#### RELATIVISTIC BRUECKNER-HARTREE-FOCK APPROACH FOR NUCLEAR MATTER

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### ABSTRACT

Starting from the full Bonn meson-exchange model for the NN-interaction an OBEP is constructed in the framework of the Thompson version of the Blankenbecler-Sugar reduction of the Bethe-Salpeter equation. The pseudo-vector coupling of the pion to the nucleon is assumed. An excellent quantitative description of the deuteron and the latest phase-shift analyses of NN-scattering is achieved. This potential is applied to the system of infinite nuclear matter in the relativistic Dirac-Brueckner approach. Due to additional strongly density-dependent relativistic saturation effects, which do not occur in conventional Brueckner theory, the empirical saturation energy <u>and</u> density of nuclear matter are reproduced. This potential may serve as a good starting point for the evaluation of the optical potential to be applied in nucleon-nucleus scattering.

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#### I. INTRODUCTION

In this contribution we will present and discuss a relativistic extension of the Brueckner theory of nuclear matter. We are aware of alternative relativistic descriptions of nuclear matter, however, as we assume that they are sufficiently well covered by other authors<sup>1</sup>, we will not discuss them here.

The principal goal of the theory of Brueckner, Bethe and Goldstone<sup>2</sup> (in short: Brueckner theory) is to understand nuclear structure in terms of the free nucleon-nucleon (NN) interaction. As we believe that this is still one of the ultimate aims of theoretical nuclear physics, we further pursue Brueckner theory in our work and not one of the alternatives in which the connection to the free NNinteraction is not clear.

The history of actual calculations in the framework of Brueckner theory is long beginning as early as 1958 in Los Alamos with a work by Brueckner and Gammel<sup>3</sup> using the Gammel-Thaler potential<sup>4</sup>. In spite of the tremendous amount of work done since then the quantitative success in explaining the ground state properties of nuclear matter and closed shell nuclei has been limited<sup>5,6</sup>.

Motivated by this fact, the first relativistic extensions of Brueckner theory were suggested and used by the Bonn group<sup>7,8,9</sup> and by Lee and Tabakin<sup>10</sup> in the early 1970's. However, in these calculations the principal problem of conventional Brueckner theory remained, namely it turned out to be impossible to reproduce the empirical saturation energy <u>and</u> density simultaneously.

The relativistic description of nuclear matter was finally substantially extended by Shakin and co-workers<sup>11</sup>, and we will call this extension the Dirac-Brueckner approach. In this work it was realized that the large scalar and vector potentials present in the nuclear medium would change the Dirac spinor representing a nucleon, a fact which was already known from mean field theory<sup>1</sup>. However, in the actual calculations performed by the Brooklyn College group<sup>11</sup> the new effect is taken into account only in lowest order perturbation theory. Single particle energies and wave functions (Dirac spinors) in the medium are not determined selfconsistently, though this is a substantial requirement in Brueckner theory. Further, this group (though they did their work around 1980) applied outdated meson-exchange potentials from the early 1970's in which - from today's point of view - partly inappropriate coupling constants and cut-off parameters were used and which were fitted to phase-shift analyses of the 1960's (i.e. from an era before the precision polarization NN-scattering data of the 1970's). The 3-dimensional reduction of the Bethe-Salpeter equation applied in that work turns out to be unsuitable for the Dirac-Brueckner approach (see Section II) when used consistently. The pseudo-vector coupling of the pion to the nucleon is also required for the Dirac approach; however, the potentials used by the Brooklyn group were constructed and fitted with the pseudo-scalar coupling. For all these reasons the results are not conclusive.

Horowitz and Serot<sup>12</sup> have recently and independently from us demonstrated how to perform the Brueckner selfconsistency in the Dirac approach both correctly and elegantly. As they intend to show relative effects only, they do not use a quantitative nuclear force in their calculations.

The work quoted so far leaves open the question as to what the Predictions of the Dirac-Brueckner approach to nuclear matter are when performed consistently and correctly.

It is the aim of this contribution to answer this question.

## II. THE RELATIVISTIC SCATTERING EQUATION

As results of a Brueckner calculation are only meaningful if a quantitative nuclear force is applied, we will first turn to the construction of the meson-exchange interaction and the relativistic scattering equation on which we will base our subsequent nuclear matter calculations.

One possible starting point for NN-scattering<sup>+</sup> is the 4-dimensional Bethe-Salpeter equation<sup>14</sup> for the scattering amplitude, which <sup>reads</sup> in operator notation:

 $T = V + V \hat{G} T$ (1)

where V is the sum of all connected two-nucleon irreducible diagrams and G the relativistic two nucleon propagator.

The solution of Eq.(1) raises formidable mathematical and numerical problems  $^{15}$ . Therefore so-called 3-dimensional relativistic re-

<sup>&</sup>lt;sup>+</sup> An alternative one is the use of time-ordered perturbation theory<sup>9</sup> as in the full Bonn model<sup>13</sup>.

ductions which can be handled more easily have been suggested. They are reviewed and discussed in the work of Woloshyn and Jackson<sup>16</sup> and in Ref.17. The basic idea is to replace Eq.(1) by a set of coupled equations:

$$T = W + WgT$$
 (2a)

$$W = V + V(\hat{G} - g)W$$
(2b)

where the propagator g is chosen such that Eq.(2a) reduces to a 3dimensional integral equation. It is common practice to leave out the second term on the r.h.s. of Eq.(2b), assuming that the old and new propagators are sufficiently close to keep that term small. This is an obvious desire as the inclusion of the full Eq.(2b) would spoil the significant simplification which is the whole purpose of the reduction.

In the present work we will choose the Thompson equation 18:

$$T(\vec{q}',\vec{q}) = V(\vec{q}',\vec{q}) - \int \frac{d\vec{k}}{(2\pi)^3} V(\vec{q}',\vec{k}) \frac{M^2}{E_k^2} \frac{\Lambda_+(1\chi\vec{k}) \Lambda_+(2\chi-\vec{k})}{2E_k - 2E_q} T(\vec{k},\vec{q})$$

with  $\pm \vec{q}(\pm \vec{q}')$  the initial (final) momenta of the interaction nucleons in the c.m. frame, M the nuclear mass,  $E_k = M^2 + \vec{k}^2$ ,  $E_q = M^2 + \vec{q}^2$ , and  $A_+^{(i)}(\vec{k})$  the positive energy projection operators of the i-th nucleon with momentum  $\vec{k}$ .

The following arguments are in favour of that equation:

(i) In the model calculations of Ref.16 the Thompson results are the closest to those gained from the full Bethe-Salpeter equation compared to all other 3-dimensional reductions which can be cast into the form Eq.(2a) and are discussed in that paper. (Note that Thompson is in fact case "F" of Ref.16 and not case "D" as stated in that work.)

(ii) Using the Thompson equation and the meson-exchange parameters of Tjon<sup>14</sup>, we almost exactly reproduce the  ${}^{1}S_{_{O}}$  phase shifts which he obtains by solving the full Bethe-Salpeter equation. (This is, in fact, also true when the Blankenbecler-Sugar<sup>19</sup> equation is used.)

(iii) Thompson (like Blankenbecler-Sugar) does not include a retardation-like term in the meson propagator, i.e. for the exchange of a scalar boson the propagator is used:

$$\frac{1}{m^2 + (\dot{q}' - \dot{q})^2}$$
(4)

where the notation is explained in Fig.l and m denotes the mass of the exchanged boson. In the former work<sup>7</sup> of the group at Bonn, a retardation-like term in the propagator was used:

$$\frac{1}{m^{2} + (\dot{\vec{q}}' - \dot{\vec{q}})^{2} - (E_{q'} - E_{q'})^{2}}$$
(5)



Fig.1. A one-boson-exchange diagram in the c.m. frame using a static propagator.

However, the correct meson-retardation is best considered in time-ordered perturbation theory, see Fig.2, in which we have the following propagator:

$$\frac{1}{\sqrt{m^{2} + (\dot{q} - \dot{q}')^{2}} (\sqrt{m^{2} + (\dot{q} - \dot{q}')^{2}} + E_{q'} - E_{q})} = \frac{1}{m^{2} + (\dot{q} - \dot{q}')^{2} + (E_{q'} - E_{q}) \sqrt{m^{2} + (\dot{q} - \dot{q})^{2}} (6)}$$

As the intermediate momenta  $\vec{q}'$  are prevailingly larger than  $\vec{q}$ , the retardation term (last term on the right in the propagator Eq.(6)) is positive in most cases, whereas in Eq.(5) it is always negative and therefore wrong. It is just an artifact of that particular 3-dimensional reduction.

Anticipating some of the nuclear matter formalism which we will

introduce in Section V, we want to indicate another fatal feature of the propagator Eq.(5). In the Dirac-Brueckner approach to nuclear matter one replaces

$$E_{q} \neq E_{q}^{*} = \sqrt{M^{*2} + \dot{q}^{2}}$$

$$E_{q}^{*}, \neq E_{q}^{*}, = \sqrt{M^{*2} + \dot{q}^{*2}}$$
(7)

and

with  $M^* < M$ .

This replacement blows up the retardation-like term in Eq.(5) and because of its negative sign enhances the propagator. As a consequence, the (attractive) second order in V is increased and by that the whole attraction in nuclear matter. However, this is an unphysical effect which only arises by taking all the details of the propagator Eq.(5) too seriously.

There is a way to estimate the medium effects on the meson propagator correctly. It is again best considered in time-ordered perturbation theory, Fig.2 and Eq.(6). The replacement (7) applied to Eq. (6) weakens the propagator which finally leads to a net repulsion in nuclear matter. In a more recent paper<sup>20</sup> we showed the inclusion of the pion-selfenergy in the propagator more than compensates the propagator effect found in Ref.9. Therefore we will use here a meson propagator without retardation or retardation-like terms.



Fig.2. A one-boson-exchange in time-ordered perturbation theory. The long dashed line indicates the states involved in the propagator.

It should be noted that the arguments (ii) and (iii) given in this section also apply to the Blankenbecler-Sugar equation, which means that it could be used equally well. Therefore the final decision for Thompson was made on purely esthetic aspects:

When using a normalization of the Dirac spinors,  $\mathsf{u}(\vec{\mathfrak{q}}),$  such that

$$u(\vec{q})^+ u(\vec{q}) = 1$$
 , (8)

which is the proper one for nuclear matter, the R-matrix version of Eq.(3) assumes the simple form:

$$R'(\vec{q}',\vec{q}) = V'(\vec{q}',\vec{q}) - P \int \frac{d^{3}k}{(2\pi)^{3}} \frac{V'(\vec{q}',\vec{k}) R'(\vec{k},\vec{q})}{2E_{k} - 2E_{q}}$$
(9)

where the prime indicates that spinors with the normalization Eq.(8) are used. P denotes the principal value. This equation has exactly the form of the familiar Lippmann-Schwinger equation in which the non-relativistic energies are replaced by relativistic ones.

The phase-shift relation for the partial wave decomposed R-matrix in an uncoupled case is

$$\tan \delta_{e} = -\frac{1}{(4\pi)^{2}} q E_{q} R_{e}^{*}(q,q)$$
 (10)

### III. THE MESON-EXCHANGE NN-INTERACTION

The kernel V of Eq.(3) should contain all irreducible meson-exchange diagrams which are relevant for NN-scattering. In the full Bonn model<sup>13</sup> such a kernel has been developed with all possible consideration, and it is explicitly used for the evaluation of the data of NN-phase-shifts and the deuteron.

As the work presented in this contribution is a <u>first</u> consistent check of the Dirac-Brueckner approach to nuclear matter, the use of the tremendous and expensive computing time, which the full kernel Would require, is not justified. Therefore we will restrict ourselves to a one-boson-exchange kernel first.

However, we construct this one-boson-exchange potential (OBEP) on the basis of the broad experience gained in developing the full model<sup>13</sup>. In particular, we choose the coupling constants and cut-off Parameters as close as possible to those determined in that comprehensive work (see Table 1). The diagrams of  $2\pi$  and  $\pi p$ -exchange are replaced by a scalar iso-scalar boson with the mass 550 MeV; in Ref. 13 it is demonstrated that this is a suitable approximation. The fit is done to the latest published phase-shift analyses of the groups of  $Bugg^{21}$  and  $Arndt^{22}$ . The deuteron and low energy scattering parameters are described accurately.

The present OBEP is in a certain sense an updating of the work in Ref.<sup>7,8</sup>, using modern coupling constants and fitting to new scattering data. However, there are also two more substantial differences: we now use a different 3-dimensional relativistic equation (see Section II) and, further, we apply the pseudo-vector (pv)

	Meson∽ mass, mα (MeV)	coupling constants and cut-offs full model <sup>a)</sup> present OBEP					
Meson		$g_{\alpha}^{L}(t=m_{\alpha}^{L})$	g <sub>α</sub> (t=0); (f/g)	Λ <sub>α</sub> (GeV)	g <sub>α</sub> (t=m <sub>α</sub> )	g <sub>α</sub> (t=0); (f/g)	Λ <sub>α</sub> (GeV)
Ť	138.03	14.4	14.08	1.3	14.6	14.27	1.3
ρ	769	0.75	0.41;	1.5	0.95	0.401;	1.3
			(6.1)			(6.1)	
ω	782.6	20	10.6	1.5	20	10.6	1.5
δ	983	2.93	1.69	2.0	4.9973	1.627	1.5
η	548.8			-	3	2.25	1.5
σ	550		~~	-	7.8749	6.729	2.0

Table l Meson Parameters

 $g_{\alpha}^{2}(t) \equiv g_{\alpha}^{2}(t=m_{\alpha}^{2}) \left[ (\Lambda_{\alpha}^{2}-m_{\alpha}^{2})/(\Lambda_{\alpha}^{2}-t) \right]^{2}$  which also defines the cut-off we apply with  $t = -(\dot{q}'-\dot{q})^{2}$ . The  $g_{\alpha}^{2}(t=0)$  coupling constant is consistent with using a cut-off of the form:  $[\Lambda_{\alpha}^{2}/(\Lambda_{\alpha}^{2}-t)]^{2}$ , which is used e.g. in Ref.15. For heavy mesons  $(\rho,\omega,\delta) g_{\alpha}^{2}(t=0)$  is the physically relevant coupling strength parameter. Note that for the present OBEP the pv-coupling of the pion is used.

a) Ref.13

coupling for the pion to the nucleon for reasons which will become <sup>a</sup>pparent in Section V. Note that this coupling differs off-shell <sup>f</sup>rom the pseudo-scalar one.

The explicit expressions for the meson-exchanges can be found in Part in the review article by Erkelenz<sup>7</sup>. Note, however, the following changes: leave out the meson retardation everywhere; as a con-<sup>seq</sup>uence of this, the  $\rho$ -exchange gets an additional term which had <sup>been</sup> published already in Ref.8 (Appendix A, Case 2, therein).

The  $\pi$ -exchange is now different from the quoted work and will be published shortly together with a comprehensive collection of all other necessary formulae.

### IV. CONVENTIONAL BRUECKNER THEORY OF NUCLEAR MATTER

The basic quantity of Brueckner theory is the reaction matrix, G, satisfying the Brueckner-Bethe-Goldstone integral equation which reads in operator notation:

$$G = V - V \frac{Q}{P} G$$
(11)

and can be written diagramatically as shown in Fig.3. V, again, denotes the NN-potential (or more general: the kernel), Q the Pauli projection operator in nuclear matter which prevents nucleons from scattering into occupied intermediate states; l/e is the two-nucleon propagator in the medium. Eq.(11) is defined in strict analo-9y to scattering the only difference being the Pauli-projector and



Fig.3. Diagramatic representation of the Brueckner integral equation and the Gmatrix. The double slash on intermediate lines indicates the channe of the NUCLE. On propagator and the Pauli-blocking in the medium.

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the change of the nucleon propagator in the nuclear medium.

The goal of Brueckner theory is to evaluate the energy per nucleon in nuclear matter as a function of the density which is in lowest order in G:

$$E/N(k_F) = \frac{1}{N} < T > + \frac{1}{2N} < G >$$
 (12)

where the first term on the r.h.s. stands for the average kinetic energy of the nucleons below the Fermi surface defined by the Fermi momentum,  $k_F$ , the second term indicates the average potential energy due to all effective two nucleon interactions. The density,  $\rho$ , is related to the Fermi-momentum by:

$$\rho = \frac{2}{3\pi^2} k_F^3 \qquad (13)$$

The explicit form of Eq.(11) is:

$$(\vec{k}_{3}\vec{k}_{4}|G(k_{F},w)|\vec{k}_{1}\vec{k}_{2}\rangle = \langle \vec{k}_{3}\vec{k}_{4}|v|\vec{k}_{1}\vec{k}_{2}\rangle - \sum_{\vec{k}_{m},\vec{k}_{n}} \langle \vec{k}_{3}\vec{k}_{4}|v|\vec{k}_{m}\vec{k}_{n}\rangle \cdot \frac{Q(k_{F},\vec{k}_{m},\vec{k}_{n})}{\varepsilon(\vec{k}_{m}) + \varepsilon(\vec{k}_{n}) - w} \langle \vec{k}_{m}\vec{k}_{n}|G(k_{F},w)|\vec{k}_{1}\vec{k}_{2}\rangle$$
(14)

where spin and isospin indices are suppressed. The single-particle energy of a nucleon in nuclear matter is defined by:

$$\varepsilon(\vec{k}_{i}) = T(\vec{k}_{i}) + U(\vec{k}_{i})$$
(15)

with  $U(\vec{k}_i) = \sum_{\substack{k_j \mid \leq k_F}} \langle \vec{k}_i \vec{k}_j \mid G(k_F, w = \epsilon(\vec{k}_i) + \epsilon(\vec{k}_j) \mid \vec{k}_i \vec{k}_j - \vec{k}_j \vec{k}_i \rangle.(16)$ 

Equation(12) is explicitly:

$$\frac{E}{N}(k_{F}) = \frac{1}{N}\sum_{\vec{k} \neq \vec{k}} \langle \vec{k} | T | \vec{k} \rangle +$$

$$+ \frac{1}{2N} |\vec{k}_{1}|, \vec{k}_{2}| \leq k_{F} \langle \vec{k}_{1} \vec{k}_{2} | G(k_{F}, w = \varepsilon(\vec{k}_{1}) + \varepsilon(\vec{k}_{2}) | \vec{k}_{1} \vec{k}_{2} - \vec{k}_{2} \vec{k}_{1})$$
(17)

where I denotes the kinetic-energy operator.

In conventional Brueckner theory<sup>2</sup> non-relativistic energies are

used, i.e.  $T(\vec{k}_i) = {\vec{k}_i}^2/2M$  (in analogy to the non-relativistic Lipp-mann-Schwinger equation of scattering) and any phenomenological NN-potential (i.e. Reid<sup>23</sup>) is suitable to be used for V.

In the first attempts for a relativistic extension of Brueckner theory (which we will subsequently call: conventional relativistic Brueckner theory), Eq.(11) was applied with the relativistic kinematical factors and energies which occured in the analogous scattering equation (e.g. Blankenbecler-Sugar or Thompson, compare Section  $II^{7,8,10}$ ). Phenomenological potentials are in general unsuitable for this approach , as at least the most popular ones like Reid<sup>23</sup> or Paris<sup>24</sup> are defined for the non-relativistic Schrödinger equation. For reasons of consistency in the relativistic approach, the relativistic field theoretic process is assumed which creates the nuclear force, and that is meson-exchange. Therefore OBEPs as well as more comprehensive meson-exchange models have been used at this stage. An OBEP consists of the sum of single meson-exchanges in which the nucleons are naturally represented by four-component free Dirac spinors,  $u(\vec{q})$ , as indicated in Fig.4, i.e. the spinors satisfy the free Dirac equation:

$$(q - M) u(\bar{q}) = 0$$
 . (18)

Within the scheme of conventional relativistic Brueckner theory unchanged free spinors are taken over into the nuclear matter calculation. Therefore the potential applied to nuclear matter is exactly the same as in NN-scattering and that is why the results are not substantially different from the non-relativistic Brueckner theory.

 υ(q̄)
 π, σ, ρ, ω
 υ(-q̄)

 υ(q̄)
 υ(-q̄)

Fig.4. One-boson-exchange diagrams contributing to NN-scattering with explicit indication of the free Dirac spinors representing the nucleons in scattering.

Figure 5a contains a survey of results obtained in the two approaches discussed so far. Only the saturation minima of the saturation curves,  $E/N(k_F)$ , are given. The squares stand for calculations usinf a free spectrum for nucleons above the Fermi surface ("gap"). The difference in the results is mainly due to a different strength of the tensor force contained in these potentials. This strength is best measured by the %-D-state,  $P_D$ , in the deuteron which the potentials give rise to. For Hamada-Johnston<sup>25</sup>, it is 6.97% (the highest point in the plot), for HM2<sup>30</sup> we have  $P_D$ =4.32% (the lowest point).

The circles indicate results in which a continuous choice is applied for the single particle spectrum of the nucleons. These results are



Fig.5a. Energy per nucleon in nuclear matter, E/N, as a function of the Fermi momentum,  $k_{\rm F}$ , obtained in lowest order Brueckner calculations for various NN potentials and different versions of Brueckner theory. The shaded square represents the empirical nuclear matter saturation. A brief history of results obtained in nuclear matter Brueckner theory is given by the small squares, circles and triangles which indicate the saturation minima of different conventional approaches explained in the text. The abbreviations for the potentials applied are given below: HJ: Hamada-Johnston<sup>25</sup>; Bethe-Johnson<sup>26</sup>; REID: Reid-soft-core-potential<sup>23</sup>; PARIS: Ref.24; HM1: Holinde-Machleidt (1)<sup>8</sup>; HM3A, HM3B: Ref.27; BG: Bryan-Gersten<sup>28</sup>; SSC: Sprung-de Tourreil super-soft-core potential C<sup>29</sup>; HM2: Holinde-Machleidt (2)<sup>30</sup>;  $\Delta$ 1,2: Ref.31;  $\Delta$ 3: Ref.32;  $\Delta$ 4: Ref.33.

generally 4-5 MeV more attractive compared to gap-calculation and appear to simulate the 3- and 4-body correlation  $^{34}$ . Finally  $\Delta(1232)$ isobars and their special medium effects are included in results symbolized by a small triangle.

It is easily seen that each type of calculation has its own Coester line (Ref.35). Though improvements with respect to the empirical area can be observed when extensions within the conventional scheme are applied (continuous choice,  $\Delta$ 's), all bands of results clearly miss the empirical range. This summarizes a long standing problem in nuclear matter theory.

### V. THE DIRAC-BRUECKNER APPROACH TO NUCLEAR MATTER

In this section we will explain and apply a special extension of conventional relativistic Brueckner theory, subsequently called the Dirac-Brueckner approach.

The basic idea of this new approach (first introduced by Shakin and co-workers<sup>11</sup>) is that, as in the mean field theory<sup>1</sup>, one realizes that the nucleons in nuclear matter are exposed to a strong common scalar and vector field and are therefore by no means free particles. Consequently, free Dirac spinors satisfying the free Dirac Eq.(18) can not be an adequate representation of a nucleon in nuclear matter. Instead, spinors obtained in a Dirac equation containing the strong common potentials should be used:

$$(\mathbf{K} - \mathbf{M} - \Sigma) \quad \widetilde{\mathbf{u}}(\mathbf{\vec{k}}) = \mathbf{0} \tag{19}$$

$$\Sigma = A(\vec{k}) + \gamma^{0}B(\vec{k})$$
 (20)

with

$$\Sigma = A(\hat{k}) + \gamma^{0}B(\hat{k})$$
 (20)

the selfenergy operator  $^{36}$ .

(Our notation is that of Ref.37.)  $A(\vec{k})$  and  $B(\vec{k})$  represent the scalar and vector potential in nuclear matter respectively.

As it turns out that the k-dependence of A and B between zero and  ${}^{k}{}_{F}$  is very weak (namely, when expanding  $A(\vec{k}) = A_{0} + A_{1}(\vec{k}^{2}/{k_{F}}^{2})$  and  $B(\vec{k}) = B_{0} + B_{1}(\vec{k}^{2}\!\!/{k_{F}}^{2})$  one gets:  $A_{1}/A_{0} \approx B_{1}/B_{0} \approx -0.05$ ), they can be assumed to be constant to a good approximation.

With  $M^* \equiv M + A$  and  $E^* \cong \sqrt{M^* + k^2}$  the Dirac spinor satisfying Eq. (19) is simply:

$$\widetilde{u}(\vec{k},s) = \sqrt{\frac{E^* + M^*}{2E^*}} \left( \frac{\overrightarrow{\sigma \cdot \vec{k}}}{E^* + M^*} \right) X_s$$
(21)

where the normalization  $\tilde{u}^{\dagger}\tilde{u}$ =1, appropriate for nuclear structure calculations, is applied, as throughout this work (compare Eq.(8)).

The selfenergy is given by Eq.(16) with

$$\Sigma(\vec{k}) = \overline{\mathfrak{U}}(\vec{k})\Sigma\widetilde{\mathfrak{U}}(\vec{k}) = U(k) \qquad . \qquad (22)$$

As  $\Sigma$  depends on the G-matrix (compare Eq.(16)) and the G-matrix depends on  $\Sigma$  via the single particle energies and the Dirac spinors defined in Eq.(21), a self-consistency is required.

Formally all formulae of conventional Brueckner theory, Eqs.(14) -(17), still apply; however, their explicit meaning may be refined:

$$|\vec{k}\rangle = \vec{u}(\vec{k})$$

$$<\vec{k}| = \vec{u}(\vec{k})$$

$$T = \vec{\chi} \cdot \vec{k} + M$$
(24)

and a term -M should be added to Eq.(17).

$$\varepsilon(\vec{k}) = \langle \vec{k} | T | \vec{k} \rangle + \langle \vec{k} | \Sigma | \vec{k} \rangle =$$

$$= \frac{MM^* + \vec{k}^2}{E^*} + \frac{M^*}{E^*} A + B =$$

$$= E^* + B \qquad (25)$$

Assuming A and B constant, they can be determined once  $\Sigma(\vec{k})$  has been evaluated for two different  $\vec{k}$ .  $(1/2)k_F$  and  $k_F$  is an appropriate choice.

Special care has to be taken for the  $\rho$ -exchange potential being derived from the Lagrangian:

$$\mathcal{L}_{NN\rho} = g\bar{\psi}\gamma^{\mu}\psi\phi_{\mu} + \frac{f}{4M}\bar{\psi}\sigma^{\mu\nu}\psi(\partial_{\mu}\phi_{\nu} - \partial_{\nu}\phi_{\mu}) \qquad (26)$$

As f/4M is a coupling <u>constant</u> which could as well be defined by  $f'/4m_{\rho}$ , where  $m_{\rho}$  denotes the mass of the  $\rho$ -meson, the M in Eq.(25) must not be replaced by  $M^*$ . If that was done incorrectly, there would be no saturation in nuclear matter, as we will see later.

The results for the Dirac-Brueckner approach are displayed in Fig.5b by full lines. The label A refers to calculations in which the OBEP presented in Section III is applied. B and C denote variations of the OBEP with an increased tensor force (by cutting off less the mNN vertex). The reason why we have performed the calculations with three different OBEPs is that the characteristic results for a certain type of many-body approach is not a point in the energy versus density plot; it is a band (a Coester band). There is not <u>the</u> nuclear force, there are several potentials, which describe the NN data equally well and nevertheless differ, namely, essentially in the strength of the tensor force. This difference leads to characteristic variations in the nuclear matter results, which always have a Coester-band structure.

Therefore, the reasonable question to be asked is whether the band characteristic for one theory is oriented such that it would pass through the empirical area. Obviously, our new approach provides additional strongly density-dependent repulsion such that the empirical result can be met.

Considering the results of the conventional approaches and their refinements, which were in part summarized in Section IV, it is clear that this is not a trivial result.

The role of the various mesons in the relativistic saturation mechanism is demonstrated in Fig.6. The effect of  $\omega,\sigma$  and  $\pi$  is



Fig.5b. Same as Fig.5a, but including the results of the present work. For curves labelled "A" the OBEP presented in Section III and Table I is applied. Label "B" and "C" refer to two variations of that OBEP with an increased tensor force. The full lines denote Dirac-Brueckner results, the dashed curves stand for calculations with free spinors ("conventional relativistic Brueckner theory").



Fig.6. "Potential energy",  $E_{pot}$ , (i.e. second term on r.h.s. of (Eg.(17)) versus Fermi momentum,  $k_F$ , for the present OBEP. Starting with the conventional result labelled "M=M", the single mesons are switched successively on to the Dirac-Brueckner approach indicated by "M=M\*". The dotted curve shows the result when the M, occuring in the Lagrangian for the  $\rho$ NN-interaction, Eq.(26), is incorrectly replaced by M\*. The short dashed curve is obtained by using the ps-coupling for the pion.

about equally strong, the  $\pi$  acting mainly through its second order contribution to the  ${}^3S_1$ -state. The  $\rho$  and especially the  $\eta$  and  $\delta$  have very little effect. The consequences of using the wrong treatment of  $\rho$ -exchange and of the ps-coupling of the  $\pi$  are also demonstrated in Fig.6. Both lead to unphysical results showing no saturation in nuclear matter.

The constant part of the scalar potential,  $A_0$ , and of the vector potential,  $B_0$ , are displayed versus the Fermi-momentum,  $k_F$ , in Fig. 7 and 8 respectively. Generally, we find a smoother density-dependence of these quantities compared to other authors.





Fig.7. Constant part of the scalar potential,  $A_0$ , versus Fermi-momentum,  $k_F$ . The full line displays the results using the present OBEP. The long and short dashed curves are from Ref.ll (using HEA) and Ref.l respectively.

Fig.8. Constant part of the vector potential,  $B_0$ , versus Fermi-momentum,  $k_F$ . The full line is obtained from the present OBEP; the dashed from the work of Ref.ll using HEA.

# VI. SUMMARY; CONCLUSIONS AND OUTLOOK

In this contribution we have constructed a one-boson-exchange potential on the basis of the latest state of the art of the mesontheory of the NN-interaction. For NN-scattering we use the three-dimensional reduction of the Bethe-Salpeter equation suggested by Thompson, which has been proven to be a good approximation to the full four-dimensional equation. The potential has been fitted to new phase-shift analyses. For the coupling of the pion to the nucleon the pv version is chosen, and this turns out to be necessary for obtaining reasonable results in the many-body system.

This potential is applied to nuclear matter in the Dirac-Brueckner approach. We avoid the drawbacks of earlier work by other authors, in which drastic approximations, outdated nuclear forces and unsuitable relativistic equations and couplings were used. Especially, we determine the single particle energies and wave functions (Dirac spinors) in nuclear matter fully selfconsistently.

It turns out that such a correctly performed Dirac-Brueckner calculation is indeed able to explain the empirical saturation properties of nuclear matter. This result is due to additional (compared to the conventional theory) strongly density-dependent repulsive effects to which the  $\sigma$ -,  $\omega$ - and  $\pi$ -exchange make essential contributions.

The successful nuclear matter results motivate further applications, e.g. the derivation of the optical potential from the Gmatrix for use in nucleon-nucleus scattering.

In spite of the encouraging findings in this contribution, there are serious questions open. We list some of them:

- (i) What are the contributions from the three- and more-body correlations in this approach?
- (ii) How do many-body forces contribute?

(iii) What are the  $N\overline{N}$ -pair corrections?

We will devote future work to some of these questions.

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