Resonant-bar detectors of gravitational wave as possible probe of the noncommutative structure of space

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We report the plausibility of using quantum mechanical transitions, induced by the combined effect of Gravitational Wave (GW) and noncommutative (NC) structure of space, among the states of a 2-dimensional harmonic oscillator, to probe the spatial NC geometry. The phonon modes excited by the passing GW within the resonant bar-detector are formally identical to forced harmonic oscillator and they represent a length variation of roughly the same order of magnitude as the characteristic length-scale of spatial noncommutativity estimated from the phenomenological upper bound of the NC parameter. This motivates our present work. We employ various GW forms that are typically expected from possible astronomical sources. We find that the transition probabilities are quite sensitive to the nature of polarization of the GW. We also elaborate on the particular type of sources of GW, radiation from which can induce transitions that can be used as effective probe of the spatial noncommutative structure.

1. Introduction

The tantalizing news of the first direct detection of Gravitational Waves (GWs) [1] has opened a new window not only for astronomical observations but also for directly looking into the structure of space-time at a length-scale never probed before. GWs are small ripples in the fabric of space-time. The present day operational GW detectors are ground-based interferometers (LIGO, VIRGO, GEO, TAMA etc.) [2]. However, the search of GWs began with resonant-mass detectors, pioneered by Weber in the 60's [3]. In the decades that followed, the sensitivity of resonant-mass detectors have improved considerably [4]. Also, the study of resonant-bar detectors is fundamental since it focuses on how GW interacts with elastic matter causing vibrations with amplitudes many order smaller than the size of a nucleus. In a bar detector it is possible to detect these tiny vibrations corresponding to just a few tens of phonons [5], and variations ΔL of the bar-length $L \sim 1$ m, with $\frac{\Delta L}{L} \sim 10^{-19}$.

Interestingly, it has long been suggested in various Gedenken experiments that a sharp localization of events in space would induce an uncertainty in spatial coordinates [6, 7] at the quantum level. This uncertainty can be realized by imposing the NC Heisenberg algebra on the operators representing phase-space variables

$$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij} \ , \quad [\hat{x}_i, \hat{x}_j] = i\theta_{ij} = i\theta\epsilon_{ij} \ , \quad [\hat{p}_i, \hat{p}_j] = 0 \ , \tag{1}$$

where θ_{ij} is the constant antisymmetric tensor, which is written in terms of the constant NC parameter θ and the totally antisymmetric tensor ϵ_{ij}. Such granularity in spatial structure have been motivated by string theoretic [8] and quantum gravity [9] results also. A wide range of theories, dubbed the NC theories, have been constructed in this framework. This includes NC quantum mechanics (NCQM) [10], NC quantum field/gauge theories [11] and gravity [12, 13]. Certain possible phenomenological consequences [14] have also been predicted. Naturally, a part of the endeavour is spent in finding the order of the NC parameter and exploring its connection with observations [15, 16, 17, 18]. The stringent upperbound on the coordinate commutator $|\theta|$ found in [16] is $\lesssim (10 \text{ TeV})^{-2}$ which corresponds to 4×10^{-40} m^2 for $\hbar = c = 1$[1]. This upperbound correspond to the length scale $\sim 10^{-20}$ m which overlaps the length scale where the first GW has been detected [1]. Thus a good possibility of detecting the NC structure of space-time would be in the present GW detection experiments as it may as well pick up the NC signature of space-time as a noise source. So we need NCQM of GW-matter interaction that can anticiapte the NC effects in GW detection events.

With this motivation, we have studied the interaction of GWs with simple matter systems in a NCQM framework in [19, 20]. Our particular interest is in the NCQM of harmonic oscillator (HO) interacting with GW because the response of a bar-detector to GW can be cast as phonon mode excitations formally identical to forced HO [5]. Thus, NCQM of the HO interacting with GW is of fundamental importance. Therefore, we investigate the

[1] In a more general NC space-time structure [8] given by $[x^\mu, x^\nu] = i\theta^{\mu\nu}$ such upperbounds on time-space NC parameter is [17] $\theta^{0i} \lesssim 9.51 \times 10^{-18}$ m^2.

transition probabilities between the ground state and the excited states of this system in the present paper by treating the combined effect of GW and spatial noncommutativity as time-dependent perturbations. We employ a number of different GW forms that are typically expected from runaway astronomical events.

2. Formulation: constructing the NC Hamiltonian

To proceed we first obtain the classical Hamiltonian appropriate for the GW-HO interaction system. This can be simply done by noting that in the proper detector frame the geodesic deviation equation for a $2-$dimensional harmonic oscillator of mass m and frequency ϖ subject to linearized GW becomes [5]

$$m\ddot{x}^j = -mR^j{}_{0,k0}x^k - m\varpi^2 x^j, \tag{2}$$

where dot denotes derivative with respect to the coordinate time of the proper detector frame[2], x^j is the proper distance of the pendulum from the origin and $R^j{}_{0,k0}$ are the relevant components of the curvature tensor in terms of the metric perturbation $h_{\mu\nu}$ defined by[3] $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu};\ |h_{\mu\nu}| << 1,$ on the flat Minkowski background $\eta_{\mu\nu}$.

The transverse-traceless (TT) gauge-choice $\left(h_{0\mu} = 0, h_{\mu\nu;}{}^{\mu} = 0, h^{\mu}_{\mu} = 0\right)$ removes all unphysical degrees of freedom (DOF) and only non-trivial components of the curvature tensor $R^j{}_{0,k0} = -\ddot{h}_{jk}/2$ appear in equation (2). Note that, equation (2) works as long as the spacial velocities are small and $|x^j|$ is much smaller than the reduced wavelength $\frac{\lambda}{2\pi}$ of GW. These conditions are collectively referred as the *small-velocity and long wavelength limit* and met by resonant bar-detectors and the Earth bound interferometric detectors, with the origin of the coordinate system centered at the detector. This also ensures that in a plane-wave expansion of GW, $h_{jk} = \int (A_{jk}e^{ikx} + A^*_{jk}e^{-ikx})d^3k/\left(2\pi\right)^3$, the spatial part $e^{i\vec{k}.\vec{x}} \approx 1$ all over the detector site. So only the time-dependent part of the GW is relevant.

The two physical DOF, referred as the $\times$ and $+$ polarizations of GW, are contained in A_{jk} and can be expressed in terms of the Pauli spin matrices as $h_{jk}\left(t\right) = 2f\left(\varepsilon_{\times}\sigma^1_{jk} + \varepsilon_{+}\sigma^3_{jk}\right)$ if z is the propagation direction. Here $2f$ is the amplitude of the GW and $(\varepsilon_{\times}, \varepsilon_{+})$ are the two possible polarization states of the GW satisfying the condition $\varepsilon^2_{\times} + \varepsilon^2_{+} = 1$ for all t.

The Lagrangian for the system (2), can be written, upto a total derivative term as $\mathcal{L} = \frac{1}{2}m\dot{x}^2 - m\Gamma^j{}_{0k}\dot{x}_j x^k - \frac{1}{2}m\varpi^2\left(x_j\right)^2$, where $R^j{}_{0,k0} = -\frac{d\Gamma^j_{0k}}{dt} = -\ddot{h}_{jk}/2$. Computing the canonical momentum $p_j = m\dot{x}_j - m\Gamma^j_{0k}x^k$ we write the Hamitonian as

$$H = \frac{1}{2m}\left(p_j + m\Gamma^j_{0k}x^k\right)^2 + \frac{1}{2}m\varpi^2\left(x_j\right)^2 . \tag{3}$$

Once we have the classical Hamiltonian we can have the NCQM description of the system[4] simply by elevating the phase-space variables $\left(x^j, p_j\right)$ to operators $\left(\hat{x}^j, \hat{p}_j\right)$ and imposing the NC Heisenberg algebra (1). But since this algebra can be mapped [19, 20] to the standard $(\theta = 0)$ Heisenberg algebra spanned by the operators X_i and P_j of the ordinary QM through the mapping $\hat{x}_i = X_i - \frac{1}{2\hbar}\theta\epsilon_{ij}P_j$, $\hat{p}_i = P_i$ so that the NCQM Hamiltonian corresponding to equation (3) can be re-expressed as [5]

$$\hat{H} = \frac{{P_j}^2}{2m} + \frac{1}{2}m\varpi^2{X_j}^2 + \Gamma^j_{0k}X_jP_k - \frac{m\varpi^2}{2\hbar}\theta\epsilon_{jm}X^jP_m - \frac{\theta}{2\hbar}\epsilon_{jm}P_mP_k\Gamma^j_{0k} = \hat{H}_0 + \hat{H}_{\rm int}. \tag{4}$$

This Hamiltonian gives the commutative equivalent description of the noncommutative system (3) in terms of the operators X_i and P_j. Since they admit the standard Heisenberg algebra, the rules of ordinary QM applies to (4) . The first two terms in equation (4) represent the unperturbed HO Hamiltonian $\hat{H}_0$. Rest of the terms are small[6] compared to $\hat{H}_0$ and can be treated as perturbations $\hat{H}_{\rm int}$.

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[2] It is the same as it's proper time to first order in the metric perturbation.

[3] As is usual, latin indices run from $1-3$. Also ; denotes covariant derivatives.

[4] Also note that it has been demonstrated in various formulations of NC general relativity [12, 13] that any NC correction in the gravity sector is second order in the NC parameter. Therefore, in a first order theory in NC space, the GW remains unaltered by NC effects.

[5] The traceless property of the GW is also required here.

[6] A term quadratic in Γ has been neglected in equation (4) since we deal with linearized gravity.

Defining raising and lowering operators $X_j = \sqrt{\frac{\hbar}{2m\varpi}}\left(a_j + a_j^{\dagger}\right)$ and $P_j = \sqrt{\frac{\hbar m\varpi}{2i}}\left(a_j - a_j^{\dagger}\right)$ in terms of the oscillator frequency ϖ, we write the time-dependent interaction part of the Hamiltonian (4) as[7]

$$\hat{H}'_{\rm int}(t) = -\frac{i\hbar}{4}\dot{h}_{jk}(t)\left(a_j a_k - a_j^{\dagger}a_k^{\dagger}\right) + \frac{m\varpi\theta}{8}\epsilon_{jm}\dot{h}_{jk}(t)\left(a_m a_k - a_m a_k^{\dagger} + C.C\right). \tag{5}$$

3. Time-dependent perturbation

We now apply the time-dependent perturbation theory to compute the probability of transition between the ground state $|0,0\rangle$ and the excited states of the 2-d harmonic oscillator. To the lowest order the probability amplitude of transition from an initial state $|i\rangle$ to a final state $|f\rangle$, $(i \neq f)$, due to a perturbation $\hat{V}(t) = F_{jk}(t)\hat{Q}_{jk}$ is given by [21]

$$C_{i\to f}(t\to\infty) = -\frac{i}{\hbar}\int_{-\infty}^{t\to+\infty} dt' \left[F_{jk}(t')\, e^{\frac{i}{\hbar}(E_f - E_i)t'}\langle \Phi_f|\hat{Q}_{jk}|\Phi_i\rangle\right].$$

Using the above result, we find that the *probability of transition survives only between the ground state* $|0,0\rangle$ *and the second excited state* and it reads

$$C_{0\to2} = -\frac{i}{\hbar}\int_{-\infty}^{+\infty} dt \left[F_{jk}(t)\, e^{\frac{i}{\hbar}(E_2 - E_0)t}\left(\langle 2,0|\hat{Q}_{jk}|0,0\rangle + \langle 1,1|\hat{Q}_{jk}|0,0\rangle + \langle 0,2|\hat{Q}_{jk}|0,0\rangle\right)\right], \tag{6}$$

where $F_{jk}(t) = \dot{h}_{jk}(t)$ contains the explicit time dependence of $\hat{H}'_{\rm int}$ and $\hat{Q}_{jk} = -\frac{i\hbar}{4}\left(a_j a_k - a_j^{\dagger}a_k^{\dagger}\right) + \frac{m\varpi\theta}{8}\epsilon_{jm}\left(a_m a_k - a_m a_k^{\dagger} + C.C\right)$ contains the raising and lowering operators appearing in equation (5). Expanding out $\hat{Q}$ for $i,j = 1,2$, we obtain the transition amplitude between the ground state $|0,0\rangle$ and the second excited state to be

$$C_{0\to2} = -\frac{i}{\hbar}\int_{-\infty}^{+\infty} dt\, e^{2i\varpi t}\left(\frac{i\hbar}{2}\dot{h}_{12}(t) + \frac{m\varpi\theta}{4}\dot{h}_{11}(t)\right). \tag{7}$$

The above equation is the main working formula in this paper. Now using the general formula (7), we can compute the corresponding transition probabilities $P_{0\to2} = |C_{0\to2}|^2$ taking various template of GW forms that are likely to be generated in runaway Astronomical events.

4. Response to Templet GW signals from possible events

We start with the simple scenario of periodic GW with sinusoidally varying amplitude and a single frequency Ω $h_{jk}(t) = 2f_0\cos\Omega t\left(\varepsilon_\times\sigma^1_{jk} + \varepsilon_+\sigma^3_{jk}\right)$. In this limiting case of an exactly monochromatic wave, the temporal duration of the signal is infinite and we get for the transition probability

$$P_{0\to2} = (\pi f_0\Omega)^2\left({\varepsilon_\times}^2 + \Lambda^2{\varepsilon_+}^2\right)\left[\delta(2\varpi+\Omega) - \delta(2\varpi-\Omega)\right]^2, \tag{8}$$

where $\Lambda = \frac{m\varpi\theta}{2\hbar} = 1.888\left(\frac{m}{10^3\rm kg}\right)\left(\frac{\omega}{1\rm kHz}\right)$ is a dimensionless parameter carrying the NC signature. Here we have used the stringent upper-bound $|\theta| \approx 4\times10^{-40}\ {\rm m}^2$ [16] for spatial noncommutativity and for reference mass and frequency used values appropriate for fundamental phonon modes of a bar detector [20] which are formally identical to the NC harmonic oscillator system considered here.

Consider periodic GW signal coming from a binary system (with quasi-circular orbit) being received by some earth-bound detector; if the orbit of the binary system is edge-on with respect to us, then we receive the + polarization of the radiation only [5] , i.e., $(\varepsilon_\times, \varepsilon_+) = (0,1)$, and in this case (8) shows that the transition probability will scale quadratically with the dimensionless parameter Λ characterized by spatial noncommutativity. Therefore such a transition will be driven by the combined perturbative effect of GW as well as spatial noncommutativity[8]

[7] C.C means complex conjugate.

[8] This corresponds to the last term in $\hat{H}_{\rm int}$ in (5).

and will occur only if the space has a NC structure. In other words, *a quantum mechanical transition induced by the linearly polarized GW from a binary system with its orbital plane lying parallel to our line of sight can be an effective test of the noncommutative structure of space.* Note that the angular frequency Ω of the quadrupole radiation is twice the angular frequency of rotation of the source [5], thus (8) also tells us that *for transition to occur we need to have a harmonic oscillator with natural frequency ϖ that matches with that of the source.* Highly accurate X-ray/radio-astronomical measurement of the frequency of orbital rotation of binary Pulsars can be used to pin-point the natural frequency of the harmonic oscillator required here.

From another binary system similar to the one considered above, but with its orbital plane perpendicular to our line of sight, both the $+$ and $\times$ polarization of the radiation will reach the detector with equal amplitude and consequently we will have a source for circularly polarized GW signal that can be generically written as $h_{jk}\left(t\right) = 2f_0\left[\varepsilon_\times\left(t\right)\sigma^1_{jk} + \varepsilon_+\left(t\right)\sigma^3_{jk}\right]$ with $\varepsilon_+\left(t\right) = \cos\Omega t$ and $\varepsilon_\times\left(t\right) = \sin\Omega t$ and constant amplitude f_0. The transition probability in this case is[9]

$$P_{0\to2} = \left(\pi f_0\Omega\right)^2\left[\left\{\left(1+\Lambda\right)\delta\left(2\omega+\Omega\right)\right\}^2 + \left\{\left(1-\Lambda\right)\delta\left(2\omega-\Omega\right)\right\}^2\right]. \tag{9}$$

Equation (9) shows a non-zero transition probability for $\Lambda = 0$, i.e. if our space has commutative structure. *Thus a transition induced by circularly polarized GW from a binary system cannot be used as a deterministic probe for spatial noncommutativity.* This feature lies with the earlier case of linearly polarized GW signals only.

In the last stable orbit of an inspiraling neutron star or black hole binary or during its merging and final ringdown, the system can liberate large amount of energy in GWs within a very short duration 10^{-3} sec $< \tau_{\rm g} < 1$ sec. Such signals are referred to as GW bursts. Supernova explosions and stellar gravitational collapse are other candidate generators. Since bursts originate from violent and explosive astrophysical phenomena, their waveform cannot be accurately predicted and only be crudely modeled as $h_{jk}\left(t\right) = 2f_0 g\left(t\right)\left(\varepsilon_\times\sigma^1_{jk} + \varepsilon_+\sigma^3_{jk}\right)$, where to be generic we have kept both components of the linear polarization. Here $g\left(t\right)$ is a smooth function which goes to zero rather fast for $|t| > \tau_{\rm g}$. A convenient choice is a function peaked at $t = 0$ with $g\left(0\right) = \mathcal{O}(1)$ so that $|h_{jk}\left(t\right)| \sim \mathcal{O}\left(f_0\right)$ near the peak. So we take a simple Gaussian $g\left(t\right) = e^{-t^2/\tau_g^2}$. Owing to its small temporal duration the burst have a continuum spectrum of frequency over a broad range upto $f_{\rm max} \sim 1/\tau_{\rm g}$ whereas the detector is sensitive only to a certain frequency window and blind beyond it. If the sensitive band-width is small compared to the typical variation scale of the signal in the frequency space, the crude choice here, instead of a precise waveform, is good enough. In terms of the Fourier decomposed modes the GW burst can thus be modeled as

$$h_{jk}\left(t\right) = \frac{f_0}{\pi}\left(\varepsilon_\times\sigma^1_{jk} + \varepsilon_+\sigma^3_{jk}\right)\int_{-\infty}^{+\infty}\tilde{g}\left(\Omega\right)e^{-i\Omega t}d\Omega, \tag{10}$$

where $\tilde{g}\left(\Omega\right) = \sqrt{\pi}\tau_g e^{-\left(\frac{\Omega\tau_g}{2}\right)^2}$ is the amplitude of the Fourier mode at frequency Ω. Using equation (10) in the general formula for transition amplitude (7) we find the probability for transition from the ground state to the second excited state induced by a GW burst is

$$P_{0\to2} \;=\; \left(2\sqrt{\pi}f_0\varpi\tau_g\right)^2 e^{-2\varpi^2\tau_g^2}\left({\varepsilon_\times}^2 + \Lambda^2{\varepsilon_+}^2\right), \tag{11}$$

where the GW Fourier mode with twice the natural frequency of the harmonic oscillator (the detector in our consideration) gets picked up. Since the burst signal duration $\tau_g \sim 10^{-2} - 10^{-3}$ sec, the maximum frequency in the Fourier spectrum can be $\Omega_{\rm max}/2\pi \sim 0.1 - 1$ kHz which partially overlaps with the sensitive bandpass for the bar-detectors. From equation (11) we again see that the $+$ polarization of the GW burst can only induce a transition if the space has a NC structure. If the polarization state of a GW signal from a given source can be anticipated since it depends largely on the orientation of the source which can be determined by observing its electromagnetic radiation and the detector geometry. *So detecting a QM transition induced by a GW burst from an appropriate source can serve as a probe of the spatial noncommutativity.*

[9]Note that here the transition probability has terms both linear and quadratic in the dimensionless NC parameter Λ. However estimate for Λ shows that for phonon modes in a bar detector which are the realization of the NC HO system considered in this paper, Λ is of the order of unity , so we cannot drop the quadratic term even though we started with a theory to first order in the NC parameter.

5. Conclusion

In conclusion we would like to convey that the considerations in the present paper suggest that the joint operation of various resonant detector groups like ALLEGRO, AURIGA, EXPLORER, NAUTILUS and NIOBE around the world in IGEC (International Gravitational Event Collaboration) [23] may possess the potential to establish the possible existence of a granular structure of our space as a by-product in the event of a direct detection of GW and therefore must be continued.

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