

## Dynamics of light clusters in fragmentation reactions

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**Summary.** — Clusters are copiously formed in heavy-ion collisions. This has been a challenging problem for transport models which are usually based on single-nucleon motions. Even in molecular dynamics approaches, clusters should be explicitly treated as quantum correlations. I will review how clusters are introduced in antisymmetrized molecular dynamics, and discuss the strong impacts of cluster correlations on reaction dynamics and observables.

### 1. – Clusters in excited nuclear systems and in heavy-ion collisions

Clusters appear in various situations in nuclear physics, in particular when a nucleus or nuclear matter is excited [1]. Some low-lying states and resonances of nuclei have solid or molecular structures composed of several clusters, such as  $\alpha$  clusters, while some other states can be explained by gas-like structures of clusters that are almost freely moving in a common potential well [2]. These cluster structures are linked with  $\alpha$  decays, molecular resonances and fission etc. An interesting point of view is what happens if the excitation energy is further raised from  $\sim 10$  MeV (of the total excitation energy), around which cluster structures typically appear in nuclei, to more than several MeV per nucleon. If a nucleus is excited by  $\sim 10$  MeV per nucleon, it is energetically possible that the nucleus is broken completely into free nucleons, but it does not actually happen. This energy domain has been studied by heavy-ion collisions intensively and extensively over the last few decades. Even at these high excitation energies, we know well that the system breaks up into clusters and fragments [3], namely into light clusters (deuterons, tritons,  $^3\text{He}$  and  $\alpha$ ) and heavier nuclei with some emitted nucleons. Thus the cluster correlations play important roles in a wide range of nuclear physics.

In head-on collisions of two nuclei at the incident energies of a few tens of MeV/nucleon and more, the system is compressed at an early stage and then it expands so that fragmentation occurs in the low-density system. The number of nucleons emitted as free particles is not very large. For example, the INDRA data for the Xe + Sn collisions at 50 MeV/nucleon [4] show that only about 10% of the total protons in the system are emitted as free particles. At higher energies, in Au + Au collisions, the FOPI data [5, 6]

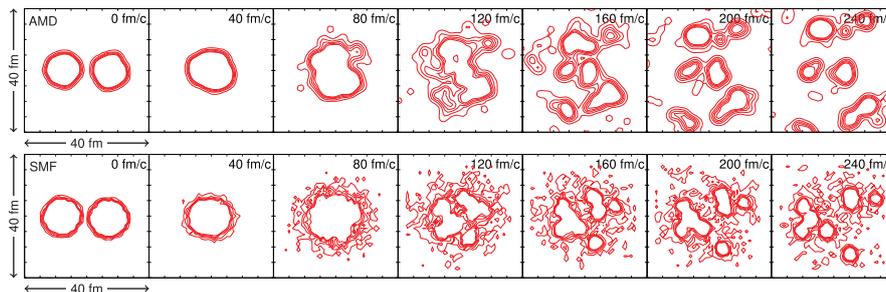


Fig. 1. – Time evolution of density profiles obtained by the AMD (upper) and SMF (lower) models, for the head-on collision of  $^{112}\text{Sn} + ^{112}\text{Sn}$  at 50 MeV/nucleon. Taken from ref. [8].

show that the free proton fraction is still 21% at 250 MeV/nucleon. On the other hand, the fraction of the protons bound in emitted light clusters ( $d$ ,  $t$ ,  $^3\text{He}$  and  $\alpha$ ) is 61%. This experimental fact suggests that cluster correlations are still very important even at these high excitation energies (or high beam energies) that are much more than the threshold energy for the system to completely break into free nucleons. The importance of clusters in highly excited systems can also be found in the data of ref. [7] which show the cluster mass fractions as functions of the excitation energy of the “vaporized” projectile nucleus.

## 2. – Approaches of nucleon-based transport models

Transport models play important roles to describe the dynamics of heavy-ion collisions. All the practically available models are more or less based on the approximate time evolution of the distribution function  $f(\mathbf{r}, \mathbf{p}, t)$  in the one-body phase space or the one-body density matrix. The effects of the mean field and the two-nucleon collisions are taken into account in general, but many-body correlations are inevitably truncated at some level. The Vlasov term in the BUU equation treats the nucleon motions in the mean field faithfully as the semiclassical approximation of the time-dependent Hartree-Fock theory. A problem is, however, that many-body correlations are missing. In particular, the deterministic time evolution of  $f(\mathbf{r}, \mathbf{p}, t)$  is not likely to describe the dynamical formation of clusters and fragments. On the other hand, molecular dynamics models may be more suitable for the description of clusters and fragments because each nucleon is represented by a wave packet that is ensured to be localized in one of the fragments at the end of the reaction. The single-particle motion, however, is not as precise as in the BUU type approaches, because the distribution is restricted to a Gaussian form in molecular dynamics approaches.

Introducing fluctuation or randomness has been a way to go beyond the above-mentioned limitations of both the BUU-type and the molecular dynamics approaches. Here I will choose two examples of such extensions: the stochastic mean field (SMF) model and the antisymmetrized molecular dynamics (AMD) model with wave-packet splitting. These models can describe fragmentation at least qualitatively as shown in fig. 1 which is a typical example of the time evolution of  $^{112}\text{Sn} + ^{112}\text{Sn}$  head-on collisions at 50 MeV/nucleon [8].

SMF is based on the BUU equation, but the fluctuation term is also taken into account [9]. The fluctuations were introduced based on the principle that the variance of the occupation of each phase-space cell should follow  $\sigma_f^2 = \bar{f}(1 - \bar{f})$ , which is equivalent

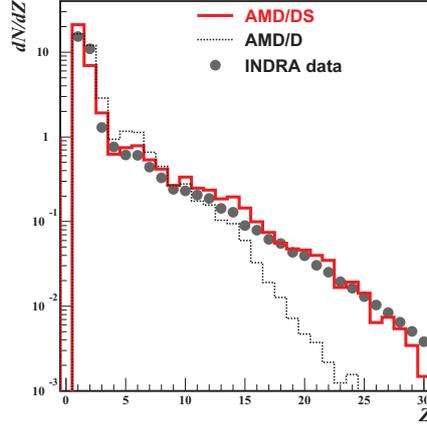


Fig. 2. – The charge distribution of the produced clusters in  $^{129}\text{Xe} + \text{Sn}$  collisions at 50 MeV/nucleon with the impact parameter  $0 < b < 4$  fm, after calculating the secondary decay of excited clusters and applying the experimental filter for the detector setup. Solid histogram (labeled AMD/DS) shows the result of AMD with the coherence time  $\tau = \tau_{\text{NN}}$ , while the dotted histogram (labeled AMD/D) shows the result with the strongest decoherence  $\tau \rightarrow 0$ . The INDRA experimental data are shown by solid points. Taken from ref. [11].

to requiring that each cell should be fully occupied ( $f = 1$ ) or empty ( $f = 0$ ). If a phase-space cell is identified with a Gaussian wave packet, SMF and molecular dynamics models are conceptually similar, but the results (as in fig. 1) can be different due to the different approximate treatments of fluctuations. More recently the fluctuation term in SMF was improved in the BLOB approach by considering the complete fluctuation in the phase space at every two nucleon collision [10]. The new BLOB calculation shows the onset of fragmentation at relatively low energies compared to the SMF case.

In this version of AMD [11, 12], the single-particle wave functions are restricted to the form of Gaussian wave packets with the centroids  $\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A$  and with a width parameter  $\nu = (2.5 \text{ fm})^{-2}$ , so that  $f(\mathbf{r}, \mathbf{p})$  is limited to the form

$$(1) \quad f(\mathbf{r}, \mathbf{p}) = 8 \sum_{j=1}^A \sum_{k=1}^A e^{-2\nu(\mathbf{r}-\mathbf{R}_{jk})^2 - (\mathbf{p}-\mathbf{P}_{jk})^2/2\hbar^2\nu} B_{jk} B_{kj}^{-1}$$

with  $\mathbf{R}_{jk} = (\mathbf{Z}_j^* + \mathbf{Z}_k)/\sqrt{\nu}$ ,  $\mathbf{P}_{jk} = 2i\hbar\nu\sqrt{\nu}(\mathbf{Z}_j^* - \mathbf{Z}_k)$  and  $B_{jk}$  being the overlap matrix of two wave packets. The time evolution is determined by the deterministic motions of the wave-packet centroids in the mean field and the stochastic collisions of two wave packets like the standard molecular dynamics models. Furthermore, the effect of the change of the shape of each wave packet is treated by giving fluctuation to the centroid (wave packet splitting).

The results of AMD with wave-packet splitting for the fragment charge distribution have been compared with the experimental data with a reasonable success [11]. In fig. 2, the AMD results are compared with data for central Xe + Sn collisions at 50 MeV/nucleon. We find a strong dependence on the choice of the coherence time  $\tau$ , namely, how long the coherence of the single-particle wave function is kept before coherence is lost due to many-body correlations. If the coherence is assumed to be lost

when the nucleon collides with another nucleon ( $\tau = \tau_{\text{NN}}$ ), the fragment yields for  $Z \gtrsim 3$  are well reproduced. However, a problem is found in the yields of light particles. The  $\alpha$ -particle multiplicity  $M_\alpha \approx 7$  is too small and the proton multiplicity  $M_p \approx 20$  is too large compared to the experimental data  $M_\alpha \approx M_p \approx 10$ . Compared to the very early version of AMD without wave packet splitting [13], the reproduction of fragment yield was improved very much in many cases by introducing wave-packet splitting [11, 12, 14]. However, a consistent reproduction of the yields of light clusters and light intermediate mass fragments has not been achieved by a single set of model parameters for all the reaction systems. This may suggest that the correlations to produce light clusters should be treated more explicitly.

The overestimation of proton multiplicity is a common problem in semiclassical transport models. To the best of my knowledge, no nucleon-based transport models have ever successfully explained the proton multiplicities of different reactions consistently. In the case of SMF, for example, the proton multiplicity is overestimated by a factor of 2 in the central Xe + Sn collisions at 50 MeV/nucleon [15].

### 3. – Explicit consideration of clusters

As we have seen examples in the previous section, it is difficult for nucleon-based transport models to describe cluster formation in heavy-ion collisions. To predict cluster observables, coalescence prescriptions have been often applied to the result of transport calculations [16], though they are not treating cluster correlations and collision dynamics consistently, *i.e.*, these prescriptions assume that correlations do not influence the time evolution of the single-nucleon distribution.

When clusters are produced in reactions, they are still surrounded by other particles. The properties of a cluster in nuclear medium were recently calculated with an inmedium Schrödinger equation by Röpke [17, 18]. A medium effect appears due to the Pauli blocking. The existence of a bound state of the cluster depends on the density of the medium and the momentum of the cluster relative to the medium. Then, in actual systems, many clusters may coexist and influence the nuclear matter properties and the collision dynamics. During the time evolution, clusters will be created and destroyed repeatedly by reactions such as  $p + n + X \leftrightarrow d + X'$ ,  $p + n + d + X \leftrightarrow \alpha + X'$  and so on.

In an extended version of BUU by Danielewicz *et al.* [19], clusters ( ${}^2\text{H}$ ,  ${}^3\text{H}$  and  ${}^3\text{He}$ ) are treated as new particle species and the equations for the distribution functions are coupled by the collision terms corresponding to various reaction channels of clusters and nucleons. A recent calculation by this model demonstrates that collision dynamics is influenced by the clusters [20]. However,  $\alpha$  clusters have not been introduced in this specific model.

In the case of AMD, recently improved versions take into account the creation of clusters in the final states of each two-nucleon collision [21, 22]. Some details of the method will be reviewed in the next section with some updates for the the most recent version. Here I will explain why a special extension is necessary in molecular dynamics to treat clusters. In fact, clusters should be already well described by the AMD wave function or eq. (1) in the ground state and low-lying excited states of nuclei [1]. When a cluster is placed in a nucleus within AMD, a reasonable value of the cluster binding energy is obtained depending on the location and the center-of-mass momentum of the cluster (see fig. 1 of ref. [23]). An important question is, however, whether such clustered states are realized with the correct probabilities during the reactions. After a two-nucleon collision, one (or both) of the scattered nucleons may accidentally form a cluster with

surrounding nucleon(s) if the wave packet centroids are close in phase space. The probability is determined by the classical phase space or the classical density of states  $\tilde{D}(E)$  for the internal degrees of freedom of a formed cluster. The density of states starts at  $E = E_{\text{gs}}$  with  $\tilde{D}(E) = 0$  for  $E \leq E_{\text{gs}}$ . Since the cluster binding energy  $|E_{\text{gs}}|$  is small due to the cancellation of the potential and kinetic energies, the bound phase-space volume  $\int_{E_{\text{gs}}}^0 \tilde{D}(E) dE$  cannot be as large as  $(2\pi\hbar)^{3(A-1)}$  corresponding to a single quantum bound state. This can be considered to be the reason why cluster correlations do not emerge dynamically with reasonable probabilities in a usual version of AMD without explicit treatment of clusters. It should be noted that irreversible stochastic processes, such as the wave packet splitting, can change the phase-space weights and improve the cluster multiplicities as in the case of the strongest decoherence ( $\tau \rightarrow 0$ ) in fig. 2. However, a fully consistent reproduction of fragmentation is difficult by adjusting only the coherence time.

#### 4. – Formulation of AMD with clusters

In molecular dynamics approaches for heavy-ion collisions, many two-nucleon collisions occur during the time evolution of a reaction. Each two-nucleon collision is treated as a random transition process. This should be interpreted as simulating quantum transitions with the rates

$$(2) \quad vd\sigma = \frac{2\pi}{\hbar} |\langle \Phi_f | V | \Phi_i \rangle|^2 \delta(E_f - E_i) \frac{p_{\text{rel}}^2 dp_{\text{rel}} d\Omega}{(2\pi\hbar)^3}.$$

In the case of AMD, both the initial state  $|\Phi_i\rangle$  and the final state  $|\Phi_f\rangle$  of this collision are AMD wave functions corresponding the Winger functions of the form of eq. (1), and  $(p_{\text{rel}}, \Omega)$  specifies the relative momentum between the two nucleons in the final state. Usually, medium modification is considered for the scattering matrix elements. The Pauli blocking is considered for the scattered nucleons.

When two-nucleon collisions are introduced in a usual way, the wave-packet momentum centroids of only the colliding two nucleons are changed by a collision (in the ‘‘physical coordinate’’ representation) [13]. This method was generalized in refs. [21, 22] to allow the possibility that each colliding nucleon may form a cluster of mass numbers  $A = 2, 3$  or 4 with some other wave packets in the final state  $|\Phi_f\rangle$ . Namely, when two nucleons  $N_1$  and  $N_2$  collide, we consider the process

$$(3) \quad N_1 + N_2 + B_1 + B_2 \rightarrow C_1 + C_2$$

in which each of the scattered nucleons  $N_j$  ( $j = 1, 2$ ) may form a cluster  $C_j$  with a spectator particle  $B_j$ . This process includes the collisions without cluster formation as the special case of  $C_j = N_j$  with empty  $B_j$  for each of  $j = 1, 2$ . The formulation was given in refs. [21, 22]. The essential ideas are repeated below with some updates for the most recent calculations.

The transition rate of the cluster-forming process is given by eq. (2) with the suitable choice of the set of final states  $|\Phi_f\rangle$ . When a cluster is formed, the corresponding wave packets are placed at the same phase-space point, *i.e.*, the cluster internal state is represented by the harmonic-oscillator  $(0s)^n$  configuration. Denoting the initial and final

states of the  $N_j + B_j$  system by  $|\varphi_j\rangle$  and  $|\varphi'_j\rangle$ , respectively, we have the transition rate

$$(4) \quad v d\sigma = \frac{2\pi}{\hbar} |\langle\varphi'_1|\varphi_1^{\mathbf{q}}\rangle|^2 |\langle\varphi'_2|\varphi_2^{-\mathbf{q}}\rangle|^2 |M|^2 \delta(E_f - E_i) \frac{p_{\text{rel}}^2 dp_{\text{rel}} d\Omega}{(2\pi\hbar)^3},$$

where  $|\varphi_j^{\pm\mathbf{q}}\rangle = e^{\pm i\mathbf{q}\cdot\mathbf{r}_j} |\varphi_j\rangle$  are the states after the momentum transfer  $\pm\mathbf{q}$  to the nucleons  $N_j$  ( $j = 1, 2$ ), and  $(p_{\text{rel}}, \Omega)$  is the relative momentum between  $N_1$  and  $N_2$  in these states. The matrix element  $|M|^2$  is essentially the same as for the usual two-nucleon collisions.

The actual situation of a two-nucleon collision requires more considerations because there are many possible ways of forming a cluster for each  $N$  of the scattered nucleons  $N_1$  and  $N_2$ . For a scattered nucleon  $N$ , we first consider the possibility that  $N$  may form a cluster with one of the nucleons  $\{B_k; k = 1, 2, \dots\}$  which have the same spin-isospin state. This spin-isospin state that is studied first is randomly decided. The cluster-formed state is denoted by  $|\Phi'_k\rangle$  which is obtained, by first changing the state to  $|\Phi^{\mathbf{q}}\rangle$  by the momentum transfer  $\mathbf{q}$  to  $N$ , and then moving the two wave packets of  $N$  and  $B_k$  to the same phase-space point without changing their center of mass. Since the different final states are not orthogonal  $\mathcal{N}_{kl} = \langle\Phi'_k|\Phi'_l\rangle \neq \delta_{kl}$ , the probability that  $N$  forms a cluster with one of  $\{B_k\}$  should be calculated as

$$(5) \quad P = \sum_{kl} \langle\Phi^{\mathbf{q}}|\Phi'_k\rangle \mathcal{N}_{kl}^{-1} \langle\Phi'_l|\Phi^{\mathbf{q}}\rangle.$$

This probability is calculated with an approximation that the many-body state is a direct product of wave packets centered at the ‘‘physical coordinates’’ [13]. With the calculated probability  $P$ , a cluster will be formed with one of  $\{B_k\}$  and, with the rest of the probability  $(1-P)$ ,  $N$  does not form a cluster with a nucleon of this spin-isospin state. The procedure is repeated for other spin-isospin states for  $\{B_k\}$ . Therefore the formation of light clusters is considered up to an  $\alpha$  particle. The factor  $|\langle\varphi'_1|\varphi_1^{\mathbf{q}}\rangle|^2 |\langle\varphi'_2|\varphi_2^{-\mathbf{q}}\rangle|^2$  in eq. (4) should be replaced by the probability determined by this prescription.

Even when the cluster formation is introduced, the many-body state is always represented by an AMD wave function which is a Slater determinant of nucleon wave packets. The time evolution of the many-body state is solved just as usual without depending on whether some of the wave packets form clusters due to collisions in the past (except for the inter-cluster binding process and the momentum fluctuation upon particle emission in the next paragraphs). A nucleon in a formed cluster may collide with some other nucleon so that the cluster is broken. It may be the case that the scattered nucleon forms the same cluster as before, so that an elastic scattering of the cluster is possible. All of these kinds of processes are based on the nucleon-nucleon scattering matrix elements, without introducing parameters to control individual channels of cluster formation. It is, of course, possible to modify the cluster formation probabilities. For example, in the calculation of ref. [22], we suppressed the overall cluster production probability when the momentum transfer is small, by a factor  $1 - \exp[-\mathbf{q}^2/(50 \text{ MeV}/c)^2]$ . In some recent calculations, however, this suppression has been turned off.

It has been realized that the correlations to bind light clusters to form heavier fragments should be taken into account on top of the usual time evolution of AMD [21]. Many of light nuclei (Li, Be etc.) have only one or a few bound states which may be regarded as bound states of internal clusters. By the same reason for the issue of light clusters, the quantum-mechanical probability of forming such a nucleus is not consistent with

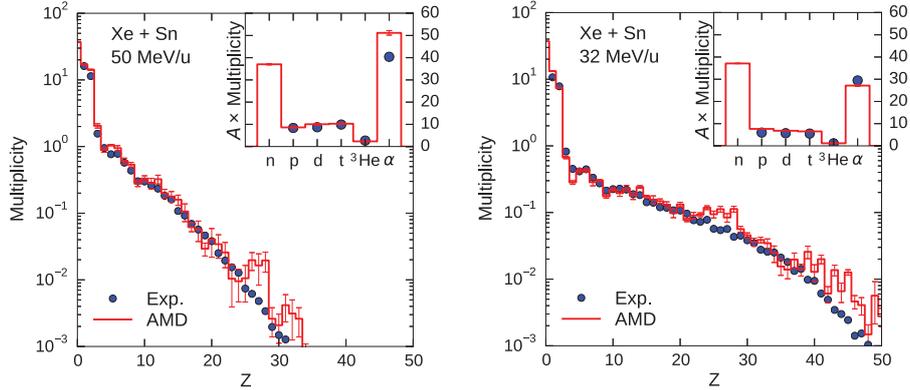


Fig. 3. – Fragment charge distribution in central Xe+Sn collisions ( $0 < b < 2$  fm) at the incident energies of 50 (left) and 35 (right) MeV/nucleon, calculated by AMD with cluster correlations. The inset shows the multiplicities of light particles multiplied by the mass number. The INDRA experimental data are taken from ref. [4].

the semiclassical phase space with which it can be formed in the standard treatment of AMD. Therefore, for a better description, inter-cluster correlation is introduced as a stochastic process of binding clusters. The basic idea is to replace the relative momentum between clusters by zero if moderately separated clusters are moving away from each other with a small relative kinetic energy. It is important how to ensure the total energy conservation.

The momentum distribution in a wave packet is also a long-standing problem in molecular dynamics approach for quantum systems. Due to the uncertainty principle, any single-particle state has a certain momentum width  $\Delta p$  as far as the particle is localized in coordinate space like a Gaussian wave packet. This is a reasonable description of nucleons in a nucleus. When a particle is emitted, however, it is reasonable to project the many-body state onto the states in each of which the emitted particle has a definite momentum. In recent versions of AMD with cluster correlations, we employ the simplest method proposed in ref. [24] with a straightforward extension to clusters. A particle (a nucleon, a cluster or a combined system of clusters) is regarded as emitted if there are no other particles within a distance in coordinate space or in the phase space. When a particle is emitted, a momentum fluctuation is randomly given to it according to the momentum distribution of the wave packet. The energy and momentum conservations are treated.

## 5. – Multifragmentation by AMD with clusters

A recent version of AMD with clusters reproduces multifragmentation data very well as in fig. 3 which shows the comparison with the INDRA data [4] for Xe + Sn central collisions at 50 and 35 MeV/nucleon. Not only the fragments with  $A \gtrsim 5$  but also the light clusters and protons are well reproduced. The fragments in other reactions such as Ca + Ca at 35 MeV/nucleon and Au + Au at several hundred MeV/nucleon are also reproduced reasonably well. With the AMD with clusters, consistent reproduction of this wide range of reactions was first achieved by the calculation in ref. [25]. The quality of the reproduction reported here is similar to ref. [25] even with more simple and natural model prescriptions.

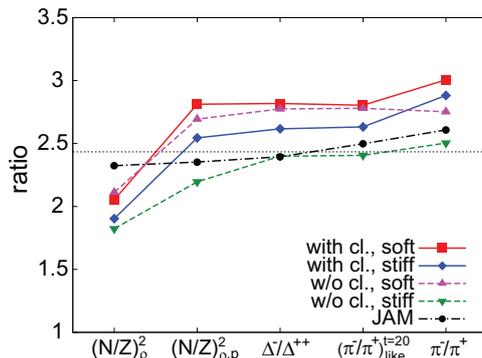


Fig. 4. – The nucleon ratios  $(N/Z)_p^2$  and  $(N/Z)_{p,p}^2$  in high-density region without and with high-momentum condition, respectively, the  $\Delta^-/\Delta^{++}$  production ratio, the pion-like ratio at  $t = 20$  fm/c, and the final  $\pi^-/\pi^+$  ratio. Each line connects the ratios for each of the five cases of calculation for central collisions of  $^{132}\text{Sn} + ^{124}\text{Sn}$  at 300 MeV/nucleon. The horizontal line represents the  $(N/Z)_{\text{sys}}^2$  ratio of the total system. Taken from ref. [22].

In these multifragmentation reactions, the emission of nucleons and light clusters are very much influenced by cluster correlations. For example, in neutron-rich systems, the neutron-to-proton spectral ratio becomes very large due to the strong cluster correlations that produce many low-velocity  $\alpha$  particles. See the results and discussion in ref. [26].

## 6. – Nucleon dynamics and pion production at 300 MeV/nucleon

Let us turn to the central collisions of neutron-rich nuclei  $^{132}\text{Sn} + ^{124}\text{Sn}$  at 300 MeV/nucleon. This system was recently studied by AMD in ref. [22] to relate emitted pions and the dynamics of neutrons and protons which carries important information of symmetry energy at high densities  $\rho \sim 2\rho_0$  [27]. The essential idea is that the  $\pi^-/\pi^+$  ratio is approximately related to the  $(N/Z)^2$  ratio of nucleons in high density region. A unique character of ref. [22] is that the effect of cluster correlations was studied as well as that of symmetry energy. A hadronic cascade model called JAM [28] was used for the treatment of  $\Delta$  resonances and pions on top of the nucleon dynamics calculated by AMD.

Figure 4 summarizes various isospin-related ratios for different AMD+JAM calculations, with soft ( $L = 46$  MeV) or stiff ( $L = 108$  MeV) symmetry energies and with or without cluster correlations in AMD. The simple JAM calculation without any mean-field term is also shown for comparison. See ref. [22] for the exact definitions of these ratios. The leftmost column represents the neutron-to-proton squared ratio  $(N/Z)_p^2$  in the high density region with  $\rho > \rho_0$ . The value of  $(N/Z)_p^2$  is lower than the  $(N/Z)_{\text{sys}}^2$  ratio of the total system because of the symmetry energy effect. The difference between soft and stiff symmetry energies, for each calculation with and without clusters, shows the degree of the sensitivity to the density dependence of symmetry energy. An important question is how this symmetry energy effect may be reflected in the final observables such as the  $\pi^-/\pi^+$  ratio that is shown in the rightmost column of fig. 4. The final pion ratio is larger than  $(N/Z)_{\text{sys}}^2$  at this incident energy, as also observed in the Au + Au experiment [6] and in other transport calculations [29-31]. The value of the final pion ratio does not actually agree with the  $(N/Z)_p^2$  ratio in the high-density region.

A new point of view was introduced in ref. [22] for the relation between the nucleon dynamics and the pion ratio, by looking at the phase space region of high density *and*

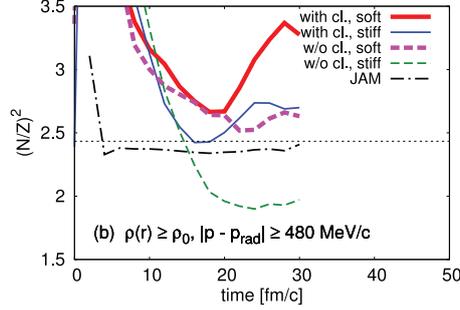


Fig. 5. – The time evolution of the squared ratio of neutrons and protons  $(N/Z)^2$  calculated for the selected nucleons that exist in the high density region  $\rho(r) \geq \rho_0$  and have high momenta  $|\mathbf{p} - \mathbf{p}_{\text{rad}}| \geq 480 \text{ MeV}/c$ . Taken from ref. [22].

high momentum. The high-momentum condition is natural because the  $\Delta$ -resonance production is possible only if enough energy is available. The second column of fig. 4 shows the  $(N/Z)_{\rho,p}^2$  ratio for the nucleons that are in the high-density and high-momentum phase-space region of  $\rho > \rho_0$  and  $|\mathbf{p} - \mathbf{p}_{\text{rad}}| > 480 \text{ MeV}/c$ , where the collective radial momentum  $\mathbf{p}_{\text{rad}}$  has been subtracted. We find that the high-momentum region is more neutron-rich in all the cases of calculations. Furthermore, this  $(N/Z)_{\rho,p}^2$  ratio well agrees with the  $\Delta^-/\Delta^{++}$  ratio of the  $\Delta$  production reaction rates of  $nn \rightarrow p\Delta^-$  and  $pp \rightarrow n\Delta^{++}$ , as shown in the third column of the figure. This agreement is confirmed by the comparison of figs. 5 and 6 which show the time evolution of these two kinds of ratios, respectively. Due to the dynamics after  $t = 20 \text{ fm}/c$ , the final pion ratio is modified from the  $\Delta^-/\Delta^{++}$  production ratio as shown in fig. 4.

In these results, effects of cluster correlations are observable in several aspects. First of all, the sensitivity to the symmetry energy is weaker with clusters than without them in the neutron-to-proton ratios,  $(N/Z)_\rho^2$  and  $(N/Z)_{\rho,p}^2$  in fig. 4. This can be understood because the cluster correlation forces some neutrons and protons to move together and therefore the symmetry forces acting differently on the neutrons and the protons are averaged out to some degree. By comparing the difference of  $(N/Z)_\rho^2$  and  $(N/Z)_{\rho,p}^2$ , we also notice that the high-momentum part is more neutron-rich when the cluster

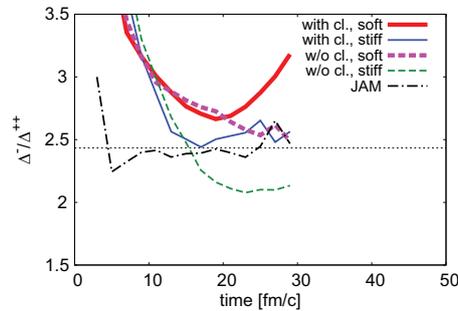


Fig. 6. – The time evolution of the  $\Delta^-/\Delta^{++}$  ratio of the  $\Delta$  production rates. The five different lines show the calculations. The horizontal line represents the  $(N/Z)_{\text{sys}}^2$  ratio of the total system. Taken from ref. [22].

correlations are turned on. A plausible reason may be as follows. If clusters are formed, they tend to have smaller velocities on average than non-clustered nucleons, as in thermal equilibrium. In addition, clusters tend to contain similar number of neutrons and protons like  $\alpha$  clusters. Therefore, due to clusters, the low-momentum part of the phase space becomes more symmetric ( $N \approx Z$ ) and thus the high-momentum part becomes more neutron-rich if the numbers of neutrons and protons are conserved. We also notice that the final pion ratio is larger than  $(\pi^-/\pi^+)_{\text{like}}$  at  $t = 20$  fm/c in the calculations with cluster correlations. This implies that the exterior region of the expanding system that the created pions have to pass through is more neutron-rich in the calculations with cluster correlations.

## 7. – Summary

Clusters are important in heavy-ion collisions, not only simply because they are emitted, but also because formation and existence of light clusters influence the global reaction dynamics and the nuclear matter properties. AMD has been extended to include cluster correlations in the final states of two-nucleon collisions. A consistent reproduction of basic characters of fragmentation by recent calculations is much more satisfactory than before. This suggests that the excited nuclear many-body systems realized in heavy-ion collisions have stronger cluster correlations than usually described by nucleon-based transport models. A recent calculation with the AMD+JAM approach [22] showed that  $\Delta$  resonances and pions are actually good probes for the neutron-proton dynamics at high densities in collisions of neutron-rich nuclei at 300 MeV/nucleon. Even at this high energy, clusters are playing important roles in collision dynamics.

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## REFERENCES

- [1] KANADA-EN'YO Y., KIMURA M. and ONO A., *Prog. Theor. Exp. Phys.*, **2012** (2012) 01A202.
- [2] FUNAKI Y., TOHSAKI A., HORIUCHI H., SCHUCK P. and RÖPKE G., *Phys. Rev. C*, **67** (2003) 051306.
- [3] BORDERIE B. and RIVET M. F., *Prog. Part. Nucl. Phys.*, **61** (2008) 551.
- [4] HUDAN S. *et al.*, *Phys. Rev. C*, **67** (2003) 064613.
- [5] REISDORF W. *et al.*, *Nucl. Phys. A*, **612** (1997) 493.
- [6] REISDORF W. *et al.*, *Nucl. Phys. A*, **848** (2010) 366.
- [7] BORDERIE B. *et al.*, *Eur. Phys. J. A*, **6** (1999) 197.
- [8] COLONNA M., ONO A. and RIZZO J., *Phys. Rev. C*, **82** (2010) 054613.
- [9] COLONNA M., DI TORO M., GUARNERA A., MACCARONE S., ZIELINSKA-PFABÉ M. and WOLTER H. H., *Nucl. Phys. A*, **642** (1998) 449.
- [10] NAPOLITANI P. and COLONNA M., *Phys. Lett. B*, **726** (2013) 382.
- [11] ONO A., HUDAN S., CHBIHI A. and FRANKLAND J. D., *Phys. Rev. C*, **66** (2002) 014603.
- [12] ONO A. and HORIUCHI H., *Prog. Part. Nucl. Phys.*, **53** (2004) 501.
- [13] ONO A., HORIUCHI H., MARUYAMA T. and OHNISHI A., *Prog. Theor. Phys.*, **87** (1992) 1185.
- [14] ONO A. and HORIUCHI H., *Phys. Rev. C*, **53** (1996) 2958.

- [15] BONNET E., COLONNA M., CHBIHI A., FRANKLAND J. D., GRUYER D. and WIELECZKO J. P., *Phys. Rev. C*, **89** (2014) 034608.
- [16] CHEN L. W., KO C. M. and LI B. A., *Nucl. Phys. A*, **729** (2003) 809.
- [17] RÖPKE G., *Nucl. Phys. A*, **867** (2011) 66.
- [18] RÖPKE G., *Phys. Rev. C*, **92** (2015) 054001.
- [19] DANIELEWICZ P. and BEARTSCH G. F., *Nucl. Phys. A*, **533** (1991) 712.
- [20] COUPLAND D. D. S., LYNCH W. G., TSANG M. B., DANIELEWICZ P. and ZHANG Y., *Phys. Rev. C*, **84** (2011) 054603.
- [21] ONO A., *J. Phys.: Conf. Ser.*, **420** (2013) 012103.
- [22] IKENO N., ONO A., NARA Y. and OHNISHI A., *Phys. Rev. C*, **93** (2016) 044612.
- [23] ONO A., *J. Phys.: Conf. Ser.*, **569** (2014) 012086.
- [24] ONO A. and HORIUCHI H., *Phys. Rev. C*, **53** (1996) 845.
- [25] ONO A., *EPJ Web of Conferences*, **117** (2016) 07003.
- [26] ONO A., *EPJ Web of Conferences*, **122** (2016) 11001.
- [27] LI B. A., *Phys. Rev. Lett.*, **88** (2002) 192701.
- [28] NARA Y., OTUKA N., OHNISHI A., NIITA K. and CHIBA S., *Phys. Rev. C*, **61** (1999) 024901.
- [29] XIAO Z., LI B. A., CHEN L. W., YONG G. C. and ZHANG M., *Phys. Rev. Lett.*, **102** (2009) 062502.
- [30] FENG Z. Q. and JIN G. M., *Phys. Lett. B*, **683** (2010) 140.
- [31] HONG J. and DANIELEWICZ P., *Phys. Rev. C*, **90** (2014) 024605.