17
   19 JXBC= ID(LTF-4)
      IF( JXBC.EQ.  0 )GO TO 17
      TRKL= SQRT( D(JXBC+3)**2 + D(JXBC+4)**2 + D(JXBC+5)**2 )
   17 PTRKA= CALCP(PTRK,TRKL,DTL,DPOPO,DPOP)
C CORRECT POORLY MEASURED TRACK MOMENTUM BY ADDING MOMENTA OF SECONDARIES
      I1= ID(LTF-5)
      IF( I1 )16,16,9
    9 KFAIL= JBIT(D(I1),10)*10
      IF( KFAIL.NE.10 )GO TO 16
      I2= I1 - 1
   10 I2= ID(I2-1)
      IF( I2 )15,15,11
   11 IFAIL= JBIT(D(I2),6) + JBIT(D(I2),13)
      IF( IFAIL.GE.1 )GO TO 10
      D(I2+2)= ABS( D(I2+2) )
      P = CALCP(0.,D(I2+2),D(I2+3),DPOPO,2.)
      I3= I2 - 1
   12 I3= ID(I3-1)
      IF( I3 )14,14,13
   13 IFAIL = JBIT(D(I3),4)
      IF( IFAIL.EQ.1 )GO TO 12
      P1= 1.0/D(I3+2)
      P = AMIN1(P,P1)
   14 PX= PX + P*COS(D(I2+8))*COS(D(I2+9))
      PY= PY + P*COS(D(I2+8))*SIN(D(I2+9))
      PZ= PZ + P*SIN(D(I2+8))
      GO TO 10
   15 PTRKB=  1.25*SQRT( PX**2 + PY**2 + PZ**2 )
   16 PTRK= AMAX1( PTRKA,PTRKB )
      RETURN
      END
```

```
  FUNCTION CALCP(P,TL,DL,DPOP1,DPOP2)
  IF( DPOP2-1.0 )1,2,2
1 CALCP= P*(DPOP1/DPOP2)
  RETURN
2 EXPO1 = -1.167
  EXPO2 = -2.167
  A = 15.88*TL**EXPO1
  SIGMK = 18.53*DL*TL**EXPO2
  CCURV= A + SIGMK
  MUONF= INT( TL/500.  )
  IF( MUONF.GE.1 )CCURV= A
  CALCP= 1.0/CCURV
  RETURN
  END
```

APPENDIX B

The conversion between a chamber filled with 100% hydrogen (at 29° K) and one which has a 47% Ne-H mix at the same temperature begins with an expression for the molar fraction of Ne in the mix.

$$\text{atomic fraction} = \frac{x}{x + 2(1 - x)},$$

where x represents the molar fraction of Ne in mix and 2(1 - x) represents the molar fraction of H_2 in mix.

$$\frac{x}{x + 2(1 - x)} = 0.47$$

$$x = 0.64.$$

The effective atomic weight of the mix can be calculated from the Ne molar fraction x, and the individual atomic weights of Ne and H.

$$A_{mix} = \frac{x\ A_{Ne} + 2(1-x)A_H}{(2-x)}$$

$$= \frac{0.64\ (20.183) + 0.72\ (1.008)}{1.36}$$

$$= 10.03.$$

Equipped with numerical values for A_H, A_{Ne}, A_{mix}, ρ_H, ρ_{Ne}, ρ_{mix} at 29°K, the range-momentum relations can be parametrized (Table II). A hydrogen-filled chamber serves as the reference from which ranges in various concentrations of neon are calculated.[5,6] Linear sections in Table II are represented by the following power law fit.

$$\text{Range (cm)} = AP^{\alpha}(\text{MeV/c}), \tag{7}$$

where $A = 3.6 \times 10^{-7}$ cm/MeV^3/c^3 and $\alpha = 3.0$.

In order to estimate ranges in the E-546 mix, the Bragg-Kleeman relations are useful.[7]

$$\frac{R_{mix}}{R_H} = \frac{N_H}{N_{mix}} \cdot \frac{1}{S_{mix}}$$

$$\simeq \frac{(\rho_H/A_H)}{(\rho_{mix}/A_{mix})} \cdot \frac{\sqrt{A_H}}{\sqrt{A_{mix}}} \tag{8}$$

$$\simeq 0.15 \pm 0.02.$$

where R_{mix} is the range in the E-546 mix, R_H is the range in 100% hydrogen fill, S_{mix} is the stopping power of mix, N_H is the number of atoms/cm^3 in hydrogen, and N_{mix} is the number of atoms/cm^3 in the mix.

$$R_{mix} = 0.15\ (3.6 \cdot 10^{-7}\ \text{cm/MeV}^3/\text{c}^3)p^3(\text{MeV/c})$$

$$\{R_{mix}(1.85 \times 10^{-2}\ \text{GeV}^3/\text{c}^3)\}^{1/3} = p(\text{GeV/c}) \tag{9}$$

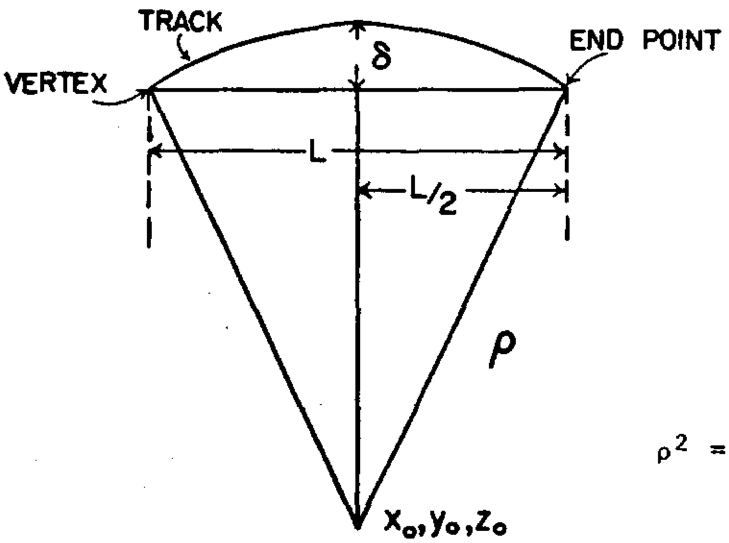
Solve Eq. (9) for (1/p)

$$\frac{1}{\{0.0185\ R_{mix}\}^{1/3}} = \frac{1}{p}.$$

Since

$$K \text{ (range)} = 1/p$$
$$= \frac{3.78}{L^{1/3}} \quad (\text{GeV}/c)^{-1}. \qquad (10)$$

Next, we derive an expression which parameterizes the radius of curvature ρ in terms of the minimum sagitta and track length.



$\delta = \bar{\delta}(\text{cm}) = 0.07$ cm

$L = L(\text{cm})$

$\rho = \rho(\text{cm})$

$$\rho^2 = (\rho - \delta)^2 + \frac{L^2}{4} \qquad (11)$$

$$\rho = \frac{\delta}{2} + \frac{L^2}{8\delta}\ . \qquad (12)$$

Substituting $\bar{\delta}$ into Eq. (12).

$$\rho = 0.035 + 1.79\ L^2$$

The particle momentum is calculated in terms of the radius of curvature in a uniform, vertical, magnetic field (30 kG) and the E-546 mix.

$$pc\ (\text{MeV}) = 0.3\ H_z \rho$$
$$= 9.0\ \rho \qquad (13)$$
$$pc\ (\text{GeV}) = 0.009\ (0.035 + 1.79\ L^2),$$

or in terms of variable K.

$$K_\delta = 1/p(\text{GeV}/c) = \frac{62.22}{L^2} \tag{14}$$

Momentum estimates in Eq. (13) are quadratic in track length, and therefore represent the upper limit for calculated momentum. Momentum estimates in Eq. (9) are slowly dependent on the track length ($\sim L^{1/3}$) and represent the lower limit for calculated momentum. In K-space we have an inverted picture of limits. K_R, calculated from range-momentum relations, represents the upper limit on curvature estimate; and K_δ, calculated from the minimum sagitta method, represents the lower limit on curvature. The correction method weights the K_δ by the slowly varying K_R and takes the square root (for reasons of dimensionality).

$$K\ (\text{estimate}) = \sqrt{K_\delta \cdot K_R}\ \ (\text{GeV}/c)^{-1} = \sqrt{\frac{252.19}{L^{2.33}}} \tag{15}$$

$$p\ (\text{estimate}) = \frac{1}{K\ (\text{estimate}) + \sigma_K}\ .$$

The error in the calculated curvature is added in order to produce a minimum estimate.

TABLE II. A Partial Range-Momentum Table For 100% H_{mix}.

Range (cm)	Momentum (MeV/c)
1.0	141
2.0	172
4.0	210
6.0	234
8.0	252
10.0	305
20.0	380
50.0	440
100.0	520
200.0	640